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A SEMI ANALYTIC ITERATIVE METHOD FOR SOLVING TWO FORMS OF BLASIUS EQUATION

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Abstract

In this paper, a semi analytic iterative method (SAIM) is presented for solving two forms of Blasius equation. Blasius equation is a third order nonlinear ordinary differential equation in the problem of the two-dimensional laminar viscous flow over half-infinite domain. In this scheme, the first solution which is in a form of convergent series solution is combined with Padé approximants to handle the boundary condition at infinity. Comparison the results obtained by SAIM with those obtained by other method such as variational iteration method and differential transform method revealed the effectiveness of the SAIM.

Keywords: Semi analytic iterative method; Blasius equation; Padé approximants

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Introduction

In recent decades, there exists interest in solving nonlinear dynamics problems in scientific and engineering phenomena using semi-analytical or numerical solution methods. The method such as, Adomian decomposition method (ADM) (Adomian, 1994), variational iterative method (VIM) (He, 2000), homotopy perturbation method (HPM) (He, 1999) and homotopy analysis method (HAM) (Liao, 2004) gained a great attention in the applications to solve nonlinear models. Some of the models considered lead to time-power series solutions which is have not contributed much to the understanding of nonlinear phenomena (Fernández, 2009). However, the applications in differential equations models are still widely used as in Jameel et al (2019), Altaie, Jameel and Saaban (2019), and Mabood et al. (2018). Some modification of the methods also presented and applied in various problems such as in Bakodah et al. (2017), Kang et al. (2017), Haq et al. (2017), Biazar and Montazeri (2019), Olumuyiwa et al. (2018), Maitama (2016), Martin (2016), Sakar and Ergören (2015), Rafiq, Ahmad and Mohyud-Din (2017), Ahmad (2018), Yin, Kumar dan Kumar (2015) and etc.

Blasius equation is a one of most important equation in fluid dynamic. Blasius equation describes the velocity profile of the fluid in the boundary layer on a half infinite interval or flat plate. Blasius equation is regarded as the first exact solution of Navier-Stoke equation where the partial differential equation of Navier-Stoke equations had been transformed into ordinary differential equation. One of the phenomena governed by Blasius equation is free convection near a vertical impermeable surface embedded in a porous medium. This problem belongs to heat transfer phenomena which have many applications in geophysical and industrial fields (Chirita, Ene, & Nicolescu, 2012).

Due to the significant of Blasius equation on sciences and engineering, many efforts had been done to solve this equation analytically and numerically at the boundary conditions of the interval. For example, Ertürk and Momani (2008) applied modified form of differential transform method (DTM) to provide the solution in the form of a convergent power series. Wang (2004) employed ADM to investigate the Blasius equation where the result had been corrected by Hashim (2006). A perturbation approach to solve Blasius equation had been done by He (2003). Meanwhile, Abbasbandy (2007) compared the numerical solution by ADM with homotopy perturbation method. Wazwaz (2007) approximate the solution of Blasius equation using VIM, which is the main reference in this article. Some recent studied on Blasius equations have been done by Asaithambi, (2016), Trujillo, Marin-

Ramirez and de Indias, C. (2018), Bougoffa and Wazwaz (2015). Zheng et al. (2017), Fazio (2016), Ogunlaran and Sagay-Yusuf (2016), Sajid et al. (2015) and Najafi (2018).

In this article, we employed the semi analytical iterative method (SAIM) proposed by Temimi and Ansari (2011a) to simulate approximate solution of Blasius equation. The SAIM has been used to find the exact and approximate solution for various differential equations problems such as nonlinear second order multi-point boundary value problems (Temimi & Ansari, 2011b), Fokker-Plank's equations (AL-Jawary, Radhi & Ravnik, 2017), nonlinear Burgers and advection-diffusion equations (AL-Jawary, Azeez & Radhi, 2018), chemistry problems (AL-Jawary & Al-Raham, 2017), thin flow problems (AL-Jawary, 2017), differential algebraic equations (AL-Jawary & Hatif, 2017), duffing equation (Al-Jawary and Al-Razaq, 2016) and some nonlinear differential equations in physics (AL-Jawary, Adwan & Radhi, 2018).

The semi analytic iterative method (SAIM)

The basic idea of SAIM can be written as (Temimi & Ansari, 2011a):

$$L\big(f(x)\big) + N(f(x)) + g(x) = 0 \tag{1}$$

with boundary conditions:

$$B\left(f, \frac{df}{dx}\right) = 0, \tag{2}$$

where x denotes the independent variable, $f(x)$ is an unknown function, $g(x)$ is known function, L is a linear operator, N is a non-linear then B is a boundary operator. L be the linear part of the differential equation but it is possible to take some linear parts and add them to N as needed. By assuming that $f_0(x)$ as an initial guess solution for the problem $f(x)$ and is the solution of the equation

$$L\big(f_0(x)\big) + g(x) = 0, \qquad B\left(f_0, \frac{df_0}{dx}\right) = 0. \tag{3}$$

The next iteration, we solve the following problem:

$$L\big(f_1(x)\big) + g(x) + N\big(f_0(x)\big) = 0, \qquad B\left(f_1, \frac{df_1}{dx}\right) = 0 \tag{4}$$

and thus, we have a simple iterative procedure which is effectively the solution of a linear set of problems,

$$L\big(f_{n+1}(x)\big) + g(x) + N\big(f_n(x)\big) = 0, \qquad B\left(f_{n+1}, \frac{df_{n+1}}{dx}\right) = 0. \tag{5}$$

We noted that, each of the $f_i(x)$ are solutions to equation (5).

Analysis of SAIM in Blasius Equations

Case I

We considered the two type of Blasius equation given by

$$f'''(x) + \frac{1}{2} f(x) f''(x) = 0, \tag{6}$$

with the initial conditions:

$$f(0) = 0, \qquad f'(0) = 1, \qquad f'(\infty) = 0. \tag{7}$$

and

$$f'''(x) + \frac{1}{2} f(x) f''(x) = 0, \tag{8}$$

with initial conditions:

$$f(0) = 0, \qquad f'(0) \ = \ 0, \qquad f'(\infty) = 1. \tag{9}$$

We applied the SAIM into (6) and (7) by choosing

$$L(f(x)) = f'''(x), \qquad N(f(x) = \frac{1}{2} f(x) f''(x), \qquad g(x) = 0, \tag{10}$$

Thus, the primary problem is

$$f_0(x) = \frac{1}{2} A e^x + x \tag{11}$$

with the initial conditions

$$f_0(0) = 0, \qquad f_0'(0) = 1, \qquad f_0'(\infty) = 0. \tag{12}$$

A general iterative procedure can be written as

$$f_{n+1}'''(x) + \frac{1}{2} f_n(x) f_{n+1}'(x) = 0 \tag{13}$$

with the initial conditions

$$f_{n+1}(0) = 0, \qquad f_{n+1}'(0) = 1, \qquad f_{n+1}'(\infty) = 0. \tag{14}$$

Approximations will be obtained by using SAIM and by used the boundary conditions given for f_0 thus, we choose $f_0(x) = \frac{1}{2} A x^2 + x$ as an initial solution, where $f_0'(0) = A$. Using equation (14), obtained the iterative solutions

$$f_0(x) = \frac{1}{2} A x^2 + x \tag{15}$$

$$f_1(x) = -\left(\frac{1}{240}\right) A^2 x^5 - \left(\frac{1}{480}\right) A x^4 + \left(\frac{1}{2}\right) A x^2 + x \tag{16}$$

$$f_2(x) = -\left(\frac{1}{5702400}\right)A^4x^{11} - \left(\frac{1}{518400}\right)A^3x^{10} + \left(\frac{11}{161280}\right)A^3x^8 - \left(\frac{1}{193536}\right)A^2x^9 + \left(\frac{11}{20160}\right)A^2x^7 - \left(\frac{1}{240}\right)A^2x^5 + \left(\frac{1}{960}\right)Ax^6 - \left(\frac{1}{48}\right)Ax^4 + \left(\frac{1}{2}\right)Ax^2 + x \quad (17)$$

$$\begin{aligned} f_3(x) = &-\left(\frac{1}{115880067072000}\right)A(\left(\frac{14}{759}\right)A^7x^{23} + \left(\frac{14}{33}\right)A^6x^{22} + \left(\frac{1379}{380}\right)A^5x^{21} \\ &+ \left(\frac{1}{6840}(-115038A^6 + 93555A^4)\right)x^{20} \\ &+ \left(\frac{1}{5814}(-1955646A^5 + 111375A^3)\right)x^{19} - \left(\frac{42171}{17}\right)A^4x^{18} \\ &+ \left(\frac{1}{4080}(20597220A^5 - 32592780A^3)\right)x^{17} \\ &+ \left(\frac{1}{3360}(288361080A^4 - 31808700A^2)\right)x^{16} + \left(\frac{6813180}{13}\right)x^{15}A^3 \\ &+ \left(\frac{1}{2184}(2894965920A^2 - 1820387520A^4)\right)x^{14} \\ &+ \left(\frac{1}{1716}(1886068800A - 20024262720A^3)\right)x^{13} - 52481520x^{12}A^2 \\ &+ \left(\frac{1}{990}(-74364998400A + 134719200000A^3)\right)x^{11} + 1496880000A^2x^{10} \\ &+ 5149267200Ax^9 + \left(\frac{1}{336}(1810626048000 - 2655584870400A^2)\right)x^8 \\ &- 63228211200Ax^7 - 120708403200x^6 + 482833612800Ax^5 \\ &+ 2414168064000x^4) + \left(\frac{1}{2}\right)Ax^2 + x \end{aligned} \quad (18)$$

$\vdots$

The aim of this work is to estimate the value of A using Padé approximants (Boyd, 1997). We noted that Padé approximants are built-in utilities in manipulation language such as MAPLE. Moreover, it is worth to mentioned that, Wazwaz (2007) and Ertürk and Momani (2008) also used the same approach to approximate the value of A. The mathematical behavior of $f(x)$ will be study by derive the approximation of $f''(0) = A > 0$. Using the boundary condition, $f'(\infty) = 0$, the diagonal approximant $[M/M]$ will be determined for $f'(x)$ where

$$
\begin{aligned}
f'(x) = -\frac{1}{115880067072000} A \Big(& \frac{14}{33} A^7 x^{22} + \frac{28}{3} A^6 x^{21} + \frac{28959}{380} A^5 x^{20} \\
& + \frac{1}{342} (-115038 A^6 + 93555 A^4) x^{19} + \frac{1}{306} (-1955646 A^5 + 111375 A^3) x^{18} \\
& - \frac{759078}{17} A^{14} x^{17} + \frac{1}{240} (20597220 A^5 - 32592780 A^3) x^{16} \\
& + \frac{1}{210} (288361080 A^4 - 31808700 A^2) x^{15} + \frac{102197700}{13} x^{14} A^3 \\
& + \frac{1}{156} (2894965920 A^2 - 1820387520 A^4) x^{13} \\
& + \frac{1}{132} (1886068800 A - 20024262720 A^3) x^{12} - 629778240 x^{11} A^2 \\
& + \frac{1}{90} (-7436998400 A + 134719200000 A^3) x^{10} + 14968800000 A^2 x^9 \\
& + 4634340404800 A x^8 + \frac{1}{42} (1810626048000 - 2655584870400 A^2) x^7 \\
& - 442597478400 A x^6 - 724250419200 x^5 + 2414168064000 A x^4 \\
& + 9656672256000 x^3 \Big) + Ax + 1
\end{aligned}
\tag{19}
$$

Table 1. A comparison value of $f''(0) = A$ between VIM, DTM and SAIM

Padé approximants	**Wazwaz (2007) (VIM)**	**Ertürk and Momani (2008) (DTM)**	**Current Aproximation (SAIM)**
[2/2]	0.5773502693	0.5773502692	0.5773502692
[3/3]	0.5163977793	0.5163977795	0.5163977795
[4/4]	0.5227030796	0.5227030798	0.5227030798
[5/5]	Complex Numbers	Complex Numbers	Complex Numbers

Table 1 show a result of numerical value of $f''(0) = A$ was obtained by using diagonal Padé approximants and the outcomes are in decent agreement with results in Wazwaz, (2007) and Ertürk and Momani (2008). Based on this proves an efficiency and accuracy of the SAIM - Padé approximations approach for solving such kind of problem. There are a similarities exact value of $f''(0) = A$ between DTM and SAIM for Padé approximations [2/2], [3/3] and [4/4]. From Table 1, we selected the approximation, $A = f''(0) = 0.5227030798$.

Case 2

Following Wazwaz (2007), in attempted to solve equations (8) and (9), we introduced a new independent variable

$$z(x) = Bf(Bx), \tag{20}$$

where, equivalent to

$$f(x) = \frac{1}{B} z\left(\frac{1}{B} x\right), \tag{21}$$

where, B is the parameter to be determined. Consequently, the equations (3) will be the same as:

$$z'''(x) + \frac{1}{2} z(x) z''(x) = 0. \tag{22}$$

with the conditions:

$$z(0) = 0,\ z'(0) = 0,\ z'(\infty) = B^2 f'(Bx) \to B^2 \text{ as } x \to \infty \tag{23}$$

which is means that $f''(0) = 1/B^3$ when we imposed $z''(0) = 1$. It is clear from (23), $B = \sqrt{z'(\infty)}$.

Applied the SAIM into (22) will yields,

$$z_0(x) = \frac{1}{2} x^2 \tag{24}$$

$$z_1(x) = -\left(\frac{1}{240}\right) x^5 + \left(\frac{1}{2}\right) x^2 \tag{25}$$

$$z_2(x) = -\left(\frac{1}{5702400}\right) x^{11} + \left(\frac{1}{161280}\right) x^8 - \left(\frac{1}{240}\right) x^5 + \left(\frac{1}{2}\right) x^2 \tag{26}$$

$$\begin{aligned} z_3(x) = &-\left(\frac{1}{6282355064832000}\right) x^{23} + \left(\frac{83}{571875655680000}\right) x^{20} - \left(\frac{5449}{125076897792000}\right) x^{17} \\ &+ \left(\frac{10033}{1394852659200}\right) x^{14} - \left(\frac{5}{4257792}\right) x^{11} + \left(\frac{11}{161280}\right) x^8 - \left(\frac{1}{240}\right) x^5 + \left(\frac{1}{2}\right) x^2 \end{aligned} \tag{27}$$

$$\vdots$$

By considered $z(x) = z_3(x)$ as the best approximation, then

$$\begin{aligned} z'(x) = &-\left(\frac{1}{273145872384000}\right) x^{22} + \left(\frac{83}{28593782784000}\right) x^{19} - \left(\frac{5449}{7357464576000}\right) x^{16} \\ &+ \left(\frac{10033}{99632332800}\right) x^{13} - \left(\frac{5}{387072}\right) x^{10} + \left(\frac{11}{20160}\right) x^7 - \left(\frac{1}{48}\right) x^4 + xx) \\ = &-\left(\frac{1}{5702400}\right) x^{11} + \left(\frac{1}{161280}\right) x^8 - \left(\frac{1}{240}\right) x^5 + \left(\frac{1}{2}\right) x^2. \end{aligned} \tag{28}$$

According to Wazwaz (2007), $z'(x)$ has a leveled off for $x > 2$. Thus, we can use $B = \sqrt{z'(\infty)}$ to estimate several values of x as show in Table 2.

Table 2. A comparison value of B and $f''(0) = 1/B^3$ between VIM and SAIM

Series approximants, x	VIM (Wazwaz, 2007)		SAIM	
	B	$f''(0)$	B	$f''(0)$
2.0	1.313034017	0.4417454320	1.313034017	0.4417454320
2.2	1.347736192	0.4084936660	1.347736192	0.4084936660
2.4	1.373000106	0.3863565574	1.373000106	0.3863565574
2.6	1.387743095	0.3741732832	1.387743095	0.3741732832
2.8	1.388836100	0.3732905625	1.388836100	0.3732905625

Table 2 show a result of numerical value of $f''(0) = 1/B^3$ was obtained by using simple series approximants and the results are in full agreement results in Wazwaz (2007). This proved the efficiency and accuracy of the SAIM approach.

From $f(x) = \frac{1}{B} z\left(\frac{1}{B} x\right)$, the series solution of second Blasius equation is given by,

$$f(x) = -\left(\frac{1}{6282355064832000}\right)\frac{x^{23}}{B^{24}} + \left(\frac{83}{571875655680000}\right)\frac{x^{20}}{B^{21}} - \left(\frac{5449}{12507689779 2000}\right)\frac{x^{17}}{B^{18}} + \left(\frac{10033}{1394852659200}\right)\frac{x^{14}}{B^{15}} - \left(\frac{5}{4257792}\right)\frac{x^{11}}{B^{12}} + \left(\frac{11}{161280}\right)\frac{x^{8}}{B^{9}} - \left(\frac{1}{240}\right)\frac{x^{5}}{B^{6}} + \left(\frac{1}{2}\right)\frac{x^{2}}{B^{3}} \quad (24)$$

where B is appximated by 1.388836100 and $f''(0) = 0.3732905625$.

Conclusion

In this article, two form of Blasius equation were solved numerically using a semi analytic iterative method. The results obtained are comparable with results from Wazwaz (2007) and Ertürk and Momani (2008). Therefore, we concluded that SAIM has a good potential to solve every types of differential equations in physical problems.

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