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Butterfly spreads: a profitable strategy?

A case on the Euro Benchmark Yield Curve

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*Each problem that I solved became a rule
which served afterwards to solve
other problems.*

Rene Descartes

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Resumo

Investimentos em obrigações são habitualmente considerados seguros, uma vez que oferecem taxas de juros fixas, pelo que é natural que sejam incluídos pelos investidores nos seus portfólios. No entanto, as variações das taxas de juros do mercado têm impacto no valor actual (ou *present value*) dos pagamentos da obrigação. Existem vários estudos feitos no sentido de tornar os portfólios imunes a variações paralelas das curvas de taxa de juro que foram, durante algum tempo, assumidas como o único tipo de variação possível. Porém, se analisarmos a evolução das curvas de taxa de juro nos últimos 15 anos, rapidamente concluímos que ocorrem diversos tipos de variações, que não podem ser classificados de paralelos.

Por um lado, tornou-se claro que era necessário imunizar os portfólios em relação a diversos movimentos da curva, não só aos mais básicos. Por outro, esta confirmação trouxe consigo uma nova oportunidade de investimento: apostas em determinados movimentos das curvas de taxa de juro, comprando e vendendo obrigações de diferentes maturidades, de acordo com as previsões de cada investidor em relação à evolução das condições do mercado, de forma a gerar lucro. É neste tipo de investimentos que se foca esta dissertação.

Existem algumas estratégias base que podem ser usadas, das quais vamos apresentar três: *Bullet*, *Ladder* e *Barbell*. As estratégias *Bullet* e *Barbell* podem ser combinadas de modo a formar a estratégia *Butterfly*, que consiste em apostas em obrigações de três maturidades diferentes (curta, média e longa), onde o valor a investir em cada uma é variável. Para cada uma das variantes da estratégia é calculado o valor adequado a cada maturidade, de acordo com o tipo de movimento da curva que é esperado pelo investidor. Nesta dissertação vamos estudar quatro variantes da estratégia *Butterfly*, e vamos aplicá-las à curvas de taxa de juro da zona Euro entre 31/12/2005 e 31/12/2014.

Dado que algumas das estratégias implicam um investimento inicial, vamos assumir sempre que o investidor não recorre a capitais próprios, mas sim, a um empréstimo, e teremos os custos a este associados em conta nos nossos cálculos. Além disso, teremos de fazer algumas estimativas, para as quais recorreremos à metodologia de Nelson-Siegel e à regressão linear.

O principal objectivo desta dissertação é verificar se, de facto, teria sido possível gerar lucros utilizando a estratégia *Butterfly* em condições reais. Tentaremos também identificar quais os cenários ou movimentos da curva de taxa de juro que geram os melhores resultados, e quais as variantes da estratégia teriam tido melhores resultados.

Palavras-chave: Obrigações taxa fixa, Curva de taxa de juro, Estratégia *Butterfly*.

Abstract

Bonds are usually considered a safe investment because the bond interest rates are fixed, so it is not surprising that they are included by many investors in their portfolios. However, when the market interest rates vary, it has an impact on the present value of the bond's interest payments. There has been extensive research in order to immunize the portfolios against small parallel changes in the yield curve, which were assumed to be the only changes the curves could suffer. However, analyzing the evolution of the curves in the last 15 years, we rapidly conclude that changes which cannot be classed as "parallel" are present.

On the one hand, it becomes clear that it is necessary to act in order to immunize the portfolios against more varied scenarios. On the other hand, a new opportunity for investment appears: betting on particular changes of the yield curve, buying and selling bonds of different maturities, according to the investors market forecasts, in order to achieve a profit. The latter, is the object study in this dissertation.

There are some basic strategies that can be used, of which we will briefly present the Bullet, Ladder and Barbell strategies. The Bullet and Barbell strategies can be combined in order to obtain the Butterfly strategy, which consists on betting on three different maturities (short, medium and long), but where the amounts invested in which vary. For each variations of this strategy, the amounts invested in each maturity are calculated, according to the type of movement expected. We will study four variations of this strategy, and we will apply them to the actual Euro yield curve from 31/12/2005 to 31/12/2014.

Given an initial investment is required to apply some of the strategies, we will assume the investor would not use his own capital, but would take out a loan instead, so we will also take the cost of this loan rates into account in our calculations. Also, some estimations are necessary, for which we will use the Nelson-Siegel methodology and linear regression.

The main objective of our work is to ascertain whether it would have been possible to generate profit using this set of strategies under real conditions. We will also be looking the scenarios or yield curve changes allow the best results, and which of the variations considered would have a better performance.

Keywords: Bonds, Yield curve, Butterfly strategy

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Chapter 1

Introduction

The European Central Bank (ECB) defines yield curves as follows (*ECB website*; n.d.):

A yield curve is a representation of the relationship between market remuneration rates and the remaining time to maturity of debt securities, also known as the term structure of interest rates.

Yield curves can be affected by many movements, and the Euro yield curve is not an exception. There may be changes in the level, steepness or curvature, or even combinations of these factors (Litterman and Scheinkman; 1991)¹. Figure 1.1 illustrates some of these movements, with examples of parallel and nonparallel movements in the Eurozone.

Different types of movements observed in the yields for the Eurozone

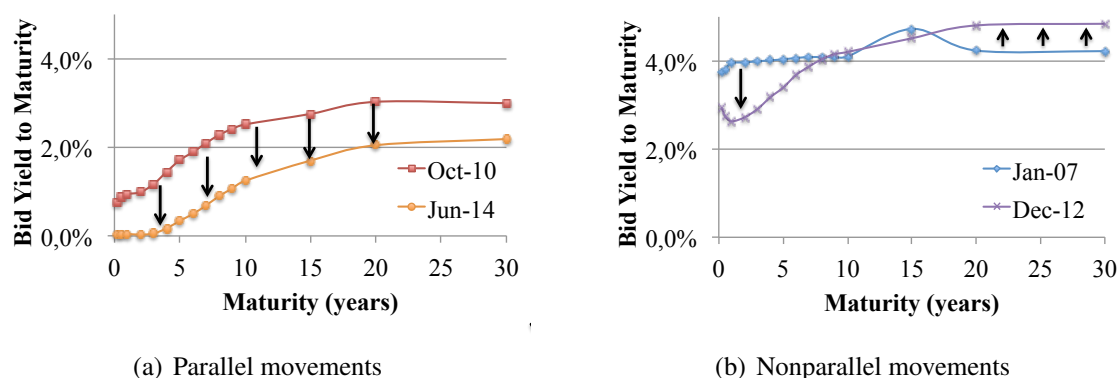


Figure 1.1: Yield curve movements in the Eurozone. On the left, a parallel shift; on the right, a twist.

¹Other works, such as (Barra; 2007; Vannerem and Iyer; 2010), use the Shift-Twist-Butterfly (STB) model, where the designation comes from the name of the movements considered. To avoid any confusion with the type of butterfly studied in this work, we have decided not to follow this naming.

Figure 1.1 shows that not all movements are simple parallel shifts, and nonparallel shifts have a major impact on fixed income portfolios: "when interest rates rise, market values of bonds go down, because bond interest rates are fixed and the present value of a bond's stream of interest payments drops", (Thornburg; 2014). Extensive research has been carried out on how to immunize a portfolio of bonds against small changes in the yield curve. However, the nonparallel shifts have also gained some importance: "the assumption that yield curves can only change by parallel shift has concerned many researchers. This concern has lead researchers to develop immunization strategies for nonparallel shifts and to examine the risks associated with nonparallel shifts" (Barber and Cooper; 1998).

Figure 1.2 exemplifies possible movements, showing the behavior of the 3-month, 5-year and 20-year curves in the Eurozone for a 12 year period.

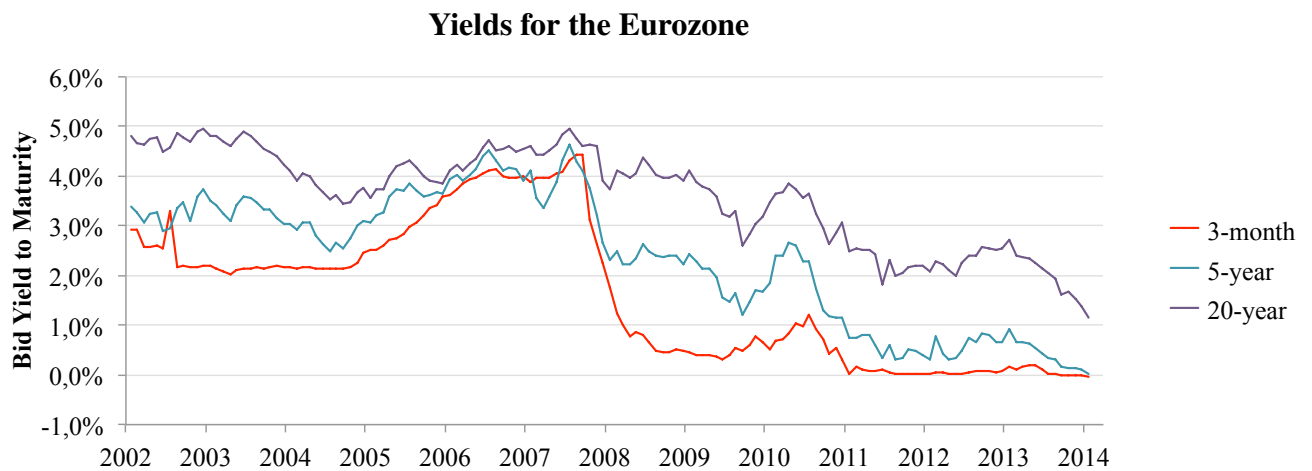


Figure 1.2: Short, medium and long-term yields for the Eurozone since 2002. Many different behaviors can be seen, proving that nonparallel movements exist.

Other authors have investigated ways of generating positive payoffs by betting on particular changes of the yield curve, using bonds of different maturities in order to achieve the desired result.

There are three basic trades/strategies (Chua et al.; 2005; Alexander and Resnick; 1985):

- Bullet strategy: investments in bonds that mature around the same date.
- Ladder strategy: involves multiple investments across different maturities.
- Barbell strategy: constructed by investing in two ends of the yield curve².

²(Chua et al.; 2005) actually defines the Barbell strategy slightly differently, similarly to what we will define as a butterfly in the next section.

Figure 1.3 illustrates these strategies.

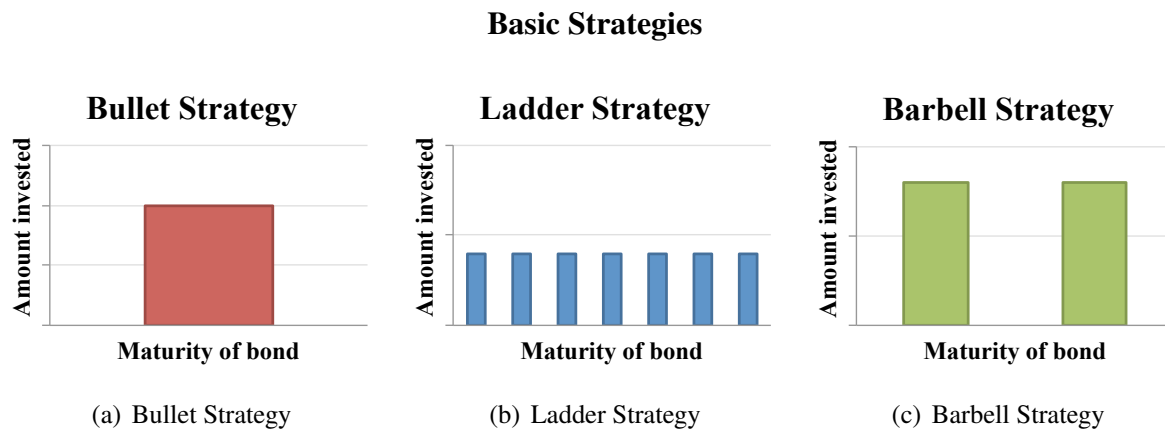


Figure 1.3: Bullet, Ladder and Barbell strategies: Bullet consists in investments maturing around the same date; Ladder implies investments across different maturities; and Barbell consists in investing heavily in two ends of the yield curve.

Another possible strategy, used by many traders and fund managers around the world, is the Butterfly Strategy³.

(Martellini et al.; 2002) define a Butterfly as "the combination of a barbell (called the wings of the butterfly) and a bullet (called the body of the butterfly). The purpose of the trade is to adjust the weights of these components so that the transaction is cash-neutral and has a $\$$ duration equal to zero."

In this work, we will study the four types of butterflies presented in (Martellini et al.; 2002), applied to the Euro yield curve, between the years of 2006 and 2014. We will try to ascertain under what conditions each type has a better performance, so that an informed investor could hypothetically profit when the right conditions happen in the market.

This dissertation proceeds as follows: Chapter 2 presents the Butterfly Strategy, the object of study in this work. Four types of butterflies will be described, as well as their main characteristics and mathematical definition. Chapter 3 includes a description of the overall method here proposed, as well as some details on the database used, and how it was modified. It also includes a brief description of the three methods considered to estimate prices, details on the chosen one (the Nelson and Siegel methodology), and the results obtained with it. Finally, a

³Butterfly strategies are particularly common in Japan, as "butterfly trading in the Japanese bond market is more profitable than that of the U. S. treasury market because of the relatively large third factor (curvature factor) in the principal component analysis of the yield movement" (Miyazaki; 2003).

more detailed explanation of the method proposed is presented. Next, Chapter 4 starts with the split of the period analyzed into three sub-periods, according to their characteristics, and then an outstanding coefficient is estimated. After this, the results of the implementation of the method are presented: first, for each sub-period, followed by the overall results and an analysis of the monthly results, in order to confirm the conclusions in the literature. Finally, the results obtained are briefly compared to those from a similar strategy using bonds with coupons. Chapter 5 concludes this dissertation, and includes a summary of the conclusions, and suggestions for future work.

Chapter 2

State-of-the-art

To develop this work, we have followed (Martellini et al.; 2002), and their mathematical definition of each type of butterfly.

Generally speaking, butterflies are composed of bonds with three different maturities: short, medium and long. The investor goes long on the short and long-term bonds (which form the barbell, or the wings), and goes short on the medium-term bond (the bullet, or body), or vice-versa. This strategy is represented in Figure 2.1.

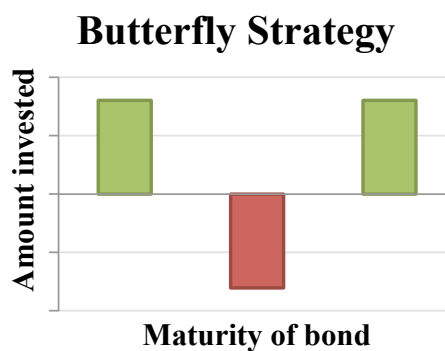


Figure 2.1: Butterfly strategy: consists in investments in bonds of three different maturities, short, medium and long-term. The investor can go long on the short and long-term bonds, and short on the medium-term bond, or vice-versa.

Before defining each type of butterfly, we list below some characteristics that are common to all four:

- $\$Duration$: the weights of each bond are adjusted so that the whole transaction has null $\$duration$. The aim is to have "a quasi-perfect interest rate neutrality when only small parallel shifts affect the yield curve" (Martellini et al.; 2002). This is guaranteed by the

following constraint, which, as we will see in the next sections, is present in the four butterflies studied:

$$q_s D_s + q_l D_l + \alpha D_m = 0 \quad (2.1)$$

where s , m and l represent the maturities and stand for short, medium and long, respectively; each q_i represents the quantity or amount invested in the i bond; D_i represents the \$duration associated to each bond; and $\alpha = q_m$, the quantity or amount of medium-term bonds sold.

- Amount invested: on the settlement date, the investor chooses the amount to allocate to the medium-term bond (which will be sold), α , and the amounts allocated to the other two bonds are calculated according to the strategy.

When the amount α to invest in the medium-term bond has been chosen, the other two amounts have to be calculated. Note that, since we will sell α , we will represent it as a negative amount. Before presenting any details, we have exemplified the problem in hand with the table below:

Bond portfolio for a Butterfly Strategy			
	Bond Price	Quantity	\$Duration
Short	P_s	q_s	D_s
Medium	P_m	$q_m = \alpha$	D_m
Long	P_l	q_l	D_l

Table 2.1: Example of a bond portfolio for a Butterfly Strategy. Only the quantity α is defined by the investor, the remaining variables either are observable in the market, or calculated.

The price and \$duration of each bond is known, so the only unknown variables are q_s and q_l , ie the quantify/weights of the short and long-term bonds.

2.1 Butterfly strategies

In the following subsections we will describe how the weights for each butterfly strategy can be determined, and summarize the key characteristics of each type.

2.1.1 Cash and \$Duration Neutral Weighting

As the name suggests, the goal of this strategy is to be \$duration neutral and have a zero initial cost. To achieve this, we will need to solve the linear system displayed below:

$$\begin{cases} q_s D_s + q_l D_l + \alpha D_m = 0 \\ q_s P_s + q_l P_l + \alpha P_m = 0 \end{cases} \quad (2.2)$$

Given there are only two variables, q_s and q_l , this is quite simple to solve, and we will obtain the following system:

$$\begin{cases} q_s = -\frac{\alpha D_m + D_l}{D_s} \\ q_l = \alpha \frac{\left(\frac{D_m}{D_s} - \frac{P_m}{P_s}\right)}{\left(\frac{P_l}{P_s} - \frac{D_l}{D_s}\right)} \end{cases} \quad (2.3)$$

2.1.2 Fifty-fifty Weighting

The main characteristic of this strategy is the fact that both wings have the same \$duration, and the overall transaction has a zero \$duration. However, contrary to the previous strategy, the Fifty-fifty weighting requires an initial cash-flow different to zero, which means there will be a cost of financing. This will be ignored for now, as it does not have an impact on these calculations, but it will be taken into account when applying the strategies.

The following system represents the constraints described above:

$$\begin{cases} q_s D_s + q_l D_l + \alpha D_m = 0 \\ q_s D_s = q_l D_l = -\frac{\alpha D_m}{2} \end{cases} \quad (2.4)$$

These constraints are put in place "to make the trade neutral to some small steepening and flattening movements" (Martellini et al.; 2002).

Solving the system above, we obtain the following:

$$\begin{cases} q_s = -\frac{\alpha D_m}{2D_s} \\ q_l = -\frac{\alpha D_m}{2D_l} \end{cases} \quad (2.5)$$

2.1.3 Maturity Weighting

To define this strategy, three additional variables need to be introduced, which represent the maturities of the three bonds considered: M_s , M_m and M_l . These variables are not unknown (once the bonds have been chosen), and have an impact on the weights calculated for this

strategy.

The Maturity weighting is a special case of the Regression weighting, presented in the next section. For now, and before entering into any details, let us define β as:

$$\beta = \left(\frac{M_m - M_s}{M_l - M_s} \right)$$

The following system represents this strategy:

$$\begin{cases} q_s D_s + q_l D_l + \alpha D_m = 0 \\ q_s D_s = -\alpha \beta D_m \\ q_l D_l = -\alpha \left(\frac{M_l - M_m}{M_l - M_s} \right) D_m \end{cases} \quad (2.6)$$

This is equivalent to the simplified system below:

$$\begin{cases} q_s = -\frac{\alpha \beta D_m}{D_s(1+\beta)} = -\frac{\alpha \left(\frac{M_m - M_s}{M_l - M_s} \right) D_m}{D_s \left(1 + \left(\frac{M_m - M_s}{M_l - M_s} \right) \right)} \\ q_l = -\frac{\alpha D_m}{D_l(1+\beta)} = -\frac{\alpha D_m}{D_l \left(1 + \left(\frac{M_m - M_s}{M_l - M_s} \right) \right)} \end{cases} \quad (2.7)$$

2.1.4 Regression Weighting

Finally, the fourth and last type of butterfly presented in this dissertation.

In order to use this strategy, we will need to define the regression coefficient, β , briefly mentioned in the previous section. β can be calculated using linear regression, to measure changes in the spread between the short wing and the body, against changes in the spread between the body and the long wing. As explained in (Martellini et al.; 2002), this coefficient is introduced to allow for the greater volatility of the short-term rates, when compared to the long-term rates.

This strategy can be mathematically defined as:

$$\begin{cases} q_s D_s + q_l D_l + \alpha D_m = 0 \\ q_s D_s \times \left(\frac{1}{\beta} \right) = q_l D_l \end{cases} \quad (2.8)$$

Which can be simplified as presented in the system below:

$$\begin{cases} q_s = -\frac{\alpha\beta D_m}{D_s(1+\beta)} \\ q_l = -\frac{\alpha D_m}{D_l(1+\beta)} \end{cases} \quad (2.9)$$

2.1.5 High-level comparison of the strategies

Now that we have presented the strategies that will be studied, and before proceeding any further, we summarize the main characteristics of the strategies presented above, in Table 2.2.

Key characteristics of the four types of butterflies

	\$Duration neutral	Zero initial investment	Same \$Duration on each wing	Neutral to some small movements
Cash and \$Duration Neutral	✓	✓		
Fifty-fifty	✓		✓	✓
Maturity	✓			✓
Regression	✓			✓

Table 2.2: Summary of the key characteristics identified in the butterflies defined previously.

These characteristics will be very helpful at a later stage, where some will be used as tests when the strategies are constructed, to guarantee the calculations performed produce reasonable results.

Chapter 3

Database and Methodology

The aim of this work is to confirm or disprove the conclusions about butterfly trades in the literature. In order to do this, we decided to build a strategy that can be briefly described: first, at time $t = 0$, a portfolio is put together, bonds are bought and sold as described in the previous chapter for each of the four kinds of butterflies, based on the same α for all. Then, a month later, the opposite transactions are performed, and the payoff at the end of each holding period is calculated.

This monthly cycle can be repeated as many times (and at any dates) as the investor wishes. Figure 3.1 systematizes the overall idea of this strategy.

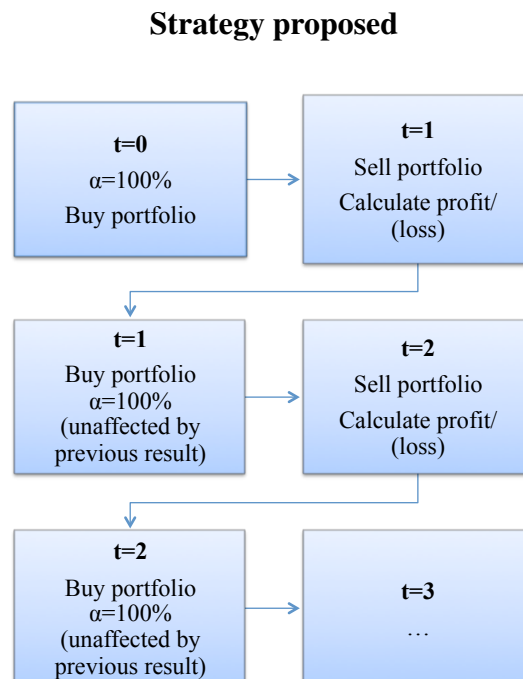


Figure 3.1: Systematization of the strategy proposed.

3.1 Database

The database used in this work consists of the Euro zero-coupon bid and ask yields to maturity (YTM), extracted from Bloomberg ("Bloomberg Generic"). The original database contained daily records, for different maturities⁴ and the maximum period available was from December 2002 to December 2014.

(Martellini et al.; 2002) propose a strategy similar to the described in the previous section, but with transactions made on a daily basis. The present dissertation aims to investigate the potential success of these strategies for longer periods, ie one month. The main reason for this choice lies with the costs, both transaction and portfolio management costs. These would eat into any profit to be generated in a single day, but will represent a smaller fraction of the result obtained if we consider monthly transactions, rather than daily. The other reason for this decision, is that we would not expect great changes in these curves from one day to the next, so it seemed reasonable to consider a smaller database.

We selected the last day of each month and constructed a "sub-database" with those. We needed to regularize the dataset: there was information missing on some of the last days of the month. In those cases, we used the information from the previous day available instead.

From the YTM's in the original database, we derived the prices, as follows (n is the maturity, in years)⁵:

$$Price = \frac{1}{\left(1 + \frac{YTM}{100}\right)^n} \times 100 \quad (3.1)$$

Looking at the results obtained, we initially observed that there were some premium bonds, but only from December 2011 and July 2012 for ask and bid, respectively, and for maturities smaller than three years. Most prices were below 100%, as expected for zero-coupon bonds. This is easily explained by the existence of some negative YTM's in the shorter-term bonds, recently.

⁴The maturities available (although not all available for the whole period studied) were: three and six months, and one, two, three, four, five, six, seven, eight, nine, 10, 15 and 20 years.

⁵Note that we have not allowed for the payment of any coupons, as we are assuming these are zero-coupon bonds.

3.2 Methodology

To put in practice the strategy described in the beginning of this chapter, there was still some data missing. In the beginning of the cycle, at the moment $t = 0$, the prices at which the bonds are sold and bought are known. However, one month later, the maturities of the bonds are one month shorter, and the prices would have changed as well.

We will follow the naming adopted in (Chua et al.; 2005): the bonds with known, observable prices will be named 'primary bonds', and the 'hypothetical bonds' will be those which prices are estimated.

To estimate the prices one month after the start of each cycle, for the hypothetical bonds, three options were considered:

- Bootstrapping
- Cubic-spline method
- Nelson and Siegel methodology

Bootstrapping This first method presents some limitations: "First, since this method does not perform optimization, it computes zero-coupon yields that exactly fit the bond prices. This leads to over-fitting (...). Second, the bootstrapping method requires ad-hoc adjustments when the number of bonds is not the same as the bootstrapping maturities, and when cash flows of different bonds do not fall on the same bootstrapping dates". See (Nawalkha and Soto; 2009) for more details.

Cubic-spline method The cubic-spline method presents some disadvantages, see (Nawalkha and Soto; 2009; Martellini et al.; 2003) for further details. One of those, is the shape of the curve obtained for the term structure: "the curve may be concave on one maturity segment and convex on the other." One can also argue the fact that the results are very sensitive to the location of the "knot points"⁶ selected is a disadvantage. Selecting many knot points to try to avoid this issue will lead to overfitting, so the number and location of these points needs to be well considered.

Nelson and Siegel method The limitations of both methods above are not a problem for this third. The Nelson and Siegel method (or its extended version, proposed in (Svensson; 1995))

⁶In this method, the term structure is divided into segments, in a set of points called "knot points".

has been adopted by various authors in their studies⁷, including the main one in this work, (Martellini et al.; 2002). For these reasons, this will be the method selected to extract the discount function and estimate the prices of the hypothetical bonds. The next section presents details on how the method works and its implementation.

3.3 Nelson and Siegel methodology

The method (or set of methods, given the abundance of variants) known as "Nelson and Siegel methodology" was initially presented by Charles Nelson and Andrew Siegel in the work (Nelson and Siegel; 1987). Their aim was the presentation of "*a simple, parsimonious model that is flexible enough to represent the range of shapes generally associated with yield curves: monotonic, humped, and S shaped*".

The authors proposed the following equation to estimate the continuous compounded spot rate:

$$R(m) = \beta_0 + \beta_1 \left[\frac{1 - \exp\left(-\frac{m}{\beta_3}\right)}{\frac{m}{\beta_3}} \right] + \beta_2 \left[\frac{1 - \exp\left(-\frac{m}{\beta_3}\right)}{\frac{m}{\beta_3}} - \exp\left(-\frac{m}{\beta_3}\right) \right] \quad (3.2)$$

In this equation, m is the maturity of the bond; $R(m)$ is the continuously compounded zero-coupon rate with maturity m ; β_0 is the limit of R when m goes to infinity, ie the asymptotic value of the term structure of the zero-coupon rates - changes to this parameter are interpreted as height changes/parallel shifts; β_1 is the limit of $\beta_0 - R(m)$ when m goes to zero, ie the short to long-term spread - this parameter can be related to slope shifts ((Nawalkha and Soto; 2009) suggests it can also have an impact on the curvature); β_2 is the curvature parameter - it generates a concave shape if it is greater than zero, and a convex shape otherwise; β_3 is a scale parameter, which measures the speed of convergence of the curve to its limit, β_0 when $m = \infty$ - a low value of β_3 will accelerate the convergence to the limit, whereas a high value will cause the opposite effect.

The impact of changes to the parameters β_0 , β_1 , β_2 and β_3 in the rates calculated are illustrated in Figures 3.2 and 3.3.

These figures also allow us to confirm that, with the right set of parameters, there is a great variety of possible yield curves shapes.

⁷See, for example, (Nawalkha and Soto; 2009; Martellini et al.; 2003; Christensen; 2002).

Sensitivity of zero coupon rates to the parameters of the Nelson and Siegel functional form

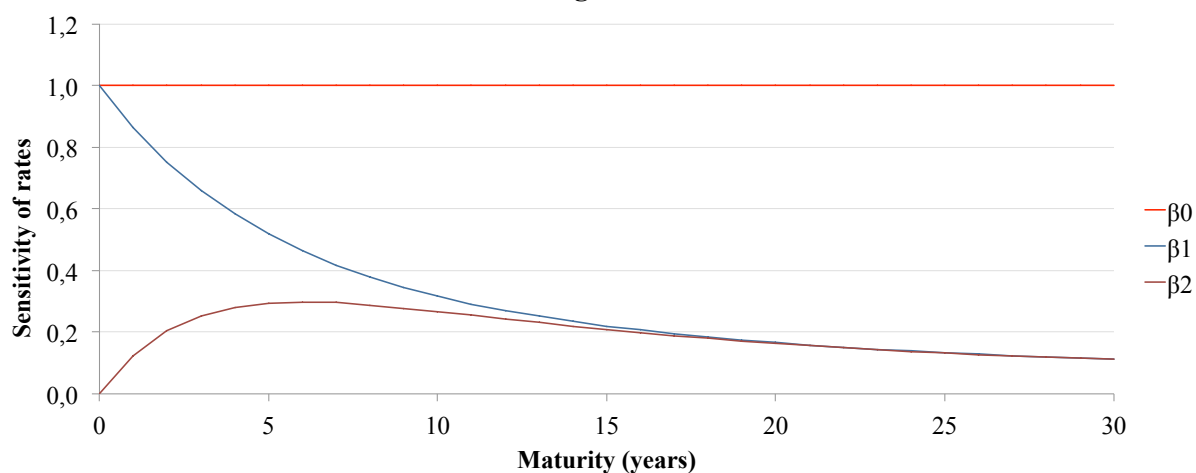


Figure 3.2: Effect of changes to β_0 , β_1 and β_2 in the Nelson Siegel model.

Sensitivity of zero coupon rates to the parameters β_3 of the Nelson and Siegel functional form

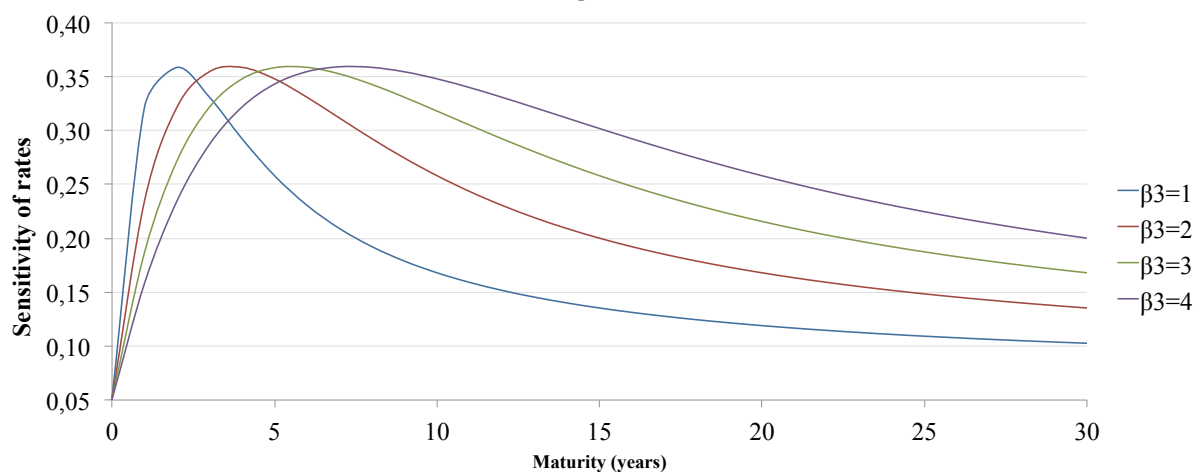


Figure 3.3: Effect of changes to β_3 in the Nelson Siegel model.

To calculate the price of a zero-coupon bond at time $t = t_0 + \frac{1}{12}$ (in years), we will need to use the following discount function:

$$d(m) = e^{-R(m) \times m} = \exp \left\{ -\beta_0 m - \beta_1 \beta_3 (1 - e^{-m/\beta_3}) + \beta_2 \left[-\beta_3 (1 - e^{-m/\beta_3}) + m e^{-m/\beta_3} \right] \right\} \quad (3.3)$$

Then, the equation 3.4 below (with continuous compounding interest) will be used. It says that the price of a zero-coupon bond maturing in m years is the discounted value of the face value of the bond, which is represented by F .

$$\begin{aligned}
 P(m) &= F \times d(m) \\
 &= F \times \exp \left\{ -\beta_0 m - \beta_1 \beta_3 (1 - e^{-m/\beta_3}) + \beta_2 \left[-\beta_3 (1 - e^{-m/\beta_3}) + m e^{-m/\beta_3} \right] \right\}
 \end{aligned}
 \tag{3.4}$$

From the monthly YTM's obtained from Bloomberg for each maturity, we will calculate the zero-coupon bonds' prices, using equation 3.1. Then, using equation 3.4, the unknown prices of the hypothetical bonds will be estimated.

This can be done with the minimum mean square error method, which minimizes the difference between the known and the estimated price. The following restraints will need to be included:

- $\beta_0 > 0$ - asymptotic value of the term structure of the zero-coupon rates must be positive.
- $\beta_0 + \beta_1 > 0$ - instantaneous short rate must be positive.
- $\beta_3 > 0$ - the convergence of the curve to its asymptotic value must be guaranteed.

Solving the minimization problem will provide the values of the four betas and allow the calculation of the hypothetical prices.

3.4 Analysis of the results of the Nelson and Siegel model

After the estimation of the parameters for each date, we will possess two sets of betas, calculated using the ask and bid prices, respectively, in order to allow for transaction costs. Using these values, the prices of the primary bonds will be estimated, and then compared to the real prices. Our aim when doing this, is to check if the estimates are reasonable before progressing any further with the work.

The following subset of maturities will be used going forward: 2, 5 and 10 years. These were the maturities analyzed in (Martellini et al.; 2002), and we decided to test our strategy using bonds with the same maturities, so that we would have results against we could compare ours. To analyze the estimation errors, we will start by plotting the prices, in order to compare

the real and the estimated prices. See Figures 3.4 and 3.5, which refer to bid and ask prices, respectively.

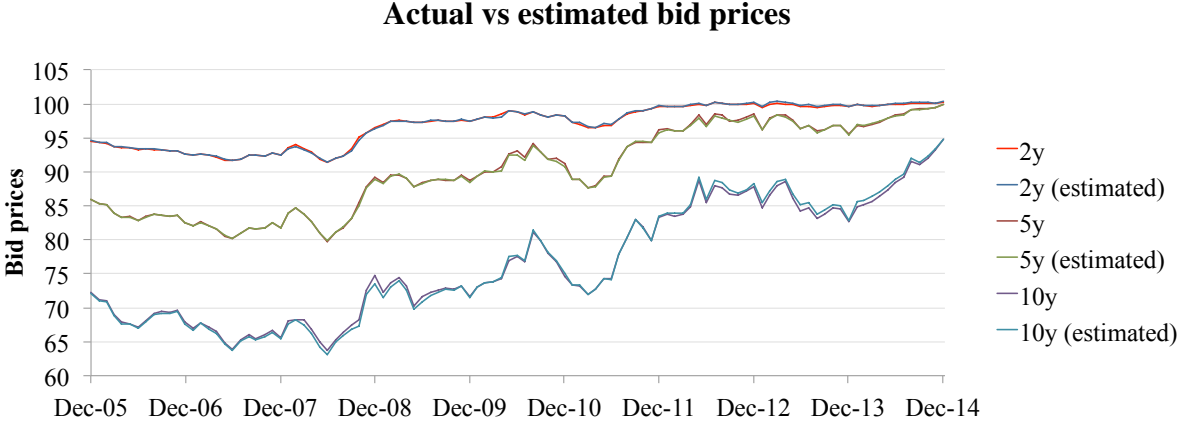


Figure 3.4: Comparison of primary and estimated bid prices.

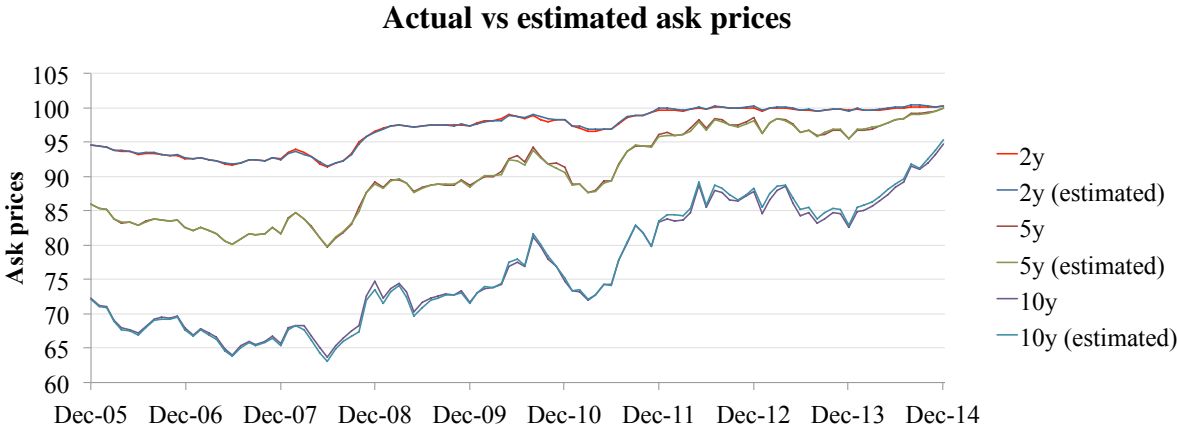


Figure 3.5: Comparison of primary and estimated ask prices.

These comparisons allow us to discard the chance of miscalculations, given the estimates are close to the actual prices. The major differences seem to happen when there are abrupt changes in the curves, not that well captured by the estimates.

Now, let us analyze the estimation errors for each maturity.

From Figures 3.5 and 3.4 we know that the errors for the bid and ask prices are similar, and their peaks happen in the same periods for each maturity. To avoid repeating very similar information, we have calculated the average of the bid and ask estimation errors, in absolute value, and represented this information in Figure 3.6.

Tables 3.1 and 3.2 summarize the information obtained from the analysis of the pricing errors.

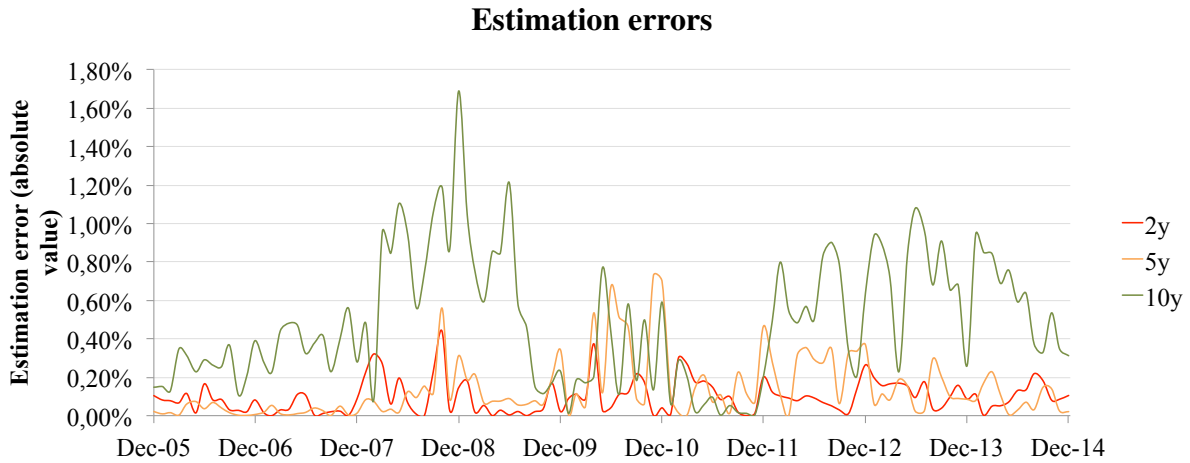


Figure 3.6: Average of the bid and ask estimation errors, in absolute value (as a percentage of the actual price).

Estimation errors - Bid prices

Maturities	2y	5y	10y
Maximum error	0,2907%	0,2051%	1,0639%
Minimum error	-0,4470%	-0,6484%	-1,7230%
Average absolute error	0,0919%	0,1253%	0,4683%
Standard deviation	0,1296%	0,1835%	0,5842%

Table 3.1: Statistics of the errors of the estimated bid prices.

Estimation errors - Ask prices

Maturities	2y	5y	10y
Maximum error	0,3963%	0,3821%	1,1620%
Minimum error	-0,4362%	-0,8418%	-1,6556%
Average absolute error	0,1050%	0,1481%	0,5087%
Standard deviation	0,1364%	0,2075%	0,6146%

Table 3.2: Statistics of the errors of the estimated ask prices.

From the empirical information in these tables, we confirm that the bid and ask errors have similar behaviors. Also, the average error and standard deviation increase with the maturity, which is not unexpected, as with greater maturity comes greater uncertainty.

Now that all the necessary data to perform the calculations proposed in the beginning of the chapter has been estimated, we can proceed.

3.5 Implementation of the Butterfly strategies

To implement the strategy proposed there is one input that one might have expected to be necessary, but has not been so far: α , the quantity to invest in the medium-term bond. For simplicity, we will assume $\alpha = 100\%$. This way, the quantities invested in the short and long-term bonds will be represented as a percentage of the quantity invested in the medium-term bond, which simplifies the calculations and makes the conclusions easier to extract.

3.5.1 Moment $t = 0$

The initial weightings for the four butterfly strategies introduced in Section 2.1 will be calculated. At this phase, the calculations use the primary/known bond prices, bid or ask, as appropriate.

As mentioned in Section 2.1.5, we will use the known characteristics of each strategy to confirm the calculations are producing the expected results. For each cycle, we will check if those characteristic are true for the weightings calculated. For example, we will confirm if all the strategies have zero \$duration and if the Fifty-fifty weighting calculated leads to the same \$duration on each wing.

Since not all the strategies are cash-neutral, it is important for the objective of the work to take into account any initial investments. We will assume the investor will not use his own capital, so he will have to take out a loan for the duration of the cycle, ie one month. This will be taken into account as follows: first, the initial investment necessary for each strategy will be calculated⁸. Then, the historical interest rates will be used, and the monthly interest rate associated with this transaction will be calculated. When the outcome of the strategies is finally derived, it will be adjusted for the financing costs, to generate the net profit.

At this point, it is also important to compare the actual and the estimated prices for the dates and maturities considered in this step. For now, we will calculate the error associated with each maturity at each date, and it will be taken into account in the next step.

⁸Again, as a percentage of the quantity invested in the medium-term strategy, α .

3.5.2 Moment $t = 1$

In order to calculate the results of each strategy after one month, we will use the betas estimated with the Nelson-Siegel method and estimate the zero-coupon prices for the maturities $m - \frac{1}{12}$. In the previous section we had calculated the errors associated with these estimates for the maturities m . One month later, although we do not know the error included in the estimates, it is reasonable to assume it is close to what was estimated in the previous month. For this reason, rather than simply using the estimated prices, we have decided to adjust them in line with the error we had estimated at the moment $t = 0$. In other words, if the estimate was 0,2% greater than the actual price at the moment $t = 0$, we will reduce the estimate at $t = 1$ by 0,2% as well.

Once we have the adjusted estimated prices, we can calculate the results from selling/buying the bonds. From these, we will deduct the initial investment and respective interest payable, when applicable, and obtain the net return, gain or loss, of each strategy.

The first cycle ends here, at $t = 1$, and a new iteration begins. This is repeated until the final settlement date is reached.

3.5.3 Moment $t = m$

When all the cycles have been completed, we will aggregate the results from each cycle and obtain the overall results for the period considered.

The next chapter will detail the results obtained using the methodology and database described so far.

Chapter 4

Results

Now that the method has been described, we shall present the results obtained.

We expect each butterfly to behave differently under different conditions but, as (Martellini et al.; 2002) say *"It is in general fairly complex to know under which exact market conditions a given butterfly generates positive or negative pay-offs"*.

We will start by splitting the period studied into "sub-periods", and see how each butterfly behaves.

4.1 Selection of the sub-periods

In order to reach conclusions regarding the efficiency of each strategy under different conditions, we will split the period considered into three "sub-periods", according to the behavior of the yield curve. Figure 4.1 illustrates the splits chosen.

We have described below the reason for this split, and introduced a naming to help us refer to these periods in the next pages. The first period, in red, goes from 31/12/2005 to 31/03/2008, where the curves are relatively stable, with a slight declining tendency. The second one, in blue, starts in 31/03/2008 and ends in 30/04/2011, and it is in this period that the greater decreases are observed, and there is great volatility on all maturities. It was during this period that the sovereign debt crisis began. The last one, in yellow, goes from 30/04/2011 to 31/12/2014, and starts with some instability, but eventually the short and medium-term curves stabilize and the long-term price increases.

Selection of the sub-periods

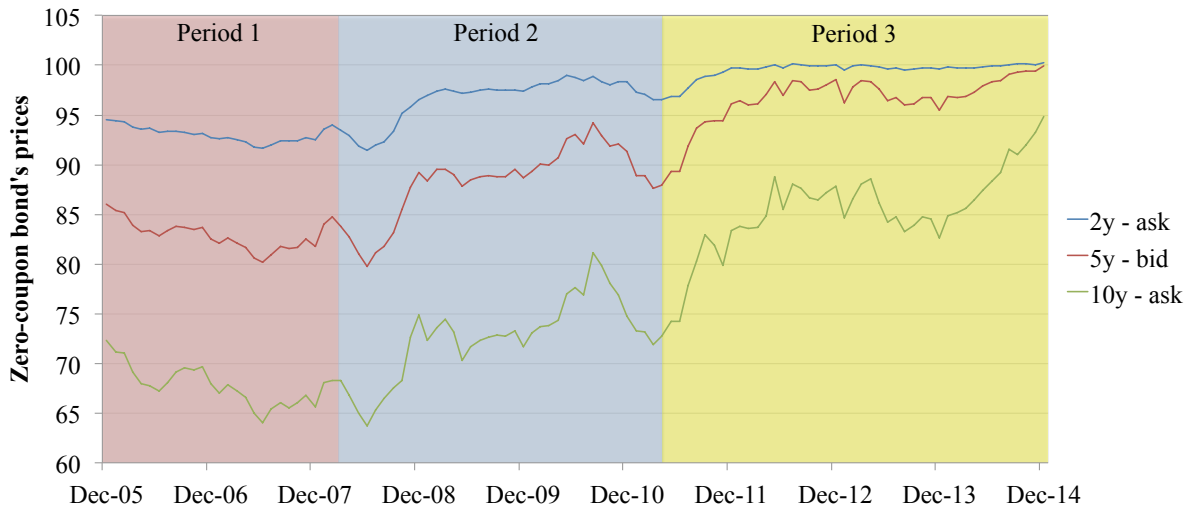


Figure 4.1: Sub-periods considered: this split was decided based on the behavior of the curves over the nine years of data studied in the dissertation.

4.2 Estimation of the regression coefficient

We have now defined the method to use, the time periods to study, and have also obtained the necessary data, apart from one input required for the Regression Weighting butterfly: as mentioned in Section 2.1.4, this butterfly requires the use of the regression coefficient.

To estimate this coefficient, we will use linear regression, relating the changes in the spread between the long wing and the body of the butterfly, with the changes in the spread between the body and the short wing. We will calculate different β for each date, using the prices from the previous 36 months.

Regression coefficients estimated

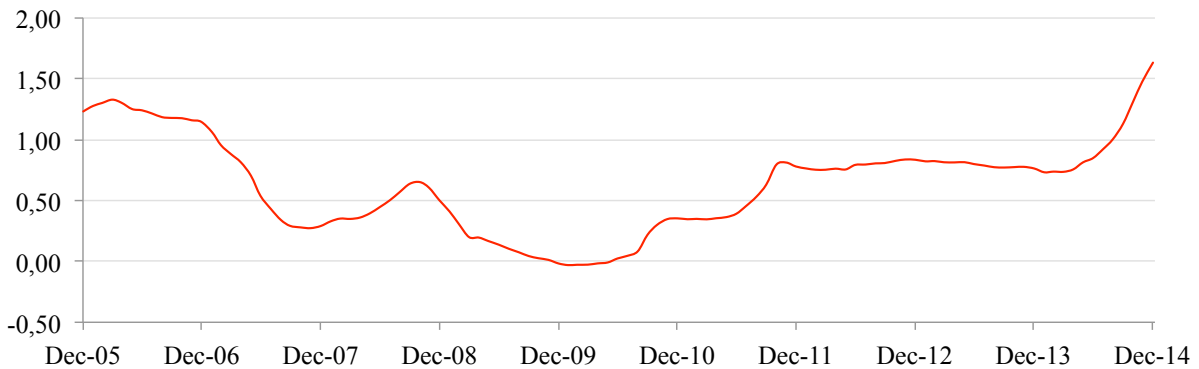


Figure 4.2: Evolution of the regression coefficient for the period considered. It becomes clear that it would not have been reasonable to assume a fixed coefficient for all the dates, and given the values calculated taken into account only the previous 36 months vary significantly.

Figure 4.2 shows the result of these calculations, ie the evolution of the β calculated over the period considered.

Given the values change substantially over the period considered, we confirm it would not have been reasonable to use the same β for the whole period.

Now that we have estimated the last outstanding parameter, we will present in the next sections the results obtained for each of the three sub-periods, and the overall result.

4.3 Empirical results

Table 4.1 summarizes the results obtained with this strategy for each sub-period, and then the total, for the whole period considered.

Empirical Results

		Cash and \$Duration N.	Fifty-fifty	Maturity	Regression
Period 1	Excluding interest	-0,8269%	2,3664%	0,7687%	1,1903%
	Including interest	-0,8269%	-0,9417%	-0,8844%	-0,9906%
Period 2	Excluding interest	-0,9977%	2,6041%	0,7982%	-0,2527%
	Including interest	-0,9977%	0,4290%	-0,2869%	-0,4452%
Period 3	Excluding interest	-1,7450%	-3,6649%	-2,7065%	-3,4913%
	Including interest	-1,7450%	-4,3711%	-3,0593%	-3,8751%
Total	Excluding interest	-3,5695%	1,3055%	-1,1397%	-2,5537%
	Including interest	-3,5695%	-4,8838%	-4,2306%	-5,3109%

Table 4.1: Empirical results for each sub-period and overall results, with and without interest.

The first period is not characterized by any major changes, as we observed previously. Overall, the prices at the start and end dates decrease, but only slightly for the short and medium-term bonds. There are some small increases and decreases along the way, but nothing drastic. The curve observed in this period is not exactly an "unchanged" curve, but it is the closest we observed. All butterflies would generate losses, when the interest due to the initial investment is taken into account in the first period.

The second is the period of greater price volatility. As we can see in the table, the results are all close to zero and the highest overall return (and the only positive one) is for the C & \$D neutral weighting.

In the third period, the last one, we can observe an overall increase of the prices. The results are negative for all four strategies, and are the worst results of the three sub-periods.

The overall results are the sum of the results from the three sub-periods, so these final results are no surprise.

Even though the overall results were not positive, there were dates with positive outcomes, just not enough to cancel out the others.

It would be useful to understand under which circumstances each strategy performs best and worst. In order to do this, we will analyze some characteristics of the set of results: extremes (overall maximum and minimum), averages and standard deviation for the four butterfly strategies.

This information is summarized in Table 4.2.

Summary of the analysis of the monthly results over the whole period

	Cash and \$Duration N.	Fifty-fifty	Maturity	Regression
Maximum	0,6945%	0,4642%	0,5402%	0,5917%
Minimum	-0,6683%	-0,8516%	-0,7594%	-0,6757%
Average	-0,0331%	-0,0452%	-0,0392%	-0,0492%
Standard deviation	0,2495%	0,2080%	0,2139%	0,2319%

Table 4.2: Statistics of the butterfly results over the whole period considered.

The statistics shown do not reveal much, since all the extremes, as well the average and standard deviation for the four strategies, seem similar. However, there is one information easy to spot, which helps to explain the overall negative results: the average return is negative for all four strategies.

From this information, it seems that the best and worst return of each strategy, even if under different circumstances for each of them, is close to the other three, so there doesn't seem to be a butterfly that is much better than the others.

However, a further analysis of the conditions where the peaks occurred might give us a different perspective.

We will now identify the settlement dates of the peaks and, following the approach in

(Martellini et al.; 2002), analyze the variation in the YTM of the three maturities on these dates, and compared them with the expected, according to the work mentioned. The data collected is in Tables 4.3 and 4.4.

Top three results for the four strategies

		Cash and \$Duration N.	Fifty-fifty	Maturity	Regression
Maxima	Result 1	0,69454%	0,46418%	0,54021%	0,59168%
	Date 1	30/04/12	30/12/08	30/11/11	30/07/10
	YTM var. 1	-12,1/-24,6/-46,8	-25/13,5/27,5	-32/-29,3/-46,8	-28,1/-48,4/-61,3
	Result 2	0,66467%	0,4152%	0,4621%	0,46729%
	Date 2	30/11/11	30/11/11	30/04/12	30/11/11
	YTM var. 2	-32/-29,3/-46,8	-32/-29,3/-46,8	-12,1/-24,6/-46,8	-32/-29,3/-46,8
	Result 3	0,46766%	0,3191%	0,3742%	0,46011%
	Date 3	31/12/10	31/12/12	31/12/10	31/12/10
	YTM var. 3	41/39,5/24,3	31,4/38,7/32,9	41/39,5/24,3	41/39,5/24,3

Table 4.3: Details of the maximum results: top three results for each strategy. Some dates are common to multiple butterflies, and one of them, 30/11/2010, is present in all four.

Bottom three results for the four strategies

		Cash and \$Duration N.	Fifty-fifty	Maturity	Regression
Minima	Result 1	-0,66826%	-0,85164%	-0,75943%	-0,67571%
	Date 1	29/04/11	29/04/11	29/04/11	29/04/11
	YTM var. 1	-5,7/-34/-23	-5,7/-34/-23	-5,7/-34/-23	-5,7/-34/-23
	Result 2	-0,65747%	-0,6079%	-0,6056%	-0,6680%
	Date 2	31/12/07	31/01/13	31/12/07	31/12/07
	YTM var. 2	-51/-56,7/-36,6	-15,2/-32,6/-24,8	-51/-56,7/-36,6	-51/-56,7/-36,6
	Result 3	-0,63465%	-0,5664%	-0,5571%	-0,5819%
	Date 3	30/09/08	30/04/10	30/04/10	31/01/13
	YTM var. 3	-82,7/-44,9/-12	-43,8/-54,6/-42,5	-43,8/-54,6/-42,5	-15,2/-32,6/-24,8

Table 4.4: Details of the minimum results: bottom three results for each strategy. Some dates are common to multiple butterflies, and the minimum occurs on the same date for all four, 29/04/2011.

Looking at the dates and variations in the YTM, we can draw some conclusions for each butterfly.

The Cash and \$Duration Neutral Weighting has maximum returns on dates when the curves

became flatter, and the minimum returns happen when the curve becomes steeper. This is in line with the expected behavior.

The Fifty-fifty Weighting is expected to produce maximum returns for steepening movements, and minimum for flattening. Even though some of the movements in the tables are not "pure" flattening or steepening movements, they generate curves with the desired shapes.

The Maturity Weighting has positive returns for flattening curves and is expected to deliver negative results for steepening curves. However, as for the butterfly above, the curve generated has the desired shape, despite the changes in the YTM.

Finally, the Regression Weighting has positive returns for flattening scenarios, and negative for steepening.

There are some other results expected that are possible to confirm with our results. An unchanged curve is expected to generate very low results. On 30/09/09, the YTM variations were: -0,35/0,19/-0,91. It is the closest we can find to an unchanged scenario. The results obtained in this date are in Table 4.5.

Results for an unchanged curve

	Cash and \$Duration N.	Fifty-fifty	Maturity	Regression
Result	0,012%	0,017%	0,015%	0,007%

Table 4.5: Results for an the closest to an "unchanged" we can find in the data analyzed. As expected, the results are very low, close to zero, for all four strategies.

Also, the Regression Weighting is expected to be quasi curve neutral for scenarios where the body is unchanged and the ratio between change in the YTM of the short and long wings is the symmetrical inverse of the regression coefficient. Table 4.6 presents the result for the Regression butterfly in such scenario.

4.4 Zero-coupon VS annual-coupon bonds

(Martellini et al.; 2002) studied different scenarios, where all the YTM's remained unchanged, varied by the same bps, or the curve got flatter/steeper. This was possible because they studied theoretical scenarios, with data constructed for the tests. In our work, we used real data, so we were not able to find scenarios equivalent to theirs. However, as mentioned in the previous

Regression butterfly - quasi curve neutral scenario

	Regression
Result	-0,023%
Date	26/02/10
Regression coefficient	-0,0278
YTM var.	-10,2/0,1/-0,3

Table 4.6: Results for a scenario where the Regression butterfly is quasi curve neutral. The Regression coefficient calculated for this date is $-0,0278$. The ratio between the short and long YTM variations is $\frac{-10,2}{-0,3} = 34$, and $-\frac{1}{34} = -0,029$, which is very close to the actual coefficient, $-0,0278$. Also, the body of the butterfly remained practically unchanged, so these are the perfect conditions for the Regression butterfly to be quasi neutral, which is verified in the results.

section, some of their conclusions match ours⁹.

Given these results come from the use of coupons, which falls outside the scope of this work, the results they presented have not been confirmed. The confirmation and further study of the results using bonds with coupons, and the application of our strategy to these are left as suggestions for future investigations.

⁹(Martellini et al.; 2002) presents two ways of measuring the performance of the strategies. In our work we have used the Total Return Measure, as the alternative (Spread Measures) cannot be applied to the Cash and \$Duration Neutral Weighting.

Chapter 5

Conclusions

Our motivation when we started studying Butterfly strategies applied to the Euro yield curve, was to understand how these would perform in our market, under real life conditions. We were not sure what we could expect.

In order to get us closer to our objective, we started by defining a strategy, identified several problems along the way, and came up with solutions for them. There were several steps that had to be taken, but we were able to build a tool that allowed us to fulfill our objective, and now it can be used under different conditions, as it was built in a way that makes it easy to use with different sets of data or different assumptions, for example.

After analyzing the results, we were able to confirm the conclusions presented in (Martellini et al.; 2002). Summarizing the results presented in the previous section, we observed that, over the period considered, the maximum monthly result obtained was for the Cash and \$Duration Neutral weighting and for a flattening movement, as expected according to (Martellini et al.; 2002). The maximum results produced by the other strategies slightly below, but all positive. There was a date (30/11/2011) where all four strategies produced one of the top three results, and there was a flattening movement on this date. All of this leads us to conclude that, under the right circumstances, all four strategies can produce positive returns. However, these strategies should not be applied over long periods, unless the investor's forecast is continuously favorable, as the results varied significantly from one cycle/month to the next. We analyzed the results of applying the strategies continuously over the 12 year period considered, and the results would be negative, as we observed in Table 4.1, in the previous section.

We now confirm that these strategies can produce profits, when correctly applies, so this work can be a basis for future investigations. The study of bonds with coupons would make it

possible to replicate the results in the study used throughout this work, (Martellini et al.; 2002), as mentioned in previous sections. It would also be interesting to study a slightly different strategy, one that allows the choice between the butterflies here presented or their inverse (going long on the medium-term bond and short on the others), according to the expected movements of the butterfly's body.

Another possibility is the study of more complex strategies, such as butterfly trades applied to forward contracts, futures, swaps and options. As the financial products evolve, many other possibilities will appear.

Getting to know the behavior of these strategies applied to a great variety of financial products would give the investors a wider range of choices, a much more consolidated knowledge base, and a maximized chance of success.

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