Departamento de Finanças

ISCTE Business School

Departamento de Matemática

Faculdade de Ciências da Universidade de Lisboa

Structural Models in Credit Risk

Carlos António Fernandes Casimiro

Dissertação

Mestrado em Matemática Financeira

2014

ISCTE Business School
Instituto Universitário de Lisboa

Departamento de Finanças

ISCTE Business School

LISBOA
UNIVERSIDADE DE LISBOA

Departamento de Matemática

Faculdade de Ciências da Universidade de Lisboa

Structural Models in Credit Risk

Carlos António Fernandes Casimiro

Dissertação

Mestrado em Matemática Financeira

Orientador: Professor Doutor José Carlos Dias

2014

Acknowledgements

I would like to express my sincere gratitude to my supervisor, Professor José Dias, who introduced me to the field of Credit Risk, for all his time and effort for my work. His guidance was crucial to the development of this thesis, which would not had been the same without our many discussions. A very big thank you to all my teachers, for all they have taught me and for allowing me to look at the financial world with new eyes. Moreover, I would like to thank all the persons that have accompanied me along this work.

Finally, I thank my parents and my family for the unconditional and constant support and for the great opportunities they have been able to provide my life with. I am sure I could have never attained many objectives of my life without their unreserved love and their huge patience.

Resumo

O principal objetivo desta tese é modelizar o risco de crédito de uma determinada instituição financeira utilizando modelos estruturais. Neste sentido, propomos a análise de dois modelos – Merton(1974) e CreditGrades – que são apresentados de acordo com a sua evolução temporal. Em cada modelo é calculada a fórmula fechada para a probabilidade de default neutra face ao risco, assim como o *credit spread* para uma empresa de referência. No entanto, antes da implementação prática dos modelos estruturais, é apresentado um referencial teórico que visa fornecer, de forma gradual, informações consideradas indispensáveis para a compreensão dos modelos em causa.

O modelo de Merton (1974) apresenta uma grande inovação que reside no modo de tratar o capital próprio de uma companhia como uma opção de compra sobre os seus ativos, permitindo assim a aplicação de métodos de avaliação de opções, tais como os modelos de Black e Scholes (1973) e de Merton (1973). As vantagens reconhecidas do modelo são não apenas a quantidade reduzida de parâmetros a estimar, como também a simplicidade de o colocar em prática. No capítulo I são também apresentadas algumas vantagens e desvantagens do modelo. O facto de o processo do valor dos ativos da empresa não ser observável no mercado constitui a maior dificuldade na implementação dos modelos estruturais. Estudos académicos propõem metodologias de estimação avançadas para determinar os parâmetros deste processo. Com efeito, um dos inconvenientes deste modelo é assumir o valor dos ativos (V_t) e a respetiva volatilidade (σ_t), como parâmetros de input ao modelo, uma vez que não são diretamente observáveis no mercado. Neste trabalho, são apresentadas duas aproximações ao modelo no que se refere à estimação dos parâmetros: uma aproximação iterativa e outra como solução de um sistema de equações não-lineares.

Em 2002, foi construído um modelo baseado na completa transparência de mercado – CreditGrades – para comparar os *spreads* modelados com os *spreads* observados no mercado e calcular a probabilidade de sobrevivência de uma determinada empresa. Construído sobre a estrutura do modelo Black e Cox (1976), o qual relativiza algumas das premissas presentes no modelo standard de Merton (1974), permite que um evento de *default* possa ocorrer antes da maturidade T (se o valor dos ativos da empresa tocar na barreira de *default*).

Outra das vantagens relevantes é o facto de a dívida financeira ser expressa por ação e estimada com base em dados financeiros provenientes de demonstrações consolidadas. Por outro lado, os poucos *inputs* do modelo são todos observáveis no mercado.

Na segunda parte deste trabalho, é apresentada uma aplicação destes *modelos* a um caso real: trata-se de um banco português que recentemente entrou em default. Pretende-se assim mostrar a probabilidade de *default* e o *credit spread* do banco em estudo, num cenário financeiro adverso, permitindo observar e analisar a adequacidade destes modelos ao mundo real. Por outro lado, o uso prévio destes modelos não teria evitado a situação de bancarrota do banco, mas daria uma boa percepção do risco de crédito ao longo do tempo.

Concluindo, o objetivo geral desta tese é informar o leitor sobre o modo possível de construir modelos de risco de crédito, dando-se um ênfase especial aos métodos práticos que um banco e/ou uma seguradora, nas respetivas áreas de corporate *banking* e atuariado, podem fazer uso, num processo de desenvolvimento de um novo modelo de *credit rating*.

Palavras-Chave: Risco de Crédito, Modelos Estruturais, Modelo de Merton (1974), Modelo CreditGrades, Probabilidade de Incumprimento, Credit Default Swap.

Abstract

The main objective of this thesis is to model the credit risk of a certain financial institution under the structural model approach. In this setting, we propose the analysis of two models - Merton (1974) and CreditGrades - which are presented according to its temporal evolution. Each model provides the closed-form formulae for the risk-neutral default probability and credit spread of a reference firm. However, before the practical implementation of the structural models, it is presented a theoretical framework that aims to provide, gradually, information considered essential to the understanding of the models under analysis.

The Merton (1974) model offers a huge innovation that lies in the way of treating a company's equity as a call option on its assets, thus allowing for applications of Black and Scholes (1973) and Merton (1973) option pricing methods. The advantages recognized for the model are not only the few parameters to estimate but also the simplicity of putting it into practice. In Chapter I, we also present some advantages and disadvantages of the model. The unobservability of the firm's assets value is a major difficulty in the implementation of structural models. Academic studies propose advanced estimation methodologies to determine the parameters of this process. In fact, one of the shortcomings of Merton's model (1974) is to assume the value of company assets (V_t) and the respective volatility (σ_t) as parameters of input to the model, since they are not directly observable in the market. In this work, we presented two approaches to the model regarding the estimation of parameters: an iterative approach and other as a solution of a system of nonlinear equations.

In 2002, it was developed a completely transparent market based model - CreditGrades - to match modeled spreads with the observed spreads and which calculate the survival probabilities of a reference firm. Built on the framework of the Black and Cox (1976) model, which relaxes some of the assumptions present in the standard Merton model, it enables a default event to occur before maturity T (if the value of company assets hits the default barrier). Others relevant advantages is the fact that the financial debt expressed on a per-share basis and estimated based on financial data from consolidated statements.

On the other hand, the inputs of the model are all observable in the market.

In the second part of this work, it is presented an application of these models to a real case, i.e. a portuguese bank that recently went into default. The aim is to show the default probability and the credit spread of the bank in study, in an adverse financial scenario, allowing to observe and analyze the adequacy of these models to the real world. Moreover, previous use of these models would not have avoided the situation of bankruptcy of the bank, but give a good insight of credit risk over time.

The general purpose of this thesis is to inform the reader on how it is possible to construct credit rating models. Special emphasis is made on the practical methods that a bank or insurance in the respective areas corporate banking sector and actuarial could make use of in the development process of a new credit rating model.

Keywords: Credit risk, Structural models, Merton (1974) model, CreditGrades model, Default probability, Credit default swap.

Contents

List of Figures

List of Tables

Chapter 1

Introduction

Credit risk has been an important issue which has been getting a growing concern among financial agents, including banking institutions, since the financial collapse that ruined Lehman Brothers. As daily practice, the banks provide financing to their clients in the form of loans, bonds, structured products among others. All this activity has to be monitored, managed and quantified. Moreover, in the last two decades, the international banking with its streams of capital and the increasing integration of financial markets, followed by economic instability of national monetary systems and the recent financial crisis, brought even more the need for rigorous measures in the calculation of capital requirements. For this reason, banks have recently allocated more resources than usual for this issue. According to the purposes set out in Basel II, capital requirements can be determined using an internal assessment of the probability of default of counterparties. This has led many researchers and practitioners to develop trading models for assessing credit risk over the last decade.

A class of models for assessing credit risk very well known in the financial literature is called structural models. Structural models use the dynamics of structural variables of a firm, such as asset and debt to measure the time of default. These models were developed from Black and Scholes (1973) and Merton (1974). Merton and Black and Scholes were pioneered in building a model of default. This class includes also the Black and Cox (1976), Geske (1977), and Vasicek (1984) models. Each of these models, built years later, so present improvements in the theoretical framework, reformulating or removing some of the unrealistic assumptions. Black and Cox (1976) introduce a more complex capital structure, with subordinated debt; Geske (1977) introduces the interest payments on the debt. Merton clarified and extended the model of Black and Scholes (1973). For a more realistic model, several assumptions were imposed. Merton assumes that the debt value of the firm is represented by a zero-coupon bond which will be due at maturity T. Based on the theory of option pricing provided by Black and Scholes (1973), the equity of a firm is a European call option on the assets of the firm with maturity T and strike price equal to the book value of liabilities. The purpose of structural models is to estimate the risk-neutral probability of firm default and therefore anticipate changes in credit quality of a firm. Thus, this model uses a default event when the value of assets is less than the value of the debt at time T. In addition to evaluating the possibility of default, the Merton (1974) model also allows calculating the credit spread on the debt.

The main advantage of the Merton (1974) model is to include option-pricing models in the estimation of default, in which they provide a necessary framework for extracting the necessary information about the bankruptcy of market prices. However, some restrictive assumptions lead to a very simplistic model, and therefore only consider five variables as inputs of the model: the face value of debt, the current value of assets, the respective volatility, debt maturity T and the risk-free rate.

One of the main problems in the implementation of structural models refers to the estimation of the variables related with the firm's assets. In the Merton model, it is difficult to estimate the value of company's assets and their volatility and it requires the application of some numerical methods. In order to overcome these difficulties, we present two approaches calculated from market value of firm's equity and the equity's instantaneous volatility suggested by Crosbie and Bohn (2003) and Vassalou and Xing (2004) and using a non-linear system of equations procedure.

Thus, for the study in question, we also present and compare the predictive performance of models presented above with other: CreditGrades. The original CreditGrades was published in 2002 by some investment banks - Deutsche Bank, Goldman Sachs, JPMorgan and the RiskMetrics Group - to compare the modeled spreads with the estimated spreads. The RiskMetrics in 2002, summarized the CreditGrades model as follows: "The purpose of the creditgrades is to establish a robust but simple framework linking the credit and equity markets". The high performance of the model in the pricing equity options and simultaneously on credit default swap (CDS) turn the model an industry benchmark, according to Currie and Morris (2002) and Yu (2005). The CreditGrades model is a version extended from the Merton (1974) model and therefore belongs to the class of structural models. Under the paper of Byström (2005) noted that the model uses the theoretical framework of the Merton (1974) model which models default probability only depending on the leverage ratio and the assets volatility of firm. In this sense, we present two possibilities to determine the survival probability: a proposal by Finger et al. (2002) which uses the closed-form formula for the survival probability and the other possibility presented by Kiesel and Veraart (2008) based on an explicit analytic formula to determine the exact survival probability.

However, the CreditGrades model differs from many structural models, this is because the main purpose of the structural models is to accurately estimate the probability of default, whereas the CreditGrades was designed to better perform the matching of credit spreads obtained by the model with the observed credit spreads on the market. On the other hand, most structural models have a great difficulty in estimating the value of company assets as well as its volatility. In CreditGrades model, the estimation methodology is based on few input parameters, which are observable in the market. In addition, this model incorporates a random default barrier to make it stochastic, which allows to include the uncertainty (which is the key feature in the reality of financial markets) at the current level of debt and a default event will happen unexpectedly. Enjoyed by many practitioners and researchers, the CreditGrades model contains an element of uncertain in recovery rates, which helps generate realistic short-term credit spreads, as yields a simple and analytic CDS pricing formula, according to Chao, Yu and Zhong (2011).

Finally, we analyze the performance of the models when they are applied in practice. In second part of this work, the models are implemented in Matlab using market data and financial data from consolidated statements to determine various credit risk measures: probabilities of default and Credit Default Swap (CDS) spreads. The models are applied to a financial institution of repute in Portugal - BES - which recently became involved in a complex situation causing the default of the bank and the consequent exit of Portuguese stock index - PSI 20. In this context, we present a brief overview of the economic situation in Portugal.

Outline of Thesis

Credit risk modeling is a wide field. In this thesis an attempt is made to shed a light on the many methods and subjects of credit risk modeling. Chapters 2 to 4 provide the fundamental understanding of credit risk modeling.

The structure of the thesis is as follows:

Chapter 2: **Financial and Mathematical Background** In order to get a better feel for credit modeling framework there are some important concepts and measures that are worth considering.

Chapter 3: **Structural Models in Credit Risk** This chapter introduces some classical firm-value models based on Geometric Brownian Motion such as the Merton model and CreditGrades model. In each of these models, it is derived the probability of default and the credit spread, as well as discussed their advantages and shortcomings.

Chapter 4: **Application to Real Case** In this chapter, we address the implementation and testing of the models presented in previous chapter. Thus, we use real data to examine the probability of default and credit spread of a portuguese bank that recently went into default.

Chapter 5: **Conclusions** Once lodged the real case are discussed, in this chapter, the conclusions regarding the implementation of the models to the real case. Some guidelines for future research will also be presented.

Chapter 2

Financial and Mathematical Background

2.1 Credit Risk

In general, the credit risk is present constantly in the lives of people, companies, financial institutions and increasingly in many countries. Credit risk, also known as default risk, is the potential loss arising from default of an economic agent to meet its contractual obligations in preestablished period of time. Credit events include default, failure to pay, loan restructuring, and others. A more common example is when a homeowner stops making mortgage payments. In this case, the risk that the bank faces is credit risk, a person acquires a loan, and the default happens when the creditor is not able to pay the promise payment of principal or interest on the loan.

The type of risk the bank is facing is exactly credit risk: the reference entity is the person that asks for the loan, and default occurs on the day the creditor declares that he is not able to honor his obligations. Other underlying risk is spread risk over the duration of the loan $[0; T]$, where T is often referred to as maturity or time horizon. Moreover, and in the current crisis observed in many countries in Europe, many lenders defaulted, this because, there are various elements that the bank does not know on the day it provides the loan. Facing the situation of uncertainty of any creditor, a bank will evaluate their creditworthiness.

Since initially the bank does not know the probability that the creditor will not meet its financial obligations, the credit risk can be generalized to the following equation:

$$Credit\,Risk = \max\left\{Actual\,Loss - Expected\,Loss; 0\right\},$$

where the actual loss is the observed financial loss. Credit risk is then the actual losses exceed expected losses. Expected losses are made up by the default probability multiplied by Exposure at Default (EAD) - is the amount that the borrower legally owes the bank - and, in turn, is again multiplied by the Loss Given Default (LGD) - is the percentage of actual loss (EAD) that the bank loses with the default of the borrower. To overcome the fact that a bank does not know the probability of default of a creditor, means that the bank develop methods for assessing credit risk. Thus, the calculation of the probability of default is done by collecting information about the lender, in order to get an idea about the likelihood of this not being able to pay the repayment of principal and interest. However, there remains two further elements of uncertainty impossible to measure: the severity of the loss and the time of default.

Throughout this study, this probability will be referred to as survival probability:

$$P_{Surv}(t) = \text{Probability that default will not occur in } [0;t].$$

Correspondingly, we will call default probability between 0 and t, as:

$$P_{Def}(t) = \text{Probability that default will occur in } [0;t].$$

For each $0 < t < T$, these probabilities can be related by the following formula: $P_{Def}(t) = 1 - P_{Surv}(t)$.

In Figure 2.1, we can see the default probabilities to a year that are observed in the market and respective correspondence to the ratings, according to the Standard & Poor's.

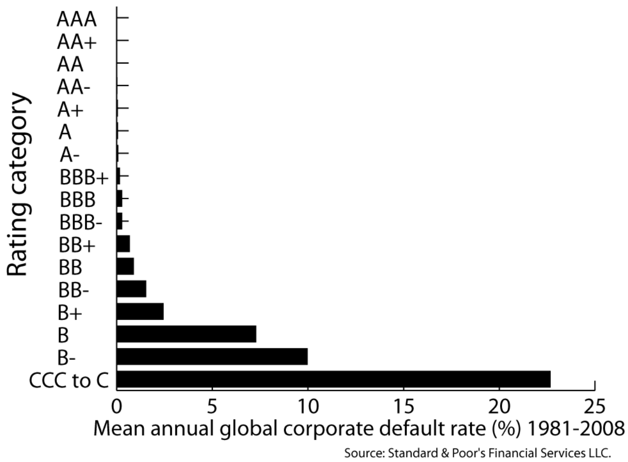

Figure 2.1: Historical average 1-year default rate for 1981-2008 following Standard&Poor's classification. Source: Standard&Poor's Financial Services LLC.

Next section focuses on the presentation of a class of models widely popular - structural models - and that it is the main target of the present study.

2.2 Credit Risk Modeling

Over the last decade, several practitioners and researchers have developed sophisticated models towards modeling the growing credit risk from important aspects of their business lines. These models allow banks to measure and manage the risk of their financial products. There are more and more reasons for the growing interest in modeling the credit risk. In recent years, the trading of financial instruments related to credit risk volume has increased exponentially. The implementation of Basel II for banks and Solvency II for insurers, has encouraged the development of internal models to set regulatory capital requirements.

2.2.1 Market price methods

The two classes of models of credit risk that are more common in the literature are the structural models and reduced-form models (also known by intensity models). Over the next chapters only structural models are studied. Structural models - also known as the value-of-the-firm approach - represent the link between equity and credit risk. These models help to provide a clear and transparent relationship between default risk and the capital structure of a firm. The main propose is to accurately estimate the default probability. However, the disadvantages that are assigned to them are the strong assumptions on the dynamics of firm's assets, $V = \{V_t; 0 \leq t \leq T\}$, in debt and how this capital is structured.

In order to better understand it, let us introduce the following simple structural model. We consider that default happens when the asset value cross the fixed level B, which corresponds to the value of the firm's liabilities within the time horizon, as shown in Figure 2.2. For the sake of illustration, we assumed that the asset value follows a geometric Brownian motion with drift 0.05 and standard deviation 0.3. We also assumed that the asset value at time $t = 0$ is $V_0 = 100$ and the level B, represented by the solid line in Figure 2.2, equals 99.95. In this particular example of structural model, the evolution of asset values default will occur after around five months.

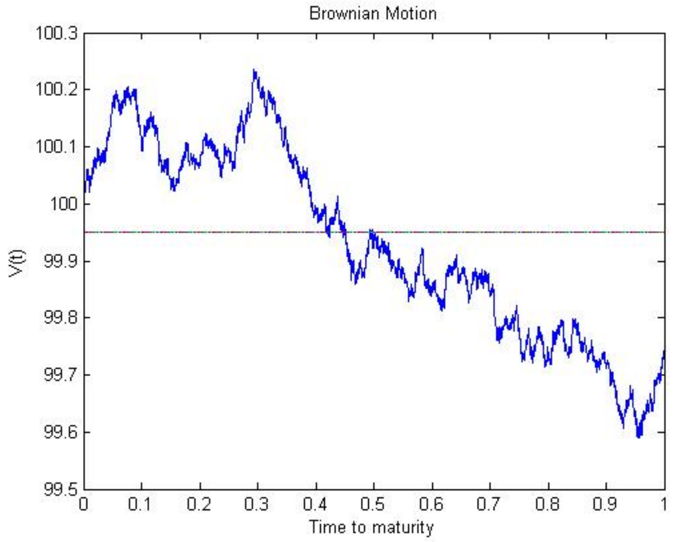

Figure 2.2: Example of a structural model ($T = 1$ year) where the dynamics of the asset value, V_t is a geometric Brownian motion ($\mu = 0.05$, $\sigma = 0.3$). Default occurs if the underlying asset hits the default barrier.

2.3 Background and overview of credit derivatives

Credit derivatives are bilateral financial contracts that transfer risk between two parties and whose payoffs are a function of the default of a specified reference entity. In many cases, credit derivatives are used to hedge, transfer, or manage credit risk and can be seen as an insurance against default. The idea is that credit risk is transferred without reallocating the ownership of the underlying asset. In general, two counterparties are involved, the protection buyer and the protection seller, which agree on a contract related to the default of the reference entity(ies). The credit derivatives market has experienced considerable growth over the past five years. We believe that the market has now achieved a critical mass that will enable it to continue to grow and mature. This growth has been driven by an increasing realization of the advantages credit derivatives possess over the cash alternative, plus the many new possibilities they present. Figure 2.3 shows the growth of the volumes of credit derivatives exchanged on the market from 2000, testifying the exponential popularity of these products. Banks and investments undertakings are using credit derivatives to hedge credit risk, reduce risk concentrations on their balance sheets, free up regulatory capital in the process and to mitigate the capital requirements imposed by the Basel II Accord. Indeed, banks use credit derivatives to hedge or assume credit risk, to enhance portfolio diversification, and to improve the management of their portfolios.

Figure 2.3: Global credit derivatives market, in trillions of U.S. dollars. *Source:* British Banker's Association.

There is a wide variety of different products which may be classified as credit derivatives. The financial product most dominant and used in the credit derivatives market has been the credit default swap (CDS) - account for more than twice as much of the market.

Credit Derivative Instrument Type	**Market Share (% Notional) at End 1999**
Credit Default Products	38%
Collateralized Loan Obligation (CLO)	18%
Asset Swaps	12%
Total Return swaps	11%
Credit Linked Notes	10%
Baskets	6%
Credit Spread products	5%

Table 2.1: Market Share of Credit Derivative Products. *Source:* British Banker's Association Credit Derivatives Report 2000.

These contracts are designed to mitigate the risk of default on credit obligations. The CDS contracts had a great use in the global crisis of 2008 and will be used to test the capability of our models to calculate the CDS spreads according with market data. A broader view of credit derivatives can be found in Das (2000). The remaining of this section will focus on this family of financial contracts.

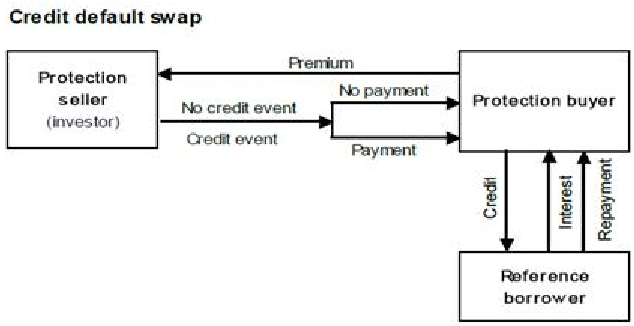

Figure 2.4: Credit Default Swap contract

2.3.1 Credit Default Swaps

In terms of credit derivatives, the "Plain vanilla" CDS quickly became the most efficient and liquid instrument for lenders, loan underwriters, bond investors, traders and portfolio managers to efficiently transfer and manage credit risk. CDS are over-the-counter (OTC) contracts. They are used to transfer credit risk of a reference entity from one party to another.

A standard CDS consists of a bilateral contract in which the protection buyer of the CDS pays a fixed premium and previously agreed to the seller until the specified maturity date of CDS or until a default occurs. When a default occurs, the protection buyer receives a payment - known as the default leg - which is the difference between face value and the recovery value of the reference entity. If no default occurs until maturity of the CDS contract, the seller pays nothing. These two cash flow streams of a CDS contract are typically named the fixed leg (the fixed periodic premium paid by the protection buyer) and the default leg (the payment contingent on the occurrence of a credit event) according to the nature of the payment. Figure 2.4 provides an illustration of the functioning of a CDS contract.

Definition 2.3.1. CDS Spreads: The CDS spread is usually a quarterly fee (in basis points) that represents the price to enter a CDS contract against the default of the reference entity, reflecting the riskiness of the underlying credit.

To understand the concept and operations of a CDS, let us suppose a protection buyer (a person or firm) purchases a zero-coupon defaultable bond of a firm with face value of protection $V = 5\ 000$ Euros, maturity $T = 10$ years and with CDS spreads of 300 basis points (bp) to be protected against the default of this bond. The protection buyer can enter a CDS contract of the same maturity of the defaultable bond.

The protection buyer thus will make annual payments of 300bp x 5 000 = 150 Euros, i.e. 150 Euros is the yearly cost of the risk the protection seller is taking. According to the situation of the bond, the payments are as follows:

- The bond does not default before maturity of CDS, the protection buyer pays annually 150 Euros and receive the face value of $V = 5\ 000$ Euros at maturity.
- The bond defaults. In this case, the protection buyer is compensated by the protection seller with the diference between V and the recovery value after default. To understand the concept of recoverable amount of a firm is necessary to have an overview of a situation of default, the creditors claiming the assets of company, such as holders of defaultable bond. The recovery rate R is the proportion of the claimed amount received in the event of default. This is usually a result of a liquidation of the company's assets and generally results in a lower amount than the par value. Historical data on the amount recovered show that R can vary between 20% and 50%, depending on the debtholders. In our example, suppose the recovery rate is $R = 40\%$ for our protection buyer (i.e. the recovery value is 2 000 Euros). The protection seller will pay an amount equal to V x $(1 - R) = 3\ 000$ Euros, lower than face value.

Next subsection, presents some concepts and mathematical symbols that are important to understand the scope of this work.

2.4 Definitions and Principles

We start by providing some basic concepts on stochastic processes. Next, we introduce some definitions and assumptions that will be used under our structural models.

Definition 2.4.1. Random Variable A random variable Z defined on a probability space $(\Omega;\mathcal{F};\mathbb{P})$ is a measurable function Z : $\Omega \to$ E.

Depending on the scenario $\omega \in \Omega$, the random variable can take diferent values; Z(ω) indicates the realization of the random variable Z, if the scenario ω happens.

Definition 2.4.2. Stochastic Process. A stochastic process $X = \{X_t, 0 \leq t \leq T\}$ is a family of random variables defined on a probability space $(\Omega;\mathcal{F};\mathbb{P})$, where t indicates the time parameter.

The functions $t \to X_t(\omega)$ attached to the outcomes are called *sample paths*, and the index t is referred to as "time". The best way to think of a stochastic process is to view it as a "random function" on the domain $[0, \infty)$, with the sample paths as its realizations.

2.4.1 Brownian Motion

Brownian motion is an important stochastic process given its great applicability to countless models of credit risk, but above all to be used as the basis for building other processes. It can be thought of as the "standard normal" process. A Brownian motion is often denoted by the letter W, since it is also known by Wiener process, who was among the first to study Brownian motion in a mathematically rigorous way.

Definition 2.4.3. Brownian Motion A stochastic process $W = W_t, t \geq 0$ is a Brownian motion (or Wiener process) if the following conditions hold:

1. $W_0 = 0$;

2. The process has stationary increments, i.e. the distribution of the increment $W_{t+s} - W_t$ over the interval $[t, t+s]$ does not depend on t , but only on the length s of the interval;

3. The process has independent increments, i.e. if $l < s \leq t < u$, $W_u - W_t$ and $W_s - W_l$ are independent random variables. In other words, increments over non-overlapping time intervals are stochastically independent;

4. For $0 \leq s < t$ the random variable $W_t - W_s$ follows a Normal distribution $N(0, t-s)$.

The paths of a Brownian motion are continuous but very irregular and can be mathematically demonstrated the infinite variations on a given compact time interval[1].

A Brownian motion can be easily simulated by discretizing time using a very small step Δt. The value of a Brownian motion at time points $\{n\Delta t, n = 1, 2, ...\}$ is obtained by sampling a series of Standard Normal $N(0,1)$[2] random numbers $\{v_n, n = 1, 2, ...\}$ and setting:

$$W_0 = 0, \quad W_{n\Delta t} = W_{(n-1)\Delta t} + \sqrt{\Delta t} v_n.$$

Figure 2.5 shows a typical Brownian motion path.

[1] For a definition see, for example, Sato (1999)

[2] Normal distribution with mean zero and standard deviation equal to one.

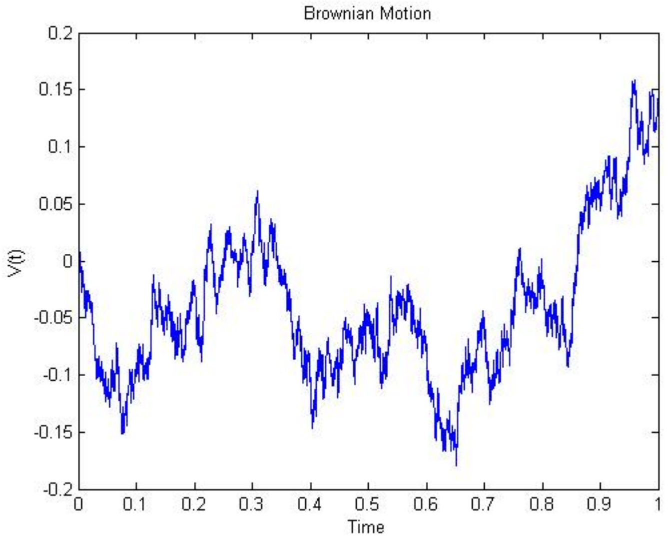

Figure 2.5: A sample path of a Brownion motion.

Geometric Brownian motion, which is constructed from a Brownian motion, is one of the most popular stochastic processes in finance, e.g. it is the basis of the Black and Scholes (1973) model for stock price dynamics in continuous time. A stochastic process $S = S_t, t \geq 0$ is a geometric Brownian motion if it satisfies the following stochastic differential equation

$$dS_t = S_t(\mu dt + \sigma dW_t), \quad S_0 > 0, \tag{2.4.1}$$

where $W = W_t, t \geq 0$ is a standard Brownian motion, μ is drift parameter, and $\sigma > 0$ is the volatility parameter. Equation 2.4.1 has only one solution (see, for instance, Bjork 1998):

$$S_t = S_0 \exp\left(\left(\mu - \frac{\sigma^2}{2}\right) t + \sigma W_t\right). \tag{2.4.2}$$

The related log-returns

$$log S_t - log S_0 = \left(\mu - \frac{\sigma^2}{2}\right) t + \sigma W_t, \tag{2.4.3}$$

follow a Normal distribution,$N(t(\mu - \frac{\sigma^2}{2}), \sigma^2 t)$. Thus S has a Lognormal distribution. Using Figure 2.6 we can graphically see the realization of a geometric Brownian motion, which is based on the sample path of the standard Brownian motion with $S_0 = 100$, $\mu = 0.05$ and $\sigma^2 = 0.3$.

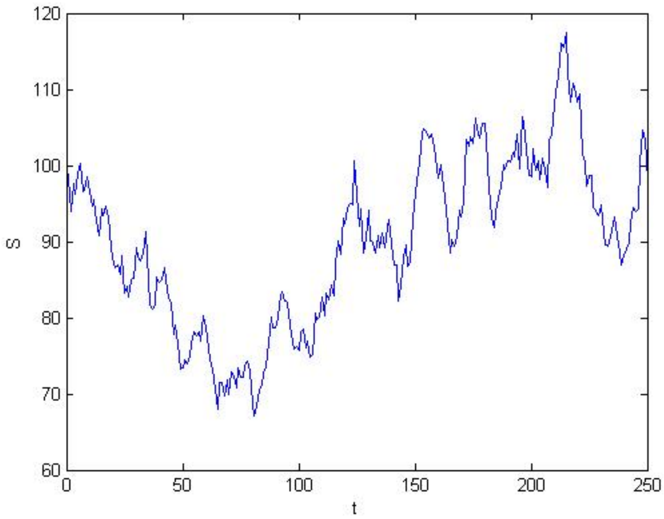

Figure 2.6: A sample path of a geometric Brownian motion with $S_0 = 100$, $\mu = 0.05$, and $\sigma^2 = 0.3$.

2.4.2 Modelling Assumptions[3]

Definition 2.4.4. Risk-Free Asset The price process $B = \{B_t, 0 \leq t \leq T\}$ is the price of a risk-free asset if it follows the dynamics

$$dB_t = r_t B_t dt, \tag{2.4.4}$$

where$B = \{B_t, 0 \leq t \leq T\}$ is called the short rate and can be either an adapted process, or a deterministic function of time. The process $B = \{B_t, 0 \leq t \leq T\}$ will be referred to as the compounded short rate.

If we interpret the risk-free asset as a bank account with short rate of constant interest r we have the discount factor:

$$D(t, T) = \exp(-r(T - t)). \tag{2.4.5}$$

From now on we will consider the short rate r to be constant, or we will directly observe the term structure $D(t, T)$ on the (bond) market. Stochastic short rate are not contemplated throughout the present study.

Definition 2.4.5. Martingale A stochastic process $X = \{X_t, 0 \leq t \leq T\}$ is a $\mathcal{F}_t$ martingale if the following conditions hold:

[3]Some of these assumptions may be relaxed or modied.

- X is adapted to the filtration $\{\mathcal{F}_t\}_{t\geq 0}$.
- $\mathbb{E}[|X_t|]<\infty$ for each t.
- For all s and t such that $s \leq t$, the following relation holds:$\mathbb{E}[X_t|\mathcal{F}_s] = X_s$.

The first condition states that at each time t, we can observe the value of X. The third condition says that the expectation of a future value of X, given the information available today, equals the present value of X. This means that a martingale has no systematic drift - fair games in gambling and absence of arbitrage in financial market models.

Definition 2.4.6. Arbitrage Let φ be a self-financing investment strategy. Let $A^{\varphi}= \{A_t^{\varphi}, 0 \leq t \leq T\}$ be the process describing the value of φ as a function of time. An arbitrage $\tilde{\varphi}$is a self financing investment strategy that makes zero investment at time zero and has net positive profit with positive probability:

$$A_0^{\tilde{\varphi}} = 0, \quad \mathbb{P}\left(A_t^{\tilde{\varphi}} \geq 0\right) = 1, \quad \mathbb{P}\left(A_t^{\tilde{\varphi}} > 0\right) > 0.$$

Definition 2.4.7. Complete Market A market is said to be complete if for every contingent claim C_T^{X} there exists a self financing investment strategy φ whose value exactly replicates the claim:

$A^{\varphi}(T) = C_T^{\mathrm{X}}$

Market completeness implies that the only price of a contingent claim C_T^{X} consistent with no-arbitrage is given by $\prod_t^{\mathrm{C}}= A_t^{\varphi}$.

Assumption 2.4.8. Perfect Markets The markets are assumed to be "perfect" and frictionless in the sense that there are no transaction costs or taxes. Assets are perfectly divisible. Investors act as price-takers (i.e. trading in assets has no effect on prices) and have equal access to information. Trading in assets takes place continuously in time. There is unlimited borrowing or lending at the riskless interest rate. There are no restrictions against short sales.

This is a standard assumption made in the literature for risk-neutral valuation of derivatives, as for example in Black and Scholes (1973) and Merton (1973).

Assumption 2.4.9. Arbitrage-Free Market. We will assume to work in a market where arbitrage opportunities are not possible. In particular, we will price any security in such a way that there is no arbitrage opportunity in the market. The arbitrage-free condition implies that with zero capital it is not possible to make any profit. It can be demonstrated that without this assumption, the market would not be in equilibrium, and correct pricing of financial instruments would not be possible.

Chapter 3

Structural Models in Credit Risk

3.1 The Merton model

The quantitative modelling of credit risk were initiated with the papers of Black and Scholes (1973) (hereafter, BS) and Merton (1974). Merton develops a framework that relates the firm's assets value to its credit risk and subsequently uses the BS option pricing formulas to price defaultable bonds and equity of the firm. This section describes the Merton (1974) model and shows how the probability of firm default can be inferred from the market valuation of firms under specific assumptions on how assets and liabilities evolve. We first resume the assumptions underlying the model and analyze the conditions of default. Then we present the formulas to price equity and debt and to calculate default probabilities and credit spreads. The shortcomings of the Merton model are discussed in section 3.1.6.

3.1.1 Assumptions and default conditions

The power of the Merton model is bought at the price of some strong assumptions. Thus Merton (1974) makes the following restrictive assumptions to develop his model[1]:

1. There are no transaction costs, bankruptcy costs, or taxes. Assets are divisible and trading takes place continuously in time with no restrictions on short selling of all assets. Borrowing and lending is possible at the same constant interest.

2. There are sufficient investors in the market place with comparable wealth levels, such that each investor can buy as much of an asset as he wants at the market price.

3. The risk-free interest rate r is constant and known with certainty.

[1] Since Merton uses the Black and Scholes (1973) methodology to price securities, Merton makes these assumptions along with some of the BS assumptions.

4. The debt is composed by a zero coupon bond.

5. The liabilities of the firm consist only of a single class of debt.

6. The evolution process of the firm's assets value V_t follows a stochastic diffusion process:

$$\frac{dV_t}{V_t} = (\mu_V - \delta)\, dt + \sigma_V dW_t, \tag{3.1.1}$$

where μ_V is the instantaneous expected rate of return on the firm's assets per unit time, δ is the payout of the firm per unit time, σ_V is the volatility of the firms assets per unit time, and dW_t is the increment to a standard Wiener process.

In addition to these assumptions, other issues have led to innovations: the use of normality in the returns distribution rather than one allowing for tail fatness; the assumption of nonstochastic interest rates; the static nature of the capital structure of the firm, and so on. However, the critical assumptions are continuous time trading and assumption 6. The total debt is treated in a very simplistic way and consists of a zero coupon bond (ZCB) and there are no other issue before maturity of the ZCB. The firm's equity consists of ordinary shares. Both debt and equity are contingent claims on the assets of the firm. The firm value or value of total assets equals the value of total debt B_t and equity E_t; in other words: $V_t = B_t + E_t$.

The ZCB has a face value of debt D, in which is paid at maturity T. A high D means that the firm is more heavily debt financed, whereas a low D means that the firm is more equity-financed. At the same time, the higher the level of debt D, the higher is the default risk of the firm, as the same underlying cash flow will now have to pay off a greater amount of debt. When the ratio of the value of assets to debt is higher than one or the value of the firm's assets at maturity exceeds D, then the firm is solvent at date T and the debt holders receive the full notional amount and the shareholders receive the residual asset value $V_T - D$. When the asset value at maturity is less than D, the firm is insolvent because it does not have sufficient assets to meet debt claims, i.e. can not make the promised debt payment and defaults. The bondholders take over the firm and receive the firm value V_T, while the shareholders receive nothing. Shareholders never have to compensate for the bondholders loss in case of default, which means that E_T can not be negative ($E_T > 0$).

To better understand the dynamics of the Merton model, figure 3.1 summarizes the model.

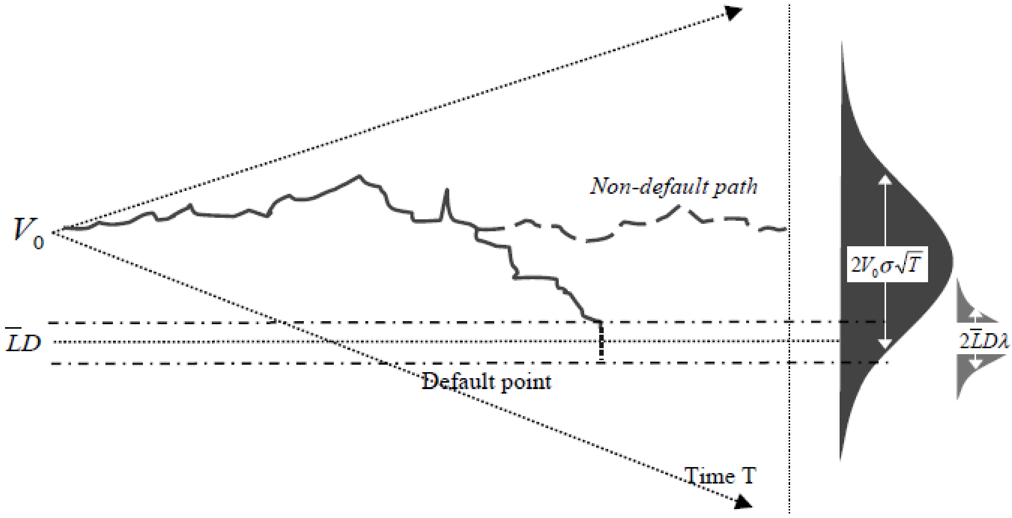

Figure 3.1: Dynamics of asset value of the Merton model. *Source:* 2002 RiskMetrics Group Inc.

Figure 3.1 illustrates the dynamics of firm assets in the Merton model. As mentioned above in the model assumptions, the total debt of the firm B_t presents a static structure over time and that the value of equity $(V_t - B_t)$ oscilates with the value of the firm's assets. In the figure are shown two possible scenarios for the firm: the bold line represents a event default, which occurs only when the firm value falls below the threshold default (LD) at maturity, such that $V_T < D$; the dotted line shows a non-default path. The shaded area below default barrier of this distribution is the probability of default.

3.1.2 Option pricing theory

Based on the assumptions and default conditions described in section (3.1.1), we want to price equity and corporate bonds issued by a firm whose assets are driven by a geometric Brownian motion. Using Merton (1974), we need the BS option pricing theory or else there would be an opportunity for arbitrage profits. To accomplish this objective, we will work under the risk-neutral probability measure.

We define the following notation E is the value of the firm's equity, B is the value of total debt and V is the value of its assets. In the Merton framework, the payoff of the equity value and debt value at time T is given by

$$E_T = max\left[V_T - D; 0\right] \tag{3.1.2}$$

$$B_T = min\left[V_T; D\right] = D - max\left[D - V_T; 0\right] = V_T - max\left[V_T - D; 0\right], \tag{3.1.3}$$

where D is the face value of a zero coupon bond.

Under the aforementioned set of simplifying assumptions, BS (1973) and Merton (1973) conclude that the firm's equity and debt can be seen as a European call option on the value of the firm with exercise price D and maturity T. The stockholders have the right but not the obligation to remain the owners of the firm by paying the exercise price D (i.e. the face value of debt). The option holder will only exercise the option if the asset value price is higher than the exercise price. We can now apply the BS option pricing formulas to determine the value of the firm's equity at time t $(0 \leq t \leq T)$. Then the equity price at time t is defined by

$$E_t = V_t\Phi(d_1)\text{-}\ D\exp(-r\,(T-t))\Phi(d_2), \tag{3.1.4}$$

where $d_1 = \frac{ln\left(\frac{V_t}{D}\right)+\left(r+\frac{\sigma^2}{2}\right)(T-t)}{\sigma_V\sqrt{T-t}}$ and $d_2 = d_1 - \sigma_V\sqrt{T-t}$. The $\Phi(\cdot)$ is the cumulative standard normal distribution function and d_1 and d_2.

Using the previous formula and knowing that, $B_t = V_t - E_t$, we can derive the value of the debt at time t,

$$B_t = V_t\Phi(-d_1)+\ D\exp(-r\,(T-t))\Phi(d_2). \tag{3.1.5}$$

On the other hand, the output value can also be obtained as the price of a riskless bond minus the price of a European put otion, then we find that debt value is also given by

$$\begin{aligned}
B_t &= D\exp(-r(T-t)) - P_t(V_t, D, T-t) && (3.1.6)\\
&= D\exp(-r(T-t)) - \left[D\exp(-r(T-t))\Phi\left(-d_2\right) - V_t\Phi\left(-d_1\right)\right] && (3.1.7)\\
&= V_t\Phi(-d_1)+\ D\exp(-r(T-t))\Phi(d_2), && (3.1.8)
\end{aligned}$$

where P_t is the price of a BS European put option, written on the value of the firm.

3.1.3 The Merton model: Estimating the asset value and asset volatility

One advantage of Merton model is to appear easy to implement in practice. However, a little thought shows two hurdles to be surmounted before the model can be applied to real-world firms - estimation of asset value and asset volatility. If all liabilities of a given firm were traded in the market, then we could easily measure the value of the firm's assets by simply adding up the market value of equity and debt. In practice, however, not all firm's debt is traded, so that we can not directly observe the market value of the firm. We present two approaches to implement the Merton model.

The iterative approach

To calculate the asset volatility σ_V we employ the iterative method proposed by Crosbie and Bohn (2003) and Vassalou and Xing (2004). This method is a relatively recent technique of calculating asset value and asset volatility and has shown considerable usefulness for better in predicting firms default probabilities. The iterative procedure has a significant advantage over the non-iterative procedure, because variability in actual market leverage is too high for the simpler approach to yield a reliable estimate of asset volatility.

Let us suppose that we would have to implement the Merton model with a one year horizon, that is our purpose would be to estimate the default probability in one year. To accomplish this task we need to estimate the asset value and volatility. The iterative procedure to estimate such unobservable variables is as follows:

1. Define a given tolerance level for convergence[2].

2. Use daily data from the past 12 months (e.g. 252 trading days) to obtain an estimate of the (historical) equity volatility σ_E. Alternatively, we may create a vector of asset pricesV_{t-a}, for $a = 0, 1, ..., 252$. The asset prices are defined as the sum of the market value of equity E_{t-a} and the book value of liabilities D_{t-a}. The market value of equity is typically defined as market capitalization and the book value of liabilities as debt in one year plus half the long-term debt. Then, set the initial value for the estimation of σ_V as the standard deviation of the log asset returns computed with the V_{t-a} vector.

3. Rearranging first the Black and Scholes (1973) equity-pricing equation for the assets value of the firm, we obtain

$$V_t = \left[E_t + D_t e^{-r_t} \Phi\left(d_2\right)\right] / \Phi\left(d_1\right), \tag{3.1.9}$$

   we use now the new σ_V in each trading day over a 12 months. This system of equations is composed by 253 equations with 253 unknowns. To compute the asset value V_{t-a} using E_{t_a} as the market value of equity and D_t as the book value of the

[2]Vassalou and Xing (2004), for instance, specify an error tolerance of 10^{-4}.

firm's liabilities of each day $t-a$, that has maturity equal to T. In that manner, we obtain a time initial serie of V_{t-a} from a time series of BS equations.

4. The next step is to calculate the standard deviation of those V_{t-a}, and the new σ_V will be used as as the input to the equation obtained in step 3 in the next iteration.

5. Repeat this procedure until the values of σ_V from two consecutive iterations converge. Our tolerance level is specified according with paper of Vassalou and Xing (2004).

For most firms, only a few iterations are necessary for σ_V to converge. Once this value is obtained, we may easily retrieve the asset value V_t through equation (3.1.9). Moreover, once daily values of V_{t-a} are estimated, we can compute the drift μ, by calculating the mean of the log asset returns of the final V_{t-a} vector.

A solution using equity values and equity volatilities

This approach to estimate the value of the firm's assets is implemented in the Merton model. Before presenting this non-iterative estimation method it is appropriate to reflect on their predictive ability. Still prior to implementing this method, we can expect some lack of effectiveness on the accuracy of the estimation of the probability of default. This is because the solution of this system of equations has a static solution and does not incorporate the dynamics of the assets value of firm, ie, the effect of leverage is not included in the estimation of parameters.

Since the market price of equity is daily observable in the market, then the market value of the firm and volatility of assets can be obtained directly using the Black-Scholes-Merton option pricing framework which recognizes equity as a call option on the underlying assets of the firm (with strike price equal to the value of debt and the same debt's maturity), that is

$$E_t = V_t\Phi(d_1)\text{- } De^{-r(\text{T-t})}\Phi(d_2), \tag{3.1.10}$$

where d_1 and d_2 are respectively given by

$$d_1 = \frac{ln\left(\frac{V_t}{D}\right) + \left(r + \frac{\sigma^2}{2}\right)(T-t)}{\sigma_V\sqrt{T-t}} \quad d_2 = d_1 - \sigma_V\sqrt{T-t}. \tag{3.1.11}$$

Moreover, the volatility of equity is related to the volatility of the underlying asset through the following local function

$$\sigma_E = \Phi\left(d_1\right)\left(\frac{V_t}{E_t}\right)\sigma_V. \tag{3.1.12}$$

Given that the market value of equity is avaliable and the equity volatility can be easily estimated, we can use these two equations to obtain an estimate of the value of assets V_t and respective volatility σ_V . This system of equations is not trivial, as we can see through the equation (3.1.10), d_1 and d_2 depend both on the asset value V_t and volatility σ_V . Thus, a numerical solution is required. Sobehart et al (2000) calculate the market value and volatility of the firm's assets from equity prices.

3.1.4 The implied credit spread of risky debt in the Merton model

Since nowadays on financial trading room is common to dealing with bonds in terms of yield rather than prices, we can obtain analytical expressions for the credit spread and the yield to maturity. The Merton's model can be used to explain risky debt yields.

Define B_t as the market price of the debt at time t and D as payment at maturity of a zero coupon-bond (ZCB). Then the yield to maturity $y_{t,T}$ is defined as the solution to:

$$B_t = De^{-y_{t,T}(T-t)}. \tag{3.1.13}$$

Rearranging the formula,

$$y_{t,T} = -\frac{ln(B_t/D)}{T-t}, \tag{3.1.14}$$

from which we can easily obtain the analytical expression of the credit spread:

$$\begin{aligned} s(t,T) &= y_{t,T} - r && (3.1.15) \\ &= -\frac{1}{T-t} ln\left(\frac{V_t\Phi(-d_1) + De^{-r(T-t)}\Phi(d_2)}{D}\right) - r && (3.1.16) \\ &= -\frac{1}{T-t} ln\left(\Phi\left(d_2\right) + \frac{V_t}{De^{-r(T-t)}}\Phi\left(-d_1\right)\right) > 0. && (3.1.17) \end{aligned}$$

Taking a closer look, we notice that the term $\frac{V_t}{De^{-r(T-t)}}$ is the inverse of the "quasi-debt ratio" $d = \frac{De^{-r(T-t)}}{V_t}$ - leverage ratio. The implied credit spread depends only on the leverage, d, the asset volatility, σ_V, and the time to maturity, $T-t$.

For $t < T$, the credit spread $s(t,T)$ is defined as the excess return on a defaultable bond. In fact, the risky bonds have a expected return higher than risk-free interest rate, ie, the yield of a corporate bond is higher than the yield of a government bond. Note also that when t tends to maturity T, the credit spread in the Merton model tends to infinity or zero, depending on whether the value of the assets at maturity is or not greater than the face value of a ZCB.

3.1.5 Default Probability

Based on this information and Merton framework, the risk-neutral probability P_D of default at time T can be calculated as:

$$P_D = \Phi(-d_2) = 1 - \Phi(d_2). \tag{3.1.18}$$

The default probability depends only on the leverage, $d = \left(\frac{V_t}{D}\right)$, the asset volatility, σ_V, the risk-free interest rate r and the time of repayment, T.

3.1.6 Advantages and disadvantages of the Merton model

The clearest advantage of the Merton model (1974) is the facility of implementation and the direct applicability of the theory of pricing European options developed by BS (1973). Thus, the Merton model allows an effective approach to assess the credit risk of a firm and to calculate the equity and debt values.

Despite its simplicity and intuitive appeal, Merton's model has several limitations:

- The most critical issue relates to the fact that the model only recognizes the firm default at maturity of debt. The behavior of the asset values of firm before maturity is not considered in the assessment of credit risk of a firm. In other words, if the value of the firm falls below the level of debt but if it is able to recover and make the payment of the debt prior to maturity, in the Merton model this fact is ignored.

- A problem that is common to all structural models, is to have as input in valuation formula, the value of company assets and the respective volatility. These variables are difficult to determine, since they are not observable in the market, such as the value of the equity of a firm. This shortcoming is overcome and studied in this work.

- In reality the capital structure of a firm is much more complicated than is assumed in the Merton model. The debt of the company is treated as a simple zero-coupon bond whose value is constant over time.

- The assumption of a flat term structure of riskless interest rates has been one of the biggest criticisms.

- Furthermore, another shortcoming of the model is the ability of the default prediction. The Merton model is unable to estimate the occurrence of jumps to the default. This is a consequence of the continuous path of geometric Brownian motion - stochastic process that to modeling the dynamic of assets value of firm. However, the assumption of continuity of the stochastic process results that default can be predicted with increasing precision as time passes.

- The assumption that liquidation/transfer of control is costless, i.e., the costs of bankruptcy are nil.

3.1.7 The empirical performance of the Merton model

The Merton model has been the target of many empirical studies, but has also been responsible for several extensions that resulted in the appearance of new models more effective. Jones et al. (1984) present the first empirical study and test the performance of the Merton model in practice. They test the ability to price corporate bonds and find that the model prices are far below the corresponding market prices. Years later, Eom et al. (2004) test five structural models and confirmed the previous study, which also reveals that bond prices were trading above their value and spreads by the Merton model. Gordon Gemmill (2002) test the performance of the Merton model with regard to spreads of the zero coupon bonds. He has shown that Merton's model works well in the particular case when zero-coupon bonds are used for funding. Campbell and Taskler (2003) find in the recent empirical work, that the levels of volatility explains well the changes in corporate bonds yield. More recently, Huang and Zhou (2008) find that the Merton model does several shortcomings in estimating CDS spreads.

In the next section, we present some extensions to the Merton model in order to address these shortcomings recognized in the literature.

3.1.8 Extensions to the Merton model

As noted earlier, the Merton model involves many simplifications and restrictive assumptions. Over the years, there has been an effort in the finance literature to overcome the shortcomings presented in the section 3.1.6. We present and discuss some extensions to the Merton model that we consider essential for a credit risk assessment model more robust, effective and better predictive ability. Thus, the extensions with regard to the capital structure of the firm, the interest rate process and the firm value process are presented.

3.1.8.1 Capital Structure

In the Merton model (1974), the firm's debt consists of a single zero coupon bond. Geske (1977, 1979) had the idea of introducing coupon bonds in the debt structure of the company, in which the payment of the coupons can be seen with a compound option and thus includes the possibility of default. In the Geske model, the debt structure is modeled with several coupon bonds. In general, shareholders have in each coupon payment date the option of making payment to bondholders, thereby ensuring the right to control the company until the maturity date of the next coupon. The shareholders make the coupon payments by issuing new equity. When the shareholders do not make a coupon payment, the firm defaults. In this case, the bondholders take over the firm and receive the assets value of firm. Geske presents a structural model with a capital structure more complex.

3.1.8.2 Fisrt-passage models

The Merton (1974) model assumes default if the assets value of the firm fall below the level of debt at maturity T. In real world, a company can default at any time and due to any financial obligation. In order to overcome this assumption, we present a new class of models called First-Passage time Models. First-Passage time Models were introduced by Black and Cox (1976), extending the Merton model, and modeled with the purpose of allowing that default can happen at any time t, and not only at the maturity date of the debt. Since there is this uncertainty of default at any time, the probability of default and the credit spread are higher in this model than in the Merton (1974) model.

On the other hand, the debt value of firm is higher than in the Merton model, the inverse situation happens with the equity value of firm. Practicioners and researchers have argued with the fact that investors have to insure against constant uncertainty of default firm.

3.1.8.3 Assets value process

The Merton model uses a diffusion process to the value of the company, as with other models proposed in this work. According to the model assumptions, the diffusion process of a company does not consider a sudden drop in the value of the company. This means that a company that is not in a difficult financial situation has a probability of default and credit spread undervalued, since any company observed in the market can suddenly be taken to default, either by external factors (which can not control) or marginal factors in the firm's assets value.

One of the approaches to overcome these problems is to include jumps in the firm's asset value process. Zhou (1997, 2001) introduced an extended Merton model with risk of random jumps in the asset value process. With this jump diffusion process a default event can occur through marginal changes in the firm's assets value (the diffusion component of V_t) or from unexpected changes in the firm value process (the jump component of V_t). In the first case, the firm value equals the default barrier at default, and, in the second case the firm value might be below the barrier at default. In Zhou's paper, the recovery rates are naturally stochastic.

One drawback of including a jump-diffusion process in structural models is that it turns the parameters estimation more difficult and are therefore less attractive for practical purposes.

3.2 The CreditGrades structural model

In this section, we describe the CreditGrades model, in which we follow the technical document of Finger (2002). The model was developed by RiskMetrics, J.P. Morgan, Goldman Sachs and Deutsche Bank to create a market benchmark in credit risk. As one of the authors of the model - RiskMetrics - the proposed model presents a more robust and realistic theoretical framework between the equity and credit market. The CreditGrades model is a structural model and hence an extension of Merton (1974) and BS (1973) model. Thus, it also assumes that the equity and debt value of the firm are modeled as an option on the asset value. The purpose of the CreditGrades model is to show the two main reasons that differ from the other structural models. First, the default probability is not determined very accurately, as the structural models. The CreditGrades model was developed in order to model credit spreads and compare with the credit spreads observed in the market. Second, the model uses approximations to the value of assets, respective volatility and drift terms, which relates these variables with other observable quantities in the market. As mentioned by Byström (2005), the CreditGrades model is a simplified version of Merton (1974) model which the probability default is only function of asset volatility and leverage ratio.

This model has also some significant advantages in yhe point of view of practical implementation because this provides a closed form solution for the pricing of credit default swaps (CDS) and, on the other hand, expresses the variables of the firm in a per share basis. Another advantage over structural models is that it overcomes the low credit sreads problem, which has been heavily criticized. This is because the firm's assets starting below the default barrier and can not retrieve only by the diffusion process. There are at least two ways to solve this problem. One proposal by Hull and White (2001) that used a time-dependent default barrier which is calibrated to market spreads. Another alternative is to incorporate two-sided jumps into the assets value process. For further study see Ozeki et al., (2011). In the CreditGrades model, the uncertainty of the default barrier may lead to the assets value reaches closer to the point of default.

Identical to the Merton (1974) model, the CreditGrades model assumes that the value of the assets of the company is driven by a geometric Brownian motion process:

$$\frac{dV_t}{V_t} = \mu_V dt + \sigma_V dW_t, \tag{3.2.1}$$

where μ_V and σ_V are two constants representing, respectively, the expected continuously compounded rate of return on the firm's assets and the volatility of the assets, and dW_t^P is a standard Brownian motion under P[3]. The model assumes that the drift rate is equal to zero, in order to maintain a constant leverage ratio.

In Merton (1974) model we discussed several limitations. One of them, is to consider that the value of company's assets evolves by a process of pure diffusion and the default barrier is fixed. In order to overcome this simplification, the CreditGrades model assumes the randomness of the default barrier by introducing a new variable - Λ - the recovery value. Thus, the default barrier can be interpreted as the amount of the company's assets remaining in the case of firm default. Additionally, the recovery rate can be different in some sectors of the industry, depending on the situation of firm default, if it is in liquidation or if the firm is default due to financial or operational problems. The random default barrier is given by

$$B_t = \Lambda D, \tag{3.2.2}$$

where D is the debt-per-share and Λ is the recovery rate. The recovery rate Λ follows a lognormal distribution with mean $\overline{\Lambda}$ and percentage standard deviation λ. The barrier is modeled as:

$$\Lambda D = \overline{\Lambda} D e^{\lambda Z - \frac{\lambda^2}{2}}, \tag{3.2.3}$$

where λ, $\overline{\Lambda} \in R^+$and Z is a standard normal random variable.

In this way, the uncertainty of the barrier is modeled as well as the level of debt. The Λ parameter is not observed with accuracy and not evolve over time. From a more practical way, this parameter is estimated by J.P. Morgan based on historical data of firms defaulted. Yu (2006) and Byström (2006) used market data to estimate the implied recovery rate and the implied standard deviation of recovery rate, minimizing the differences between the theoretical and empirical credit spreads.

With the uncertainty of the recovery rate, the default barrier can be hit unexpectedly, once the default event occurs when the assets of the firm touch in barrier for the first time.

[3]The physical measure P is a probability measure. The most common applications are seen in statistical estimations from the hedging of portfolios. The risk-neutral measure is very important in the world of mathematical finance. Under the risk-neutral measure, the expected value of the financial derivatives is discounted at the risk-free rate r.

This represents a major improvement over the Merton model.

3.2.1 Survival Probabilities

Approximated Survival Probability

The survival probability (Lardy et al., 2000) of a firm is based on the firm's ability to pay its total debt service, i.e. the asset values probability of not reaching the default barrier before time t. Based on the above assumptions, the closed-form approximation for the survival probability P_t under the CreditGrades model is

$$P_t = \Phi\left(-\frac{A_t}{2} + \frac{log(d)}{A_t}\right) - d\Phi\left(-\frac{A_t}{2} - \frac{log(d)}{A_t}\right), \tag{3.2.4}$$

where

$$d = \frac{V_0 e^{\lambda^2}}{\bar{\Lambda} D}, \tag{3.2.5}$$

and

$$A_t^2 = \sigma^2 t + \lambda^2. \tag{3.2.6}$$

The formula for approximating of the survival probability equation (3.2.4), presents some drawbacks, as regards the temporal boundary. In other words, at period of time $]-\Delta t, 0]$, there is a possibility of non-zero probability of default. Under Finger (2002), this condition may be related with some modeling assumption, more specifically the lognormality the barrier recovery rate at the default.

Exact Survival Probability

For the reason given above, the alternative closed-form solution for computing the survival probability is provided by Kiesel and Veraart (2008), which corrects the formula given in Finger (2002). In a practical way, the difference between the two approaches of the survival probability is residual. The exact survival probability at time t is given by

$$P_t = \Phi_2\left(-\frac{\lambda}{2} + \frac{ln(d)}{\lambda}, -\frac{A_t}{2} + \frac{ln(d)}{A_t}; \frac{\lambda}{A_t}\right) - d\Phi_2\left(\frac{\lambda}{2} + \frac{ln(d)}{\lambda}, -\frac{A_t}{2} - \frac{ln(d)}{A_t}; -\frac{\lambda}{A_t}\right), \tag{3.2.7}$$

where d and A_t are defined as in equations (3.2.5) and (3.2.6), respectively, and

$$\Phi_2(a, b; \rho) = \int_{-\infty}^{a} \int_{-\infty}^{b} \frac{1}{2\pi\sqrt{1-\rho^2}} \exp\left(-\frac{1}{2}\left(\frac{x^2 - 2\rho xy + y^2}{1-\rho^2}\right)\right) dxdy \tag{3.2.8}$$

is the cumulative bivariate normal distribution.

3.2.2 Credit Spreads

For a given constant free-risk interest rate r, the default probability given by equation (3.2.7) and the specific recovery rate R, we can express the continuously compound credit spread c^* as

$$c^* = r(1-R)\frac{1-P(0)+e^{r\xi}(G(t+\xi)+G(\xi))}{P(0)+P(t)e^{-rt}-e^{r\xi}(G(t+\xi)-G(\xi))}, \tag{3.2.9}$$

where $\xi = \frac{\lambda^2}{\sigma^2}$, and the function G is given by Rubinstein and Reiner (1991):

$$G(u) = d^{z+\frac{1}{2}}\Phi(-\frac{log(d)}{\sigma\sqrt{u}} - z\sigma\sqrt{u}) + d^{-z+\frac{1}{2}}\Phi(-\frac{log(d)}{\sigma\sqrt{u}} + z\sigma\sqrt{u}), \tag{3.2.10}$$

with $z = \sqrt{\frac{1}{4}+\frac{2r}{\sigma^2}}$.

Additionally, it is necessary to specify the meaning of two variables to avoid confusion. The parameter R is the expected recovery rate of the specific debt of a firm, while $\overline{\Lambda}$ is the expected average recovery rate of all classes of debt. The specific recovery rate R is lower than the $\overline{\Lambda}$ to cover more classes of debt.

3.2.3 Implementation of the CreditGrades Model

To implement the CreditGrades model, there are several parameters that need to be calibrated. Some variables are estimated from market data, such as the assets value of firm at the initial time $t = 0$, the volatility of the assets and the debt-per-share.

The debt-per-share is obtained from financial data using consolidated statements. However, first we need to determine the total amount of debt of the firm, which includes short-term and long-term borrowings and half the sum of the other short and long term liabilities. Non-financial liabilities correspond to accounts payable, deferred taxes and reserves are not included in model. The shares used to calculate the debt-per-share are common shares plus preferred shares.

The number of common shares are directly observable on the Bloomberg but can also be obtained by dividing the market capitalization by stock price. Similarly, the preferred shares are calculated by dividing the book value of preferred shares by the price of common stock on the date of reporting of book value. However, these shares are limited at the half the number of common shares.

In this model, the distance-to-default is obtained from the Itô's lemma that relates the equity and assets volatilities. The default threshold is given by:

$$\eta = \frac{1}{\sigma} ln(\frac{V_t}{\Lambda D}). \tag{3.2.11}$$

Based on the analysis of the behavior of the distance-to-default, Finger (2002) argue that the best approach to the initial value of the assets is

$$V_0 = S_0 + \overline{\Lambda} D, \tag{3.2.12}$$

where S_0 is the current stock price. According to the assets value formula, the relationship between equity and asset volatilities is given by

$$\sigma_V = \sigma_E \frac{S_t}{S_t + \overline{\Lambda} D}. \tag{3.2.13}$$

Thus, the equations of the leverage ratio (3.2.5) and (3.2.6), we have the following change

$$d = \frac{S_0 + \overline{\Lambda} D}{\overline{\Lambda} D} e^{\lambda^2}, \tag{3.2.14}$$

and

$$A_t^2 = (\sigma_E \frac{S_t}{S_t + \overline{\Lambda} D})^2 t + \lambda^2, \tag{3.2.15}$$

which allows a closed-form solution for the survival probability given by equation (3.2.4).

The mean of recovery rate $\overline{\Lambda}$ and the percentage standard deviation λ are estimated empirically and in a subjective manner, ie, several rating agencies (such as, J.P Morgan and Standard & Poor's) published empirical study based on a historical database of companies defaulted. The CreditGrades model proposes $\overline{\Lambda}$=0.5 and λ=0.3.

Chapter 4

Application to a Real Case

In this chapter, we address the model implementation and testing. We summarize our empirical results on testing the structural credit risk models, based on the stochastic process Geometric Brownian Motion to observe default probabilities and credit spreads. In order to accomplish this, we provide examples of how the credit models presented can be used to analyze and monitor changes in the credit riskiness of particular firms.

4.1 BES bank

The Banco Espírito Santo (BES) has a long history of contribution to the economic, social and cultural development of Portugal. It is known for being one of the largest financial institutions operating in Portugal and the largest national bank in the PSI-20, with a market share of 20.3% and 2.1 million customers. However, the end of first half of 2014 (July 31, 2014) the supervisor - Bank of Portugal - announced a historic loss of € 3.5 billion, driven by provisions for impairments of € 4.3 billion. With growing daily depreciation in the stock market, the regulatory minimum capital requirement of BES was 5%, below of the threshold set by regulators, i.e. 7% under Capital Requirements Directive (CRD). Thus, given that the minimum capital ratios were not secured and a growing concern about the financial strength of the Espírito Santo Group (GES), the bank was not authorized to obtain credit from their lenders. On 10 July, the Espirito Santo International (ESI) estimates a bankruptcy request, while the rating agency Moody's lowered the rating on three levels of the Espírito Santo Financial Group (ESFG), to Caa2 from B2. Regarding the bank in the table below we can see the latest revision of the rating agency Moody's in long-term debt and deposit ratings of Banco Espírito Santo, S.A.

Firm	Rating	Rating Date	Industry
BES	Ba3	11 Jul 2014	Financial Services

Table 4.1: Rating BES by Moody's. *Source*: Bloomberg

Moody's justified the rating cut, which was already off the scale of investment ('junk'), with the rise of the credit risk of ESFG. The position of the shareholders of the bank in this process were highly damaged financially, because, on July 31, Euronext announced that BES would leave the PSI-20 index next August 8, which led bank shares to be worth zero euros. Moreover, as happens in the eventual liquidation of bank, the bondholders are the first to be reimbursed. This scenario is not on the table in the case of BES because European legislation does not currently impose losses on senior creditors (senior debt). On 31 July, the debt reaching maturity in 2019 is trading at 92% of the nominal value, ie the one to which the bank agrees to repay at maturity.

After heavy falls of shares BES on market, on August 1, equity was trading in the € 0.12 under figure 4.1.

Figure 4.1: Historical equity prices of BES bank. *Source:* Bloomberg.

Similary, the volume of total equity decreased sharply in recent months, as is seen in Figure 4.2.

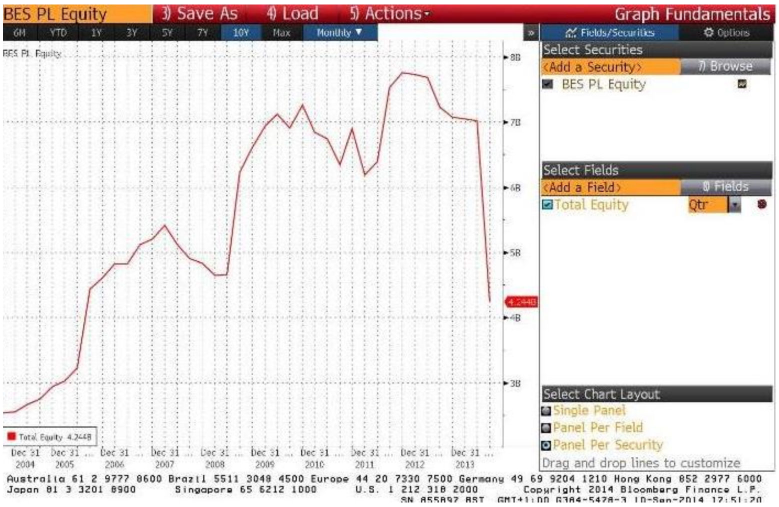

Figure 4.2: Historical volume of total equity of BES bank. *Source:* Bloomberg

A few weeks after the presentation of results on bank's solvency and the constant devaluation of assets led to the bankruptcy of bank. As such, the regulatory and supervisory authorities intervened to protect the situation of depositors and credits from banks separating the current insolvent bank into two bank (the bank "bad" and "good" bank), ie all toxic assets (the high volume of debt) were kept in the current bank BES and other assets considered "goods" (e.g. customer deposits, credits from banks and insurance Tranquilidade) are integrated in the Novo Banco together with a new presidency led by Vítor Bento. This Novo Banco present all capital requirements required to have been capitalized with funds from Troika[1].

Since this is a very recent case and given the complexity of the structure of the Espírito Santo Group (ESG), the complete responsibilities are not yet defined. However, currently the prior president of the BES - Ricardo Salgado - is released after having paid a bail of 3 millions. On 31 July, Bank of Portugal issued a statement where he admitted to criminal practice on the BES. Among others, the crimes under current investigation addressed mainly by regulatory and supervisory authorities (Bank of Portugal and Securities Market Commission - CMVM) are the fraudulent management and illicit financing.

[1]The Troika is made up of three elements, the European Commission (EC), the European Central Bank (ECB) and the International Monetary Fund (IMF). Troika team assessed that the real accounts Portugal to define the financing needs of the country. They negotiated and evaluated the financial rescue program for Portugal was composed by Jürgen Kröger (European Commission), Poul Thomsen (International Monetary Fund) and Rasmus Rüffer (European Central Bank).

4.2 Macroeconomic Framework

Despite the continuation of a trend of recovery, the first half of 2014 was marked by an increase in overall economic activity below expectations. This was especially visible in the Eurozone, which should be recorded in the second quarter, a variation of PIB growth slightly higher than that observed in the first three months of the year (0.2% compared with previous year). The activity in this economy remained penalized by the persistence of a strong euro, with adverse impacts on external demand and industrial activity. Additionally, and despite some signs of stabilization, credit to non-financial private sector remained in decline. With annual inflation at 0.5% the European Central Bank (ECB) announced in May, a reduction of reference interest rates, taking the interest rate on the main refinancing operations to 0.15% and the interest rate on the deposit facility[2] to -0.1%. The monetary authority also announced new measures to support the financing of economic activity. In this context, the 3-month Euribor fell from 0.287% to 0.207% in the first half, while the bonds yield on 10-year fell from 1.929% to 1.245%. The euro depreciated 0.7% over the same period to EUR / USD 1.369.

In Portugal, after a fall of 0.6% in first quarter, the PIB is expected to record a very slight expansion in second quarter, still penalized by the temporary drop of industrial activity. Already private consumption and activity in the services extended the recent trend of recovery. The yield of OT's to 10 years decreased from 6.13% to 3.65% in the first half, with the Portuguese Treasury aimed at accessing long-term debt markets, with emissions at 5 and 10 years, in euros and dollars. Despite a gain of 3.7% in the first six months of the year, the PSI-20 fell by 10.6% in second quarter, penalized by unfavorable developments in the financial sector.

4.3 Case studies: data and methodology

To identify estimation issues that are typical for the three models considered in chapter 3 we perform one case study. In this case study, we calculate CDS spreads and probability of default of a firm based on various input parameter sets that are estimated from that firm's market and balance sheet data. The next section presents the results.

[2]The deposit rate of the European Central Bank (ECB) is the interest rate that the ECB remunerates for deposits that banks hold at the central bank. On 5 June 2014, the ECB decided to lower this rate to -0.1%. This means that banks will have to pay to put their deposits in the ECB.

4.3.1 Data

Since BES is a Portuguese bank that has recently entered into default, we decided to analyze the performance of the models under study. To determine the input parameters for the models we use a simple methodology described below. To apply this methodology, we need approximately 252 trading days of equity prices and balance sheet data. All data was collected from Bloomberg. For each trading day between 28 June 2013 and 30 June 2014, we take the bank closing price of equity, the market capitalization, equity preferred, and the outstanding amount of short and long term debt of the bank if available. As a proxy for the risk-free interest[3] rate we take the 12-month Euribor daily rates. In this sense, as the models in study require interest rate with continuous capitalization, so we proceed to the respective change. In the CreditGrades model we take the mean recovery rate ($\overline{\Lambda}$) and standard deviatio of global recovery (λ) from empirical studies in literature in which it is assumed that $\overline{\Lambda} = 0.50$ and $\lambda = 0.30$.

4.3.2 Methodology

This section describes the methodology to determine the input parameters for models under study from the acquired data. We use a fast and simple methodology such that we can focus on estimation issues to the input parameters.

Firm and Diffusion Parameters

In the previous chapter, we presented the Merton(1974) model in which we identify the unobservability of the firm's assets value process as the main problem in applications of structural models. It requires that the diffusion parameter (σ_V) are determined from other data.

As mentioned above - in section 3.1.3 - the Merton (1974) model we applied two distinct implementations. The estimation of the parameters were estimated using the following methodology: we construct a daily time series of asset value using the balance sheet equation (assets V_t is equal to capital E_t more liabilities X_t). According to this equation the total assets of a firm equals the sum of total debt and total equity. On each trading day, we add the firm's market capitalization as total equity and the book value of debt as total debt to construct a time series of daily assets value. From this time series, we calculate the daily log returns of the firm's assets value to determine the parameters[4].

[3]Note that the risk-free interest rate is only used to discount the cash flows and not as drift in the firm's assets value process.

[4]The equity prices have a lognormal distribution, thus we take the log-returns of the time series of the firm's assets, such thatσ_V is normally distributed

The alternative methodology was also implemented in the Merton model, consists in resolution of the system of non-linear equations of equity value and respective volatility - described in equations (3.1.10) and (3.1.12). Solving this system of equations is not trivial, so are given two initial conditions for the asset value of firm and asset volatility. The initial assets value equals the book value of debt more the firm's market capitalization in initial time $t = 0$. The starting point of the assets volatility of firm are estimated based on standard deviation of daily log returns of firm's values. Another starting point often seen in the financial literature is to consider the local function given by equation (3.1.12) that relating the asset volatility with the observable equity volatility. The equity volatility is calculated from the standard deviation of daily log returns of the stock prices of equity. In order to get a better precision, we choose a error tolerance of 10^{-13}.

The CreditGrades model has a particularity that makes it more effective and accurate in the estimation of these parameters: it treats the variables in a per share basis. However, the fundamental accounting equation remains valid. In the initial assets value of firms at time $t = 0$, we have the current stock price (S_0) more the default threshold, i.e., the average recovery value of debt per share ($\overline{\Lambda}D$). The debt-per-share D is computed by dividing the liabilities by the number of shares. The various items that make up these responsibilities were already presented in section 3.2.3. The methodology applied to estimation of the assets volatility does not differ from that used in the Merton model, ie, through local function that relates the equity and asset volatility.

4.3.3 Empirical results

The models discussed in this paper were programmed in Matlab. After running the models, whose market and accounting information is between June 28, 2013 and June 30, 2014, produce the default probabilities to one-year and the credit spread for each model at the date June 30, 2014. Note that these are the most important outputs to measure the financial health of the financial institution concerned. The default probability of short-term may be viewed as forward-looking and take into account the firm's liabilities. In this sense, provide an essential aid as a quantitative measure of solvency of the financial institution, which uses current market information. The credit spread is a risk measure associated with a credit situation of the financial institution, it is normal for an institution in financial difficulty to show larger credit spreads due to the compensation of risk involved. The case study is about the BES, and the period of analysis coincides with the breakdown of its financial situation in which the reasons were mentioned in the previous chapter. The results produced by the structural models can be summarized in the following table:

Structural models	Default Probability	CDS Spreads (in basis points)
Merton (1974) - EqSolve	0%	0bps
Merton (1974) - Iterative	25.03%	76bps
CrediGrades model	30.46%	254bps

Table 4.2: Results of structural models for the BES.

Merton (1974) model

In general, we expected to obtain these results. The methodology for estimating the firm's value - using the system of nonlinear equations of the equity value and respective volatility - has some problems which we explain in the next chapter. However, in order to analyse the evolution of the default probability and credit spread, we calculate these risk indicators for the three previous quarters.

Quarters	Default Probability	CDS spread
Sep/13	0,000000000069057	0,000000000000074
Dec/13	0,000000023979961	0,000000000045853
Mar/14	0,000000068687869	0,000000000173229
Jun/14	0,000000000000000	0,000000000000000

Table 4.3: Quarterly results of Merton (1974) model using the system of nonlinear equations for the BES.

These results are counterproductive, since the fact that the value of the default probability and the CDS spread are both very small, we were already expecting. However, these two risk measures have an intuitive behavior, i.e., evolve with the deterioration of financial conditions of BES. Moreover, at the date June 30, 2014, the default probability should be higher when compared to previous quarters, but that is not true in these results, which seems a counterproductive result, as we can see by the figure below.

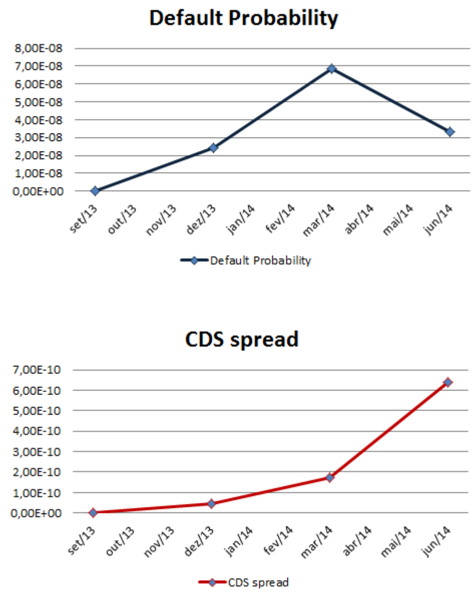

Figure 4.3: Time-series of the probability of default and the CDS spread by a system of equations of the Merton (1974) model.

If we compare the results obtained by Merton (1974) model - according to the non-linear system of equations - with the actual events, we see that the default probability does not reflect a very credible assessment of the true financial condition of the bank. Regarding the credit spread, although results are very low and without adherence to reality, the behavior of the CDS spread is is in line with the events of the bank, i.e., registering higher values when the BES is deteriorating.

As a basis of comparison, we also calculated for the previous quarters the default probability and the CDS spread, adopting the Merton (1974) model with the iterative approach. The results can be seen in the table below.

Quarters	**Default Probability**	**CDS spread**
Sep/13	15.05%	28bps
Dec/13	5.70%	8bps
Mar/14	6.62%	13bps
Jun/14	25.03%	76bps

Table 4.4: Quarterly results of Merton (1974) model using the iterative procedure for the BES.

From June 2013 until the ending of year, the risk of default BES tends to decrease slightly over time, however already at this stage revealed serious concerns (according to our model with a default probability 15.05%). For this reason, since the end of 2013 that the the Bank of Portugal has been closely watching the accounts of BES and alert the bank about the growing trend of debt and impairment charges related to the business. Although the record value of debt has been announced following the presentation of the accounts from BES for the first half of 2014, in the first quarter, BES presented their accounts with a prejudice never before reported by the bank. This was the turning point, i.e., it was from the first quarter of the year that the markets began to pay more attention to the degradation of BES, recording historical depreciation in the stock market. It is in this scenario that the default probability and credit spread given by the Merton (1974) model, under the iterative approach, reflects better the reality.

In contrast with the non-linear system of equations approach, the probability of default and CDS spreads calculated by iterative procedure are more realistic, given the notorious and recognized ability to predict default. In this work, these advantages can be verified from Table 4.2, which shows the difference of estimation methods of the firm's variables through the results. Another widely advantage recognized in the literature (Vassalou and Xing (2004) and Crosbie and Bohn (2002)) is the fact that the estimation of firm's value are calculated only using observable market information. On the one hand, yields more accurate estimates and on the other allows to evaluate the credit risk is taken almost to the moment, being possible to generate time series of the default probability and CDS spread. The main reason for the success of this iterative method applied to the Merton (1974) model is revealed in the next chapter.

CreditGrades model

Of all the structural models presented in this work, the CreditGrades model is the one with default values of probability and CDS spreads more reliable and accurate. The reason focuses on capital structure more complete, and therefore closest to the reality of the market, the input parameters are all observable in the market and the default threshold is volatile, thus incorporating the uncertainty of the market when the liabilities of a firm.

In general, it is difficult to assess whether a model overestimates or underestimates the default probability. However, we could be induced in error - once the default probability and CDS spread is larger than the other model under study - the CreditGrades model overestimates these risk measures, once through the models do not know the behavior of the stock the second half of 2014.

Rating Agencies	**30-06-2014**	**31-07-2014**
S&P	BB	B3
Moody's	Ba3	B-
Fitch	BB+	-

Table 4.5: Comparison of the ratings of the three largest rating agencies. *Source:* Investor Report - 30 June 2014 and Banco Espírito Santo Special Report.

But comparing with the market view concerning the financial condition of the BES, we can observe that the model CreditGrades predicted more accurately the credit risk of the bank. On the other hand, we can still compare the estimation of credit spread by our model with the implicit risk of CDS spread estimated by Bloomberg. Although the time series does not have the most logical and intuitive path of the credit spread of a bank nearly bankrupt, the residual differences lies in 9 basis points, as we can see in the figure below.

Figure 4.4: Relationship between stock prices and implicit risk in the CDS spread calculated by Bloomberg. *Source:* Bloomberg

In the CreditGrades model we decided to make the parallelism of the closed-form solutions to compute the survival probability using the exact and the approximate formula. On the other hand, we change the parameter of the model - the Standard Deviation of Global Recovery - which is preset in our model and assumes a value of 0.3 (such as in the theoretical document of the CreditGrades model), for a Deviation of Global Recovery of 0.1. The reason for this, lies in the work of Veraart (2008), which justifies the change of this parameter, stating that the financial sector is heavily regulated, so λ may be lower. The results of this parameter change in exact and approximate default probability, are as follows:

Parameter λ	**Exact Default Prob.**	**Approximate Default Prob.**	**CDS spread**
λ=0.3	30.465%	43.054%	254bps
λ=0.1	15.680%	16.400%	59bps

Table 4.6: Change of Global Recovery Standard Deviation (λ=0.3 to λ=0.1) in the exact and approximate survival probability of the CreditGrades model.

These changes in the volatility of the recovery rate, leading to significant changes in default probabilities and distance themselves from the results of Merton (1974) model, according iterative procedure. On the other hand, it seems that the higher the volatility of the recovery rate, the higher differences between closed-form solutions of the default probability.

Chapter 5

Conclusions

The aim of this work is to implement and analyze the adequacy of structural models to real life, in order to make the link between the theoretical models (often developed by academics) and the applicability to any one quoted company in the market (i.e., the professional reality). The models presented in this work - Merton (1974) model and CreditGrades model - are classics in the literature of structural models. In the Merton (1974), we use model used two approaches for estimating the firm's variables to solve the main problem of structural models, ie, the firm's variables are not directly observable in the market. We use the solution of the non-linear equations system, of the equity value and respective volatility, and the iterative approach proposed and studied by Vassalou and Xing (2004), Crosbie and Bohn (2002).

Concerning the approach of nonlinear equations system, this does not produce consistent and accurate values for the default probability and credit spread. This is because it considers only the instant and can not incorporate market dynamics, i.e., the differences in market leverage over time. This is also confirmed by Duan (1994), which mentions some drawbacks of this approach: First, consider the constant equity volatility over time and independent of assets value and time; Second, the equation of the equity value is redundant, since it is only considered to derive the second equation; Third, the traditional approach does not provide distribution functions or confidence intervals for the estimates of the variables of the firm (V_t and σ_V). For a broader knowledge of this approach, see Duan (1994) and Ronn and Verma (1986).

There are some papers of researchers (Crouhy, Galai, Mark (2000), Deliandes and Geske (2003) and (Ho Eom, Helwege &Huang, 2004)), which claim that the default probability calculated by structural models is seen as an upper bound of the objective probability of default. The reason for this is related to market leverage. When the market leverage decreases suddenly, the asset volatility increases given that the company is more exposed to credit risk. It will tend to be overestimated and the same happens the default probability. In the opposite scenario, when the market leverage suddenly grow, the asset price volatility, as well as the default probability is underestimated, since the firm's debt is falling, so the credit risk of the firm decreases. These little reasonable and consistent results are given by the formula (3.1.13). The iterative procedure has a significant advantage over non-iterative procedure by the reasons given above, i.e., the variability of market leverage is great for a model does not incorporate this dynamic, since it is an essential factor to produce an reliable estimate of asset volatility.

The CreditGrades model presents a closed-form solution to calculate the survival probability using two distinct formulas. In 2002, Finger presented an approximate formula and years later Veraart and Kiesel (2008) corrected and provided the exact probability of survival. According to Finger (2002), the difference between the exact and approximate formula is small. However, in our application to the BES, we find the significant differences between the formulas of the default probability. With the change in volatility of the recovery rate, we obtain probabilities far below those calculated by Standard Deviation of Global Recovery, as well as the CDS spread that came down from 254bps to 59bps.

This work provides an important insight into the monitoring of the model CreditGrades during the financial crisis, even more, being this the model that more accurately estimated the indicators of credit risk. On the other hand, provides a survey of the most important factors that are missing in the others structural models. Lastly, still provides a clear reference guide to the pricing of CDS, since this model is used by many professionals for the purpose of negotiation and evaluation of results of their portfolios.

5.1 Further Studies

Throughout this work three models were presented for assessing and measuring the credit risk of a given quoted firm. Some of these models have assumptions which restrict its performance and thus yields some unrealistic values.

In this regard, we present one of the improvements suggested in the Merton (1974) model, but now for the CreditGrades model. This extension to the original CreditGrades model presented by Finger (2002) and Stamicar and Finger (2006), is to include the possibility of jumps in the dynamic process of the firm's assets. The presence of jumps in the firm value is modeled by Lévy processes, with the purpose of describing the discontinuous dynamic of asset value in a certain company. In this scenario, this new extension has almost closed formulas for the equity pricing options and for the calculation of the survival probability. Moreover, since the original model is widely used by researchers to make the pricing CDS, such as Chu Yu and Zhong (2011), it would be interesting to investigate this new approach to assess the effect of jumps on the value of the company, on the short-term credit spreads and equity volatility skew. A good guideline to implement this extension is given by Ozeki, Yamazaki, Umezawa and Yoshikawa (2011).

Moreover, the models presented in this work assume that the interest rate r is constant over time. As we know, this is another unrealistic assumption of structural models. As such, and in order to improve the performance of the original model CreditGrades, we suggest including a process of stochastic interest and adopting the valuation model for noncallable and callable corporate bonds, according to the paper of Kim (1993). Thus, it would be possible to further analysis and a more consistent and realistic comparison between market risk and credit risk, and thus the default probability and credit spread would make a report of the most appropriate market risk.

There are many other topics of interest to investigate and improve the structural models, however we believe that these two proposals make them more practical models and above all more realistic in the eyes of financial markets.

Bibliography

[1] Black and Scholes (1973). "The Pricing of Options and Corporate Liabilities", Journal of Political Economy, Volume 81, pp. 637-659.

[2] Benjamin Yibin Zhang, Hao Zhou and Haibin Zhu (2006), Explaining Credit Default Swap Spreads with the Equity Volatility and Jump Risks of Individual Firms, Review of Financial Studies, Vol. 22, No. 12, pp. 5099-5131.

[3] Bystrom, H. (2006), CreditGrades and the iTraxx CDS index market. *Financial Anal. J.*, 62(6), pp. 65–76.

[4] Cao, Charles, Fan Yu, and Zhaodong Zhong, (2011), Pricing Credit Default Swaps with Option-Implied Volatility, Financial Analysts Journal 67.

[5] Christopher C. Finger (2002), CreditGrades: Technical Document, RiskMetrics Group, Inc., New York.

[6] Crouhy, Galai, Mark, (2000), A comparative analysis of current credit risk models, Vol. 24, pp. 59-117.

[7] Delianedis and Geske. (February 2003). Credit Risk and Risk Neutral Default Probabilities: Information About Rating Migrations and Defaults. *EFA 2003 Annual Conference Paper No. 962.*

[8] Duan (1994), Maximum Likelihood Estimation Using Price Data of the Derivative Contract, Journal Mathematical Finance, Vol. 4, No. 2, pp. 155-167.

[9] Duffie and Wang, (2004), Multi-Period Corporate Failure Prediction with Stochastic Covariates, National Bureau of Economic Research, No. 10743, http://www.nber.org/papers/w10743.

[10] E. Philip Jones and Scott P. Mason and Eric Rosenfeld, (1984), Contingent Claims Analysis of Corporate Capital Structures: An Empirical Investigation, Journal of Finance, Vol. 39, No. 3, pp. 611-625.

[11] Einarsson (2008), Credit Risk Modeling, Technical University of Denmark, IMM-PHD: ISSN 0909-3192.

[12] Fan Yu (2006), How profitable is capital structure arbitrage?, Financial Analysts Journal, Vol. 62, No. 5, pp. 47-62.

[13] Fischer Black and John C. Cox (1976), Valuing Corporate Securities: Some Effects of Bond Indenture Provisions, Journal of Finance, Volume 31, 351-367.

[14] Geske, Robert, "The Valuation of Corporate Liabilities as Compound Options", Journal of Financial and Quantitative Analysis, Vol. 12, No. 4, (November 1977), pp. 541-552.

[15] Hillegeist, Keating, Cram and Lundstedt, (March 2004), Assessing the Probability of Bankruptcy, Review of Accounting Studies, Vol. 9, No. 1, pp. 5-34.

[16] Ho Eom, Helwege & Huang, (2004), Structural Models of Corporate Bond Pricing: An Empirical Analysis, Vol. 17, No. 2, pp. 499-544.

[17] Joseph Ogden (1987), Determinants of the Ratings and Yields on Corporate Bonds: Tests of the Contingent Claims Model, Journal of Financial Research, Vol. 10, No. 4, pp. 329-339.

[18] Kealhofer and Kurbat, (2001), Predictive Merton models, www.risk.net/data/Pay_per.../0202_kealhofer.pdf.

[19] Kiesel, Rüdiger and Veraart, Luitgard A. M. (2008) A note on the survival probability in CreditGrades. Journal of Credit Risk, Vol. 4, No. 2, pp. 65-74.

[20] Hull, John, "Options, Futures, and Other Derivatives", Prentice Hall 4th Edition.

[21] Lando (2004), Credit Risk Modeling: Theory and Applications, Princeton University Press, New Jersey.

[22] Lardy, J.-P., Finkelstein, V., Khuong-Huu, P. and Yang, N. (2000). Method and System for Determining a Company's Probability of No Default, Internal document, JPMorgan.

[23] Löffer, Gunter and Peter N. Posch, 2011, Credit Risk Modeling Using Excel and VBA, John Wiley & Sons, ISBN: 978-0-470-03157-5.

[24] Longstaff and Gandhi, (January 2010), Counterparty Credit Risk and The Credit Default Swap Market, http://www.defaultrisk.com/pp_crdrv183.htm.

[25] Merton (May 1974), On the pricing of Corporate Debt: The Risk Structure of Interest Rates, Robert Merton, Journal of Finance, Vol. 29, No. 2, pp. 449-470.

[26] Merton's Model, Credit Risk and Volatility Skews, Journal of Credit Risk, Vol. 1, No. 1, pp. 3-28, Winter 2004/05.

[27] Oldrich A. Vasicek (March 1984), Credit Valuation, Working Paper, KMV Corporation.

[28] Owen (1956), Tables for computing Bivariate Normal Probabilities, The Annals of Mathematical Statistics, Volume 27, No. 4 , pp. 1075-1090.

[29] Peter Crosbie and Jeffrey R. Bohn (2003), "Modeling Default Risk. Modeling Methology.", Technical Document, Moody's KMV.

[30] Philipp Schönbucher (2003), Credit derivatives pricing models: Models, pricing and implementation. John Wiley & Sons.

[31] Ronn and Verma (1986), Pricing Risk-Adjusted Deposit Insurance: An Option-Based Model, Journal of Finance, Vol. 41, issue 4, pp. 871-95.

[32] Schoutens and Cariboni (July 2009), Lévy Processes in Credit Risk, John Wiley & Sons.

[33] Sobehart and Stein, (2000), Validation methodologies for default risk models.

[34] Vassalou and Xing, (April 2004), Default Risk in Equity Returns, Journal of Finance, Vol. 59, No. 2, pp. 831-868.

[35] The Analysis of KMV-Merton Model in Forecasting Default Probability, Science and Engineering Research, 2002.

[36] Credit risk modeling and CDS valuation, An analysis of structural models, Master thesis J.A.G. van Beem, April, 2010.