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## On the cosmology of massive bigravity

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**Summary.** — In this note we briefly review the present status of cosmology in massive bigravity. The bottom line is that no stable FLRW cosmology exists at the leading order, showing a breakdown of cosmological perturbation theory much earlier than in GR. A possible way out could be a non-linear treatment.

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### 1. – Introduction

One of the most compelling problems in the theories of standard cosmology is related to the present acceleration of the Universe. The simplest explanation consists of adding to the action of General Relativity (GR) a (cosmological) constant  $\Lambda$  tuned to fit with experimental data exacerbating, if possible, the old cosmological constant problem [1]. An alternative approach is to look for theories of gravity that modify GR in order to get a weaker interaction at large distances, but still retaining the good short distances behaviour of GR (see [2] for a recent review). Since GR turns out to be the unique non-linear theory of a massless spin two field, any deviation will introduce new degrees of freedom (dof). These extra dof therefore have to kick in only at cosmological scales and need to be screened below the Solar System scale [3]<sup>(1)</sup>.

An obvious modification of GR is to promote the massless spin two mediator to a massive one, hence the name of *massive gravity* (for a review see [5]). A massive spin two field propagates 5 dof and this statement should be retained also when non-linear self-interactions are introduced into the action. For a long time it was believed that a fully non-linear theory of massive gravity could not exist due to a 6<sup>th</sup> ghost mode that

<sup>(1)</sup> Recently, shadows have been casted on the trustability of the Vainshtein mechanism [4].

usually appears at higher orders in perturbations or around non-trivial backgrounds [6]. Only very recently it has been realized that it is actually possible to construct several potentials which propagate 5 dof [7], avoiding the extra ghost mode. Among the Lorentz invariant candidates, the only one known that allows for a Minkowski background, and is therefore a non-linear completion of the Fierz-Pauli action [8], is the dRGT model [9,10].

To write down a self-interaction term for the metric, it is mandatory to introduce another  $(0, 2)$  tensor field  $f_{\mu\nu}$  that allows for the unique diff invariant construction  $X_{\nu}^{\mu} = g^{\mu\alpha} f_{\alpha\nu}$ ; this term will be the building block of the massive gravity potential. The extra tensor  $f_{\mu\nu}$  can be considered as a fixed, God given object like an aether, or dynamical with its own Ricci scalar in the action. In the last case we are in the framework of bigravity theories [11-14] where at linear level a massive and a massless spin two field propagate. It should be stressed that in massive gravity the mass scale of the graviton is set by hand and typically when considered as an effective field theory the resulting cutoff is much lower than the Planck mass. Thus, in order to be a fully predictive theory, an ultraviolet completion is needed.

Cosmological solutions strongly indicate that the bigravity formulation works better compared to the fixed second metric one. Indeed in the second case spatially flat homogenous Friedmann-Robertson-Walker (FRW) solutions simply do not exist [15] and even allowing for open FRW solutions [16], strong coupling [17] and ghostlike instabilities [18,19] develop.

This note will focus on the cosmological evolution in the bigravity approach [20-22], showing that the only consistent and reasonable background solutions turn out to be unstable at the leading order in cosmological perturbation theory [23].

## 2. – Massive bigravity: the model

The action for massive bigravity takes the following form:

$$(1) \quad S = \int d^4x \left\{ \sqrt{g} [M_{pl}^2 (R(g) - 2m^2 V) + L_M] + \sqrt{f} \kappa M_{pl}^2 R(f) \right\},$$

where  $R(g)$  and  $R(f)$  are the corresponding Ricci scalars of  $g_{\mu\nu}$  and  $f_{\mu\nu}$  and  $V = \sum_{n=0}^4 a_n V_n$  is the ghost free potential [9,10] written in terms of the symmetric polynomials  $V_n$  of  $Y_{\nu}^{\mu} \equiv (\sqrt{X})_{\nu}^{\mu}$ .

Matter is minimally coupled with  $g$  only and is described by  $L_M$ . The constant  $\kappa$  controls the relative size of the strength of gravitational interactions in the two sectors, while  $m$  sets the scale of the graviton mass. In the limit  $\kappa \rightarrow \infty$ , the second metric gets frozen to a prescribed background value making contact with the fixed second metric formulation.

## 3. – Massive bigravity: cosmology

To account for the present acceleration of the Universe through massive gravity, we need to assume that the scale of the graviton mass  $m$  is of the order of the present Hubble size, *i.e.*  $m \sim H_0 \sim 10^{-33}$  eV. We do not consider the case where  $m$  is not related to  $H_0$ , so that a cosmological constant is still necessary to fine tune with  $m$  and give the

observed value [24]. Let us assume that both metrics are homogeneous and isotropic

$$(2) \quad \begin{aligned} ds^2 &= g_{\mu\nu} dx^\mu dx^\nu = a^2(\tau) (-d\tau^2 + dr^2 + r^2 d\Omega^2), \\ \widetilde{ds}^2 &= f_{\mu\nu} dx^\mu dx^\nu = \omega^2(\tau) [-c^2(\tau) d\tau^2 + dr^2 + r^2 d\Omega^2], \end{aligned}$$

and define the Hubble parameters as  $\mathcal{H} \equiv a'/a$ ,  $\mathcal{H}_\omega \equiv \omega'/\omega$  and the ratio  $\xi$  between the two scale factors  $\xi \equiv \omega/a$ .

Solutions fall into two branches depending on how the constraint coming from the Bianchi identities is realized. The physically interesting branch gives  $c = \mathcal{H}_\omega/\mathcal{H}$ . We will not discuss here the other branch where  $\xi$  is constant, the effect of modification amounts to an effective cosmological constant and perturbations are strongly coupled [20, 23].

The Hubble parameter  $\mathcal{H}$  is given by

$$(3) \quad \frac{3\mathcal{H}^2}{a^2} = 8\pi G \rho + m^2 (6 a_3 \xi^3 + 6 a_2 \xi^2 + 3 a_1 \xi + a_0),$$

and  $\xi$  turns out to be determined by the following algebraic equation

$$(4) \quad \xi^2 \left( \frac{8 a_4}{\kappa} - 2 a_2 \right) + \xi \left( \frac{6 a_3}{\kappa} - a_1 \right) + \frac{a_1}{3 \kappa \xi} + \frac{2 a_2}{\kappa} - 2 a_3 \xi^3 - \frac{a_0}{3} = \frac{8\pi G}{3 m^2} \rho.$$

Late time cosmology is the sought quasi de Sitter phase. Indeed, when matter is so diluted that  $G\rho \ll m^2$ , eq. (4) gives an almost constant value for  $\xi$  and  $c = 1$ . Clearly, from (3), late time acceleration requires that  $m^2 \sim H_0^2$ .

At early times instead, under the above assumption, the rhs of equation (4) gets very large until the very present epoch where ‘‘Dark energy’’ starts to dominate. This implies for consistency that  $\xi$  can only be very large or very small if we take the parameters  $a_i$  and  $\kappa$  of  $\mathcal{O}(1)$  avoiding fine tuning. Solutions with  $\xi \gg 1$ , *i.e.*  $\xi \sim (\rho/\rho_\Lambda)^{1/n}$  where  $n$  depends on the dominating power of  $\xi$  and  $\rho_\Lambda = 3H_0^2/(8\pi G)$ , will spoil early time cosmology since the additional ‘‘source’’ term in (3) is comparable with the contribution from matter. The only exception is for a specific choice of the parameters  $a_i$  ( $a_3 = 0$ ,  $4a_4 - \kappa a_2 > 0$ ) that allows for a cancellation of the modification respect to GR at the background level. Nevertheless, in this very special case, the value of  $c$  is negative. Thus, starting from  $c < 0$ , to approach the quasi de Sitter era with  $c = 1$  at late time, the metric  $f$  must cross  $c = 0$  where  $R(f)$  blows up<sup>(2)</sup>. In conclusion, solutions with  $\xi \gg 1$  are not very interesting.

On the other hand, when  $\xi \ll 1$ , *i.e.*  $\xi \sim (\rho_\Lambda/\rho)$ , a sensible early time cosmology is possible, since the additional source term in (3) is subdominant and  $c = 4 + 3w > 0$  for standard matter equation of state  $w$ . This is then the only viable branch of background solutions that is worth to further study.

Cosmological perturbations around the small  $\xi$  branch can be studied in the usual way, dividing the fluctuations in tensor, vector and scalar representations of the rotation group. Tensor and vector perturbations turns out to be free of instabilities; the scalar sector instead is rather problematic [23]. Two scalars propagate and they can be identified with the suitable gauge invariant built from  $g_{ij} = a^2 \delta_{ij}(1 + 2F_1(t, \vec{x}))$  and

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<sup>(2)</sup> Surprisingly, in a very recent paper [25] it was shown that the singularity seems to do not affect cosmological perturbations, at least at linear order.

$f_{ij} = \omega^2 \delta_{ij}(1 + 2F_2(t, \vec{x}))$ . At the leading order, the scalar perturbation of the physical metric (the one coupled with matter) behaves like in GR; on the contrary, the one associated to the second metric develops a gradient instability inside the horizon, both in radiation and matter dominated era. This exponential instability signals the breakdown of the standard perturbation theory, for any fixed co-moving momentum  $k$  inside the horizon, at very early time. As a result, the prediction for the matter contrast (formally the same as in GR at early times) is unreliable.

At a first glance, this instability could be attributed to the strong asymmetry between the two metrics, since only  $g$  is sourced by matter, and that gives a background solution where the physical scale factor  $a$  is much bigger than the other one, *i.e.*  $a \gg \omega$ . Introducing an additional matter sector minimally coupled with the second metric <sup>(3)</sup>, the situation does not get much better [27]. In this case the rhs of equation (4) becomes  $\frac{8\pi G}{3m^2}(\rho_1 - \xi^2 \rho_2/\kappa)$  and a background solution where the two scale factors are comparable ( $\xi \sim 1$ ) is indeed possible. However, though the pressure provided by the second matter stabilises  $F_2$  (its dynamics becomes similar to GR), sub horizon instability persists for the extra scalar perturbation which is present in this case. This suggests that massive bigravity has an intrinsic exponential instability for FLRW cosmological solutions, irrespectively of the (a)symmetry in the matter coupling.

#### 4. – Conclusions

In this note we summarised the current status of cosmology in massive bigravity theories. The picture that emerges is that even if FLRW solutions exist, cosmological perturbations cannot be trusted due to a gradient instability in the scalar sector inside the horizon for any co-moving momentum  $k$ . One could speculate that some sort of cosmological Vainshtein mechanism [28] exists and the instability is solved through nonlinearities in the perturbations of the second metric. Even in the presence of such a mechanism, the prize to pay will be rather high, the impossibility to use cosmological perturbation theory.

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#### REFERENCES

- [1] WEINBERG S., *Rev. Mod. Phys.*, **61** (1989) 1.
- [2] CLIFTON T., FERREIRA P. G., PADILLA A. and SKORDIS C., *Phys. Rep.*, **513** (2012) 1; arXiv:1106.2476 [astro-ph.CO]; JOYCE A., JAIN B., KHOURY J. and TRODDEN M., arXiv:1407.0059 [astro-ph.CO].
- [3] VAINSHTEIN A. I., *Phys. Lett. B*, **39** (1972) 393.
- [4] KALOPEL N., PADILLA A., SAFFIN P. and STEFANYSZYN D., arXiv:1409.3243 [hep-th].
- [5] DE RHAM C., arXiv:1401.4173 [hep-th].
- [6] BOULWARE D. G. and DESER S., *Phys. Lett. B*, **40** (1972) 227.

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<sup>(3)</sup> Coupling the same matter to both the metrics, besides a violation of the equivalence principle, will reintroduce the BD ghost mode [26].

- [7] COMELLI D., CRISOSTOMI M., NESTI F. and PILO L., *Phys. Rev. D*, **86** (2012) 101502; arXiv:1204.1027 [hep-th].
- [8] FIERZ M. and PAULI W., *Proc. R. Soc. London A*, **173** (1939) 211.
- [9] DE RHAM C. and GABADADZE G., *Phys. Rev. D*, **82** (2010) 044020; arXiv:1007.0443 [hep-th]; DE RHAM C., GABADADZE G. and TOLLEY A. J., *Phys. Rev. Lett.*, **106** (2011) 231101; arXiv:1011.1232 [hep-th].
- [10] HASSAN S. F. and ROSEN R. A., *JHEP*, **07** (2011) 009; arXiv:1103.6055 [hep-th]; HASSAN S. F. and ROSEN R. A., *Phys. Rev. Lett.*, **108** (2012) 041101; arXiv:1106.3344 [hep-th]; HASSAN S. F. and ROSEN R. A., *JHEP*, **02** (2012) 126; arXiv:1109.3515 [hep-th]; HASSAN S. F., ROSEN R. A. and SCHMIDT-MAY A., *JHEP*, **02** (2012) 026; arXiv:1109.3230 [hep-th]; HASSAN S. F. and ROSEN R. A., *JHEP*, **04** (2012) 123; arXiv:1111.2070 [hep-th].
- [11] ISHAM C. J., SALAM A. and STRATHDEE J. A., *Phys. Rev. D*, **3** (1971) 867; SALAM A. and STRATHDEE J. A., *Phys. Rev. D*, **16** (1977) 2668.
- [12] DAMOUR T. and KOGAN I. I., *Phys. Rev. D*, **66** (2002) 104024.
- [13] BEREZHIANI Z., COMELLI D., NESTI F. and PILO L., *Phys. Rev. Lett.*, **99** (2007) 131101; hep-th/0703264 [hep-th].
- [14] COMELLI D., CRISOSTOMI M., NESTI F. and PILO L., *Phys. Rev. D*, **85** (2012) 024044; arXiv:1110.4967 [hep-th].
- [15] D'AMICO G., DE RHAM C., DUBOVSKY S., GABADADZE G., PIRTSKHALAVA D. and TOLLEY A. J., *Phys. Rev. D*, **84** (2011) 124046; arXiv:1108.5231 [hep-th].
- [16] GUMRUKCUOGLU A. E., LIN C. and MUKOHYAMA S., *JCAP*, **11** (2011) 030; arXiv:1109.3845 [hep-th].
- [17] KHOSRAVI N., NIZ G., KOYAMA K. and TASINATO G., arXiv:1305.4950 [hep-th].
- [18] DE FELICE A., GUMRUKCUOGLU A. E. and MUKOHYAMA S., *Phys. Rev. Lett.*, **109** (2012) 171101; arXiv:1206.2080 [hep-th].
- [19] DE FELICE A., GUMRUKCUOGLU A. E., LIN C. and MUKOHYAMA S., *JCAP*, **05** (2013) 035; arXiv:1303.4154 [hep-th].
- [20] COMELLI D., CRISOSTOMI M., NESTI F. and PILO L., *JHEP*, **03** (2012) 067; arXiv:1111.1983 [hep-th].
- [21] VON STRAUSS M., SCHMIDT-MAY A., ENANDER J., MORTSELL E. and HASSAN S. F., arXiv:1111.1655 [gr-qc].
- [22] VOLKOV M. S., *JHEP*, **01** (2012) 035; arXiv:1110.6153 [hep-th].
- [23] COMELLI D., CRISOSTOMI M. and PILO L., *JHEP*, **06** (2012) 085; arXiv:1202.1986 [hep-th].
- [24] DE FELICE A., NAKAMURA T. and TANAKA T., arXiv:1304.3920 [gr-qc]; DE FELICE A., GÜMRÜKÇÜOĞLU A. E., MUKOHYAMA S., TANAHASHI N. and TANAKA T., *JCAP*, **06** (2014) 037; arXiv:1404.0008 [hep-th].
- [25] LAGOS M. and FERREIRA P. G., arXiv:1410.0207 [gr-qc].
- [26] YAMASHITA Y., DE FELICE A. and TANAKA T., arXiv:1408.0487 [hep-th].
- [27] COMELLI D., CRISOSTOMI M. and PILO L., *Phys. Rev. D*, **90** (2014) 084003; arXiv:1403.5679 [hep-th].
- [28] BABICHEV E. and CRISOSTOMI M., *Phys. Rev. D*, **88** (2013) 084002; arXiv:1307.3640; BABICHEV E. and DEFFAYET C.D., *Class. Quantum Grav.*, **30** (2013) 184001; arXiv:1304.7240 [gr-qc].