

Mathematical Modelling of Performance and Wear Prediction of PDC Drill Bits: Impact of Bit Profile, Bit Hydraulic, and Rock Strength

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Abstract

The estimation of Polycrystalline Diamond Compact (PDC) cutters wear has been an area of concern for the drilling industry for years now. The cutter's wear has been measured practically by pulling the bit out for evaluation at the surface. It is important to find the right time for tripping out as this helps to avoid the fishing job and reduces the operational cost significantly. The prediction of the drilling performance is based on the interaction of cutter and rock. Several authors focused on the cutter-rock interface but only a few researchers tried to model the wear of the PDC bit cutters. The aim of this research is to understand the relationships between the rate of penetration (ROP) and the drilling variables per each foot, and then determine the overall bit efficiency for the whole drilling operation. A new mathematical model is derived to predict the PDC bit performance by considering the factors that were already not taken into account. These factors include rock strength, bit design, and bit hydraulic. The model investigates the effect of these parameters to estimate the abrasive cutters wear on the inner and the outer bit cones by deriving modified equations to calculate the mechanical specific energy (MSE), torque, and depth of cut (DOC) as a function of effective blades (EB). The model is used to forecast the bit cutters wear conditions in four wells in the oil fields located in Libya, which were drilled with three different PDC's sizes. The model enables the results to be compared to the actual bit cutters wear measured for inner and outer cones. The results are found that are well in agreement with the actual field data obtained in bit records.

Keywords

PDC cutters wear; Mechanical specific energy; Depth of cut; Effective blades; ROP.

1. Introduction

Drill bits are the main tools that penetrate the formation down to the planned production zone. An efficient PDC bit should have good drillability by achieving a sufficient ROP, and it should be durable to be used in other wells. Both ROP and bit life rely directly on bit design, drilling parameters, and rock properties (Ersoy and Waller, 1995, Sinor and Warren, 1987).

Ziaja and Miska (1982) presented a mathematical model for the diamond core bit to estimate rock strength index, rock abrasiveness index, and ROP. The model also provided an equation to measure the bit dullness by assuming gradual bit wear while drilling.

Warren and Sinor (1987) developed a single PDC cutter model to predict the cutter temperature, force, and wear. The model assumed constant ROP and considered the mechanical design parameters, and results were compared to the laboratory drilling test for four different bits design.

Kuru and Wojtanowicz (1988) proposed a model that derived from torque and ROP equations to prevent early damage of PDC bit by evaluating the bit condition and detecting the rock change while drilling. The approach required data from bit geometry and its dull condition to set up software to optimize the drilling parameters.

Understanding the breakage process generated at the cutter – rock interface is the key for interpretation of the drilling parameters. There are considerable work in the literature that focused on studying the cutter – rock interface but not to determine the PDC bit wear. Jones (1990) presented an improved cutter configuration to optimize the cutter- rock interface area according to the total volume of diamond that penetrate the formation along the bit radius. Detournay and Defourny (1992) studied the cutter- rock interaction as a function of two processes: cutting of the rock and the friction underneath the cutters. They developed a model, which was used as a guide to understand the drilling process by assuming that the cutting component of the torque and the weight on bit (WOB) are both proportional to the depth of cut (DOC).

Wojtanowicz and Kuru (1993) proposed a model to control the performance of a spherical PDC bit cutters in order to maintain the balance between the WOB distribution and bit wear uniformity. The results were obtained by adjusting empirical constants while drilling in homogenous formation to match bit recorded data from offset well. The model assumed that WOB, rotary speed (RPM), mud flow are constant and the model also ignored the cutting angle.

Gerbaud et al. (2006) developed a new cutter-rock interface by taking into account the latest technology of the PDC cutters shape and investigated its effects on the ROP, bit stability, and bit wear. Detournay et al. (2008) presented a model to extend their work of 1992. The model introduced the contact length, a measure of bit dullness, and the contact strength to the frictional process. The model was developed under the same assumption that the drilling response of PDC bit followed a linear constraint between torque, WOB, and DOC.

Tulu and Heasley (2009) investigated numerically a 3D single cutter to analyse the cutter-rock interaction and compared their work with a single cutter on a laboratory scale. The objective of their work was to analyse the vertical and horizontal forces based on the cutting depth, however, they ignored the effect of mud hydraulic. Gouda et al. (2011) developed a mathematical model to estimate the PDC cutters wear by including the torque obtained from drill pipe, string stabilizer, and the bit. The model was valid for certain back and side rack angles.

Yahiaoui et al. (2011) studied the cutter-rock interaction, by testing six different manufacturers' cutters in the laboratory scale to compare the wear rate. The tests were carried out under atmospheric conditions in which no drilling fluid was pumped into the contact surface. Patil and Teodoriu (2013) studied the cutter-rock interaction and introduced a mathematical model to analyse the stick-slip vibration (Alkaragoolee, 2018) by considering the PDC cutter wear.

Liu et al. (2014) introduced an analytical model coupled with real-time gamma-ray data to predict the PDC bit wear. The model assumed that the volume loss of cutters is proportional to WOB. Chen et al. (2014) presented a new cutter-rock interaction to analyse the cutting force. They concluded that the cutting arc length plays a major role in the cutting force calculation. Doshvarpassand et al. (2017) conducted the experimental work to evaluate the effects of cutter-rock interaction on the cutting action of different sizes of PDC cutters. They found that the effect of the cutters edge may differ based on the rock type.

Micro- and nano-scratch tests wear applied to estimate the wear volume of the PDC cutters diamond layers. The experimental results were compared to the results obtained from wear models in the past (Abbas and Hassanpour, 2018).

Yang et al. (2019) presented a model as a combination of mechanical specific energy, principal component analysis, and wavelet analysis to decide when to pull the bit out of the hole and

predict cutters wear of the Kymera PDC bit. The model suggested a constant wear increment with the depth.

To evaluate the bit performance accurately, the bit profile needs to be considered along with the cutter-rock interface. This has been proved by Glowka (1987) who developed a method to predict the cutter force and bit performance. He confirmed that the bit profile can significantly affect PDC wear. His model was based on the assumption that cutter wear will be uniform regardless of its location.

A number of researches aimed to design the crown shape of the PDC bit by employing the principles of equal wear, equal volume of rock removed and power (Glowka, 1985, Hibbs and Flom, 1978). However, they ignored the effect of cutter-rock interaction; therefore, they did not efficiently reflect the force and the cutting wear. Wang et al. (2018) studied the cutter-rock interaction and concluded that the bottom hole shape of the PDC bit is one of the essential factors that affect the force and the wear of PDC cutters.

Most of the available models in the literature are not so efficient, albeit are helpful, as they do not address the real operating conditions of the rig site. These models were developed using a laboratory scale and under atmospheric conditions. This paper proposes a new model that can be applied on rig site and it is a function of all drilling parameters that include the rock-bit interaction, bit profile, rock strength, and bit hydraulic as shown in (Fig.1). The results enable the model to compare the cutters wear on both inner and outer cones with the actual bit wear according to the standard bit record form. Full description of the analyses of the drilling parameters is given by adjusting some factors for rock hardness and mud hydraulic, rock friction, and DOC to understand prediction of the bit performance.

Fig.1 PDC cutters wear prediction model.

1.2 Computational Model of PDC Bit Life

The design of the PDC profile was based on the assumption of equal wear and equal volume and energy (Wang et al., 2018). However, the distribution and density of cutters along the bit profile are not the same. Therefore, the degree of wear is not the same for inner and outer cutters due to the rotating radius. In this work, bit profile is considered and the analyses are divided into inner and outer cones.

The force applied on PDC is transferred to all cutters, and the cutter breaking mechanism differs because of bit design and cutters location, therefore, the analyses of the forces on the single cutter as proposed in previous work done by Glowka (1985), Hibbs and Flom (1978) is not accurate. Huang et al. (2017) added that the force conditions are more related to cutting arc length and wear degree. In this model, the drilling impedance (DRIMP) as a wear evaluation index will be used instead of using cutting forces.

The previous studies (Gouda et al., 2011, Pryhorovska, 2017, Wojtanowicz and Kuru, 1993) carried out on the analysis of rock-bit interaction suggested good hole cleaning. Besides, Kuru (1990) confirmed that the diamond compact material loses strength at a temperature above 350°C resulting from high friction generated at the interface. Therefore, drilling fluid has to be pumped in during drilling operations to maintain and keep the operation temperature below 350°C. Additionally, it was shown experimentally that the wear rate of PDC cutter was much greater for dry shearing than wet cutting (Gray, 1967). Hence, to address the real conditions of the drilling operation, this model considers the influence of the bit hydraulic.

It has been confirmed that abrasive wear is a function of DOC; therefore, to control DOC of PDC cutters, the volume of rock removed and the torque generated have to be controlled (Sinor et al., 2001). DOC is modified in this work based on the active cutters for both cones.

Tian et al. (2015) concluded that the drilling efficiency and the volume wear rate of PDC bit is dependent on the cutting angle, while Hankins et al. (2015) proved that the back and side – rack angles have the minor effect on the performance of drilling. As a result, this model ignores the effect of back and side – rack angles and consider the cutting angle.

Hareland and Rampersad (1994), Maurer (1966), and Teale (1965) used Eq. 1 which gives the theoretical volume of the removed rock per revolution.

$$(V_r)_t = \frac{T}{MSE} = 2\pi \sum_{i=n}^i A_C \times R \dots\dots\dots (1)$$

Where $(V_r)_t$ is the theoretical volume of removed rock (in³), T is the torque (lb_m – in), MSE is the mechanical specific energy (psi), A_C is the cutting area of the cutter (in²), and R is the radial or the distance from cutter to bit centre (in).

MSE is defined as the ratio between the input energy to the volume of removed rock. One of the major causes of error for this method in estimating the MSE is that the rock hardness is not considered because of the variety of rock strength (Tveit and Berg, 2016). Apart from the

energy needed to crush the rock, the energy which is required to transport the drilling cuts underneath the PDC bit to the surface should be considered as well (Mohan et al., 2009).

In this work, MSE is correlated with rock hardness and mud hydraulic as shown in Eq. 2.

$$MSE_{mod.} = MSE_R + MSE_H \dots\dots\dots (2)$$

The modified MSE for rock hardness is presented in Eq. 3 as follows:

$$MSE_R = MSE \times \text{Hardness Coefficient} \dots\dots\dots (3)$$

Where, $MSE_{mod.}$ is the modified mechanical specific energy (psi), MSE_R is the correlated mechanical specific energy for the rock hardness (psi), and MSE_H is the correlated mechanical specific energy for the mud hydraulic (psi).

$$\text{Hardness Coefficient} = \frac{MSE}{\text{Hardness}} \dots\dots\dots (4)$$

Jogi and Zoeller (1995) proposed that the rock hardness can be estimated by use of Eq. 5.

$$\text{Hardness} = \frac{WOB \times RPM}{ROP \times D_b} \dots\dots\dots (5)$$

Where, Hardness (psi), WOB is the weight on bit (lb_m), RPM is the rotary speed (rpm), ROP is the rate of penetration (ft/hr), and D_b is the bit diameter (in).

Kerr (1988) introduced that the specific hydraulic energy can be expressed by the hydraulic horsepower efficiency as follows:

$$MSE_H = HSI = \frac{HP}{D_b^2} \dots\dots\dots (6)$$

Where, HSI is the bit hydraulic efficiency (psi), HP is the bit hydraulic horsepower (lb).

HP and the bit pressure drop (P) can be estimated by applying Eqs. 7 and 8 as proposed by Kerr (1988).

$$HP = \frac{P \times Q}{1714} \dots\dots\dots (7)$$

Where, P is the bit pressure drop (psi), and Q is the flow rate (gpm).

$$P = \frac{Q^2 \times MW}{TFA^2} \dots\dots\dots (8)$$

Where, MW is the mud weight (lb/gal), and TFA is the bit total flow area (in^2).

The torque also is correlated with rock friction in this work as stated by Jogi and Zoeller (1995).

The modified torque is given in Eq. 9.

$$T_{\text{mod.}} = T \times \mu \dots\dots\dots (9)$$

Where, $T_{\text{mod.}}$ is the modified torque (lb_m – in), and μ is the rock friction (unitless).

$$\mu = \frac{T}{\text{WOB} \times D_b} \dots\dots\dots (10)$$

By rearranging and substituting the parameters, Eq. 1 become:

$$\frac{\text{Hardness} \times T}{\text{MSE} \times \text{WOB} \times D_b} \times (V_r)_a = 2\pi \sum_{i=n}^i A_C \times R \dots\dots\dots (11)$$

$$\frac{\text{Hardness} \times T \times D_b}{\text{MSE} \times \text{DRIMP}} = 8 \sum_{i=n}^i A_C \times R \dots\dots\dots (12)$$

Where, DRIMP is the wear evaluation index (lb/in).

The actual volume of removed rock as a function of ROP can be defined as reported by Jogi and Zoeller (1995) in Eq. 13.

$$(V_r)_a = \frac{\pi D_b^2}{4} \times \frac{\text{ROP}}{\text{RPM}} \dots\dots\dots (13)$$

Where, $(V_r)_a$ is the actual volume of rock removed per revolution (in³).

DRIMP can be estimated using Eq. 14 as proposed by De Reynal (2011).

$$\text{DRIMP} = \frac{\text{WOB}}{\text{DOC}} \dots\dots\dots (14)$$

Where, DOC is the depth of cut (in).

Sinor et al. (2001) defined the depth of cut per revolution as a function of ROP and RPM as stated in Eq. 15.

$$\text{DOC} = \frac{\text{ROP}}{\text{RPM}} \dots\dots\dots (15)$$

Chen et al. (2014) modified Eq. 15 by including the number of blades as follows.

$$\text{DOC} = \frac{\text{ROP}}{\text{RPM} \times N_b} \dots\dots\dots (16)$$

Where, N_b is the number of blades.

The PDC bit is designed with a plurality of cutters mounted on every blade. Once weight applied on the bit and the bit started rotating, the cutter dragged by torque to cut a layer of rock. Based on that, DOC is the depth to which cutter penetrates the formation as shown in Fig. 2. In this work, DOC is calculated based on a modified method suggested in Spread (2017) (see Eq. 17). The objective of this technique is to estimate the effective blade (EB) which is defined as

how partially the cutters are involved in the drilling mechanism by computing the active cutters width to the blade length at a given ROP. Then DOC is calculated for the inner and outer cones using Eq. 17.

$$DOC = \frac{ROP}{RPM \times N_b \times EB} \dots\dots\dots (17)$$

Where, EB is the effective blades (unitless).

Fig.2 Diagram of PDC drill bit cutting element D_c , DOC , A_c , r_o , r_i , C_t , and C .

EB can be calculated using Eq. 18 as a function of the total cutters' width on the blade (C_t) in both cones (see Fig. 2).

$$EB = \frac{C_t}{L_b} \dots\dots\dots (18)$$

Where, C_t is the total of cutters width (in), and L_b is the blade length (in).

L_b can be determined according to the bit design as seen in Fig. 3. The cone height (G) is determined as shown in Fig. 3.

Fig.3 Updated bit selection chart (Bourgoyne et al., 1986).

The inner and outer radius as shown in Fig. 4 can be determined using Eqs. 19 and 20:

$$r_i = \frac{2}{3} \times r_b \dots\dots\dots (19)$$

$$r_o = \frac{1}{3} \times r_b \dots\dots\dots (20)$$

Where, r_b is the bit radius (in), r_i is the inner cone radius (in), and r_o is the outer cone radius (in).

Then, L_b can be calculated in both cones using Eqs. 21 and 22 as shown in Fig. 4.

$$L_b(\text{inner}) = [(r_i)^2 + (G)^2]^{0.5} \dots\dots\dots (21)$$

$$L_b(\text{outer}) = [(r_o)^2 + (G)^2]^{0.5} \dots\dots\dots (22)$$

Fig.4 Inner and Outer Cones (Brandon et al., 1992).

The cross-sectional area of the cutting (A_c) is dependent on the depth of cut and cutter arc length, taking into account the distribution of all cutters within the radial (Chen et al., 2014, Glowka, 1985).

Chen et al. (2014) stated that the arc length of cutters varies according to its location in the cone and the shape of the cutting area as shown in Fig. 5. Therefore, the cutting area in this model will be calculated as a function of cutter width (C) instead of arc cutter length ($A_c = DOC * C$) as described in Fig. 2 and Eq. 23.

Fig.5 Various cutters geometry due to different locations.

Gouda et al. (2011) reported that the cutting force is obtained as follows.

$$F_c = \frac{A_c}{\cos \phi} \times \delta = \frac{DOC \times C}{\cos \phi} \times \delta \dots\dots\dots (23)$$

Where, F_c is the cutting force (lb), ϕ is the cutting angle ($^\circ$), and δ is the hardness or the cutting force per unit area (psi).

The cutter width can be calculated as suggested in Spread (2017) using Eq. 24.

$$C = 2 \times [DOC \times (D_c - DOC)]^{0.5} \dots\dots\dots (24)$$

Eq. 25 is developed based on Eq. 23 by including the number of cutters to compute DOC and C for inner and outer cones.

$$DOC \times C \times N_c = \frac{WOB}{Hardness} \times \cos \phi \dots\dots\dots (25)$$

Where, N_c is the number of cutters.

In the cutting process using PDC bit, the mechanism of the cutting element at the bottom hole is complex. To facilitate the analyses of the cutting forces, based on bit design, the cone height is assumed to be 3 inch for the 8.5" and 12.25" PDC bits, and 4 inches for the 16" PDC bit as follows:

- Both 8.5" PDC are designed with a parabolic profile with shallow cone. Therefore, the cone height value represents the option 6 as shown in Fig. 3, which is within the range of $1/8 D_b < G$
- 16" PDC is designed with a parabolic profile with a medium cone. Therefore, the cone height value represents option 5 as shown in Fig. 3, which is within the range of $1/8 D_b < G \leq 1/4 D_b$.

- 12.25" PDC is designed with a parabolic profile with a medium cone. Therefore, the cone height value represents option 5 as shown in Fig. 3, which is within the range of $1/8 D_b < G \leq 1/4 D_b$.

Knowing the value of G and L_b , the cutting angle can be easily estimated. Given WOB, RPM, torque, N_b , and density of cutters in both cones; gPROMS software is used to numerically solve the model and find the cutter width C and DOC for both inner and outer cones by using Eqs. 24 and 25. Then, DOC is modified by the use of Eq. 17.

It has been reported that the abrasive wear occurred caused by low DOC, while high DOC caused an impact wear (Van Quickenberghe et al., 2006). The wear occurs while the bit is rotating but not cutting (Mensa-Wilmot et al., 2003). This was confirmed by Gouda et al. (2011) who stated that no torque leads to wear. On this basis, this model neglects the cutting force on the wear effect and heat generated between the cutters – rock interaction to estimate the bit wear.

Maurer (1966) stated that Eq. 26 gives the energy of rotation per revolution:

$$\text{Energy}_{\text{rot.}} = 2 \pi \times T \times \text{RPM} \dots\dots\dots (26)$$

The correlated MSE then is defined as follows in Eq. 27 which then be substituted in Eq. 12.

$$\text{MSE} = \frac{\text{Energy}_{\text{rot.}}}{(V_r)_a} \dots\dots\dots (27)$$

Maurer (1966) defined the torque due to friction as a function of the wear cutter area with zero cutting force as shown in Eqs. 28.

$$T = \text{Cutting Force} + \mu \times \sigma \times D_b \times A_w \dots\dots\dots (28)$$

Where, A_w is the wear cutter area (mm^2).

In this work, A_w is estimated based on what was proposed by Detournay (1993) as follows:

$$A_w = L \times r_c \dots\dots\dots (29)$$

Where, r_c is the cutter radius (mm), and L is the wear cutter height (mm).

By substituting Eq. 29 into Eq. 28, a new Eq. 30 is obtained to calculate the torque as a function of L .

$$T = 2 \times \mu \times \sigma \times r_b \times r_c \times L \dots\dots\dots (30)$$

Finally, L can be estimated by taking into account the number of cutters on both inner and outer cones using Eqs. 26, 27, and 30.

The overall methodology is shown in the flow chart in Fig. 6. The steps of the modeling process are as follows:

Step 1: A collection of the required data, and apply the model equations using Eqs. 17, 18, 24 and 25 to determine the cutter-rock contact variables on inner and outer cones.

Step 2: The volume of removed rock is estimated on both cones using Eq. 13 and rock strength is determined based on cutting percentage as described in section 1.4.

Step 3: DRIMP, MSE, and torque are calculated based on Eqs. 9, 12 and 14 in both cones to show the influence of bit forces on bit wear.

The model has been applied to four vertical wells in the oil fields of Libya. The candidate wells were drilled by using three different sizes of sharp PDC bits and one used PDC bit. The formation stress (O') is estimated according to the cutting percentage of the lithology that was already provided in the well logging data (see section 1.4). The model equations are used to determine the PDC cutters wears by calculating the height of cutters wear using the analyses of the drilling parameters which determined as an average for every foot, then the estimated cutters wear is compared to the actual cutters wear obtained in the bit record.

Fig.6 Flow chart illustrating the proposed technique.

1.3 Assumptions

The following assumptions were made in this work to estimate the abrasive cutters wear:

- It is assumed all cutters that located in the inner cone have equal wear, equal volume, and energy (Wang et al., 2018). The same is also applied for the outer cone.
- DOC and cutting area A_C were assumed constant for all cutters in every cone.
- The model considered the PDC cutters as round in shape as the most used shape.
- Blades have the same length and are assumed to be straight to calculate L_b for both cones (see Fig.4, Eqs. 21, and 22). Also, the space between cutters is neglected.
- Radial location of cutters is equal to the average distance for both inner and outer. The inner radial (R_i) is the distance between cutter position at the middle of the inner cone to the bit centre ($\frac{r_i}{2}$), and the outer radial (R_o) is the distance between cutter position at the outer cone middle to the bit centre ($\frac{r_o}{2} + r_i$) as shown in Fig. 5.

1.4 Unconfined Compression Strength as a Measure of Formation Strength

The model calculates the formation strength which is derived from the unconfined compressional strength (UCS) values as shown in Table 1 and the formation cutting percentage as shown in Figs. 7, 14, 22 and 27. The standard lithology column was broken down into sections where the formation strength is measured.

- The formation was divided into different sections as described and shown in Figs. 7, 14, 22 and 27.
- The formation strength was estimated by multiplying the cutting fraction with the values of UCS shown in Table 1.

Rock Type	UCS, MPa
Sandstone	70
Limestone	25
Shale	20
Calcilutite	15
Anhydrite	25
Clay	2
Dolomite	70
Salt	12
Chert	180
Marble	100

Table 1. UCS for various rocks (Ragan, 2009).

2. Results and Discussions

Based on the methodology described in the above sections, the results of analysis for the different wells of 1, 2, 3 and 4 are presented, in ranking order, in this part.

2.1 Well Number 1

The 8.5" PDC bit with 8 blades and 13 mm cutter size ran at 12720 ft to 14343 ft to drill through Lower Sirte, Upper salt, and Mid anhydrite, which mainly consist of shale, limestone, anhydrite, and salt (see Fig. 7). The bit is pulled out because there was a reduction in penetration rate (PR) and dulled in the bit record for 1-1 (a measure of bit ware- 1mm lost in inner and 1mm lost in outer cone out of 8mm).

Fig.7 Estimation of the Lower Sirte formation strength.

Table 2 summarizes the obtained average of ROP, UCS, DOC, EB, and DRIMP values. DRIMP in both cones which reflect the bit efficiency condition increase with UCS and decreases with ROP for Lower Sirte, Upper salt, and Mid anhydrite rocks. Fig. 8 shows the profiles of ROP, WOB, and RPM along the drilling depth obtained from well logging. The figure also correspondingly shows the trends of EB, DRIMP, and L that were calculated using the model. It can be seen from Fig.8 that EB for both inner and outer cones is increased with ROP while WOB and DRIMP show an inverse proportional to ROP among the whole interval.

Rock Type	Thick, ft	UCS, MPa	Time, hr	ROP, ft/hr	DOC, mm		EB, %		DRIMP, T/mm	
					Inner Cone	Outer Cone	Inner Cone	Outer Cone	Inner Cone	Outer Cone
Lower Sirte	640	22.4	84.6	7.6	0.18	0.45	14.7	12.9	57.4	24.0
Upper salt	840	15.7	67.0	12.5	0.22	0.55	17.9	15.6	43.6	17.7
Mid anhydrite	143	26.2	33.2	4.3	0.16	0.39	12.8	11.1	81.6	32.9

Table 2. 8.5" PDC bit - rock type with estimated drilling parameters.

The bit took 84.6 hours to drill 640 ft while the bit drilling in Lower Sirte rock (see Table 2), and the results indicated that the bit received damage in the early stage of the run. ROP has reached 33 ft/hr at depth of 12729 ft during drilling in 100% shale with WOB of 12 klbs and RPM of 85 rpm (see Figs. 8b and 8c). 29.6% of the inner cone effective blade (EB I) were in contact with rock (see Fig. 8d), and as a result, the wear evaluation index in the inner cone (DRIMP I) measured to 22 T/mm as shown in Fig. 8e. While in the outer cone effective blades (EB O) was reduced to 25% and wear evaluation index in the outer cone (DRIMP O) was estimated to 9 T/mm with no damage was estimated using the model equations in both cones as shown in Fig. 8f.

However, as the PDC bit penetrated into layers of 50% limestone with 50% shale at 12736 ft, the ROP dropped because EB I was reduced to approximately 14.6% indicating the reductions on cutters width involved in drilling mechanism. In addition, the corresponding DRIMP I increased to 50.5 T/mm as shown in Fig. 8e that cause a damage in the inner cone (L I) which calculated to 0.87 mm lost (see Fig. 8f). While in the outer cone DRIMP O increased to reach 20.6 T/mm with minor damage. As the rock structure was changed to Upper salt and rock strength reduces to 15.7 MPa at 13360 ft (see Table 2), the DRIMP reduced to 43.6 and 17.7 for inner and outer cones, respectively due to increment in EB of 18 and 15.6 % in both cones as a function of DOC. Accordingly, the bit turns to be more aggressive and this is accompanied by an increment in the cutting area and rock removed. This is the case for both inner and outer cones.

At 12750 ft depth, the ROP of the bit decreased from 5.87 to 2.6 ft/hr. Using the model equations, the L I was calculated to 1.58 mm lost at that depth as shown in Fig. 8f, where around 11% of EB I was involved in drilling as shown in Fig. 8d. On the other hand, the wear in the outer cone (L O) was calculated to about 0.0045 mm lost (see Fig. 8f). The ROP is raised to 20 ft/hr as the bit started drilling into Upper salt rock at depth 13360 ft where the rock strength is decreased to an average of 15.7 MPa (see in the second row in Table 2). According to the model estimation, the EB I and EB O were 22% and 19%, respectively.

At depth of 14200 ft during drilling in Mid anhydrite with the corresponding rock strength of 26.2 MPa, the PDC bit struggled and ROP dropped to only 4.3 ft/hr, while DRIMP jumped to reach high average values of 81.6 T/mm in the inner cone compared to 33 T/mm in the outer cone (see the third row in Table 2). Based on the above discussions and observations, keeping the bit in the hole with a sharp increase of DRIMP at the depth of over 14200 ft may cause more severe damage to the inner cone, so the benefit of pulling the bit out is obvious at this depth.

Fig.8 Trend of drilling parameters along the depth of drilling and corresponding estimated drilling parameters for well number 1.

2.2 Well Number 2

The 16" PDC bit, designed with 6 blades - double rows and 16 mm cutter size was run at a depth of 2014 ft to 3536 ft to drill through Miocene, Oligocene, and Upper Eocene. The interval consists of limestone, clay, dolomite, sand, and marble. The bit was pulled out of the hole at the depth of 3536 ft after severe damage at the outer cone and dulled for 1-4 as reported in the bit record.

Fig.9 Estimating of the stress of Miocene, Oligocene, and Upper Eocene.

Rock Type	Thick, ft	UCS, MPa	Hardness lb/sq.mm	Time, hr	ROP, ft/hr	DOC, mm		EB, %		DRIMP, T/mm	
						Inner Cone	Outer Cone	Inner Cone	Outer Cone	Inner Cone	Outer Cone
Miocene	646	20.6	18.9	25.0	25.7	0.51	0.83	28.9	60.4	9.8	17.5
Oligocene	600	54.7	26.7	29.1	20.6	0.40	0.68	25.2	52.9	11.3	19.0
Upper Eocene	276	31.3	100.4	30.7	8.9	0.24	0.40	19.3	40.8	31.8	41.1

Table 3. 16" PDC bit - rock type with estimated drilling parameters.

The application consists of formations of Miocene, Oligocene, and Upper Eocene rocks with the UCS of 20.6, 54.7 and 31.3 MPa, respectively (see Table 3). In addition to the rock strength,

the interval is heterogeneous as shown in Fig. 9 which create several challenges for the PDC to achieve a good ROP. The results of the model show that the PDC bit had a total wear flat height in the outer cone L O greater than the actual wear after drilling 1522 ft in 84.7 hr (see Fig. 10).

The analysis of Table 3 reveals that the bit operated in good conditions and spent over 50 hours to drill nearly 1200 ft including Miocene (646 ft) and Oligocene (600 ft). At the depth of 3260 ft, WOB slightly increased as shown in Fig. 11b where the bit started to require more energy to maintain the same ROP. This is accompanied by a drop of EB to 28.6 and 60 %, and that led DRIMP to raise to 5.8 and 12.3 T/mm in both inner and outer cone respectively.

A sudden increase in WOB from 5.4 to 9 tone was applied to the bit to increase the ROP at 2187 ft (see Fig. 11b), which resulted in a drop of 35% in ROP (see Fig. 11a) due to formation cutting changed from 85% clay to 75% limestone as shown in Fig. 9. Accordingly, the bit outer cone showed a much stronger response as compared to the inner cone, DRIMP O was nearly doubled its value with damage; L O estimated to equal 1mm lost (refer to Fig. 11f). The bit design has a close influence on wear, DOC showed more response in the outer cone because of the cutting angle that resulted in an increase of cutting area. Based on that EB I of 25% out of total cutters width were involved in drilling (see Fig. 11d), while EB O was doubled of the inner as shown in Fig. 11d. This is corresponding to the DRIMP values obtained in both cones. By using the information of the Oligocene rock strength and rock hardness (see Table 3), the UCS was increased from 20.6 to 54.7 MPa; while ROP was decreased from 25.7 to 20.6 ft/hr with no extra wear recorded in both cones as shown in Fig. 11f. This has occurred as DRIMP increased to an average of 10% in both cones (i.e. from 9.8 to 11.3 T/mm).

Approximately 31 hours were spent to drill 276 ft along Upper Eocene rock (see Table 3). However, the same period of time was taken to drill 600 ft in Oligocene rock using the same PDC bit. Nevertheless, the formation strength was reduced from 54.7 to 31.3 MPa as shown in Table 3. The closeness of this result may suggest that ROP is insensitive to rock hardness. The DOC and EB in both cones as shown in Table 3 decreased dramatically. This is related to ROP with a noticeable increment in DRIMP in both cones until the end of the 16" section.

Wear measurement was made at every foot. However, it is logical to consider the wear when the bit was pulled out of the hole. To determine the bit dull condition, the wear model equations were used. L O was calculated to 5.3 mm as compared to 4.0 mm as an actual dull grading (see Fig. 11f). While the bit record presented that the bit is dulled to 1 mm lost in the inner cone as

compared to 1.4 mm lost L I estimated by model equations. Fig. 10 shows the real photos of damage of the 16" PDC bit at the surface. The difference between the actual and the estimated wear as can be seen later in Fig.18 would be justified to the assumptions made in this model.

Fig.10 Damage of the 16" PDC bit.

Fig.11 Trend of drilling parameters along the depth of drilling and corresponding estimated drilling parameters for well number 2.

2.3 Well Number 3

The 8.5" PDC bit contributed from 6 blades and 19 mm cutter size was used to drill 248 ft into Upper Sirte formation with the average ROP of 11.8 ft/hr (see Table 4). The formation mainly consist of shale. Hydro - Guard mud, a clay free designed, was pumped into this section as a solution for reactive shale. The bit was pulled out of the hole at depth of 10673 ft to check or change the bottom hole assembly and dulled in the bit record for 3-4.

Fig.12 Estimating of the Upper Sirte rocks strength.

Top, ft	BTM, ft	Thick, ft	Rock Type	UCS, MPa	Hardne ss lb/sq.m m	Time, hr	ROP, ft/hr	DRIMP, T/mm	
								Inner Cone	Outer Cone
10425	10673	248	Upper Sirte	19.54	281.9	20.97	11.83	19.81	12.37

Table 4. 8.5" Used PDC bit - rock type with estimated drilling parameters.

The PDC drilled till depth of 10629 ft under controlled conditions, with stable DRIMP along an approximate of 70% interval of shale as shown in Fig. 12. The average ROP estimated to 93 ft/hr and DRIMP calculated to 15.1 and 9.4 T/mm in the inner and outer cones, respectively as shown in Fig. 13e. During the section from 10629 to 10635 ft, and as the bit penetrated into traces of calcilutite rock (see Fig. 12), the driller decided to reduce WOB and increase RPM as shown in Figs. 13b and 13c. As a result, the average ROP declined to 87 ft/hr and DRIMP was suddenly raised to 24.5 and 15.2 T/mm on both cones which reflect the poor bit efficiency with a drop in the effective blade in both inner and outer cones of 21.2 and 23 %, respectively (see Fig. 13d). At depth of 10658 ft, ROP dropped from 8.8 to 3.6 ft/hr with an increases in DRIMP because of the 15% increment of hardness from 18.3 to 22.5 T/mm in inner cone and from 11.4 to 14 T/mm in the outer (see Fig. 13e).

The model has assumed that the bit is sharp, and hence the cutting angle, DOC, and wear in both cones were calculated as summarised in Table 5.

	Cutting angle, °	DOC, mm	L, mm
Inner cone	43.1	0.422	0.340
Outer cone	25.8	0.650	0.450

Table 5. 8.5" Used PDC bit – cutting angle for inner and outer cones.

The results of Table 5 show that the outer cone is more responsive to cutting force caused by the radial distance to the bit centre, which was assumed constant for all cutters. The bit which used to drill in this section was a reused bit and maybe of partial integrity, i.e. cutter elements may be lost. However, the bit was evaluated at the surface, for 3-4 in the bit record, and concluded that the PDC has experienced more damage as compared to the results obtained from the model. This explained the difference obtained in the comparison between the estimated bit wear and the actual bit wear as shown later in Fig. 18.

Fig.13 Trend of drilling parameters along the depth of drilling and corresponding estimated drilling parameters for well number 3.

2.4 Well Number 4

The 12.25" PDC bit which designed with 6 blades and 16 mm cutter size was used to drill in 12.25" hole. The bit drilled 112 ft in Algata formation and 1588 ft in Gir formation to the final planned depth of 6650 ft. The bit was pulled out from the hole at 6650 ft and dulled for 3-X. (The actual dull grading of the outer cone is unknown).

Fig.14 Estimating of the Algata formation strength.

Top, ft	BTM, ft	Thick, ft	Rock Type	UCS, MPa	Hardness lb/sq.mm	Time, hr	ROP, ft/hr	DRIMP, T/mm	
								Inner Cone	Outer Cone
4950	5062	112	Algata	14.46	99.85	2.68	41.79	21.11	11.45
5062	6650	1588	Gir	21.27	136.94	35.66	44.53	23.63	12.87

Table 6. 12.25" PDC bit - rock type with estimated drilling parameters.

The selected 12.25" PDC drilled 1700 ft in 38.3 hours through interbedded formation of calcilutite, dolomite, shale, and some layers of chert as shown in Fig.14. The results of the model show that the bit has pulled out in very good condition with no abrasive wear in both cones as shown in Fig. 15f.

The high values of DOC are an indication of the impact wear resulted from sudden change of rock strength, and that can be seen in Fig. 15d for both inner and outer cones. This is evidence from the DOC which is reached to a value of approximately 3 mm for the outer cone. The bit is considered as efficient if the bit has low DRIMP and ROP is improved. The previous results for wells 1, 2 and 3 have demonstrated that the ROP should exhibit an inverse relationship with DRIMP, while in well 4, this is not the case and ROP is directly proportional to DRIMP in both inner and outer cones (see Figs. 15a, 15e, 16 and 17). This is clearly against the principles of the abrasive wear model. Additionally, considering the nature of the trend of ROP vs. DOC, ROP was decreased instead of rapidly increasing with DOC (see Figs. 16 and 17). These phenomena strengthen the idea that the wear which occurred on the 12.25" PDC is not abrasive wear but it is an impact wear. Accordingly, bit wear cannot be estimated by the model equations, and that can be clearly shown with poor comparison between the estimated and actual wear in both cones as summarised later in Fig.18.

Fig.15 Trend of drilling parameters along the depth of drilling and corresponding estimated drilling parameters for well number 4.

Fig.16 Average. DOC - ROP and DRIMP (Inner Cone).

Fig.17 Average. DOC - ROP and DRIMP (Outer Cone).

Fig.18 compares the obtained results of the calculated wear using the model and actual cutters wear recorded in the field. The model estimates the abrasive cutters wear in wells 1, 2 and 3 based on the assumptions made earlier for the development of the model. The good agreements are found between the estimated and the actual cutters wear for wells 1 and 2. There is a considerable difference between the actual and the determined cutters wear in wells 3 and 4. This is attributed to the case as the PDC bit in well 3 was not a new bit, so the bit may lost partially cutter elements during drilling in an offset well before rerun again to drill the 8.5" section in well 3. Also, the wear occurred to the bit drilled the 12.25" section in well 4 was a result of impact wear and not because of abrasive wear.

Fig.18 Comparing the actual and the estimated bit wear for wells 1, 2, 3, and 4.

3. Conclusion

The following conclusions can be drawn in this work:

1. The model presented in this paper is developed to aid PDC bit design and evaluation. The model also serves as a methodology to identify additional factors such as bit design and bit hydraulic that were not included in the past.
2. The concept of equal cutter wear and volume across the bit face reported in the literature is incomplete. This model does not consider the single cutter test, and accordingly, the model enables to estimate of the cutters wear in the inner and outer cones.
3. One of the most important outcomes of this work is the inclusion of the cutter width and the estimation of the effective cutters per blade which are involved in the drilling mechanism. This principle can account for designing a PDC bit with varied wear.
4. Effective blades, as well as DOC, are shown to be inversely proportional to the cutting angle.
5. The actual bit cutters wear for wells no 1 and 2 were in good agreement with the estimated wear by the model equations based on the model assumptions.
6. The actual bit cutters wear for wells no 3 and 4 exceeds the estimated bit wear. The bit ran in well 3 was a used bit, while; the damage occurred to bit drilled the 12.25" section in well 4 was due to impact wear (see Fig. 18).
7. This model has given satisfactory results only for sharp PDC bits and can only be applied to estimate the abrasive cutters wear. In addition, DRIMP changed in agreement with the bit life and is mainly depend on the bit geometry.
8. The proposed model in this paper is valid and can be applied for homogeneous and heterogeneous formation under the assumptions made in this work. Furthermore, the input data are standard and readily available or easy to obtain.
9. It would be most desirable if the model can be extended to apply to different shapes of cutters such as oval cutters. This is the ongoing research and will look into this in the future.

Nomenclature

A_C = Cutting area of the cutter, in²

A_w = Wearflat area, in²
 C = Cutter width, in²
 C_t = Total of cutters width, in
 D_b = Bit diameter, in
 D_c = Cutter diameter, in
DOC = Depth of cut, in
DRIMP = Wear evaluation index, lb/in
DRIMP I = Wear evaluation index in inner cone, lb/in
DRIMP O = Wear evaluation index in outer cone, lb/in
EB = Effective blades, unitless
EB I = Effective blades in inner cone, unitless
EB O = Effective blades in outer cone, unitless
 F_c = Cutting force, lb
 g = gage height, in
 G = Cone height, in
HP = Bit hydraulic horsepower, lb
HSI = Bit hydraulic efficiency, psi
 L = Wear bit height, mm
 $L_b(\text{inner})$ = Inner blade length, mm
 $L_b(\text{outer})$ = Outer blade length, mm
 $L I$ = Wear bit height in inner cone, mm
 $L O$ = Wear bit height in outer cone, mm
 L_b = Blade length, in
MSE = Mechanical specific energy, psi
 MSE_H = Correlated mechanical specific energy for mud hydraulic, psi
 $MSE_{\text{mod.}}$ = Modified mechanical specific energy, psi
 MSE_R = Correlated mechanical specific energy for rock hardness, psi
MW = Mud weight, lb/gal
 N_b = Number of blades, unitless
 N_c = Number of cutters, unitless
 P = Bit pressure drop, psi
PR = Penetration rate, ft/hr
 Q = Flow rate, gpm

R = Distance from cutter to bit centre, in
 R_i = Inner radial distance, in
 R_o = Outer radial distance, in
 ROP = Rate of penetration, ft/hr
 RPM = Rotary speed, rpm
 r_b = Bit radius, in
 r_c = Cutter radius, in
 r_i = Inner cone radius, in
 r_o = Outer cone radius, in
 T = Torque, lb_m – in
 TFA = Bit total flow area, in^2
 $T_{mod.}$ = Torque, lb_f – in
 UCS = Unconfined compressional strength, psi
 $(V_r)_a$ = Actual volume of rock removed, in^3
 $(V_r)_t$ = Theoretical volume of rock removed, in^3
 WOB = Weight on bit, lb_m
 δ = Hardness or the cutting force per unit area, psi
 \emptyset = Cutting angle, °
 σ = Formation stress, psi
 μ = Rock friction, unitless

Acknowledgment

The financial support from ministry of higher education in Libya is gratefully acknowledged.

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