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## REAL-TIME AD CLICK FRAUD DETECTION

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# REAL-TIME AD CLICK FRAUD DETECTION

A Thesis

Presented to

The Faculty of the Department of Computer Science

San José State University

In Partial Fulfillment

of the Requirements for the Degree

Master of Science

by

Apoorva Srivastava

May 2020

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The Designated Thesis Committee Approves the Thesis Titled

REAL-TIME AD CLICK FRAUD DETECTION

by

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May 2020

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ABSTRACT  
REAL-TIME AD CLICK FRAUD DETECTION

by Apoorva Srivastava

With the increase in Internet usage, it is now considered a very important platform for advertising and marketing. Digital marketing has become very important to the economy: some of the major Internet services available publicly to users are free, thanks to digital advertising. It has also allowed the publisher ecosystem to flourish, ensuring significant monetary incentives for creating quality public content, helping to usher in the information age. Digital advertising, however, comes with its own set of challenges. One of the biggest challenges is ad fraud. There is a proliferation of malicious parties and software seeking to undermine the ecosystem and causing monetary harm to digital advertisers and ad networks. Pay-per-click advertising is especially susceptible to click fraud, where each click is highly valuable. This leads advertisers to lose money and ad networks to lose their credibility, hurting the overall ecosystem. Much of the fraud detection is done in offline data pipelines, which compute fraud/non-fraud labels on clicks long after they happened. This is because click fraud detection usually depends on complex machine learning models using a large number of features on huge datasets, which can be very costly to train and lookup. In this thesis, the existence of low-cost ad click fraud classifiers with reasonable precision and recall is hypothesized. A set of simple heuristics as well as basic machine learning models (with associated simplified feature spaces) are compared with complex machine learning models, on performance and classification accuracy. Through research and experimentation, a performant classifier is discovered which can be deployed for real-time fraud detection.

**Keywords – Digital marketing, Online advertising, Click fraud detection, Ad Spam, Real-time fraud detection**

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## **1 INTRODUCTION**

In the past few decades, the use of the Internet has grown exponentially. The growth of the web-based online advertising industry has created many new opportunities for lead generation, brand awareness, and electronic commerce for advertisers. A large volume of this content is offered for free. This is true even for content that we used to pay for, such as newspapers, blogs, advertiser's site (Macy's etc.). Naturally, content creators need to make up for the absence of income by finding a new revenue stream. This revenue stream is Internet advertising. By showing relevant ads to their visitors, and having them click on those ads, content creators are able to convert traffic into an effective revenue stream, while also delivering value to the users. In the online marketplace, page views, forms submissions and clicks often result in money changing hands between advertisers, ad networks, and web site publishers. Figure 1 shows an advertising ecosystem, with various entities and their relationships. Online advertising has become the greatest source of revenue for many Internet giants such as Google, Yahoo!, Bing, Facebook, Twitter, etc [1]. These search engines and content destinations make revenue by selling ad views and ad clicks, which are generated along with the search results or presented alongside their content.

Digital advertising can be broadly categorized into two types: brand advertising and performance advertising. In brand advertising, a marketer is essentially looking to promote their brand by reaching a broad audience. Even before the advent of digital marketing, brand advertising has been the dominant form of advertising on non-digital channels, such as Television, Print Media, billboards and hoardings. In this form of advertising, the views matter, as the advertisers are trying to reach as many users as possible in their given marketing budget. The pricing models for brand advertising are predominantly pay-per-view, where the advertisers are charged each time a user views their ad online. Facebook Ads is a great example of a brand advertising ad network.

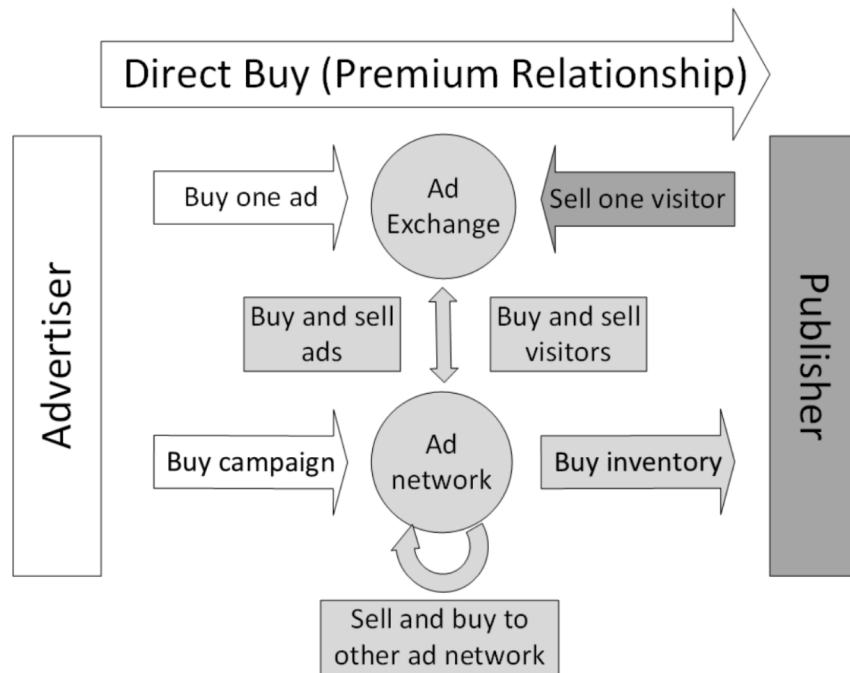


Fig. 1: The Advertising Ecosystem [2]

In performance advertising, a marketer is trying to drive an outcome: such as online shopping purchases from their website or offline purchases from their physical stores, lead generation to create a pool of high-value users, mobile App downloads, etc. Performance advertising tends to be highly targeted and relatively more contextually useful to the users. In this form of advertising, the advertisers are typically unwilling to pay for each ad view, since brand creation and propagation is not the intent, performance is. Thus, performance advertising tends to use a pay-per-click (PPC) model for pricing, where advertisers pay for each ad click on the publishers' websites. Google Search Ads is a great example of a performance advertising network. Other search engines such as Yahoo and Bing also offer performance advertising and pay-per-click pricing. These search engines take the mantle of matching a user's search terms to the right ads (aka relevance), and then charge for each ad click.

Both these forms of digital advertising are susceptible to ad fraud. In brand advertising, the predominant ad fraud that occurs is the impression fraud. This is particularly true of ad networks such as Facebook Ad Network (FAN) or Google Display Network which place ads on third-party publisher websites (say random blog.blogwebsite.com). Here, malicious publishers try to garner fraudulent ad impressions.

In performance advertising, the predominant ad fraud is the click fraud. In this fraud, malicious players try to generate fraudulent ad clicks, thereby leading the ad network to charge the advertiser incorrectly. Fraudsters also seek to take advantage of new opportunities to conduct fraud against these parties with the hope of having some money illegitimately change into their own hands. Generation of such invalid clicks, either by humans or software with the intention to make money or to deplete a competitor's budget, is known as click fraud. This ultimately threatens the fundamental economics of online advertising, as advertisers are forced off of auctions, and in general, good content can no longer be supported by advertising. Combating Click Fraud requires significant investment in resources and large-scale detection systems, as click fraud bots constantly change and evolve in response to detection.

Click fraud is very demotivating for advertisers. Click fraud drives up advertising costs for the businesses [2]. It skews the statistics and analytical data the advertisers rely on to make effective marketing decisions. These decisions could include things such as which search keywords to bid for ads on, which campaigns to run longer due to a better return on investment, how much marketing budget should be spent on digital marketing.

The vast majority of click fraud originates from an advertiser's competitors. Click fraud can help the competitors waste an advertiser's pay-per-click marketing budget, thus

driving up their marketing costs (cost per sale / acquisition – CPA). Many ad networks allow setting daily ad budget upper limits and caps, which means that click fraud can lead to the daily budget being exhausted and the advertiser's ads stopping altogether. In an auction scenario, this can in turn reduce the advertising costs for the competitors, as they do not need to outbid the advertiser in question. Webmasters and publishers are the next big source of origin for ad click fraud. These publishers have an incentive to click on ads on their website to generate short-term income, at the cost of advertisers. Fraud rings are also an important and growing source of ad click fraud. These large groups of people specifically target certain ad networks to generate large amounts of revenue for themselves. They might use human click farms, or a huge array of automated programs and botnets.

A wide variety of research has been carried out on this highly sensitive and lucrative topic. Major ad networks have dedicated research teams which have looked into this problem. Broadly speaking, the approaches can be classified as logged-event-based (prior activity information) and client-side based (blocking malware on legitimate users'machines).

Most of the approaches in use today perform an offline classification of ad click fraud. Fraud detection usually relies on complex machine learning models, working in high-dimensional feature spaces. Additionally, the data sets tend to be massive. Big ad companies might see multiple hundreds of millions of ad clicks per day. Thus, training these models and using these models for lookups tends to be quite costly. As a result, ad click fraud is predominantly done offline. This, however, limits the ad networks' or advertisers' response. If the fraud could be detected in real-time, ad networks and advertisers could be better positioned to investigate the perpetrators and take active measures.

In addition, there is a bigger set of raw features available in real-time. Most ad companies limit the amount and nature of data they log for each click. This has to do with their users' privacy and data retention policies. For instance, many HTTP headers might not be logged, or IP addresses anonymized before logging. As a result, machine learning models that could be built offline work with a subset of available data. If the fraud model training and lookups could be done in real-time, while processing the clicks, there is a greater possibility of detecting fraud clicks which might be undiscoverable offline. Unfortunately, most techniques used today typically do not have this level of performance.

There is also a trade-off between fraud model performance versus precision and recall. It is easy to build models with very low cost, but these would typically suffer from poor precision and/or recall. A fraud detection system which appropriately optimizes for low cost, while delivering good precision and recall is highly desirable. This is the subject of this thesis.

This report details the research and analysis work for real-time ad click fraud detection. It starts with a literature survey of the state-of-the-art of ad fraud detection, in section 2. The survey provides an overview of the techniques used for detecting fraud, along with the performance, wherever mentioned in the associated literature. This is followed by a description of datasets used for research and experimentation, as well as the associated dataset construction in section 3. A detailed exploratory data analysis has been carried out and articulated in section 4. The training and testing system design has been discussed in section 5. A state of the art model based on multi-level perceptrons is presented in section 6 as the baseline. Heuristic rules have been explored in section 7. Following that, machine learning models and associated features are presented in section 8. The results of various models are presented in section 9. Finally, conclusions and potential future work have been discussed in section 10.

## 2 LITERATURE SURVEY

This section presents a survey of published papers in reputed journals and conference proceedings on the topic of ad click fraud detection. There are a wide variety of ad click fraud detection methods proposed in prior work. This section examines the results and success these methods have exhibited.

The section is organized as shown in Fig 2. Section 2.1 will analyze the research done for logged data-based approach. This will consider logged event activity related to ad network events, advertiser website events and publisher events. Section 2.2 will describe the studies done on client-based mitigation.

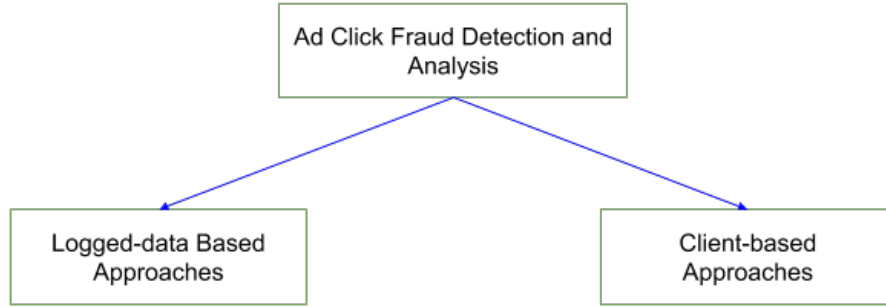


Fig. 2: Survey Section Organization

### 2.1 Logged-events Based Approaches

A large number of published works use what could be broadly characterized as logged-events based approaches. The basic idea here is to keep detailed ad impressions, ad clicks and advertiser website visit logs, and identify or learn patterns signifying malicious ad clicks.

As one of the earliest works on ad click fraud detection, Li Chen et. al. [3] focused on crowd-sourcing click fraud attacks. In this scheme, a malicious party sources a set of bad actors who could be paid to click on ads on a given publisher website, or ads of a



particular advertiser across publishers. Li Chen et. al. performed a clustering analysis to detect crowdsourcing click frauds. Their approach uses multiple click descriptors such as ClickID, UserIP, SearchQuery, Cookies, AdvertiserID, ClickTime and so on. They concluded that specific advertisers get predominantly targeted by such attackers. They also found out that IP addresses are a good identity mechanism to identify malicious clickers. They defined crowdsourcing fraud click behavior based on 1) Denseness, 2) Moderateness and 3) Concentricity.

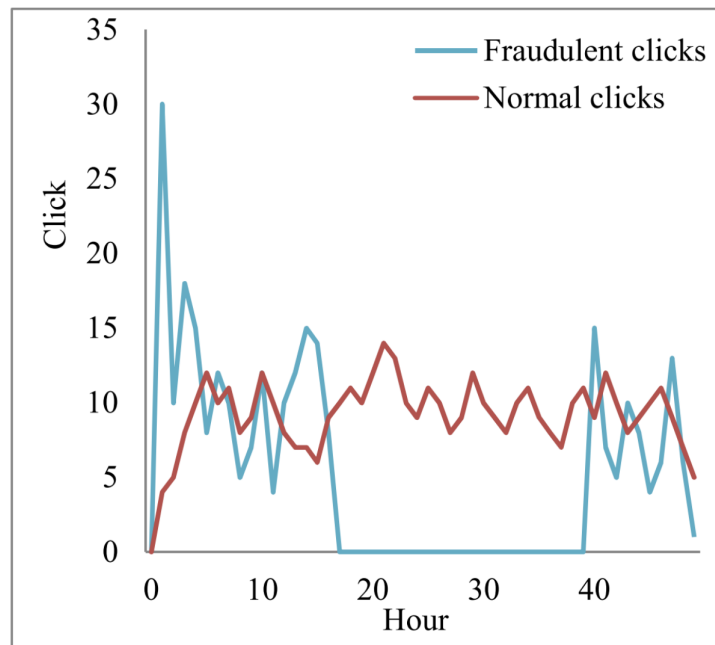


Fig. 3: Click Concentricity helping identify Ad Fraud [3].

Denseness pertains to the entropy between AdvertiserId and UserIP. If, for example, few IP addresses account for a large share of a given advertiser's clicks, there's a denseness concern indicative of ad click fraud. Similarly, fraudulent ad clicks from crowdsourcing tend to occur during short time periods, as opposed to regular users' traffic. Thus, if the data shows a large number of clicks in a short time duration out of the ordinary, the data demonstrates a concentricity pattern and is indicative of ad click fraud.

Based on these features, the study uses clustering analysis to detect click fraud. For this clustering, Li Chen et. al. [3] used a bipartite graph between `AdvertiserId` and `UserIP`, and detected groups of IPs deemed to pertain to the fraud operation. Members in a malicious group click the same advertisers' ads over a short time period. The model leveraged this property and can effectively detect these malicious cliques. On simulated data sets, Li Chen et. al. have demonstrated 65% to nearly 100% recall, along with 82% precision on detecting crowdsourcing click fraud on simulated data sets [3]. Additionally, their technique could label 398 million records in 61 minutes, thus incurring a cost of 9.196  $\mu$ s per classification.

Matthieu Faou et. al. [4] used a remarkably different approach to combat ad click spam. They monitored the activities of an ad clicking malware over multiple months, and created a map of fraudulent actors involved in the malicious activity. They identified these actors using social network analysis, and then asserted these actors could be pressured and influenced to disrupt the click fraud emanating from them. To identify these actors, Faou et. al. analyzed the ad click redirect chains deeply, looking at referrer URLs. They studied the redirection chains emanating from infected computers, to understand the value chains and actors' relationships in real world. With this analysis, they were able to create a social graph among the actors and to determine the key actors ripe for disrupting the malware. The study shows that fraud perpetration tends to be concentrated to a few actors. As such, if these actors could be disrupted, their impact could be significantly curtailed. Such an approach is good from a long-term perspective. Referral URLs and request origins, however, could be good signals useful for real-time fraud detection.

Brendan Kitts et. al. [5] proposed a different approach. They used bot signatures for click fraud detection. They mention that typical ecosystem measures such as Spiders List, IAB Robots and public domain IP blacklists are insufficient to combat sophisticated bots perpetrating ad click fraud. Their work recommends creating a common pool of bot

signatures. A bot signature is defined as a set of logged HTTP headers corresponding to the weblog (sequence of requests received) of a service under attack where the bot has been identified positively, along with the probability of each request originating from that bot.

According to the study, bot signatures have multiple benefits: (a) They are readily sharable between security organizations, enabling quicker mitigation of click fraud threats. (b) They create a good dataset for Ad Networks to test and evaluate their safety systems, and (c) They allow companies to use labeled data to ensure that their system is accurately detecting fraudulent traffic. Brendan Kitts et. al. [5] used techniques for creating 8 simulated bots, which create simulated fraudulent traffic of various kinds. They used different algorithms, such as user click frequency, presence of cookie, IP blacklist, cluster of users, user ad clicks sequence count, user keyword clicks count for bot filtration. Frequency capping as a mitigation mechanism (and correspondingly, number of clicks per IP as a feature) works well. Mature bots, however, were able to circumvent detection and prevention despite these frequency capping filters. Hence, they trained a decision tree to utilize multiple features in order to determine whether the traffic is from a bot or from a human.

The approach by Mehmed Kantardzic et. al. [6] uses multi-model evidence fusion for detecting click fraud in real-time. The basic idea is to rely on multiple sensors indicative of ad click fraud, and then fuse the signals from them to make a higher accuracy detection. The paper identifies each independent sensor component as a data mining module. Each of these sensors yield identification signals of low precision. The output of these sensors is then fused and correlated, to achieve better precision. Fusing the data lets it provide complete and timely assessments of situations and threats, as well as their significance. They've termed their system as a collaborative click fraud detection and prevention (CCFDP) system. Three modules have been used to find fraudulent clicks.

There is (1) a rule/heuristics based module, (2) a click map module, and (3) an outlier detection module. The sensors span across server as well as client side. Each click is first scored independently by each module. Scores are later combined using the Dempster-Shafer evidence theory [7]. The results of the analysis estimated the average precision of identifying click fraud as 64%. This is despite the fact that there was only 53% highest average precision among the individual modules. The paper has not discussed performance characteristics associated with the CCFDP system. Given that it is combining scores across a few weak classifiers, it is expected that the CPU-time would be reasonably low.

Haitao Xu et. al. [8] have proposed an approach that enables advertisers to detect the ad click spam at their end, instead of relying on ad networks to filter out such information. This helps the advertisers evaluate the ad campaign ROI better. In their system, each click is classified as either fraudulent, casual, or valid. The technical basis of their work is passively scrutinizing user engagement on the advertisers' websites and identifying visiting clients as true users. Haitao Xu et. al. [8] recommended a click fraud detection system with two components: (1) a proactive functionality test and (2) a passive examination of browsing behavior. The functionality test is used to check if the client is authentic (a browser or a bot). It assumes that most clickbots have limited capabilities and functionality, when compared to regular browsers. For instance, a bot might not be able to set some cookie, or a value in browser local-storage. When a device fails the functionality test, all the clicks originating from that device would be considered fraudulent. The second component uses browsing behavior to identify if the user is human or bot. For instance, a bot would likely not engage in a human-like behavior of checking out various products on an e-marketing website after an ad click. When both these checks pass, the corresponding click is validated. They have also proposed the concept of a "casual click". A casual click is defined as a click done by a real user, with no intention of

making a transaction on the advertiser website. If the device passes the functionality test, but doesn't show much engagement on the advertiser website, it is marked as a "casual click". To test the proposed system, they built a prototype and deployed it on a large production web server at an advertiser. They ran ad campaigns at a major ad network for 10 days. Their experimental results show that the approach can detect much more fraudulent clicks than the ad network's in-house detection system. On an independent dataset, they demonstrated a 93.9% precision and 94.6% recall. The detection is based on firing an ajax request with a challenge code for Javascript. They observed an average round-trip time of 200 ms at the clients' end to facilitate this.

Research done by D. Antoniou et. al. [9] extends the concept of click fraud. They characterized ad click fraud as a pattern or sequence of ad clicks aimed towards changing the regular function of a website, with a goal to produce some specific results. The paper focuses on two types of ad click frauds: 1) Inflationary click fraud: publishers clicking on ads on their own websites, and 2) Competitive click fraud: advertisers clicking on competitors' ads to exhaust their budgets and bring their ROI down. The paper proposed an algorithm that uses splay trees for: a) tracking publisher web pages, to detect which web pages encounter bursts using visit frequency, and b) tracking actors' IPs, to determine which ones might be malicious, based on their click frequency. The proposed solution in the paper attempts a real-time click fraud detection using efficient data structures. Their paper hasn't discussed the performance characteristics, however, aside from asymptotic complexities.

One of the first works on an advanced malicious CloudBot detection is done by Yu Guo et. al. [10]. They used raw traffic analysis to study the CloudBot traffic. Their detection approach is based on using machine learning techniques with multi-layer features. They divided the work into three segments. Firstly, a sampling method is followed to sample across IP and time, and arrive at a more practically viable smaller

dataset sample. With this, they were able to extract user characteristics over a significantly large period of time, while making sure there is sufficient information to identify cloudbots. Secondly, they used multi-layer traffic features, to encompass the primary differences between human traffic and cloudbots. Lastly, a method is used for detecting CloudBots accurately in close to real-time. The feature set suggested in the paper includes basic features, such as the #peer nodes, #packets, #bytes, #flows, their statistical characteristics, OS fingerprint information, network packet TTL related features, port related and application layer features. The models are then evaluated by measuring the precision (P) and recall (R) rates of identifying IPs belonging to cloud bots. Based on the experiments, Random Forest models outperform other algorithms, with a high precision of 93.3%, along with a 94.3% recall. Decision-tree gives the second best results and logistic regression is the next best algorithm. The results show that Random Forest based detection methods can provide good classification characteristics. The performance characteristics haven't been discussed in the paper.

Riwa Mouawi et. al. [11] proposed a profoundly different approach. The fraud detection is typically carried out by trust services managed either by the ad networks or by the advertisers. Typically, there is no collaboration between the two sides. Both the parties have different interests, sometimes at loggerheads with each other. Riwa Mouawi et. al. [11] recommend a neutral third party, trusted by both sides, to overcome this problem. They crowd-sourced data from various (network, publisher, advertiser) triplets and combined the data across these. 1) Per-publisher % of duration-suspicious clicks, 2) Per-publisher #Clicks, 3) Per-publisher #UniqueIPs / #Clicks, 4) #Conversions/#Clicks per-publisher, 5) Time-Variance of #Clicks.

Multiple different machine learning classifiers were tried out: Artificial Neural Network (ANN) classification, K-nearest Neighbours (KNN) classification and Support Vector Machine (SVM) classification. In their evaluations, KNN performed the best with

98% precision. SVM was the next best classifier, with nearly 96% precision, and it was followed by ANN with nearly 93% accuracy. Their fraud detection classifiers had a small false positive rate.

Leyi Song et. al. [12] developed a three stage click detection / click spam filtration system, on large scale data. The stage-wise filtering system architecture described in this paper has three major components of the research: 1) Stage 1 consists of heuristics-based filters to detect the most obvious invalid clicks with very high confidence. These filters identify the most obvious malicious players: heavy hitters and frequent clickers; 2) Stage 2 consists of classification-based filters. It determines more complex clicks and uses a labeled training set, and 3) Stage 3 consists of a clustering-based filter, to identify groups of similar publisher websites involved in malicious behavior. The research analyzed distance between cluster centroids and diversity in queries to segment malicious groups. They observed that there is a greater diversity in queries from normal clicks, compared to the ones originating in malicious groups. Song et. al. [12] presented their results to a group of ad click fraud experts, and claimed a 97% click-fraud prediction precision.

Table 1: Performance Characteristics in Surveyed Literature

Reference	Precision	Recall	Prediction Time
[3]	0.82	0.946	9.1959 $\mu s$
[6]	0.64		200 ms (client-side)
[8]	0.939		
[11]	0.98		
[12]	0.976		337 $\mu s$

Table 1 summarizes the performance characteristics from reviewed literature. This summary establishes that machine learning approaches tend to have better precision characteristics. Additionally, the CPU-time for classification varies widely, depending on the technique. Even for server-side approaches, the fraud classification could easily consume upto hundreds of microseconds. While per-request this might look small, this

adds up to a lot of compute cost and resources in real-world scenarios, which might deal with huge number of queries per second. As a result, much of the practical deployments tend to be offline.

## **2.2 Client-based Approaches**

There are other researchers who have taken the route of identifying the ad-click fraud right at the source, as opposed to looking through logged events at the ad service, or the advertiser website. The overall approach is to identify and mitigate automatically operating malicious programs (malware, bots, etc) operating on users' machines and clicking ads maliciously.

The research paper by Iqbal et. al. [13] explores a fraud detection technique, termed FCFraud, which could be included in an operating system's anti-malware software. It is based on correlating HTTP traffic and input hardware events, such as from mouse and keyboard. With these inputs, they claim to be able to detect the fraudulent processes in user machines which automatically click on ads silently in background mode. To prevent click-fraud, they look for programs running in background which mimic browser functions and send click traffic. They also utilize URL and network features to differentiate ad-click versus non-ad-click traffic. They use a RandomForest algorithm on features put across hardware events and click urls to automatically classify the ad requests. In experimental evaluations, their system successfully detected all the malicious processes running in the background. The authors engineered FCFraud to blocked these programs from accessing the network and hence stymied the botnets. The FCFraud system is able to classify 99.6% ad requests correctly from all user processes and was 100% successful in finding the fraudulent processes.

Another study by Rodrigo Alves Costa et. al. [14] explores mitigating click fraud based on the use of clickable CAPTCHAs. They proposed that a Certifier entity should provide credentials/certificates to users after they have passed a CAPTCHA test. It



enables the network advertiser to distinguish valid clicks, performed by humans, from ordinary clicks originated in overall traffic. The certifier can then attest to the validity of clicks from this user later on, which can then strengthen the probability of identifying click fraud. It has two main factors: a) Certifier can be either cookie based authentication method or a service which acts as an authentication authority, and b) Ticket authentications. To distinguish humans and computers, clickable Class II CAPTCHAs seem to have the best performance.

Kantardzici et. al. [15] analyzed the user activity by combining information from both client and server side. The paper utilizes various data fusion techniques such from 1) Direct sources 2) Indirect sources, and 3) History data. These sources are merged to record all events about the click traffic. This way, they provide a fraud-score in real-time, and are able to thwart malicious ad clicks at their source.

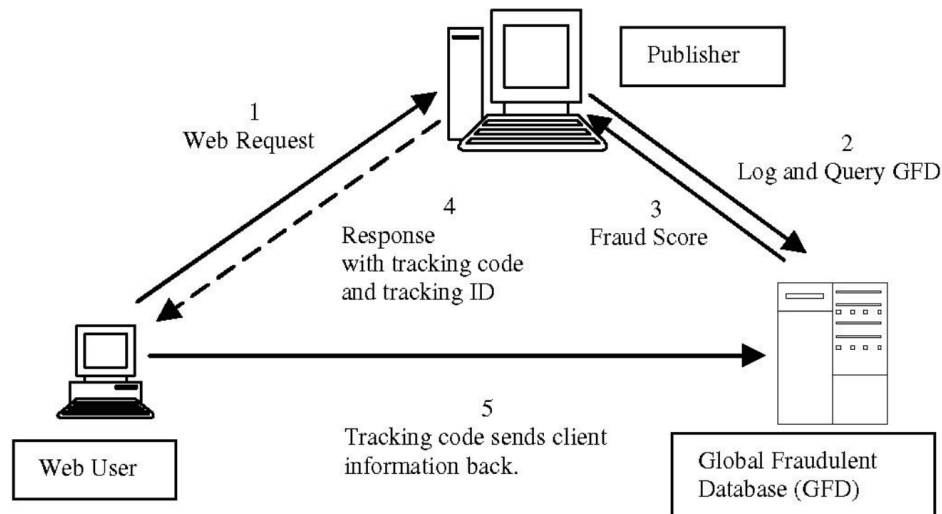


Fig. 4: Data Collection Setup from Kantardzic et. al. [15].

They discovered a high percentage of ad click fraud even with the most popular search services such as Google. The approach still needs more work, however. In particular, there were security, scalability and privacy concerns that need to be addressed.

### 3 DATASETS

In this section, we describe how we construct the datasets to use for analysis, training and evaluation. The dataset construction is based on the original TalkingData datasets [16] hosted on Kaggle. This dataset has been widely used by various research papers to study ad click fraud [TODO: add citations]. In the competition posted at Kaggle, the objective was to predict whether a given set of click ids each converted or not, based on the training data. Low-probability of conversion for a click can be a great basis to qualify a click as a spammy/fraudulent click.

#### 3.1 Original datasets

The original datasets on Kaggle have test and train CSV files. The columns contained in the training CSV are as follows:

Table 2: TalkingData Training Data CSV Schema

Column	Description
ip	IP address of the click (obfuscated)
app	app id for marketing
device	device type id of user mobile phone
os	os version id of mobile phone
channel	channel id of mobile ad publisher
click_time	timestamp of click (UTC)
attributed_time	timestamp of app download following the click, if any
is_attributed	prediction target, indicating if the app was downloaded

The test CSV doesn't have `attributed_time` and `is_attributed` columns. Instead, it contains another column: `click_id`, to hold a click identifier.

#### 3.2 Construction

To enable data evaluation, the original test dataset cannot be used as it is. This is because it doesn't have the labeled outcome column `is_attributed`. Therefore, the train CSV file in the original dataset is split into a test and train dataset. This is what is

used for all data analysis, training and evaluation. The original train CSV is split  $3/4^{\text{th}}$  into a new training dataset, and  $1/4^{\text{th}}$  into a new testing dataset.

This, however, has another problem. The clicks with conversions are too sparse. As mentioned in section 4.1, the conversion rate of clicks is very low. If a random predictor is used to predict conversions based on the conversion probability, the resulting classifier accuracy is too high. Thus, the test dataset creation has to be more sophisticated. The “new testing dataset” should be further sampled, so the resulting dataset contains 50% converting and 50% non-converting clicks. The complete construction is shown in Figure 5.

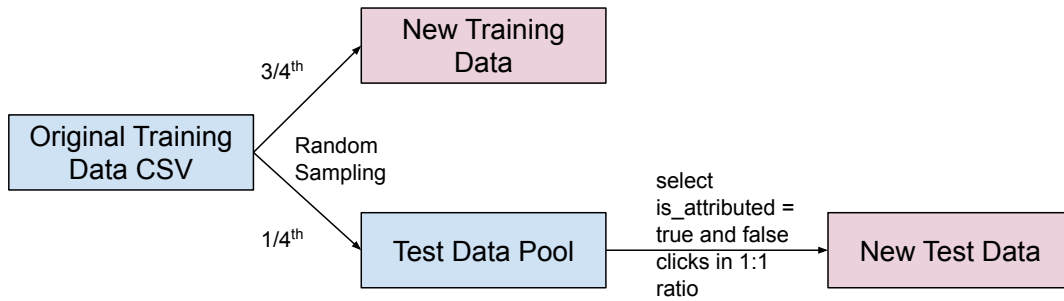


Fig. 5: Dataset Construction.

This dataset construction yields a test-set which is very similar to the one used for the competition on Kaggle. There too, the clicks have a 50–50 distribution on attributed and non-attributed. This helps test approaches locally quickly, before running models on the massive test datasets and evaluating using Kaggle.

### 3.3 Dataset hosting

The original datasets are very massive, and difficult to work with. Thus, these were sampled to make them easy to work with locally. Local sampled datasets greatly speed up the execution velocity and modeling iterations.

At the same time, certain nuances can only be captured by looking at the entire data. For instance, in section 7.2.3, the heuristic uses the notion of a session. Session-based rules can only be effective if sessions can be accurately identified in their entirety. Sampling, however, prevents that from happening, and yields much smaller sessions. In such cases, some jobs were run in a cloud environment, to allow pulling data based on database queries close to the source of data.

The datasets were thus, additionally, hosted in a Google Cloud Platform project, using BigQuery. BigQuery is a big-data analysis solution available on Google Cloud Platform. It allows automatically parsing the CSV datafiles from the original TalkingData datasets, and creating tables. The tables can be queried using a GUI interface provided by the BigQuery UI, enabling running analysis quickly. It also provides SQL-like query interface on the data, to perform complex queries.

The BigQuery interface is also useful for splitting the data into component sub-tables. As described in section 3.2, it is important to split the original training dataset into the real test and training datasets used by the project. Through BigQuery, it is very easy to perform the splits. For instance, if we wanted to split a table `tablename` into two tables, one with  $3/4^{\text{th}}$  and the other with  $1/4^{\text{th}}$  of data randomly:

```
SELECT *
FROM `ad_fraud.Fraud.orig_train_dataset` WHERE
MOD(FARM_FINGERPRINT(CAST(click_time AS STRING)), 4)
in (0, 1, 2) -> new_train_dataset

SELECT *
FROM `ad_fraud.Fraud.orig_train_dataset` WHERE
MOD(FARM_FINGERPRINT(CAST(click_time AS STRING)), 4) = 3
-> new_test_dataset
```

Similarly, for creating the final test set with 1:1 attributed and unattributed clicks, a query similar to the following could be done:

```
SELECT *
FROM `Fraud.new_test_dataset`
WHERE is_attributed = 1

UNION ALL

SELECT *
FROM `Fraud.new_test_dataset`
WHERE is_attributed = 0 and RAND() < 0.00247
-> final_test_dataset
```

## 4 EXPLORATORY DATA ANALYSIS

In this section, we describe the exploratory data analysis (EDA) carried out on the TalkingData dataset [16].

### 4.1 Conversion Sparsity

Conversions are very sparse in the TalkingData datasets [16]. In fact, only 0.2479% of ad clicks in the training dataset have attributed conversions. Thus, even a random predictor, using 0.2479% as the prediction probability for conversion, shows high precision. On the test dataset, this predictor had an accuracy of 98.1805%. Similarly, on such a dataset, a predictor which always predicts a click as “not converted” would achieve an over 99% precision. This is why the test dataset has to be crafted more carefully. The actual test dataset will include a significant percentage of clicks which were attributed. This would help in measuring the deltas between models much better and provide a better prediction accuracy baseline.

As mentioned in section 3.2, finally a test dataset consisting of 1:1 attributed and unattributed ad clicks is used. On running the aforementioned random predictor on this dataset, the accuracy is 49.49%. In fact, if the predictor always predicted “fraud” or “not fraud”, the theoretical accuracy would be 50%. This accuracy should be considered baseline for all the models. Each legitimate model is expected to have a better than a 50% prediction accuracy.

Note that this has a side-effect of understating the precision. In real-world traffic, one would not expect to see conversions on most clicks. In this case, however, the converting clicks have been amplified. Additionally, in the real-world traffic, it is easier to identify the non-converting clicks. Thus, a 90% precision on the modified test set might actually translate to a much higher precision on a real-world traffic dataset. As such, the precision numbers stated in the experiments to follow should be treated as under-estimates.

## 4.2 Clicky IPs

In the dataset, ‘clicky’ IPs are quite common. There are many IPs with a large number of clicks. At the same time, many of these IPs with a large number of clicks have abysmal conversion rates (see Table 3).

Table 3: Clicky IP Addresses (with  $\#clicks \geq 100$ ) vs. Non-Clicky IPs

	<b>Clicky</b>	<b>Non-Clicky IPs</b>
# Clicks	90196132	2283967
# Conversions	117040	112208
Conversion Rates	0.13%	4.91%

As is evident from the data, there is a substantial difference in conversion rates between clicky and non-clicky IPs. Non-clicky IPs have a conversion rate close to 5%, which is quite practical [17]. Clicky-ness is a major indicator of conversion rates. Therefore, clicky-ness should be considered as an important feature in model training. Heuristic models can rely on this to identify IPs which are more likely to convert.

At the same time, the clicky IPs contribute more than 50% of conversions, hence can’t be all considered fully spammy. This is partly because at least some of these clicky IPs represent scenarios where a single IP spans across a large number of users. For instance, the IP could be that of a router in a workspace with many dozens of users. In such scenarios, it is completely possible to have a large number of clicks and a substantial volume of conversions. The challenge is in being able to separate out these two kinds of IPs.

## 4.3 Relationship between #Clicks and Conversion Rates

While highly clicky users convert at a far lower rate, in general there is also a relationship between number of clicks and the observed conversion rate. For legitimate

users, it is expected that the conversion rate would go down as the number of clicks increase. For instance, a user might click on an app download ad multiple times, but might ultimately download the app only once. Figure 6 shows the average conversion rates observed in the training data set for IPs with the corresponding number of clicks. Note that there are many users (IPs) which only have a single click, but have a very high (96%) probability of a conversion. This is again something a heuristic model could utilize in its rules. Similarly, this points to the number of clicks being a significant estimator of conversion rates for other kinds of models as well.

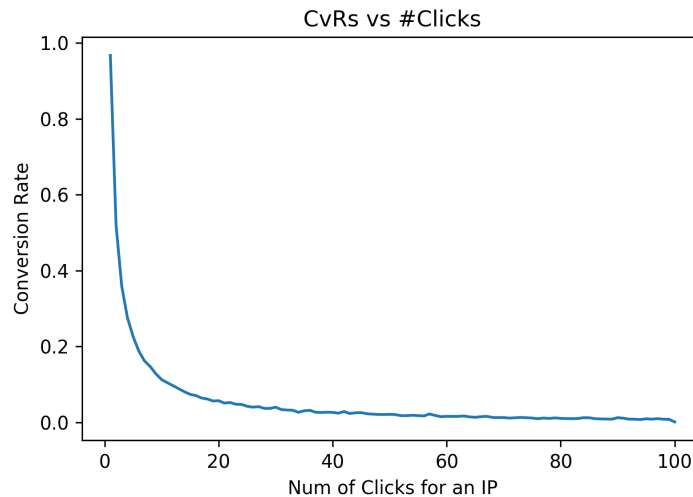


Fig. 6: IPs' Conversion Rates by Number of Clicks.

#### 4.4 Apps and Conversion Rates

There are 700+ app ids in the dataset, numbered starting from 1. Apps conversion rates across these ids vary very significantly. Figure 7 shows a visual representation of conversion rates by app ids. There are certain apps which have a nearly 100% conversion rate, whereas certain other apps which have a close to 0% conversion rate. This indicates that app is an important feature. For rate-based models, `app_cvr` can also be used as a



compact feature, instead of a categorical `app` feature. Heuristic models can close in on the high-CvR `app` ids, and declare their ad clicks as converted with a high probability.

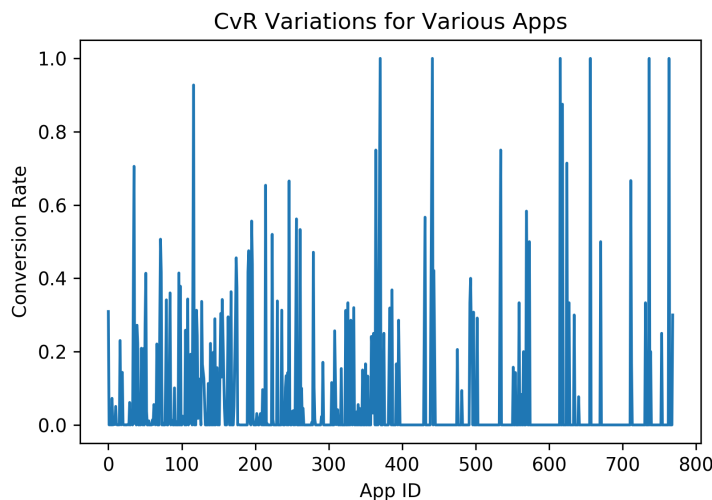


Fig. 7: Conversion Rates for Various App Ids.

#### 4.5 Conversion Rates by Device Types

Conversion rates can vary significantly across different types of devices users might have. For instance, it is well known that different device operating systems might have very different ad performance for users on different os. In this dataset, there are 800 distinct OS IDs, numbered from 1 to 800. While there are only 5-6 dominant operating systems in real world, in the dataset description, these have been explained as representing each OS version. Their conversion rate performance is shown in Figure 8. For most OS, the conversion rates are close to zero. But there are a select few OS where conversion rates are quite high. At the same time, most other OS have mid-range conversion rates. Due to this, we don't expect OS features to have a strong distinguishing power by themselves. They might, however, be quite potent when paired with other features.

Devices also show some variation in conversion rates across them. There are 3475 different device ids in the dataset. Figure 9 shows a histogram of CvR variation across

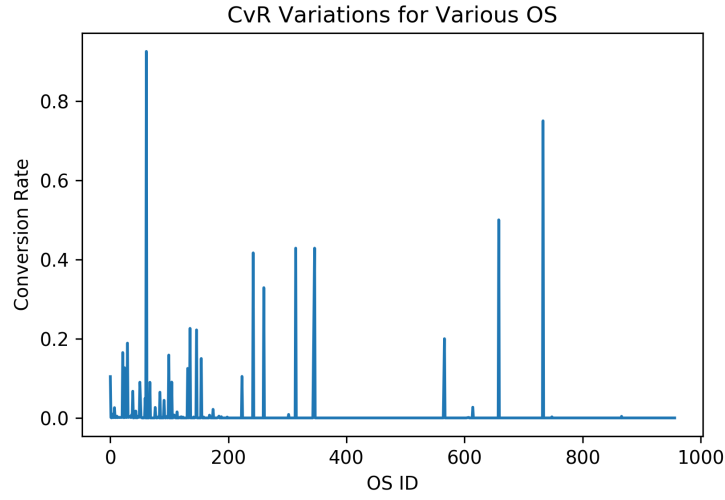


Fig. 8: Conversion Rates for Various OS Ids.

these devices. Here again, the differences in conversion rates is lower, with most conversion rates well away from 1.0. Therefore, it is expected that device features, by themselves, won't have a high distinguishing power, unless they are paired with other features.

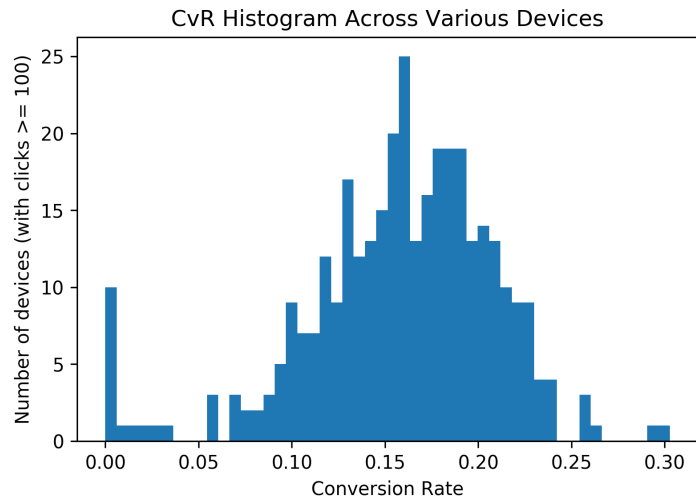


Fig. 9: Histogram of Conversion Rates Across Various Devices (with  $clicks \geq 100$ )

## 4.6 Channel Conversion Rates

On the channel dimension, the conversion rates do not show significant variation. Figure 10 shows the range of conversion rates observed across channels. On the y-axis, % of total conversions covered by channels falling in those conversion rate buckets is presented. This shows that most of the conversions come from channels with very low conversion rates. There are, however, a small number of channels with significant conversion rates. As such, the expected utility of channel ids to identify convertible clicks is low. Yet, it can not be fully ignored either, due to the presence of a few channel ids with significant conversion rates. Semantically, channels are very important to ad fraud detection. Channels represent publishers, and publishers have a strong revenue-incentive to fake ad clicks.

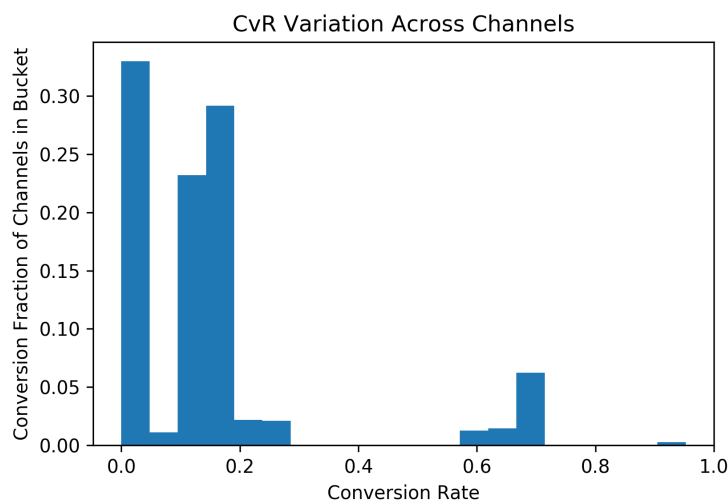


Fig. 10: % of Conversions in Channels Against the Channel CvRs

## 4.7 Conversion Rate Time Series

Conversion rates vary significantly across the hour of day. Figure 11 shows a timeseries of hourly conversion rates, superimposed with click volumes. Conversion rates in peak hours are nearly double of conversion rates in non-peak hours. The peaks of

conversion rates and clicks coincide. Interestingly, there are a few hours with fewer clicks but relatively higher conversion rates as well. This indicates that conversion rate by hours, or the hour of day, should be an important feature used for fraud detection.

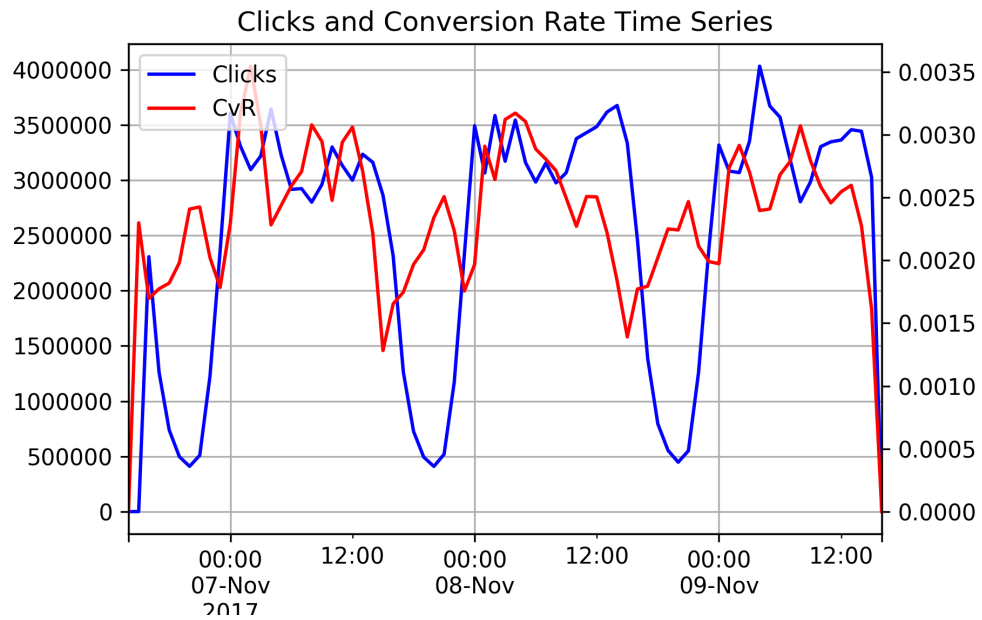


Fig. 11: Conversion Rate Time Series

## 5 SYSTEM DESIGN

This section describes the system used to perform model training and evaluation. The system developed provides a seamless way to train and test multiple models and their variants, and is capable of running well locally as well as on a cloud platform. The overall system design is provided in Figure 12.

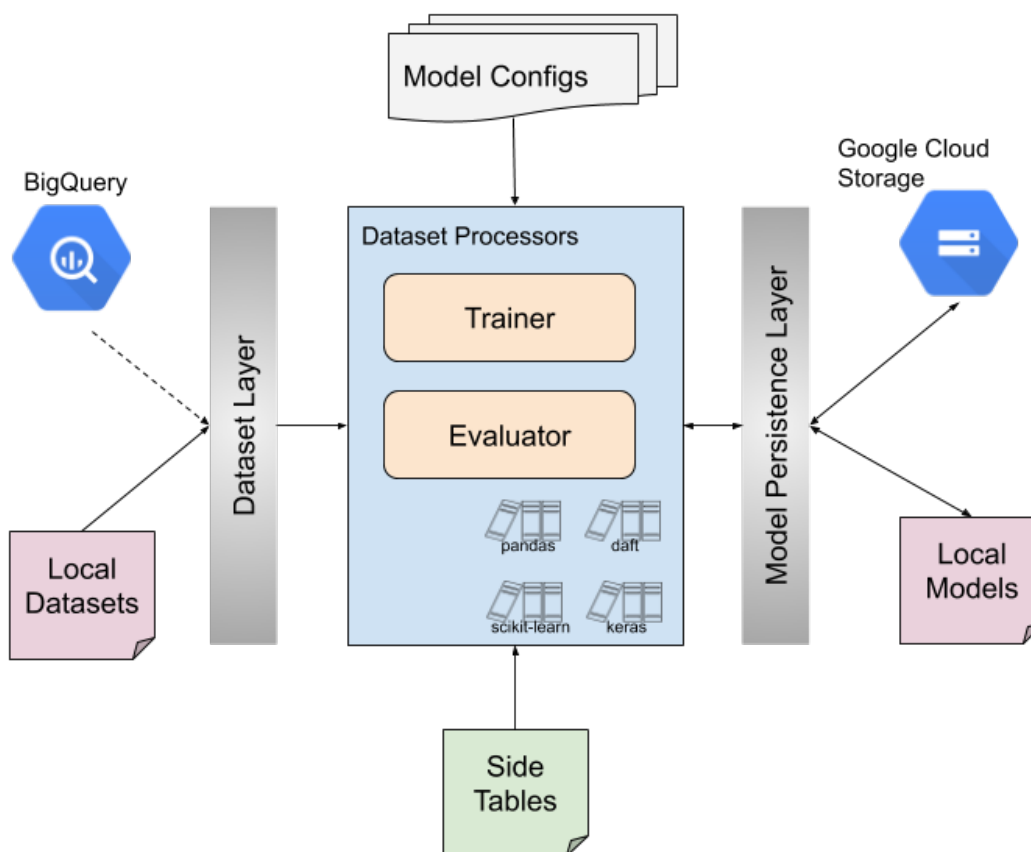


Fig. 12: System Design

At the core, the system has two `DatasetProcessors`. These processors work to process specific datasets, for a specified job. Two different implementations of the dataset processors: `ModelTrainer` and `ModelEvaluator` provide the mechanisms to train and evaluate the models respectively. These data processors rely on a dataset layer to pull

the data from appropriate sources. These sources could be either the local datasets, or tables hosted on BigQuery. The latter is useful when running the processors in a cloud environment. The dataset layer is capable of performing random sampling of the input data at the specified rates.

The data processors load model and variant configurations from JSON files. These files provide information about model tuning parameters to use, as well as the features to add, drop and transform. A common scheme of specifying (*model*, *variant*) has been used, to have an easy way of referencing the various model settings combinations. Based on the features, the data processors at times might load ‘Side Tables’. These are pre-computed pieces of data side-loaded by the data processors to speed up calculations. For instance, for CvR-space models, all the dimensional CvRs are side-loaded and joined with the datasets. These ‘Side Tables’ are generated by various Jupyter notebooks offline.

Finally, the data processors also access and modify models in a persistence layer. Model persistence is important as it allows training a model once while allowing testing it multiple times on different datasets. Notably, a model trained in a cloud environment can be tested locally on a smaller test dataset, and vice-versa. Two persistence mechanisms were used: Google Cloud Storage and local files. These can be used interchangeably across both the cloud and local environments, to allow rapid experimentation.

## 6 MULTI-LEVEL PERCEPTRONS

As the first step in research work, a multi-level perceptron network was attempted for the classification problem. This was to establish a baseline for precision, accuracy and performance against which other, more simplified methods could be compared. Machine learning models, based on MLPs and Neural Networks, are being increasingly used in the industry for classification problems, including ad fraud detection. Multi-level perceptrons, also popularly known as neural networks, have gained a lot of prominence in classification tasks over the last decade. As such, they serve the purpose of establishing a baseline performance on the given dataset at hand.

A perceptron models a single neuron, and is combined together to form larger neural networks. Mathematically, neural networks are able to learn a mapping function from inputs to output, and have been proven to be a very good universal approximation method [18]. Their predictive capability originates from their multi-layered structure. With this structure, they are capable of learning and representing features at various different scales. Multiple variants of the MLPs were attempted in various configurations, in order to get reasonable performance and accuracy.

### 6.1 MLP Construction

For the best performing model, the feature space used for MLPs was based on one-hot encoding of all categorical fields in data (device, os, channel, app). Additionally, the click-time was expressed into two features: click\_day and click\_hour. IP addresses were dropped from the data altogether. The MLP, thus, had 1379 input features in the training dataset. Two hidden layers were constructed, one with 48 perceptrons and the other with 24. Both these layers were set to have a hyperbolic tangent (tanh) as the activation function. Finally, since a binary classification is desired, an output layer consisting of one perceptron was constructed, with a sigmoid activation function. The Adam optimizer [19]

was used to minimize the binary cross-entropy loss [20]. In terms of training data, the classifier was trained with about 1% of available training set, using random sampling. To perform the training, 3 rounds were performed (2 rounds had a large loss, more than 3 rounds didn't yield much better results incrementally).

## 6.2 Performance

This model was able to provide a 95.43% precision on the test dataset. It also scored very well on Kaggle - at 95.114% precision, comparing well with the leading submissions on Kaggle for this dataset. Thus, it could be considered to be a good representative of neural network based approaches.

The training time for the model was fairly large. On an average, the model required 284  $\mu s$  per click to be trained. This is significant in the context of the dataset at hand, as it translated to multiple minutes on a very small sample (1%) of data. The performance for prediction tasks was better, requiring 9.63  $\mu s$  per classification. These results are summarized in Table 4. When compared with some of the results in the literature survey in Table 1, the performance of MLPs matches within 1  $\mu s$  to the approach followed by Li Chen et. al. [3], which is the best performance among the literature surveyed. Yet, this means that for any meaningful traffic load going into hundreds of thousands of ad requests per second, running such a model in a real-time mode would be costly. The model would not just be looked up, but would also need to be continually trained to adapt to the new traffic patterns.

Table 4: Multi-Level Perceptrons' Performance

Metric	Value
Test-set Precision	0.9543
Kaggle Precision	0.95114
Training Time per Click	284 $\mu s$
Prediction Time per Click	9.63 $\mu s$



These results are referred to as a baseline for other methods explored. From here on, the emphasis is on exploring simplified classification models, with the aim to get a better trade-off between precision and training/prediction time.

## 7 HEURISTICS

The first few attempts at simplifying the classification task to achieve better performance were based on heuristics. Heuristic models are an age-old way of performing classification tasks. These models mimic natural methods of classification. These methods might be mildly or significantly imperfect, but the hope is that they provide a solid classification scheme when various heuristic rules are grouped together. Heuristic models tend to be nearly fully explainable – it is easy to understand why a certain label was predicted vs. not. They require little to no training. Execution-wise, heuristics just apply the rules to incoming data, and tend to be extremely fast.

Heuristic models rely on a number of rules executed on the dataset. As an example, a rule can say: predict a click as converted if  $num\_ip\_clicks = 1$ . These rules are learnt by understanding the problem space, as well as data characteristics, very deeply. As such, they tend to be highly domain-dependent, as well as highly data-dependent. This is quite different from most machine learning models, where the models themselves follow a uniform framework, and feature engineering is the real work. Despite machine learning gaining prominence in the last decade, heuristic models are still very popular. Their explain-ability and deep knowledge of data are a major asset.

### 7.1 Sessionization

Some of the heuristics explored use the concept of ‘sessions’. A session can be defined as contiguous click activity (potentially spanning multiple ad clicks across multiple apps and channels) originating from a single IP address. Additionally, a session must be constrained to emanate from a given set of  $(os, device)$ . OS and Device could act as a proxy for identifying a single user within an IP address used by multiple users. While not perfect (ideally, one would use a Cookie Id or a logged-in User Id), it significantly increases the number of single-user activity sessions available to analyze. Various

session-level metrics can be defined and used as features for heuristics as well as even in machine learning models.

## **7.2 Heuristics Explored**

In this section, various heuristics for the problem at hand are described. Note that these heuristics may not generally apply to any ad click dataset for conversion or fraud-click detection. Instead, these are very specific to the problem+datasets at hand. Nevertheless, these heuristics have the potential to demonstrate strong wins in prediction accuracy. The remainder of this section discusses the various heuristics explored and used.

### *7.2.1 High attribute conversion rate*

The dataset at hand has a generally low conversion rate. The probability of any random ad click being attributable is lower than 1%. At the same time, there are a few signals which show strong conversion potential. For instance, if it is known that a given IP address only has one click, the data analysis (Figure 6) shows that it is over 98% likely for this click to get attributed. This heuristic exploits this fact. It looks at all the attribute values for a given ad click, to see if there is any attribute which has a fairly high observed conversion rate on training data. If so, it declares the click has converted. Note that this is only workable because the typical conversion probability if a random click is very, very low. If, on the other hand, it was meaningfully high (even, say, 10–20%), this heuristic would incur significant error rate. Additionally, this heuristic also provides a fairly high rate of false positives too. Yet, the overall precision of this heuristic was 98%, which is very high.

### *7.2.2 IP's clicks too high in an hour*

This is a negative heuristic. On finding IPs which have significant bursts of traffic in a given hour, these IPs'clicks can be marked as non-converted for that hour. This could be despite the fact that other rules predict a positive conversion outcome.

### 7.2.3 *Session's last click is convertible*

A user might legitimately perform multiple ad clicks in a given session. This could be because the user might be researching and exploring whether the app meets their needs and has the appropriate features. In this process, they might perform various searches, interact with ads at various times, before they ultimately download the app. If it was known that an IP address belongs to a single user (or a single household), the prior ad clicks can be discounted for conversions, and instead the conversion can be attributed to the last click in the session. This heuristic requires understanding and segmenting the user sessions.

## 7.3 **Heuristic Combination**

Different heuristics might provide conflicting positive and negative signals for a few clicks. That makes it difficult to provide a single answer for a click. Additionally, some signals might carry more weight than others – so a positive signal from one heuristic might be overridable by a negative signal from another heuristic with a higher weight. To capture these nuances, a heuristic combination method was built. Each heuristic specifies the predictive positive signal, based on its conviction. These predictive signals are then merged and clipped to arrive at the final answer. This provides a flexible framework to express and work with a large number of potential heuristics. The combination method is described in Algorithm 1.

## 7.4 **Heuristic Results**

The various heuristics' precision and recall is provided in Table 5. Coverage or recall here is defined by the % of converting clicks which the rule can identify (and, notably, not on the base of all clicks).

On their own, the heuristics did not yield very favorable results. This is because heuristics in this dataset have a high bar on their precision to be useful for classification.

---

**Algorithm 1** Heuristic Combination Algorithm

---

```
1: procedure COMPUTEANDCOMBINEHEURISTICS(Data, Heuristics, Weights)
2:   Results  $\leftarrow \emptyset$ 
3:   for each d  $\in$  Data do
4:     credits  $\leftarrow 0$ 
5:     for each h  $\in$  Heuristics do
6:       score  $\leftarrow h(d)$ 
7:       credits  $\leftarrow credits + Weights_h * score$ 
8:     if credits  $\leq 0$  then
9:       prediction  $\leftarrow 0$ 
10:    if credits  $\geq 1$  then
11:      prediction  $\leftarrow 1$ 
12:    Results  $\leftarrow Results + [prediction]$ 
13:  return Results
```

---

Table 5: Heuristics' Performance

Heuristic	Precision	Coverage
High Conversion Rate Attributes	0.98	0.1737
Single-click Sessions Convertible	0.1467	0.1998
Last Click Convertible	0.3853	0.112

The model effectiveness depends on the rules' precision. If the dataset has a low natural positives rate, it would typically require rules with more than 50% precision for the rules to be effective. The relationship is described in the equation below.

$$\Pr(positiveLabel) + Precision_{heuristic} > 0.5 \quad (1)$$

where:  $\Pr(positiveLabel)$  = Probability a random observation has a positive label, and  $Precision_{heuristic}$  = Precision of positive predictions by the heuristic. In the dataset,  $\Pr(positiveLabel)$  is 0.25%, which is really low. Because of this, only those heuristics help which have more than a 50% precision. Only one such heuristic was found: the high conversion rate heuristic described in section 7.2.1. A few other heuristics were identified, which, while would be typically quite effective, were stymied by the low random

observation positive probability. Some very high-precision heuristics are possible to identify, but they suffer from coverage problems.

There are many negative heuristics available as well, such as bursty-IPs, etc. Given the dataset, however, these are not very effective. Due to a low  $\Pr(\text{positiveLabel})$ , most clicks are anyway likely to be negatively identified as non-converting. As such, the negative heuristics do not add much value. For instance, while the ‘Clicky-IP’ heuristic clearly identifies a much higher average conversion rate in non-high-click IPs, it leaves out 50% of conversions which are done by high click IPs.

At the same time, heuristics do yield interesting insights into simplistic features and classifiers which can yield good results. Heuristics here have demonstrated a much higher precision than what’s expected of a random observation probability, which means they carry a positive signal. Yet, it is evident that they might not ultimately lead to good models. A model trained with these heuristics put together could only surmise a 62% precision on the test dataset. This result compares closely with multi-model evidence fusion work done by Mehmed et. al [6], who also used multiple weak sensors and combined data across them, and obtained a 64% precision. Yet, 62% is a weak precision for a classifier, and leads to a noisy output. The expectation is to be able to classify most of the clicks correctly, and heuristics fell short of doing that.

## 8 MACHINE LEARNING MODELS

After the heuristics, a few machine learning models were attempted. The idea was to keep a small feature set, and get models with reasonable accuracy (close to 90% precision) and good performance suitable for real-time applications (one order of magnitude improvement over MLPs). This section describes the various machine learning models attempted, along with their various variants, configuration settings and features.

### 8.1 Features

Even though the original dataset is limited in the number of features provided, a variety of features can be extrapolated from the given dataset. These features have the potential to add more distinguishing power in the models. A comprehensive list of features used in machine-learning models across the various trials is provided in Table 6. A subset of these features was chosen for various different types / variants of these models. In the next few sub sections, details on various different types of models and variants are provided.

#### 8.1.1 Feature Descriptions

Many of the features described above are self-explanatory, though some aren't. These other features are briefly described below.

**click\_day and hour\_of\_day:** These have their typical meanings. One important point to note is that these have been used in both categorical and non-categorical fashion in different model variants.

**ip\_num\_clicks:** Number of clicks observed from the given IP. To cover the entire timeline spectrum, these counts have been done on both the training and test data put together. As seen in EDA Section 4.3, number of clicks observed for an IP is an important signal to consider.

Table 6: Machine Learning Model Features

Feature	Description
app	App Id of the Ad Click
device	Device Id of the Ad Click
os	OS Id of the Ad Click
channel	Channel Id of the Ad Click
hour_of_day	Click's Hour of Day
click_day	Click's Day
ip_num_clicks	IP's #Clicks in Train+Test Period
one_click_ip	If the IP only has 1 Ad Click
ip_num_click_cvr	Avg CvR of IPs having this IP's #Clicks
app_cvr	Avg CvR of Ad Click's App Id
os_cvr	Avg CvR of Ad Click's OS Id
device_cvr	Avg CvR of Ad Click's Device Id
channel_cvr	Avg CvR of Ad Click's Channel Id
hour_cvr	Avg CvR of all clicks in Ad Click's Hour
ip_day_hour_click_count	Count of clicks grouped by ip-day-hour
ip_app_click_count	Count of clicks from an IP for an App Id
ip_app_os_click_count	Count of clicks from an IP on an OS for an App Id
ip_app_channel_variance_click_day	Variance in click days for an (IP, App, Channel)

**one\_click\_ip:** Related to above, is a boolean feature. The signal provided by ip\_num\_clicks dissipates rapidly after 1 click. This feature helps capture this particular aspect.

**ip\_day\_hour\_click\_count:** Helps look at the IP's burstiness in a given hour. Bustry-ips in a given hour are less likely to lead to conversions.

**ip\_app\_click\_count:** Attempts to capture spam perpetrated by 'competitors', by helping identify IPs which fire a lot of clicks for a given app.

**ip\_app\_os\_click\_count:** Goes one level deeper on the above signal. Some IPs are multi-user IPs, and this one helps identify specific devices or users the fraud ad clicks might be coming from.



**ip\_app\_channel\_variance\_click\_day**: Attempts to identify if most clicks from an IP for an App on a Channel occur within a short timeframe. The overall idea is that, if this variance is high, the user is performing lots of clicks on a regular basis, without any intent to download the app.

## 8.2 Base Models

As the most simple modeling method, the raw signals in the data were directly used as features for various types of ML models. These models have been referred to as “base models” in this thesis. IP addresses were dropped, but other dimensions were used as non-categorical features. Click time was dropped but replaced by hour\_of\_day. This way, the set of features was kept very small - only 6 features used for training and classification.

Multiple types of ML models were used. From literature survey, gradient boosting model, Linear-SVC and K-NN stand out. Additionally, Naive-Bayes was also explored, being a very simple modeling technique. All these different ML models were trained using various random samples of training data. The reason behind varying the training sample sizes was to optimize for precision while minimizing over-fitting.

Table 7: Base Models’ Performance

Model	Sample	Precision	Trn. cost ( $\mu$ s)	Pred. cost ( $\mu$ s)
Linear-SVC	0.001	0.5003	5.9643	0.0595
Linear-SVC	0.01	0.5000	8.1099	0.0475
Naive-Bayes	0.01	0.5469	0.1908	0.1364
Naive-Bayes	0.001	0.5445	0.2131	0.1363
k-NN	0.001	0.5000	2.2873	30.9784
G-Boost	0.001	0.7105	14.8722	0.3632
G-Boost	0.01	0.5933	22.9355	0.3520

The results for base models are summarized in Table 7. Even with such simplistic features, some good insights can be drawn. LinearSVC and k-NN performed poorly,

achieving 50% precision. In fact, looking at their outputs, they nearly always predicted zeroes (except for 1-2 clicks in both cases): which means that those clicks did not convert. This is because of the way these two models work, and the nature of features provided to them. k-NN, for instance, relies on looking at the 'k' nearest neighbors in the feature space for a given click being predicted. The distance used here was L2 distance, with a  $k = 5$ . A L2 distance would mean that device=10 and device=11 are devices with close similarity, compared to device=10 and device=100. This clearly isn't true in this dataset, however. Features like device are categorical. So, L2 distance is meaningless in this space. LinearSVC also relies on a similar mechanic. Even Naive-bayes's classification is weak due to this reason, though it achieves a slightly better precision at 54.69%.

On the other hand, gradient-boosting performance was the best among the models attempted. With a 0.1% training data sample, a 71% precision is obtained. This is despite the fact that the features have been treated as continuous and not categorical. This is because gradient boosting does not rely on a notion of L2 distance, and is instead attempting to perform gradient descent from labels to learn which labels are likely to yield positive outcomes. Gradient boosting model's performance, however, went down significantly when 10X training data was provided. This is suggestive of over-fitting. The base gradient boosting model with 71% precision classifies better than the heuristics (refer back to Table 5). The model's prediction time is only  $0.32 \mu s$  per click. When compared to MLP model's prediction time of  $9.63 \mu s$  per click, this is an order of magnitude lower. Yet, the precision is not as high as we had set out to look for.

### **8.3 One-hot Encoded Feature Space**

In Section 8.2, LinearSVC and k-NN performed poorly because the feature space was not conducive for distance calculations. This is typically avoided by transforming categorical features into a one-hot encoding. In one-hot encoding, each feature-value becomes a distinct feature. For instance, if the device feature can take values from 1 to

100, it would be transformed into 100 features, representing  $device_1, device_2, \dots, device_{100}$ . Now, for each click, only one of these 100 features would have a value of 1 or true, while all others would have zeroes. In such a space, distance functions begin to become more useful. Now a click from device=1 is similarly distant from a click from device=10 versus another click from device=100, assuming all other dimensions are identical.

One-hot encoding variants were created for each type of model. All categorical features: os, device, app and channel were transformed to be one hot encoded. As in Section 8.2, IP addresses were excluded and click\_time was replaced by hour\_of\_day. In addition, ip\_num\_clicks was also included, to represent distance on IP addresses based on a very identifying feature such as number of clicks (see Section 4.3). In total, this transformed the feature-space into 5589 dimensions. The results are summarized in Table 8.

Table 8: One-hot Models' Performance

Model	Sample	Precision	Trn. cost ( $\mu$ s)	Pred. cost ( $\mu$ s)
Linear-SVC	0.001	0.6304	108.4241	70.7381
Linear-SVC	0.01	0.6330	211.9565	76.1893
Naive-Bayes	0.001	0.6964	126.7783	114.2180
Naive-Bayes	0.01	0.6140	296.6669	104.8078
k-NN	0.0001	0.0860	500.8176	11509.5484
G-Boost	0.001	0.5010	17.3676	63.5168
G-Boost	0.01	0.6535	31.5229	66.1342
G-Boost	0.02	0.5417	39.7130	64.7975

Naive-Bayes is the best model in this feature space. It benefits from meaningful L2-distances. With a training sample of 0.1%, sufficient learning is achieved and it provides a nearly 70% prediction precision. While LinearSVC improves as well when compared to the base feature space, the improvement is not as much as for NaiveBayes.

This is because the surface separating the converted and non-converted clicks in this hyperspace might not be express-able as a plane. NaiveBayes can handle this scenario better, compared to LinearSVC.

k-NN, on the other hand, deteriorates and becomes much worse. This is, however, attributable to k-NN's smaller training sample. k-NN models are very bulky, as they need to store each data point in a high-dimensional space. These models were trained here in a 5589-dimensional space. This required us to reduce the training sample massively, but it significantly ended up affecting the precision. Even with such a small sample, k-NN models took 11.5 ms to lookup for each click, which is extremely costly. Sampling was needed even while performing evaluations (unlike any other ML model tried in any feature-space). It can be concluded that one-hot feature space is not conducive to k-NN.

Gradient-boosting shows interesting trends. Working in a much bigger feature-space, the precision drops sharply for the 0.1% training sample. This indicates that the decision trees find it difficult to classify clicks when the feature space is very large. As expected, however, the precision improves when this model is provided more data. More input is required to have the decision trees converge across a larger feature-space. Beyond a point, however, over-fitting becomes a concern again.

Across all the models in one-hot encoding feature space, the performance trends are very clear. There is a significant cost paid for working in a high-dimensional feature space such as this. When compared to performance on base features as shown in Table 7, the prediction and training costs have all gone up by over 100X. Clearly, while most models improve in precision in this space, their utility for real-time classification of ad fraud is limited due to their cost.

A common issue with one-hot feature spaces is about relative weights. When features are one-hot encoded, one has to reason about distance between features from the same raw feature (eg.  $L2(device_1, device_5)$ ) versus distance between features across different

raw features (eg.  $L2(device_1, os_{20})$ ). This creates a challenge for models which use distance functions, as opposed to loss functions, such as NaiveBayes and LinearSVC. On the other hand, such a space is very conducive for classifiers meant for categorical spaces, and even neural networks tend to work better with such spaces. In fact, the MLPs evaluated in Section 6 also use one-hot encoded features, and provide a 95% precision.

## 8.4 CvR Space

So far, NaiveBayes and LinearSVC models have demonstrated better cost performance when working with a small number of features. If one could create a feature space with small number of features which these models could exploit better, there is a good chance that a better precision could be achieved at a small computation cost. This is the motivation behind exploring the CvR space.

The original dataset predominantly consists of categorical features. If these categorical features are used as is, this leads to lower precision (see ‘base’ variants’ performance in Table 7). This is because most models assume a continuous feature axis per feature. Thus,  $ip = 100$  and  $ip = 101$  are treated as almost identical, even though these really don't have much in common, apart from a happenstance of their values being close.

One way to deal with categorical features without blowing up the feature space is to use conditional probability coding [21]. The basic idea of conditional probability (CP) coding is to replace each categorical feature with its conditional probability of predicting the label value. Since the training labels for ad fraud detection are binary (0 = non-converting click, 1 = converting click), CP-coding is possible. Each categorical variable (os, device, channel) is replaced by their CP-coding. eg.  $os = 10$  is replaced by  $\Pr(is\_attributed = 1 \mid os = 10)$ . Additionally, the method is applicable to non-categorical variables too. For instance, it could be applied to ‘hour of day’ as well, by replacing it with the probability of conversion in a given ‘hour of day’. The result of these transforms is a “CvR Space”. This is a n-dimensional space, where each axis represents the

conditional conversion rate (CvR) of a particular feature. The expectation is that converting and non-converting clicks can be separated out in this space easily. For instance, converting clicks will gather closer to the point  $(1, 1, \dots, 1)$ , whereas the non-converting clicks will gather closer to  $(0, 0, \dots, 0)$ . Results for various model trials with CvR space are listed in Table 9.

Table 9: Models' Performance in Conversion-Rate Space

Model	Sample	Precision	Trn. cost ( $\mu s$ )	Pred. cost ( $\mu s$ )
Linear-SVC	0.001	0.6258	0.7550	0.0599
Linear-SVC	0.01	0.6440	0.8243	0.0519
Linear-SVC	0.05	0.6477	0.8678	0.0546
Naive-Bayes	0.001	0.8996	0.1599	0.1365
Naive-Bayes	0.01	0.8997	0.1624	0.1354
k-NN	0.001	0.6093	7.6525	54.2434
k-NN	0.01	0.6868	108.3013	294.7478
G-Boost	0.001	0.6909	8.4141	0.4233
G-Boost (1pc)	0.01	0.6816	9.7152	0.4092

CvR-space shows good improvement in Naive-Bayes model's precision, going up to 90%. Naive-Bayes also shows good performance from a cost perspective, requiring only about  $0.26 \mu s$  to train per click, and  $0.136 \mu s$  to lookup for each click. The model was also used to predict the labels for the Kaggle test dataset. Kaggle score was quite high too, at 89.33% precision. The model precision and training/prediction costs fit within the real-time classifier parameters laid out earlier.

In addition, CvR space provides improvements for other models too. LinearSVC exhibits similar precision as the one-hot encoding feature space (64.77% here vs 63.30% in one-hot encoding space), but with significantly reduced prediction cost ( $0.0546 \mu s$  here vs  $76 \mu s$  for one-hot encoding). The precision performance is attributable to a similar concern on CvR-space not being separable by a plane to classify converting and non-converting clicks well. Gradient-boosting also keeps its 69% precision, but at a

reduced training cost when compared to the base variant. Gradient-boosting model required to be tuned differently for the different training sample sizes, to get optimal precision.

Finally, k-NN shows improvements in classification accuracy, particularly when working with a larger sample size, taking precision to 68.68%. This is because, in this space, the distance functions are the most meaningful. Distances are now normalized across feature-dimensions, leading to better distance measurement. A big caveat with k-NN, however, is its cost. As can be seen, when the training sample size grew by 10X, there were significant increases in training and prediction costs as well. Training cost grew by more than 10X. The model size was large as well, making it impractical to train the model over even bigger samples, without utilizing a large number of machines.

## **8.5 IP Aggregate Features**

So far, IP addresses have not been used in the models. The number of distinct IPs are over 100K, and there are IP addresses in test sets (both the synthetic test set, as well as the Kaggle private test set) which are not present in training dataset. Additionally, IP addresses have not been provided in their usual 32-bit format with their subnets, etc. IPs are basically categorical in this dataset, and no information could be derived by just looking at the IP addresses (presumably, for privacy reasons). This limits their utility, as some potentially useful features such as subnets, geo-location, etc cannot be obtained.

There is potential to aggregate various kinds of information on IP-addresses, however. Similar to the sessionization introduced in Section 7.1, various statistics can be computed on aggregates where IP is at least one of the dimensions. Many of these features have been mentioned in Table 6. The machine learning models were trained with these additional aggregate features, to see if IP addresses could provide useful classification signal as well. Note that these features were treated as continuous, instead of categorical. The results have been tabulated in Table 10.

Table 10: Models' Performance with IP Aggregate Features

<b>Model</b>	<b>Sample</b>	<b>Precision</b>	<b>Trn. cost (<math>\mu</math>s)</b>	<b>Pred. cost (<math>\mu</math>s)</b>
Linear-SVC	0.001	0.5425	64.1757	0.0645
Linear-SVC	0.01	0.5425	73.3807	0.0813
Naive-Bayes	0.001	0.5749	0.2711	0.2409
Naive-Bayes	0.01	0.5744	0.2737	0.2391
k-NN	0.001	0.5860	138.3072	98.1994
G-Boost	0.001	0.5425	6.2003	0.4083
G-Boost	0.01	0.5722	15.8315	0.3642

The results show that there is some signal in these ip-aggregate features, but overall the precision is lower than the other models. This implies these features could provide minor improvements in precision only.



## 9 RESULTS

In this thesis, multiple experiments were conducted to evaluate the utility-cost tradeoffs for classification, with the goal of identifying prediction methods suitable for real-time ad fraud detection. The precision and performance metrics for various methods have been provided in Table 4 for MLP, Table 5 for heuristics, Table 7 for ML models with base features, Table 8 for models with one-hot encoded features, Table 9 for models in CvR space and Table 10 for ML models working with IP-aggregates as features. Note that, as mentioned in Section 4.1, these precisions are strict lower-bounds of the true precision, as they have been computed on a skewed test-set, to allow judging the model performance better.

Based on the experiments, machine learning methods based on neural networks / multi-layer perceptrons have the best precision at 95.43% among the models attempted. These models have a prediction time at  $9.63 \mu s$  per click. The model is costly to train as well, requiring  $284 \mu s$  per click, comparing unfavorably against most other models attempted. Put together, due to the higher training and prediction cost, the model's utility for real-time classification is limited. Note that the timing method used accounts only for the model fitting and model prediction times, and ignores the data preparation overhead. Given the nature of features used by MLP (one-hot encoded), the data preparation phase is also compute-intensive, and adds on top of the training and prediction time.

Most of the models and feature-spaces capped out at about 70% precision for classification times less than  $1 \mu s$ . This level of precision provides only a weak classifier for ad fraud clicks in real-time. Gradient-boosting, widely quoted in literature, provided 71% precision at best in the experiments, working with the base features. Naive-Bayes, with the exception of CvR space, provided 69.64% accuracy with the one-hot feature space. k-NN provided the best accuracy of 68.68% working in the CvR space, though the model size was large and the classification time was close to  $300 \mu s$ . k-NN performed

poorly with large feature spaces: specifically had more than 11ms of prediction time in one-hot encoding space. Linear-SVC only provided 64.47% at best, with the CvR space.

The best classifier for real-time use, among the experiments, was found to be the Naive-Bayes classifier in the CvR space, providing 89.97% precision. The classifier also had smaller training and prediction costs, requiring only 0.16 $\mu$ s and 0.13 $\mu$ s respectively. The model prediction error is close to 2X of the MLP model, while the prediction cost is less than 1/70<sup>th</sup> of the MLP model (see Table 4 for comparison).

These top models were also uploaded to Kaggle to evaluate on independent test datasets. They had identical performance on Kaggle, matching the precision results obtained over the test sets within 1%. The Naive-Bayes classifier in CvR space scored at 89.33% on Kaggle. This also shows that the test dataset construction in Section 3.2 is effective, and mimic'ed the Kaggle independent test dataset well.

Put together, the results show that machine learning models are effective in predicting clicks likely to have conversions, and can produce more than 90% precision. Conversely, these models are also able to predict which clicks are highly likely to be fraudulent, with close to 0 probability of conversion. It has also been demonstrated that it is possible to build models which have low cost, using less than 1 $\mu$ s per click for training and prediction, and 90% precision. Such models are suitable for real-time ad click fraud classification.

## 10 CONCLUSIONS AND FUTURE WORK

Based on the results above, it can be concluded that real-time detection of ad click fraud is possible, using simple classifiers such as Naive-Bayes. Even on this unbalanced dataset, it is possible to achieve a 90% precision in identifying convertible clicks, while keeping compute costs below  $1\mu s$  per click.

There are some other interesting approaches worth exploring in the future. Gradient-boosting on top of weaker predictive models could be tried. Gradient boosting could also be attempted with more session-based signals: such as first click in session, or last click in session. Some of the heuristics demonstrate reasonable precision, and could be used to create new features which could then be used in various ML models. Other datasets could be explored, where the data might be more meaningful: for instance with real IP addresses. These would open up many more features useful for analysis: such as the geographical location, sub-networks, etc.

More study should be done to build tiered models. Tier 1 could perform real-time ad click fraud detection with low cost. Output from tier 1 models could be used to perform incidence response and investigation. Tier 2 models could be more exhaustive, and run offline. Tier 2 models could also learn from the outcome of Tier 1 models, and thereby be able to perform better. Such a system would be able to amalgamate the best properties of real-time and offline classification systems.

Availability of public and labeled datasets for ad click fraud is limited. This limits the experimentation and research which could be done. There is, however, a possibility of creating simulated ad click fraud, and melding it with regular website traffic. This could help evaluate the efficacy of ad click fraud detection under specific types of ad click fraud attacks: such as botnets, or human clickers. This could also open up many more interesting features, such as referrer URLs and HTTP headers, along with user ids such as Cookies.

Ultimately, ad click fraud detection is a moving target. As ad networks and advertisers catch up with the state-of-the-art in ad click fraud, more and more malignant players keep inventing new methods to circumvent fraud prevention. Thus, it is imperative to continue the research in this space and continue to catch up to the ad click fraud.

## Literature Cited

- [1] D. Liu, S. Xu, L. Chen, and C. Wang, “Some observations on online advertising: A new advertising system,” in *2015 IEEE/ACIS 14th International Conference on Computer and Information Science (ICIS)*, pp. 387–392, June 2015.
- [2] X. Li, Y. Liu, and D. Zeng, “Publisher click fraud in the pay-per-click advertising market: Incentives and consequences,” in *Proceedings of 2011 IEEE International Conference on Intelligence and Security Informatics*, pp. 207–209, July 2011.
- [3] X. Jiarui and L. Chen, “Detecting crowdsourcing click fraud in search advertising based on clustering analysis,” in *2015 IEEE 12th Intl Conf on Ubiquitous Intelligence and Computing and 2015 IEEE 12th Intl Conf on Autonomic and Trusted Computing and 2015 IEEE 15th Intl Conf on Scalable Computing and Communications and Its Associated Workshops (UIC-ATC-ScalCom)*, pp. 894–900, 2015.
- [4] M. Faou, A. Lemay, D. Décary-Héту, J. Calvet, F. Labrèche, M. Jean, B. Dupont, and J. M. Fernande, “Follow the traffic: Stopping click fraud by disrupting the value chain,” in *2016 14th Annual Conference on Privacy, Security and Trust (PST)*, pp. 464–476, 2016.
- [5] B. Kitts, J. Y. Zhang, A. Roux, and R. Mills, “Click fraud detection with bot signatures,” in *2013 IEEE International Conference on Intelligence and Security Informatics*, pp. 146–150, 2013.
- [6] M. Kantardzic, C. Walgampaya, and W. Emara, “Click fraud prevention in pay-per-click model: Learning through multi-model evidence fusion,” in *2010 International Conference on Machine and Web Intelligence*, pp. 20–27, 2010.
- [7] G. Shafer, “A mathematical theory of evidence,” *International Journal of Approximate Reasoning*, vol. 79, July 2016.
- [8] H. Xu, D. Liu, A. Koehl, H. Wang, and A. Stavrou, “Click fraud detection on the advertiser side,” in *Computer Security - ESORICS 2014*, vol. 8713, pp. 419–438, September 2014.
- [9] A. Dimitris, M. Paschou, E. Sakkopoulos, E. Sourla, G. Tzimas, A. Tsakalidis, and E. Viennas, “Exposing click-fraud using a burst detection algorithm,” pp. 1111–1116, June 2011.

- [10] Y. Guo, J. Shi, Z. Cao, C. Kang, G. Xiong, and Z. Li, "Machine learning based cloudbot detection using multi-layer traffic statistics," in *2019 IEEE 21st International Conference on High Performance Computing and Communications; IEEE 17th International Conference on Smart City; IEEE 5th International Conference on Data Science and Systems (HPCC/SmartCity/DSS)*, pp. 2428–2435, August 2019.
- [11] R. Mouawi, M. Awad, A. Chehab, I. H. E. Hajj, and A. Kayssi, "Towards a machine learning approach for detecting click fraud in mobile advertizing," in *2018 International Conference on Innovations in Information Technology (IIT)*, pp. 88–92, 2018.
- [12] L. Song, X. Gong, X. He, R. Zhang, and A. Zhou, "Multi-stage malicious click detection on large scale web advertising data," *CEUR Workshop Proceedings*, vol. 1018, pp. 67–72, January 2013.
- [13] M. S. Iqbal, M. Zulkernine, F. Jaafar, and Y. Gu, "Fc fraud: Fighting click-fraud from the user side," in *2016 IEEE 17th International Symposium on High Assurance Systems Engineering (HASE)*, pp. 157–164, 2016.
- [14] R. A. Costa, R. J. G. B. de Queiroz, and E. R. Cavalcanti, "A proposal to prevent click-fraud using clickable captchas," in *2012 IEEE Sixth International Conference on Software Security and Reliability Companion*, pp. 62–67, 2012.
- [15] M. Kantardzic, C. Walgampaya, B. Wenerstrom, O. Lozitskiy, S. Higgins, and D. King, "Improving click fraud detection by real time data fusion," in *2008 IEEE International Symposium on Signal Processing and Information Technology*, pp. 69–74, 2008.
- [16] TalkingData, "Talkingdata adtracking fraud detection challenge." <https://www.kaggle.com/c/talkingdata-adtracking-fraud-detection/data>, 2018.
- [17] AppTweak, "Average app conversion rate per category [2020]." <https://www.apptweak.com/asoblog/average-app-conversion-rate-per-category-2019>, 2020.
- [18] G. Lewicki and G. Marino, "Approximation by superpositions of a sigmoidal function," *Appl. Math. Lett.*, vol. 17, pp. 1147–1152, 12 2004.

- [19] D. Kingma and J. Ba, “Adam: A method for stochastic optimization,” *International Conference on Learning Representations*, 12 2014.
- [20] S. Mannor, D. Peleg, and R. Rubinstein, “The cross entropy method for classification,” in *Proceedings of the 22nd International Conference on Machine Learning*, ICML ’05, (New York, NY, USA), p. 561–568, Association for Computing Machinery, 2005.
- [21] A. Radianto and AirbnbEng, “Designing machine learning models.” <https://medium.com/airbnb-engineering/designing-machine-learning-models-7d0048249e69>, July 2015.