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**On the Coefficient of Variation as a Criterion for Decision under Risk**

**James C. Cox and Vjollca Sadiraj**

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## 1           **On the Coefficient of Variation as a Criterion for Decision under Risk\***

2  
3           Weber, Shafir, and Blais (2004) report a meta-analysis of data for animal and human  
4 decision making that uses the coefficient of variation (CV) as a measure of “risk sensitivity.”  
5 They argue that CV, defined as the ratio of standard deviation (SD) to expected value (EV), is a  
6 good predictor of animal and human risk sensitivity. Weber et al. (2004, pgs. 432 – 434) review  
7 results from the analysis of animal data in Shafir (2000) in which the proportion of animals  
8 choosing certain (sure thing) rewards rather than risky rewards is regressed on the CV of risky  
9 rewards. For comparison, Weber, et al. (pg. 434) report results from another regression in which  
10 the proportion of sure thing choices is regressed on the EV of risky rewards. They conclude that:  
11 “...neither SD nor EV predicts risk sensitivity in isolation. Their ratio, however, in the form of  
12 the CV does so very well.”

13           Weber et al. (2004) report that a large advantage of CV over SD in fitting data occurred  
14 with data from environments in which decision makers had to learn about the outcome  
15 distribution of different options by trial and error sampling and personal experience. Arguably,  
16 such environments might better be described as involving uncertainty rather than risk (Knight,  
17 1921). And different models of decision making might apply in environments characterized by  
18 uncertainty than in environments characterized by risk. Weber et al. (pg. 443), however, also  
19 offered their model as a solution to the implausibility problems for expected utility theory that  
20 follow from the concavity calibration critique that applies to theories of decision *under risk*  
21 (Rabin 2000). In response, we ask whether CV can provide either a credible normative criterion  
22 for decision making under risk or a useful descriptive model of decisions under risk in two  
23 experiments designed to test its distinctive properties. We also ask whether CV provides a useful  
24 descriptive model of decisions in an experiment designed to test a concavity calibration  
25 supposition (with described options) similar to the one discussed by Rabin (2000); this is a  
26 direct response to claims made about the CV model by Weber et al.

1 Weber et al. (eq. 1, p. 433) write the utility of an option with random return  $X$  as a linear  
 2 function of the utility of its expected value,  $u(\text{EV}(X))$  and its risk  $R(X)$ . They note that, for a  
 3 quadratic utility function, the measure of risk is the variance,  $\text{Var}(X)$ . They argue for use of the  
 4 dimension-invariant  $\text{CV}(X)$ , instead of  $\text{Var}(X)$  or  $\text{SD}(X)$ , as the measure of risk.

5 The CV measure only provides a measure of risk that is distinct from SD when it is  
 6 applied to distributions with distinct EVs. If application of CV were to be restricted to  
 7 comparisons across gambles with the same EV then it would produce a measure that was  
 8 identical to SD except for a scale factor. Thus we consider gambles with distinct EVs. Consider  
 9 two risky choice options with random returns,  $X$  and  $Y$ . If for an individual  $j$  one has  
 10  $u_j(\text{EV}(X)) - b_j \text{CV}(X) < u_j(\text{EV}(Y)) - b_j \text{CV}(Y)$  (1)

11 then option  $Y$  is more likely to be chosen than option  $X$  by individual  $j$  (Weber et al., pgs.  
 12 433, 443).

13 We first ask whether choosing among risky alternatives according to the CV criterion (as  
 14 in statement (1)) would promote the self-interest of the decision maker. In other words, we ask  
 15 what are the normative properties of the CV criterion. Subsequently, we examine the descriptive  
 16 power of the CV measure of risk sensitivity in three experiments.

17

### 18 **Normative properties of the CV Measure of Risk Sensitivity for Decision under Risk?**

19 We here show that CV cannot provide a credible normative criterion for choosing between non-  
 20 degenerate lotteries because it is inconsistent with preference for more to less of the reward  
 21 medium (or “positive monotonicity” of preferences). Let a risk averse agent’s choices over  
 22 lotteries that involve gains be characterized by some  $b > 0$  and some positively monotonic  
 23 function  $u(\cdot)$  over expected payoffs, as in statement (1). Let positive numbers  $\ell, k$ , and  $n > 2$  be  
 24 given. Let  $L_n$  denote the binary lottery that yields payoff of  $\ell/p$  with probability  $p$  and payoff

1 of  $n\ell/(1-p)$  with probability  $1-p$ . Similarly,  $L_{n+k}$  denotes the binary lottery that yields  
 2 payoff of  $\ell/p$  with probability  $p$  and payoff of  $(n+k)\ell/(1-p)$  with probability  $1-p$ .

3

4 **Proposition 1.** Let positive  $\ell, k$ , and  $n > 2$  be given. For all  $b > 0$ , there exists  
 5  $p \in [0.5, 1)$  such that the individual is more likely to choose lottery  $L_n$  than lottery  $L_{n+k}$   
 6 according to the CV criterion.

7 Proof. See Appendix 1.

8

9 Note that the only difference between lotteries  $L_n$  and  $L_{n+k}$  is that the high outcome in  
 10  $L_{n+k}$  is larger than the high outcome in  $L_n$  by the payoff amount  $k\ell/(1-p)$ . Yet lottery  
 11  $L_n$  is more likely to be chosen than  $L_{n+k}$  according to the CV criterion. This is  
 12 inconsistent with positive monotonicity of preferences.

13 As an example, let  $\ell = 1, n = 2$  and  $k = 1$ . Suppose that an agent's coefficient  $b$  in the  
 14 CV preference criterion (see statement (1)) is found to satisfy the inequality (\*)  
 15  $b > 6[u(4) - u(3)]$ . Then application of Proposition 1 with  $p = 1/2$  implies that option  $L_2$  is  
 16 more likely to be chosen than option  $L_3$  where: (a) option  $L_2$  is the binary lottery that pays 2 or 4  
 17 with equal probabilities; and (b) option  $L_3$  is the binary lottery that pays 2 or 6 with equal  
 18 probabilities. It can be easily verified that  $EV_{L_2} = 3, EV_{L_3} = 4, CV_{L_2} = 1/3$ , and  $CV_{L_3} = 1/2$ . So,  
 19 according to the CV measure of risk sensitivity,

$$20 \quad U(X_{L_2}) - U(X_{L_3}) = u(3) - b/3 - u(4) + b/2 > 0 \quad (2)$$

21 where the last inequality follows from statement (\*). Hence the CV criterion implies that the  
 22 agent is more likely to choose option  $L_2$  than option  $L_3$ . But this conclusion is inconsistent with  
 23 the self-interest of any agent that is not satiated in the reward medium because the only difference

1 between the two lotteries is that the high payoff equals 4 in option  $L_2$  and 6 in option  $L_3$ . It is  
 2 easiest to think of these numbers as referring to amounts of money but that interpretation is not  
 3 necessary for our argument. If the payoffs *are* amounts of money then whether they are numbers  
 4 of dollars or millions of dollars or some other unit of account is irrelevant to the argument.

5 Another example (considered also in the next section) which illustrates the CV model's  
 6 inconsistency with positive monotonicity is as follows. Let option C be a sure payoff of 10.50.  
 7 Option D pays 0 with probability 4/10 and pays 22 with probability 6/10. The expected payoff in  
 8 option D is 13.2 whereas the CV of this option is 0.82. Suppose that an agent weakly prefers  
 9 option C to option D. In that case, according to the CV criterion the agent has revealed that most  
 10 likely

$$11 \quad u(10.5) \geq u(13.2) - b \times 0.82 \quad (3)$$

12 Next consider options J and K. Option J pays 10 with probability 19/20 and pays 20 with  
 13 probability 1/20 and has EV of 10.5 and CV of 0.21. Option K pays 10 with probability 19/20 and  
 14 pays 74 with probability 1/20. It has EV of 13.2 and CV of 1.06. Note however that inequality  
 15 (3) implies that

$$16 \quad \begin{aligned} u(EV(K)) - bCV(K) &= u(13.2) - b \times 1.06 \\ &< u(13.2) - b(0.82 + 0.21) \leq u(10.5) - b \times 0.21 = u(EV(J)) - bCV(J) \end{aligned}$$

17 Therefore, weak preference for option C over option D implies that option J is more likely to be  
 18 chosen than option K. But this is inconsistent with positive monotonicity of preferences because  
 19 the only difference between the two options is a high payoff of 20 in option J and 74 in option K.

20 These examples demonstrate why CV cannot provide a credible normative criterion for  
 21 decision under risk because it implies in some contexts that agents prefer lower payoff to higher  
 22 payoff.

23

24

25

## 1                    **Can CV Provide a Descriptive Model for Decision under Risk?**

2    Although the CV criterion is inconsistent with positive monotonicity of payoff preferences,  
3    perhaps it is nevertheless consistent with how agents actually make decisions. The literature that  
4    reports experiments on decision under risk includes some reports of inconsistencies with first  
5    order stochastic dominance, which is an implication of positive monotonicity.<sup>1</sup> And some of these  
6    experiments involve real decisions among risky alternatives with non-trivial salient money  
7    payoffs rather than reports of hypothetical preferences.<sup>2</sup> Previous experiments do not provide  
8    support for the type of transparent violation of payoff (or consequence) monotonicity  
9    demonstrated above for the CV criterion. But they were not designed to test the specific  
10    implications of the CV criterion. Experiment 1 provides such a test.

11

### 12    Experiment 1

13    Consider again the second example of properties of the CV criterion in the preceding section. The  
14    reasoning takes the form: (a) if an agent weakly prefers option C to option D then (b) the CV  
15    model implies that the agent is more likely to choose option J than option K, between which the  
16    only difference is a 1/20 probability of getting 20 in option J and 74 in option K. But what if  
17    almost no one were to actually prefer option C to option D? If that were the case, then the  
18    example might not be compelling. We here report an experiment with these four options. In the  
19    experiment, the payoff numbers represent amounts of money in 2009 U.S. dollars.

20            Subjects were informed that the experiment included two decision tasks and that a coin  
21    would be flipped to determine which task paid money. Subjects were first shown options C and D  
22    and asked to choose one option, by circling C or D, or to record indifference (“I don’t care”) by  
23    circling I. Subjects were told that if they circled I then the experimenter would choose option A or  
24    option B for them. After making their decisions about options C and D, subjects were shown  
25    options J and K and asked to choose one option or record indifference.

1           Before making any decisions, subjects were invited to inspect the coin that would be  
2 flipped to select which decision would be paid. They were shown the box containing four white  
3 balls and six red balls that would be used to determine payoffs in option D. Before making their  
4 option J and K decisions, subjects were shown the box containing 19 white balls and one orange  
5 ball that would be used to determine payoffs for options J and K. They were informed that, after  
6 making their choices, they would be called one at a time to observe the coin flip that would select  
7 one of their two decision pages for payoff and that they would themselves each draw one ball  
8 from the appropriate box (except for an option C choice). They were informed that they would be  
9 invited to inspect the coin and the contents of a box immediately before these “urns” were used to  
10 determine an individual’s payoff. Subject instructions and decision pages (or response forms) for  
11 experiment 1 are included in the appendix.

12           Experiment 1 included 66 volunteer subjects who were students at Georgia State  
13 University. 37 of the subjects chose option C and one chose indifference between options C and  
14 D. Recall that the CV criterion predicts that all of these 38 subjects will more likely choose  
15 option J than option K, so the majority of them should be observed to choose option J. Data  
16 reject this hypothesis. All of these 38 subjects chose option K rather than option J. (In fact, all 66  
17 subjects in the experiment chose option K.) Therefore, the CV model does not provide an  
18 empirically valid descriptive model for decisions by subjects in this experiment. In contrast,  
19 theories of decision under risk such as expected utility theory, dual theory of expected utility  
20 (Yaari, 1987), and rank dependent expected utility theory (Quiggin, 1993), are consistent with  
21 data for all of the subjects in this experiment. These theories predict that all agents will more  
22 likely choose option K rather than option J, regardless of whether they choose option C or option  
23 D, because the theories incorporate positive monotonicity of preferences in the reward medium.

24

25

26



1 Design of Experiment 2

2 In experiment 2 subjects were asked to choose between two options or record indifference on  
3 each of six decision pages. One of the six decision pages of each subject was randomly chosen for  
4 money payoff by rolling a die in the presence of the subject. Experiment 2 instructions and  
5 decision pages (or response forms) are included in the appendix. The decision pages were given  
6 to each subject in an independently drawn random order. Subjects had access to all six decision  
7 pages while making each decision.

8 Before making their decisions, subjects were invited to inspect the die that would be  
9 rolled to select which decision would be paid. They were shown the bingo cage containing balls  
10 numbered 1,2,..., 10 that would be used to determine lottery payoffs. They were informed that,  
11 after making their choices, they would be called one at a time to observe the die roll that would  
12 select one of their six decision pages for payoff and that they would observe selection of a ball  
13 from the bingo cage if they had chosen the lottery on the selected decision page. They were  
14 informed that they would be invited to inspect the die and the contents of the bingo cage  
15 immediately before these “urns” were used to determine an individual’s payoff.

16 Table 1 reports the six decision tasks in experiment 2 that were presented to the subjects  
17 in lettered tables, one table to a page. Each decision table contained the two options in a single  
18 row of Table 1. In the “Table Q” decision task, for example, a subject is asked to choose between  
19 \$7 for sure and the binary lottery that pays \$2.50 if ball 1 is drawn from the bingo cage or pays  
20 \$7.50 if ball 2, 3,...,9, or 10 is drawn. Table 1 reports the EV and CV of option B in each of six  
21 decision tables. (The subjects were *not* given the EV and CV values.) Note that the EV of option  
22 B in each table equals the sure payoff in option A of that table. The CV measure increases  
23 monotonically with decision table, from a low of 0.21 for Table Q to a high of 2.50 for Table T.  
24 Therefore, if the CV measure is consistent with subjects’ decision making under risk then their  
25 frequency of choosing the sure amount of payoff should be correlated with the CVs of options B  
26 in these tables since

$$1 \quad u(EV(A)) - bCV_A - u(EV(B)) + bCV_B = bCV_B \quad (4)$$

2

3 Data from Experiment 2

4 Different subjects participated in experiment 2 than in experiment 1. Experiment 2 included 50  
 5 volunteer subjects who were students at Georgia State University. Figure 1 shows graphs of the  
 6 proportions of subjects who chose the safe option and the CVs of the lotteries in six tables in the  
 7 experiment. The proportion of subjects who chose the sure thing shows no clear relationship to  
 8 the CVs of options B in different tables. The rank correlation coefficient is 0.143. The Spearman  
 9 test reports a p-value of 0.787, so the null hypothesis that proportions of choices of the sure thing  
 10 and CVs of the risky lotteries are independent is *not* rejected. We also ran a linear regression of  
 11 the observed proportions of choices of the sure options on the CVs of the paired risky options.  
 12 The OLS-estimated coefficient for the CV variable is 0.043 (p-value is 0.591) and the constant is  
 13 0.632 (p-value is 0.004). So again, the CV model has no explanatory power for these data as  
 14 revealed by the p-value of 0.591.

15 The Spearman test, the graphs in Figure 1, and the linear regression use aggregate data.  
 16 Perhaps the effects of subject heterogeneity mask the effects of CV on individuals' willingness to  
 17 take risks. To address this question, we ran logit and probit regressions on individual data. The  
 18 Linear CV column of Table 2 for experiment 2 data reports estimates from a clustering logit  
 19 analysis of the data in which observations for each individual subject comprise a "group" of  
 20 observations.<sup>3</sup> The explanatory variables are CV of the risky lotteries and a constant. The  
 21 estimated parameter  $\hat{b}$  of the coefficient on CV is not significantly different from 0. Hence,  
 22 according to the logit analysis of data, CV has no explanatory value for individual subjects'  
 23 choices in experiment 2. Probit estimation leads to the same conclusion.

24 Perhaps subject heterogeneity, that cannot be captured by the individual-specific error  
 25 terms in clustered data analysis, masks the explanatory power of CV in the results reported in

1 Table 2. We next ask whether adding questionnaire data to the econometric model changes the  
 2 conclusions about the insignificance of CV. We conducted logit and probit (with individual  
 3 subjects' clusters) analyses of data using CV and 15 measures of subjects' individual  
 4 characteristics as right-hand variables that are additive to CV. These 15 measures are provided by  
 5 subjects' responses to the questionnaire included in the appendix. In these data analyses,  
 6 coefficients on some of the subject-characteristic variables (such as age, "opportunity seeking,"  
 7 and "get jittery") are significant but coefficients on CV are not; the estimated values of  $\hat{b}$  and  
 8 their p-values (in parentheses) are 0.152 and (0.137) for probit and 0.240 and (0.180) for logit.  
 9 We also conducted probit and logit analyses in which subjects' coefficients for CV are estimated  
 10 as a function of the subject characteristic variables (using the approach in Harrison and Rutstrom,  
 11 2008). Again, logit and probit analyses do not detect a significant effect of the CV of risky  
 12 lotteries on subjects' choices.

13

#### 14 *Application of An Alternative Model*

15 A natural question to ask is whether an alternative theory of decision under risk can rationalize  
 16 data from experiment 2. A more pointed question is whether an alternative theory that  
 17 incorporates positive monotonicity can rationalize the data. We apply rank dependent theory of  
 18 decision under risk to the data. For lotteries in which all payoffs are positive, the income model  
 19 of rank dependent expected utility theory (Quiggin, 1993) and cumulative prospect theory  
 20 (Tversky and Kahneman, 1992) are equivalent; hence we label this income model simply as RD,  
 21 for "rank dependent." For binary lotteries, this RD utility functional can be written in a simple  
 22 form, as follows. Consider two binary lotteries, X and Y. Lottery X has probability  $p_X$  of the  
 23 larger payoff  $x_L$  and probability  $1 - p_X$  of the smaller payoff  $x_S$ . Lottery Y has probability  $p_Y$   
 24 of the larger payoff  $y_L$  and probability  $1 - p_Y$  of the smaller payoff  $y_S$ . The probability

1 transformation function for agent  $j$  is  $h_j(\cdot)$  and the utility of payoff function is  $u_j(\cdot)$ . With RD  
 2 theory, lottery X is preferred to lottery Y by agent  $j$  if <sup>4</sup>

$$3 \quad h_j(p_X)u_j(x_L) + [1 - h_j(p_X)]u_j(x_S) > h_j(p_Y)u_j(y_L) + [1 - h_j(p_Y)]u_j(y_S) \quad (5)$$

4 We follow common practice in applications of RD theory to data by using a parametric  
 5 model with probability transformation and utility of payoff functions given by  
 6  $h_j(p) = w(p) = p^\theta / (p^\theta + (1-p)^\theta)^{1/\theta}$  and  $u_j(x) = x^r$ , for all  $j$  (Tversky and Kahneman,  
 7 1992). Option A in each pair of options in experiment 2 is a sure thing (or “degenerate lottery”)  
 8 with payoff  $x_A$ . Hence with the parametric RD model, option A is preferred to risky option B by  
 9 agent  $j$  if

$$10 \quad \Delta U_j^{RD} = [(x_A)^{r_j} - (x_S)^{r_j}] - w(p_B)[(x_L)^{r_j} - (x_S)^{r_j}] > 0 \quad (6)$$

11 The above inequality and the nature of payoffs and probabilities chosen in our  
 12 experiment imply the following hypothesis with respect to the explanatory power of the RD  
 13 model. First, it can be verified straightforwardly that if the estimated  $\hat{r}$  is not significantly  
 14 different from 0 then this model has no explanatory power. Second, recall that a characteristic of  
 15 our experimental design is that  $x_A - x_S = p_B(x_L - x_S)$ . Therefore, if the estimated  $\hat{\theta}$  is not  
 16 significantly different from 1 and the estimated  $\hat{r}$  is also not significantly different from 1 then  
 17 the RD model has no explanatory power. Logit analysis of data from experiment 2 with the RD  
 18 model is reported in the RD Model column of Table 2. Estimates of  $\hat{\theta}$  and  $\hat{r}$  are significantly  
 19 different from 0 (with p-values = 0.000). We report chi-square tests for nonlinearity of the  
 20 probability transformation and utility of payoff functions. The hypothesis that  $\hat{\theta} = 1$  is rejected  
 21 (p-value is 0.008). The joint hypothesis that  $\hat{\theta} = 1$  and  $\hat{r} = 1$  is also rejected (p-value is 0.000).  
 22 Therefore data from experiment 2 can be rationalized by the RD model. Probit estimation leads to  
 23 the same conclusions.

1

2 *Implications of Data from a Concavity Calibration Experiment*

3 Experiments 1 and 2 were designed to test distinguishing features of the CV model. Referees  
 4 have suggested that such tests can be viewed as ones “designed to make the theory fail.” Perhaps  
 5 so, but this is a common characteristic of tests of other theories of decision under risk. For  
 6 example, tests for Allais paradox and related phenomena are important precisely because they are  
 7 designed to test implications of the most central distinguishing characteristic of expected utility  
 8 theory, the independence axiom. But it is nevertheless an interesting question to ask whether data  
 9 from experiments designed to test other decision theories have implications for the CV model.  
 10 We here use data from an experiment designed to test a version of Rabin’s (2000) calibration  
 11 proposition. This seems especially appropriate since Weber et al. (pg. 443) offered their model as  
 12 an alternative to models called into question by this calibration proposition.

13 Cox et al. (2009) reports data from experiments designed to test the small-stakes risk  
 14 aversion suppositions in calibration propositions for several prominent theories that represent risk  
 15 aversion with nonlinear transformation of payoffs or nonlinear transformation of probabilities (or  
 16 both). One of those experiments, designed to test the version of the concavity calibration  
 17 proposition reported in Cox and Sadiraj (2006), uses the following choice options. Subjects  
 18 choose between a certain amount  $x$  and a 50/50 lottery that pays  $x-100$  or  $x+110$  for each  
 19 value of  $x$  in  $\{3K, 9K, 50K, 70K, 90K, 110K\}$ , where  $K = 1,000$ . In this experiment, money  
 20 payoffs were in “contingent euros,” which were received only with low probability if a  
 21 subsequent casino bet by the experimenter paid off.<sup>5</sup> The CV criterion says that option B is more  
 22 likely to be chosen than option A if

$$23 \quad u(\text{EV}(\text{B})) - u(\text{EV}(\text{A})) - b[\text{CV}(\text{B}) - \text{CV}(\text{A})] > 0 \quad (7)$$

24 Let option B denote the certain payoff  $x$ . Let option A denote the 50/50 lottery. The expected  
 25 value of option A is  $x+5$  whereas the expected value of option B is of course  $x$ . Hence, the

1 expected value of option A is always larger than the expected value of option B, for all positive  
2  $x$ .

3 The CVs of option A and option B are, respectively,  $105/(x+5)$  and 0. Clearly, option  
4 A's CV is decreasing in  $x$ , hence so is the likelihood of choosing option B according to the CV  
5 criterion. The data are inconsistent with this. We observed mostly non-decreasing proportions  
6 (0.41, 0.40, 0.48, 0.50, 0.54, 0.53) of option B choices as  $x$  varied from 3,000 to 110,000.  
7 Figure 2 shows values of CV and proportions of option B (the certain payoff option) across  
8 choice pairs. We also ran OLS and nonlinear regressions of the observed proportions of option B  
9 choices on CV(A) and the scaled certain amounts of payoff  $z = x/10,000$ . The OLS-estimated  
10 coefficient for the CV variable is -3.36 and it is marginally significantly different from 0 (p-value  
11 = 0.075) but its sign is the opposite of the one predicted by the CV criterion for risky choices on  
12 the gain domain.

13 We analyzed data at the individual level using logit and probit estimators for the CV  
14 model. The empirical specification comes from the power function form,  $u(EV) = EV^r$ , which  
15 implies

$$16 \quad \Delta U^{CV} = x^r - (x+5)^r + bCV(x) \quad (8)$$

17 We call this the Nonlinear CV Model. Recall that in our experiment  $x$  takes values from  
18 {3K, 9K, 50K, 70K, 90K, 110K}, where K = 1,000, whereas the difference in expected value  
19 of the binary lottery and the sure amount of money is simply 5. For comparison with  
20 other estimations, we also estimated the linear specification of the CV model used above  
21 and in Weber, et al. (2004), that is  $\Delta U^{CV} = bCV(x)$ . The Linear CV Model and  
22 Nonlinear CV Model columns of Table 2 for data from the concavity calibration experiment  
23 report estimates of  $\hat{b}$  that are negative and insignificant whereas the CV model implies that they

1 should be significantly positive. We conclude that the CV criterion has no explanatory value for  
 2 data from the concavity calibration experiment.

3 For the RD model, the specification comes from power functions  $u(y) = y^r$  for payoffs  
 4 and  $w(p) = p^\theta / (p^\theta + (1-p)^\theta)^{1/\theta}$  for probabilities, which implies

$$5 \quad \Delta U^{RD} = x^r - w(0.5)(x+110)^r - (1-w(0.5))(x-100)^r \quad (9)$$

6 The RD Model column of Table 2 reports significantly positive estimates of  $\hat{\theta}$  and  $\hat{r}$ , as implied  
 7 by RD theory. We conclude that the RD model can rationalize data from the calibration  
 8 experiment whereas the CV model cannot.

9

### 10 **Conclusion**

11 We conclude that the CV criterion does not provide a credible normative criterion for decision  
 12 under risk because it is inconsistent with positive monotonicity of preferences for payoffs. The  
 13 CV criterion does not provide a useful descriptive model of subjects' decision making in the risky  
 14 environment of experiments 1 and 2 designed to test the criterion's distinctive properties. In  
 15 contrast rank dependent theory of decision under risk, that incorporates positive monotonicity of  
 16 preferences for payoffs, is consistent with data from experiments 1 and 2. Finally, the CV  
 17 criterion does not provide a useful descriptive model for subjects' decisions in an experiment  
 18 designed to test a supposition used in concavity calibration propositions for prominent theories of  
 19 decision under risk. This last test is a direct response to the claim that the CV criterion provides a  
 20 solution to the problems for plausibility of theories of decision under risk that arise from the  
 21 concavity calibration critique.

### Endnotes

- 1 \* We wish to thank Jerome R. Busemeyer, Michael Lee, and anonymous reviewers for helpful  
2 comments and suggestions on earlier versions. The National Science Foundation provided  
3 support for this research (grant numbers IIS-0630805 and SES-0849590).
- 4 1. Let  $F$  and  $G$  be two cumulative probability distributions of payoffs defined on the set  $\Omega$ .  
5 Strict first order stochastic dominance of  $F$  over  $G$  (Hadar & Russell, 1969; Rothschild &  
6 Stiglitz, 1970) is defined by:  
7 (i)  $F(x) \leq G(x)$ , for all  $x \in \Omega$ ; and (ii)  $F(x_k) < G(x_k)$ , for at least one  $x_k \in \Omega$ .
- 8 2. See, for example, Loomes, Starmer, and Sugden (1992).
- 9 3. Stata software for logit analysis does not use the seven observations in which subjects chose  
10 indifference rather than strict preference for option A or option B (coded as 0 and 1). Hence the  
11 parameter estimates are based on 293 observations.
- 12 4. The original exposition of rank dependent expected utility theory (Quiggin, 1993) orders  
13 payoffs from lowest to highest. We here follow recent convention that orders payoffs from  
14 highest to lowest for both that theory and other rank dependent theories such as cumulative  
15 prospect theory.
- 16 5. The experiment design and contingent payoff protocol are discussed in greater detail in Cox et  
17 al. (2009). The six other experiments reported in that paper use ordinary money payoffs rather  
18 than contingent payoffs.
- 19



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**Table 1. Options A and B in Six Decision Pages in Experiment 2**

Table	Option A	Option B					
		Low Payoff \$	Low Payoff Balls	High Payoff \$	High Payoff Balls	EV	CV
Q	\$7	\$2.50	1	\$7.50	2, 3, 4, 5, 6, 7, 8, 9, 10	\$7	0.21
S	\$27	\$2.50	1,2,3	\$37.50	4, 5, 6, 7, 8, 9, 10	\$27	0.59
U	\$35	\$2.50	1, 2, 3, 4, 5	\$67.50	6, 7, 8, 9, 10	\$35	0.93
W	\$31	\$2.50	1, 2, 3, 4, 5, 6, 7	\$97.50	8, 9, 10	\$31	1.40
R	\$22	\$2.50	1, 2, 3, 4, 5, 6, 7, 8	\$100.00	9, 10	\$22	1.77
T	\$15	\$2.50	1, 2, 3, 4, 5, 6, 7, 8, 9	\$127.50	10	\$15	2.50

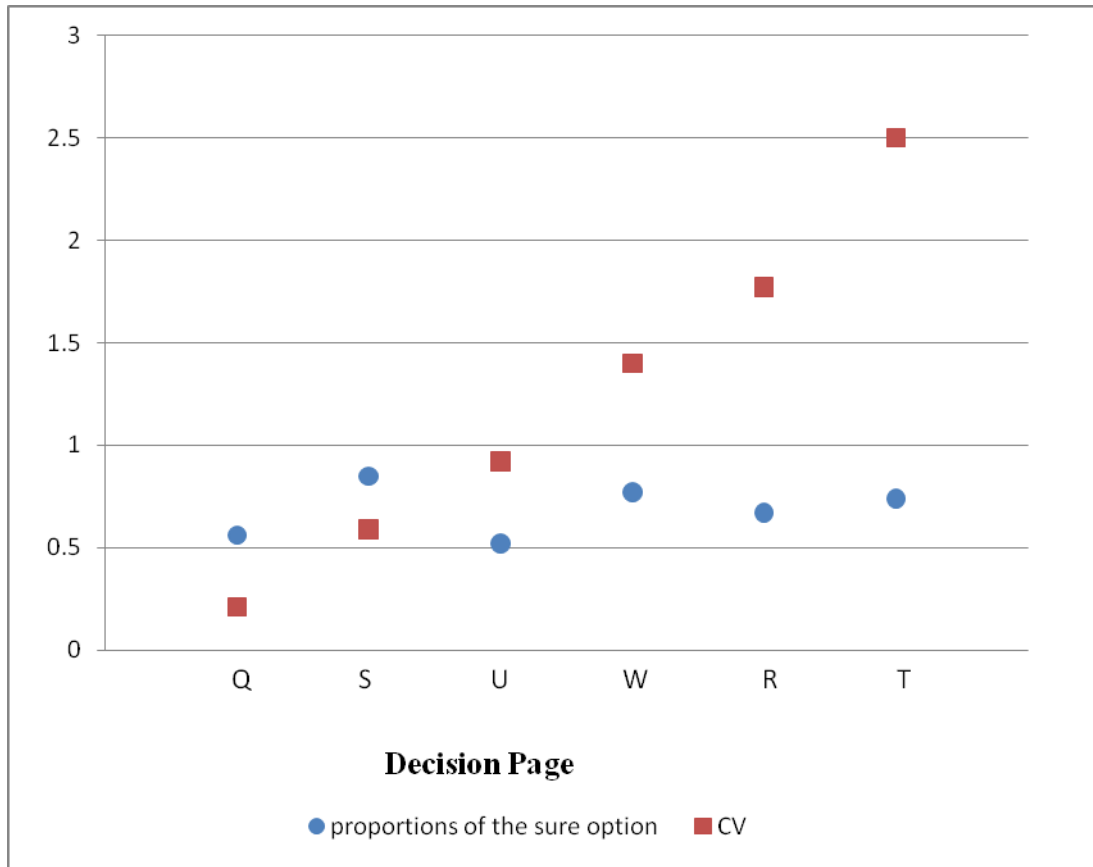
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2**Table 2. Logit Analysis of Data**

Parameter Estimates and Statistics	Experiment 2		Concavity Calibration Experiment		
	Linear CV Model	RD Model	Linear CV Model	Nonlinear CV Model	RD Model
$\hat{b}$	0.198 (0.211)		-12.639 (0.354)	- 9.381 (0.562)	
$\hat{\theta}$		0.739*** (0.000)			1*** (0.000)
$\hat{r}$		0.428*** (0.000)		0.581 (0.286)	0.686*** (0.000)
constant	0.557** (0.008)		0.011 (0.969)		
Log-likelihood	-180.83	-182.55	-166.72	-166.70	-166.93
Chi-square test of $\theta = 1$		6.97** (0.008)			0.00 (0.999)
Chi-square test of $r = 1$		60.88*** (0.000)		0.59 (0.442)	7.64** (0.006)
Chi-square test of $\theta = 1$ and $r = 1$		67.41*** (0.000)			
Nr of observations	293	293	242	242	242

3 Linear CV Model:  $\Pr(\text{sure}) = F(bCV + c)$ . Nonlinear CV Model:  $\Pr(\text{sure}) = F(\text{sure}^r - EV^r + bCV)$ . RD Model:4  $\Pr(\text{sure}) = F(\text{sure}^r - L - w(p)(H^r - L))$ . p-values are reported in brackets. Linear CV Model columns report estimates using standard  
5 STATA (probit command, cluster(id)). Other columns contain estimates as in the Harrison approach since STATA does not contain code for  
6 these models. \* denotes p-value < 0.05. \*\* denotes p-value < 0.01. \*\*\* denotes p-value < 0.001

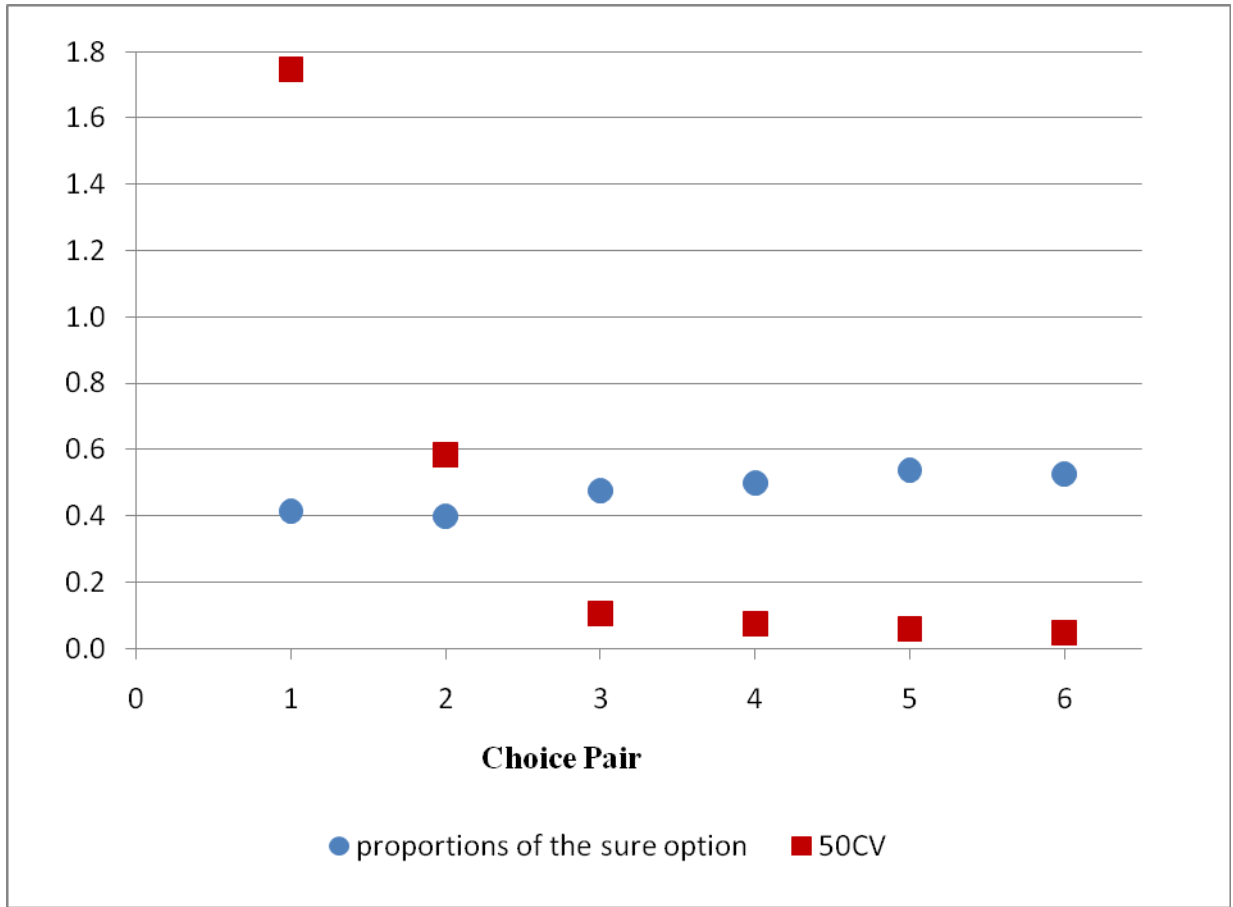
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**Figure 1. Choices of the Sure Option and Values of CV in Experiment 2**

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**Figure 2. Choices of the Sure Option and Values of CV in the Calibration Experiment**

### Appendix 1. Proof of Proposition 1

To show the result start by noting that the CV of lottery  $L_n$  is given by

$$CV = \sqrt{\frac{\ell^2 / p + n^2 \ell^2 / (1-p)}{(n+1)^2 \ell^2}} - 1 = \frac{|p-1/(n+1)|}{\sqrt{p(1-p)}} \quad (\text{a.1})$$

It can be easily checked that for  $p > 1/(n+1)$ , which follows from  $p \in [0.5, 1)$ , and  $n > 2$ , the CV of gambles  $L_n$  is increasing in  $n$ . According to the CV measure of risk sensitivity (Weber et al., pgs. 433, 443), probability of choosing  $L_n$  is

$$\Pr(L_n) = F\left(u(EV_{L_n}) - bCV_{L_n} - u(EV_{L_{n+k}}) + bCV_{L_{n+k}}\right) \quad (\text{a.2})$$

where  $F: R \rightarrow [0, 1]$ , increasing and  $F(0) = 0.5$ . Substituting expressions for EV and CV yields

$$u(EV_{L_n}) - bCV_{L_n} = u((n+1)\ell) - b \frac{|p-1/(n+1)|}{\sqrt{p(1-p)}} \quad (\text{a.3})$$

and

$$u(EV_{L_{n+k}}) - bCV_{L_{n+k}} = u((n+1+k)\ell) - b \frac{|p-1/(n+1+k)|}{\sqrt{p(1-p)}} \quad (\text{a.4})$$

Therefore, in statement (a.2), we get that

$$\begin{aligned} \Pr(L_n) &= F\left(u((n+1)\ell) - u((n+1+k)\ell) - b\left[CV_{L_n} - CV_{L_{n+k}}\right]\right) \\ &= F\left(-\Delta u + \frac{b}{\sqrt{p(1-p)}} \frac{k}{(n+k+1)(n+1)}\right) \end{aligned} \quad (\text{a.5}) \text{Note that}$$

$\Pr(L_n) > 1/2$  if the expression on the right-hand-side of (a.5) is positive, which is equivalent with

$$\frac{bk}{(n+1)(n+1+k)\Delta u} \geq \sqrt{p(1-p)} \quad (\text{a.6})$$

For any given  $n, k, u(\cdot)$ , and  $b$ , the left hand side of the last inequality (a.6) is a finite number, say  $C$ . The right-hand side has limit 0 for  $p$  close to 1. So, we can always find a  $p$  such that  $\sqrt{p(1-p)}$  is less than  $C$ . Therefore,  $L_n$  is more likely to be chosen than option  $L_{n+k}$  for such  $p$ .

1  
2 **Appendix 2. Subject Instructions and Decision Pages for Experiment 1**

3  
4 Subject Instructions

5  
6 **No Talking:** Now that the experiment has begun, we ask that you do not talk. If you have a  
7 question, please raise your hand and the experimenter will approach you and answer your  
8 question in private.

9  
10 **Money Payoff:** You will be paid an amount determined by your decisions in this experiment and  
11 the outcomes from flipping a coin and picking a colored ball from a box.

12 The experiment proceeds as follows. First, you choose your preferred option in each of  
13 two tables. Second, which of the two tables will be selected for money payoff will be determined  
14 by flipping a coin. The side of the coin that lands up determines which one of your decision tables  
15 pays money.

16 Note that only one of your two decisions will be selected for money payoff by flipping a  
17 coin; thus you should decide which option you prefer in each table **independently** of your choice  
18 in the other table.

19  
20  
21 Heads Choice Page

22  
23 If the coin lands **heads** then you will be paid for your choice in Table 1. Please decide whether  
24 you prefer Option A or Option B. If you don't care which option you get, then you can choose  
25 Option I (which stands for "I don't care").

26  
27 In Table 1, if you choose Option A, then your payoff is **\$10.50** (if Table 1 is selected by coin  
28 flip).

29  
30 If you choose Option B (and Table 1 is selected by coin flip) then your payoff is determined by  
31 picking a ball from a box that contains **4** white balls and **6** red balls. Your payoff from choosing  
32 Option B is:

33 *\$0 if you pick one of the 4 white balls;*

34 *\$22 if you pick one of the 6 red balls.*

35  
36  
37 *Please mark your choice by circling A, B, or I in the My Choice column.*

38  
39 **Table 1**

Option A	Option B	My Choice
<b>\$10.50</b>	<b>\$0</b> if you draw one of the <b>4</b> white balls or <b>\$22</b> if you draw one of the <b>6</b> red balls	<b>A B I</b>

1

2 Tails Choice Page

3

4 If the coin lands **tails** then you will be paid for your choice in Table 2. Please decide whether you  
 5 prefer Option X or Option Y. If you don't care which option you get, then you can choose  
 6 Option I (which stands for "I don't care").

7

8 If you choose Option X (and Table 2 is selected by coin flip) then your payoff is determined by  
 9 picking a ball from a box that contains **1** orange ball and **19** white balls. Your payoff from  
 10 choosing Option X is:

11 *\$10 if you pick one of the 19 white balls;*12 *\$20 if you pick the 1 orange ball.*

13

14 If you choose Option Y (and Table 2 is selected by coin flip) then your payoff is determined by  
 15 picking a ball from a box that contains **1** orange ball and **19** white balls. Your payoff from  
 16 choosing Option Y is:

17 *\$10 if you pick one of the 19 white balls;*18 *\$74 if you pick the 1 orange ball.*

19

20 *Please mark your choice by circling X, Y, or I in the My Choice column.*

21

22 **Table 2**

23

Option X	Option Y	My Choice
<b>\$10</b> if you pick one of the <b>19</b> white balls or <b>\$20</b> if you pick the <b>1</b> orange ball	<b>\$10</b> if you pick one of the <b>19</b> white balls or <b>\$74</b> if you pick the <b>1</b> orange ball	<b>X Y I</b>

24

25

26

27



## Appendix 3: Subject Instructions, Decision Pages, and Questionnaire for Experiment 2

### Subject Instructions

You will be paid an amount determined by your decisions in this experiment and the outcomes from rolling a die and drawing a numbered ball from a bingo cage.

The experiment proceeds as follows. First, you choose your preferred option in each of six tables. Second, which of the six tables will be selected for money payoff will be determined by rolling a six-sided die in your presence. The number that ends “up” on the die determines which one of your decision tables pays money.

Note that only one of your six decisions will be selected for money payoff by rolling the die; thus you should decide which option you prefer in each table **independently** of your choice in other tables.

You are asked to choose either option A or option B in each of six tables.

The example below shows the type of choice table that will be included in the experiment.

In Table 1, if you choose Option A, then your payoff is \$54 (if this table is selected by rolling the die).

If you choose Option B (and this table is selected by rolling the die) then your payoff is determined by drawing a ball from a bingo cage that contains 10 balls with 10 different numbers.

Your payoff from choosing Option B is:

**\$5** if the bingo cage selects **ball 1, 2, or 3**;

**\$75** if the bingo cage selects **ball 4, 5, 6, 7, 8, 9, or 10**.

You indicate your choice in the third column. If you prefer Option A to Option B then circle A. If you prefer Option B to Option A then circle B. In case you are indifferent between the two options, you circle I. If I is circled then the experimenter will choose Option A or Option B for you.

1

**Table 1**

Option A	Option B	My Choice
\$54	\$5 if ball from {1, 2, 3} or \$75 if ball from {4, 5, 6, 7, 8, 9, 10}	A B I

2

3

4 Decision Pages

5

6 The response forms used in the experiment consist of six decision pages. Each decision  
7 page contains one of the following tables, summary instructions for the choice task in the  
8 table, and explanation of the lotteries on that page. Full text of the decision pages is  
9 available upon request to the authors. The decision pages were given to individual  
10 subjects in independently drawn random order. Subjects had access to all six decision  
11 pages while making their decisions on every page. One decision page for each subject  
12 was independently, randomly selected for money payoff.

13

14

**Table Q**

Option A	Option B	My Choice
\$7	\$2.50 if ball from {1} or \$7.50 if ball from {2,3,4,5,6,7,8,9,10}	A B I

15

16

**Table S**

Option A	Option B	My Choice
\$27	\$2.50 if ball from {1, 2, 3} or \$37.50 if ball from {4, 5, 6, 7, 8, 9, 10}	A B I

1

2

3

**Table U**

Option A	Option B	My Choice
\$35	\$2.50 if ball from {1, 2, 3, 4, 5} or \$67.50 if ball from { 6, 7, 8, 9, 10}	A B I

4

5

6

**Table W**

Option A	Option B	My Choice
\$31	\$2.50 if ball from { 1, 2, 3, 4, 5, 6, 7 } or \$97.50 if ball from { 8, 9, 10 }	A B I

7

8

9

**Table R**

Option A	Option B	My Choice
\$22	\$2.50 if ball from { 1, 2, 3, 4, 5, 6, 7, 8 } or \$100.00 if ball from { 9, 10 }	A B I

10

11

**Table T**

Option A	Option B	My Choice
\$15	\$2.50 if ball from { 1, 2, 3, 4, 5, 6, 7, 8, 9 } or \$127.50 if ball from { 10 }	A B I

1 Questionnaire (with anonymous, coded responses)

2

3 Thank you very much for participating in our decision experiment. We would like to ask you a  
4 few questions. Your privacy is protected because your name will not appear on this questionnaire  
5 or on your decision tables.

6

7

8

9

**Information about you:**

10

11

1. What year are you in school?

12

Freshman \_\_\_\_ Sophomore \_\_\_\_ Junior \_\_\_\_ Senior \_\_\_\_ Grad. \_\_\_\_

13

14

2. What is your intended or declared major ? \_\_\_\_\_

15

16

3. What is your age? \_\_\_\_\_

17

18

4. What is your gender? Female \_\_\_\_ Male \_\_\_\_

19

20

5. What is your race?

21

Asian \_\_ Black/African American \_\_\_\_ White \_\_ Other \_\_\_\_ Prefer not to respond \_\_\_\_

22

23

6. What is your height? Feet \_\_\_\_ Inches \_\_\_\_

24

25

7. Are you a smoker? Yes \_\_\_\_ No \_\_\_\_

26

27

8. What is your birth order?

28

Oldest child in your family? \_\_\_\_ Youngest child? \_\_\_\_ Middle child? \_\_\_\_

29

30

31

32

**General Questions:** Please answer the following questions on a scale of 1-10, where 1 is  
33 **Strongly Disagree** and 10 is **Strongly Agree**. Please **circle** the number that represents your best  
34 answer.

35

36

37

1. I seek opportunities for doing things that I never did before.

38

39

1 2 3 4 5 6 7 8 9 10

40

41

Strongly disagree

Strongly agree

42

43

44

2. I don't worry about the consequences of what I do.

45

46

1 2 3 4 5 6 7 8 9 10

47

48

Strongly disagree

Strongly agree

49

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36

3. I never get lucky breaks.

**1 2 3 4 5 6 7 8 9 10**

Strongly disagree

Strongly agree

4. I frequently get jittery and worry about things.

**1 2 3 4 5 6 7 8 9 10**

Strongly disagree

Strongly agree

5. I proceed with care in most endeavors.

**1 2 3 4 5 6 7 8 9 10**

Strongly disagree

Strongly agree

6. I always assess the prospects and risks before starting a new activity.

**1 2 3 4 5 6 7 8 9 10**

Strongly disagree

Strongly agree

7. I tend to do dangerous things without adequate precautions.

**1 2 3 4 5 6 7 8 9 10**

Strongly disagree

Strongly agree