

1941. Using negative velocity feedback controller to reduce the vibration of a suspended cable

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Abstract. The system of suspended cable with mixed excitation forces is controlled in this paper by using negative linear velocity feedback controller. The equations of motion of this system contain quadratic and cubic nonlinearities. The multiple scale perturbation technique is applied to determine the response of the nonlinear system near the simultaneous sub-harmonic and combined resonance case of this system. The stability of the obtained numerical solution is investigated using frequency response equations. The effect of different parameters on the vibrating system are investigated and reported.

Keywords: suspended cable, negative velocity feedback, mixed forces, stability.

1. Introduction

Suspended cables are lightweight, flexible structural elements used in numerous applications in mechanical, civil, electrical, ocean, and space engineering due to their capability of transmitting forces, carrying payloads, and conducting signals across large distances. At the same time, the suspended cable is a basic element of theoretical interest in applied mechanics and an archetypal model of various phenomena in dynamics. Arafat and Nayfeh [1] studied the motion of shallow suspended cables with primary resonance excitation. The method of multiple scales is applied to study nonlinear response of this suspended cables, its stability and the dynamic solutions. Some interesting work on the nonlinear dynamics of cables to harmonic excitations can be found in the review articles by Rega [2, 3]. Zheng et al. [4] considered the super-harmonics and internal resonance of a suspended cable with almost commensurable natural frequencies. Zhang and Tang [5] investigated the chaotic dynamics and global bifurcations of the suspended inclined cable under combined parametric and external excitations. Chen and Xu [6] investigated the global bifurcations of the inclined cable subjected to a harmonic excitation leading to primary resonances with the external damping by using averaging method. Benedettini et al. [7] studied nonlinear oscillations of a four-degree-of-freedom model of the suspended cables under multiple internal resonance conditions. They have established that the response of suspended cables near or away from the first crossover can be large and exhibit very complex behavior due to the simultaneous presence of multiple internal resonances involving several in-plane modes and out-of-plane modes.

Kamel and Hamed [8] analyzed the nonlinear behavior of an elastic cable subjected to harmonic excitation near the simultaneous principle primary and internal resonance using multiple scale method. The numerical solutions and chaotic response of the nonlinear system of elastic cable for different parameters are also studied. Abe [9] investigated the accuracy of nonlinear vibration analyses of a suspended cable, which possesses quadratic and cubic nonlinearities, with 1:1 internal resonance. The nonlinear dynamics of suspended cable structures have been studied with 2:1 internal resonances by the authors [10, 11]. Chen et al. [12] studied the bifurcations and chaotic dynamics of the parametrically and externally excited suspended elastic cable. A sample suspended cable, representing a physical model is considered as the case study, and non-collocated

feedback, based on active transverse control, is considered as a final application of the state observer. Also, active feedback control for cable vibrations is studied by Ubertaini [13] for analytical and numerical models. A suitable dimensional analytical Galerkin model is derived to investigate the effectiveness of the feedback control, which represents the final application of the state observer.

Wang and Zhao [14-16] applied different methods to investigate the nonlinear response of the suspended cable with three-to-one internal resonance, and numerical simulations are used to illustrate the chaotic dynamics of the cable. They also extended the previous work to consider the out-of-plane motion of a shallow suspended cable [17]. The three-to-one internal resonance between the third and the first symmetric in-plane modes and the one-to-one internal resonance between the third symmetric in-plane mode and the third symmetric out-of-plane mode are taken in to account. The case of the primary resonance of the first symmetric mode is also considered. Sofi and Muscolino [18] studied the dynamics of suspended cables with small sag-to-span ratios carrying an array of moving oscillators with arbitrarily varying velocities. Numerical results demonstrate that, despite the basis functions are continuous; the improved series enables to capture with very few terms the abrupt changes of cable profile at the contact points between the cable and the moving oscillators. Wang and Rega [19] obtained the 3D nonlinear equations of motion of the suspended cable with moving mass via the Hamilton principle, and its transient linear planar dynamics is investigated. They studied the transient response of the suspended cable subjected to a sequence of masses moving with constant velocity. The numerical results show that the effect of the velocity on the maximum mid span displacement is very small. Huang et al. [20] obtained the nonlinear ordinary differential equations (ODEs) by the Galerkin method from the nonlinear partial differential equations (PDEs) to describe the forced vibration of the coupled structure of a suspended-cable-stayed beam. The multiple-scale method is employed to solve the ODEs. They found that there are typical jumps and saturation phenomena of the vibration amplitude in the structure and the structure may present quasi-periodic vibration or chaos, if the stiffness of the cable stays membrane and frequency of external excitation are disturbed. The control strategies proposed in the literature often involve the application of control command to various parts of the crane, such as the cables [21-23]. Hamed and Amer [24] studied an active vibration controller for suppressing the vibration of the non-linear composite beam subjected to parametric excitation force in the presence of 1:2 internal resonances. The numerical results show that the saturation control of steady state vibrations is efficient. Al-Qassab et al. [25] investigated the dynamic behavior of an elastic horizontal and inclined cable with a moving mass along its length. Kim and Chang [26] analyzed the free vibration and presented a dynamic stiffness matrix for an inclined cable. F. Xu et al. [27] introduced an analyzing method and initial experimental method to study coupling dynamic characteristics of cable-robot system. The experimental results validate the theoretical vibration characteristics, including frequency, vibration amplitude, and acceleration.

The aim of this work is to control a two degree of freedom nonlinear differential equations of a suspended cable having quadratic and cubic nonlinearities subjected to mixed excitations via a negative velocity feedback controller. The method of multiple scales perturbation [28] is applied to solve the nonlinear differential equations describing the controlled system up to second order approximations. The behavior of the system is studied applying Runge-Kutta fourth-order method. The stability of the proposed analytic nonlinear solution near the simultaneous sub-harmonic and combined resonance case is studied numerically. The stability of the system is investigated applying frequency response equations. The effect of different parameters on the steady state responses of the vibrating system is studied and discussed from the frequency response curves.

2. Mathematical reduction from PDEs to ODEs

Following Benedettini et al. [7], we express the dimensional non-planar equations of motion for a shallow suspended elastic cable hanging at fixed supports and caused by axial excitation and horizontal load as shown in Fig. 1, is as follows:

$$\begin{aligned}
 m \frac{\partial^2 \hat{u}_1}{\partial \hat{t}^2} + 2\hat{c}_1 \frac{\partial \hat{u}_1}{\partial \hat{t}} - H \frac{\partial^2 \hat{u}_1}{\partial \hat{x}^2} &= \frac{EA}{L} \left(\frac{\partial^2 \hat{u}_1}{\partial \hat{x}^2} + \frac{d^2 \hat{\psi}}{d\hat{x}^2} \right) \\
 &\times \int_0^L \left[\hat{F}_1 \cos \hat{\Omega}_1 t + \frac{\partial \hat{u}_1}{\partial \hat{x}} \frac{d\hat{\psi}}{d\hat{x}} + \frac{1}{2} \left(\left(\frac{\partial \hat{u}_1}{\partial \hat{x}} \right)^2 + \left(\frac{\partial \hat{u}_2}{\partial \hat{x}} \right)^2 \right) \right] d\hat{x}, \\
 m \frac{\partial^2 \hat{u}_2}{\partial \hat{t}^2} + 2\hat{c}_2 \frac{\partial \hat{u}_2}{\partial \hat{t}} - H \frac{\partial^2 \hat{u}_2}{\partial \hat{x}^2} &= \frac{EA}{L} \frac{\partial^2 \hat{u}_2}{\partial \hat{x}^2} \\
 &\times \int_0^L \left[\hat{F}_1 \cos \hat{\Omega}_1 t + \frac{\partial \hat{u}_1}{\partial \hat{x}} \frac{d\hat{\psi}}{d\hat{x}} + \frac{1}{2} \left(\left(\frac{\partial \hat{u}_1}{\partial \hat{x}} \right)^2 + \left(\frac{\partial \hat{u}_2}{\partial \hat{x}} \right)^2 \right) \right] d\hat{x} + \hat{F}(\hat{x}) \cos \hat{\Omega} \hat{t},
 \end{aligned} \tag{1}$$

where, for a sag-to-span ratio $\hat{b}/L \leq 1/8$, the static equilibrium profile $\hat{\psi}(x)$ of the cable may be sufficiently approximated by a parabola. Hence:

$$\hat{\psi}(x) = 4\hat{b} \frac{\hat{x}}{L} \left(1 - \frac{\hat{x}}{L} \right), \quad H = \frac{mgL^2}{8\hat{b}}. \tag{2}$$

where, $\hat{u}_1(\hat{x}, \hat{t})$ and $\hat{u}_2(\hat{x}, \hat{t})$ denote the in-plane (in-plane; i.e., the plane defined by the initial static configuration of the cable) and out-of-plane displacements at position \hat{x} at time \hat{t} , m is the mass per unit length, \hat{b} is the cable sag, L is the cable span, E is Young's modulus, A is the area of the cross section, \hat{c}_i are viscous damping coefficients, and g is the gravitational acceleration.

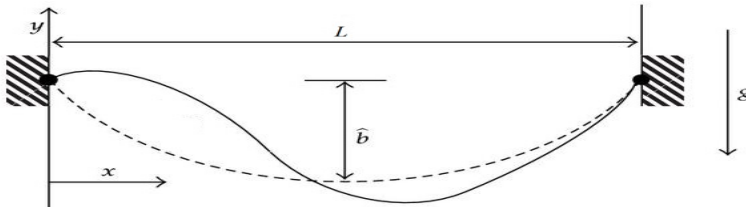


Fig. 1. A schematic diagram of a shallow suspended cable

The boundary conditions are:

$$\hat{u}_1 = \hat{u}_2 = 0 \text{ at } x = 0 \text{ and } x = 1. \tag{3}$$

We introduce non-dimensional quantities defined by:

$$x = \frac{\hat{x}}{L}, \quad u_i = \frac{\hat{u}_i}{L}, \quad b = \frac{\hat{b}}{L}, \quad \psi = \frac{\hat{\psi}}{\hat{b}}, \quad t = \gamma \hat{t}, \tag{4}$$

into Eqs. (1) and (2), where the characteristic time $1/\gamma$ will be chosen at the end of the analysis, and obtain the following non-dimensional non-planar equations of motion:

$$\begin{aligned}
 \ddot{u}_1 + 2c_1 \dot{u}_1 - u_1'' &= (\alpha u_1'' + b\psi'') \int_0^1 \left[F_1 \cos \Omega_1 t + b\psi' u_1' + \frac{1}{2} (u_1'^2 + u_2'^2) \right] dx, \\
 \ddot{u}_2 + 2c_2 \dot{u}_2 - u_2'' &= \alpha u_2'' \int_0^1 \left[F_1 \cos \Omega_1 t + b\psi' u_1' + \frac{1}{2} (u_1'^2 + u_2'^2) \right] dx + F(x) \cos \Omega t,
 \end{aligned} \tag{5}$$

where $\psi(x) = 4x(1 - x)$ and the boundary conditions are:

$$u_1 = u_2 = 0 \text{ at } x = 0 \text{ and } x = 1. \quad (6)$$

The overdot and prime indicate the derivatives with respect to t and x , respectively, and:

$$\gamma^2 = \frac{H}{mL^2}, \quad \alpha = \frac{EA}{H} = \frac{8\hat{b}EA}{mgL^2}, \quad F_1 = \frac{L\hat{F}_1}{H}, \quad F = \frac{L\hat{F}}{H}, \quad c_i = \frac{L^2\gamma}{H} \hat{c}_i, \quad \Omega_1 = \frac{\hat{\Omega}_1}{\gamma}, \quad \Omega = \frac{\hat{\Omega}}{\gamma}. \quad (7)$$

In order to analyze the nonlinear responses of this system, we first discretize Eq. (5) using the Galerkin procedure. To this end, we express the in-plane and out-of-plane displacements u_1 and u_2 as:

$$u_1(x, t) = \phi_n(x)x(t), \quad u_2(x, t) = \varphi_n(x)y(t), \quad (8)$$

where the $\phi_n(x)$ and $\varphi_n(x)$ are the modes shapes of the linearized problem, $x(t)$ and $y(t)$ are generalized coordinates. Substituting Eq. (8) into Eq. (5), the two-degree-of-freedom nonlinear equations can be obtained as follows:

$$\ddot{x} + 2\varepsilon^2 c_x \dot{x} + (\omega_x^2 + 2\varepsilon^2 \Omega f_1 \cos \Omega_1 t)x + \varepsilon \alpha_2 x^2 + \varepsilon \beta_2 y^2 + \varepsilon^2 \gamma_2 x^3 + \varepsilon^2 \eta_2 x y^2 = 0, \quad (9)$$

$$\ddot{y} + 2\varepsilon^2 c_y \dot{y} + (\omega_y^2 + 2\varepsilon^2 \Omega f_2 \cos \Omega_1 t)y + \varepsilon \alpha_3 x y + \varepsilon^2 \gamma_3 y^3 + \varepsilon^2 \eta_3 x^2 y = 2\varepsilon^2 \Omega F \cos \Omega t, \quad (10)$$

where x and y refer to the vertical (in-plane) and horizontal (out-of-plane) displacements, respectively, c_x and c_y are viscous damping coefficients, $0 < \varepsilon \ll 1$ is a small parameter, ω_x and ω_y respectively, represent the natural frequencies associated with the anti-symmetric or symmetric in-plane and out-of-plane modes, $\alpha_i, \gamma_i, \eta_i, \beta_2$ ($i = 2, 3$) are nonlinear coefficients, Ω, Ω_1 are the excitation frequencies, f_1, f_2 are the parametric excitation force amplitudes of the cable and F is the external excitation force amplitude of the cable.

3. Perturbation analysis

Our attention is focused on a suspended cable excited by harmonic, parametric and tuned excitation forces. The two-degree-of- freedom differential equations describing the nonlinear dynamics of the suspended cable [12] can be investigated as:

$$\ddot{x} + 2\varepsilon c_x \dot{x} + (\omega_x^2 + 2\varepsilon \Omega f_1 \cos \Omega_1 t)x + \varepsilon \alpha_2 x^2 + \varepsilon \beta_2 y^2 + \varepsilon \gamma_2 x^3 + \varepsilon \eta_2 x y^2 = 2\varepsilon \Omega (F \sin \Omega t + Q \cos \Omega_2 t \sin \Omega_3 t) + R_1, \quad (11)$$

$$\ddot{y} + 2\varepsilon c_y \dot{y} + (\omega_y^2 + 2\varepsilon \Omega f_2 \cos \Omega_1 t)y + \varepsilon \alpha_3 x y + \varepsilon \gamma_3 y^3 + \varepsilon \eta_3 x^2 y = 2\varepsilon \Omega (F \cos \Omega t + Q \cos \Omega_2 t \cos \Omega_3 t) + R_2, \quad (12)$$

where $R_1 = -\varepsilon G_1 \dot{x}$ and $R_2 = -\varepsilon G_2 \dot{y}$ are the control forces which added to the system, Ω_2, Ω_3 are the excitation frequencies, Q is the tuned excitation force amplitude of the cable and G_1, G_2 are the positive constants (gains).

Assuming the solution of Eqs. (11) and (12) in the forms:

$$x(\varepsilon; t) = x_0(T_0, T_1) + \varepsilon x_1(T_0, T_1) + O(\varepsilon^2), \quad (13)$$

$$y(\varepsilon; t) = y_0(T_0, T_1) + \varepsilon y_1(T_0, T_1) + O(\varepsilon^2), \quad (14)$$

where, $T_n = \varepsilon^n t$, ($n = 0, 1$), T_0 and T_1 are the fast and slow time scales respectively.

The derivatives will be in the form:

$$\frac{d}{dt} = D_0 + \varepsilon D_1 + \dots, \quad (15)$$

$$\frac{d^2}{dt^2} = D_0^2 + 2\varepsilon D_0 D_1 + \dots \tag{16}$$

where $D_n = \partial/\partial T_n, n = 0, 1$.

Substituting from Eqs. (13) and (14) into Eqs. (11) and (12) and equating the same power of ε we have:

$$\varepsilon^0: (D_0^2 + \omega_x^2)x_0 = 0, \tag{17}$$

$$(D_0^2 + \omega_y^2)y_0 = 0, \tag{18}$$

$$\varepsilon^1: (D_0^2 + \omega_x^2)x_1 = -2D_0 D_1 x_0 - 2c_x D_0 x_0 - 2\Omega x_0 f_1 \cos \Omega_1 t - \alpha_2 x_0^2 - \beta_2 y_0^2 - \gamma_2 x_0^3 - \eta_2 x_0 y_0^2 + 2\Omega F \sin \Omega t + 2Q \Omega \cos \Omega_2 t \sin \Omega_3 t - G_1 (D_0 x_0), \tag{19}$$

$$(D_0^2 + \omega_y^2)y_1 = -2D_0 D_1 y_0 - 2c_y D_0 y_0 - 2\Omega y_0 f_1 \cos \Omega_1 t - \alpha_3 x_0 y_0 - \gamma_3 y_0^3 - \eta_3 y_0 x_0^2 + 2\Omega F \cos \Omega t + 2Q \Omega \cos \Omega_2 t \cos \Omega_3 t - G_2 (D_0 y_0). \tag{20}$$

The general solutions of Eqs. (17) and (18) can be written in the form:

$$x_0(T_0, T_1) = A_0(T_1) \exp(i\omega_x T_0) + cc, \tag{21}$$

$$y_0(T_0, T_1) = B_0(T_1) \exp(i\omega_y T_0) + cc, \tag{22}$$

where A_0, B_0 are complex functions in T_1 and cc denotes the complex conjugate functions. Substituting Eqs. (21) and (22) into Eqs. (19) and (20), we get:

$$\begin{aligned} (D_0^2 + \omega_x^2)x_1 = & [-2i\omega_x c_x A_0 - 2i\omega_x A_0' - 3\gamma_2 A_0^2 \bar{A}_0 - 2\eta_2 A_0 B_0 \bar{B}_0 - i\omega_x G_1 A_0] e^{i\omega_x T_0} \\ & - [\alpha_2 A_0^2] e^{2i\omega_x T_0} - [\gamma_2 A_0^3] e^{3i\omega_x T_0} - [\beta_2 B_0^2] e^{2i\omega_y T_0} \\ & - [\alpha_2 A_0 \bar{A}_0 + \beta_2 B_0 \bar{B}_0] - [\Omega f_1 A_0] e^{i(\Omega_1 + \omega_x) T_0} - [\Omega f_1 \bar{A}_0] e^{i(\Omega_1 - \omega_x) T_0} - [i\Omega F] e^{i\Omega T_0} \\ & - [\eta_2 A_0 B_0^2] e^{i(\omega_x + 2\omega_y) T_0} - [\eta_2 A_0 \bar{B}_0^2] e^{i(\omega_x - 2\omega_y) T_0} - \left[\frac{i}{2} \Omega Q\right] e^{i(\Omega_2 + \Omega_3) T_0} \\ & + \left[\frac{i}{2} \Omega Q\right] e^{i(\Omega_2 - \Omega_3) T_0} + cc, \end{aligned} \tag{23}$$

$$\begin{aligned} (D_0^2 + \omega_y^2)y_1 = & [-2i\omega_y c_y B_0 - 2i\omega_y B_0' - 3\gamma_3 B_0^2 \bar{B}_0 - 2\eta_3 B_0 A_0 \bar{A}_0 - i\omega_y G_2 B_0] e^{i\omega_y T_0} \\ & - [\gamma_3 B_0^3] e^{3i\omega_y T_0} + [-\alpha_3 A_0 B_0] e^{i(\omega_x + \omega_y) T_0} + [-\alpha_3 A_0 \bar{B}_0] e^{i(\omega_x - \omega_y) T_0} \\ & + [-\eta_3 B_0 A_0^2] e^{i(\omega_y + 2\omega_x) T_0} - [\eta_3 B_0 \bar{A}_0^2] e^{i(\omega_y - 2\omega_x) T_0} - [\Omega f_2 B_0] e^{i(\Omega_1 + \omega_y) T_0} \\ & - [\Omega f_2 \bar{B}_0] e^{i(\Omega_1 - \omega_y) T_0} + [\Omega F] e^{i\Omega T_0} + \left[\frac{1}{2} \Omega Q\right] e^{i(\Omega_2 + \Omega_3) T_0} + \left[\frac{1}{2} \Omega Q\right] e^{i(\Omega_2 - \Omega_3) T_0} + cc. \end{aligned} \tag{24}$$

Eliminating the secular terms of Eqs. (23) and (24) to get bounded solutions, then the general solution of the resulting equations obtained as:

$$\begin{aligned} x_1 = & A_1 e^{i\omega_x T_0} + \frac{1}{3\omega_x^2} [\alpha_2 A_0^2] e^{2i\omega_x T_0} + \frac{1}{8\omega_x^2} [\gamma_2 A_0^3] e^{3i\omega_x T_0} - \frac{1}{(\omega_x^2 - 4\omega_y^2)} [\beta_2 B_0^2] e^{2i\omega_y T_0} \\ & - \frac{1}{\omega_x^2} [\alpha_2 A_0 \bar{A}_0 - \beta_2 B_0 \bar{B}_0] - \frac{1}{(\omega_x^2 - (\Omega_1 + \omega_x)^2)} [\Omega f_1 A_0] e^{i(\Omega_1 + \omega_x) T_0} \\ & - \frac{1}{(\omega_x^2 - (\Omega_1 - \omega_x)^2)} [\Omega f_1 \bar{A}_0] e^{i(\Omega_1 - \omega_x) T_0} - \frac{1}{(\omega_x^2 - \Omega^2)} [i\Omega F] e^{i\Omega T_0} \\ & - \frac{1}{(\omega_x^2 - (\omega_x + 2\omega_y)^2)} [\eta_2 A_0 B_0^2] e^{i(\omega_x + 2\omega_y) T_0} \\ & - \frac{1}{(\omega_x^2 - (\omega_x - 2\omega_y)^2)} [\eta_2 A_0 \bar{B}_0^2] e^{i(\omega_x - 2\omega_y) T_0} \\ & - \frac{i\Omega Q}{2(\omega_x^2 - (\Omega_2 + \Omega_3)^2)} e^{i(\Omega_2 + \Omega_3) T_0} + \frac{i\Omega Q}{2(\omega_x^2 - (\Omega_2 - \Omega_3)^2)} e^{i(\Omega_2 - \Omega_3) T_0} + cc, \end{aligned} \tag{25}$$

$$\begin{aligned}
 y_1 = & B_1 e^{i\omega_y T_0} + \frac{1}{8\omega_y^2} [\gamma_3 B_0^3] e^{3i\omega_y T_0} + \frac{1}{(\omega_y^2 - (\omega_x + \omega_y)^2)} [-\alpha_3 A_0 B_0] e^{i(\omega_x + \omega_y) T_0} \\
 & - \frac{1}{(\omega_y^2 - (\omega_x - \omega_y)^2)} [\alpha_3 A_0 \bar{B}_0] e^{i(\omega_x - \omega_y) T_0} \\
 & - \frac{1}{(\omega_y^2 - (\omega_y + 2\omega_x)^2)} [\eta_3 B_0 A_0^2] e^{i(\omega_y + 2\omega_x) T_0} \\
 & - \frac{1}{(\omega_y^2 - (\omega_y - 2\omega_x)^2)} [\eta_3 B_0 \bar{A}_0^2] e^{i(\omega_y - 2\omega_x) T_0} - \frac{\Omega f_2 B_0}{(\omega_y^2 - (\Omega_1 + \omega_y)^2)} e^{i(\Omega_1 + \omega_y) T_0} \\
 & - \frac{1}{(\omega_y^2 - (\Omega_1 - \omega_y)^2)} [\Omega f_2 \bar{B}_0] e^{i(\Omega_1 - \omega_y) T_0} + \frac{1}{(\omega_y^2 - \Omega^2)} [\Omega F] e^{i\Omega T_0} \\
 & + \frac{\Omega Q}{2(\omega_y^2 - (\Omega_2 + \Omega_3)^2)} e^{i(\Omega_2 + \Omega_3) T_0} + \frac{\Omega Q}{2(\omega_y^2 - (\Omega_2 - \Omega_3)^2)} e^{i(\Omega_2 - \Omega_3) T_0} + c.c.
 \end{aligned} \tag{26}$$

From the Eqs. (25) and (26), the deduced resonance cases are:

- (i) Primary resonance: $\Omega \cong \omega_1, \omega_2$;
- (ii) Sub-harmonic resonance: $\Omega_1 \cong 2\omega_x, 2\omega_y$;
- (iii) Internal resonance: $\omega_x \cong r\omega_y, r = 1, 2$;
- (iv) Combined resonance: $\Omega_2 \pm \Omega_3 \cong \omega_x, \omega_y$;

(iv) Simultaneous resonance: Any combination of the above resonance cases is considered as simultaneous resonance.

4. Stability analysis

The stability of the considered system is investigated at the worst resonance case (confirmed numerically), which is the simultaneous sub-harmonic and combined resonance case, where $\Omega_1 \cong 2\omega_x, \Omega_2 + \Omega_3 \cong \omega_y$. Consider the resonance conditions $\Omega_1 \cong 2\omega_x + \varepsilon\sigma_1$ and $\Omega_2 + \Omega_3 \cong \omega_y + \varepsilon\sigma_2$, where σ_1 and σ_2 are called the detuning parameters and eliminating the secular terms from the first approximations of Eqs. (23) and (24) leads to solvability conditions as:

$$-2i\omega_x c_x A_0 - 2i\omega_x A_0' - 3\gamma_2 A_0^2 \bar{A}_0 - 2\eta_2 A_0 B_0 \bar{B}_0 - i\omega_x G_1 A_0 - \Omega f_1 \bar{A}_0 e^{i\sigma_1 T_1} = 0, \tag{27}$$

$$-2i\omega_y c_y B_0 - 2i\omega_y B_0' - 3\gamma_3 B_0^2 \bar{B}_0 - 2\eta_3 B_0 A_0 \bar{A}_0 - i\omega_y G_2 B_0 + \frac{1}{2} \Omega Q \bar{A}_0 e^{i\sigma_2 T_1} = 0. \tag{28}$$

Using the polar form:

$$A_0 = \frac{1}{2} a_1 \exp(i\zeta_1), \quad B_0 = \frac{1}{2} a_2 \exp(i\zeta_2), \tag{29}$$

where a_1, a_2 and ζ_1, ζ_2 are the steady state amplitudes and phases of the motions respectively. Substituting from Eq. (29) into Eqs. (27) and (28) and equating imaginary and real parts, we obtain:

$$a_1' = -c_x a_1 - \frac{1}{2} G_1 a_1 - \frac{\Omega f_1}{2\omega_x} a_1 \sin\theta_1, \tag{30}$$

$$\zeta_1' = \sigma_1 - \frac{3\gamma_2}{4\omega_x} a_1^2 - \frac{\eta_2}{2\omega_x} a_2^2 - \frac{\Omega f_1}{\omega_x} \cos\theta_1, \tag{31}$$

$$a_2' = -c_y a_2 - \frac{1}{2} G_2 a_2 + \frac{\Omega Q}{2\omega_y} \sin\theta_2, \tag{32}$$

$$a_2 \zeta_2' = a_2 \sigma_2 - \frac{3\gamma_3}{8\omega_y} a_2^3 - \frac{\eta_3}{4\omega_y} a_2 a_1^2 + \frac{\Omega Q}{2\omega_y} \cos\theta_2, \tag{33}$$

where, $\theta_1 = \sigma_1 T_1 - 2\zeta_1$ and $\theta_2 = \sigma_2 T_1 - \zeta_2$ are phases of motion.

For steady-state solutions, we put $a'_n = \zeta'_n = 0$, $n = 1, 2$ to Eqs. (30)-(33) we get the following:

$$-c_x a_1 - \frac{1}{2} G_1 a_1 - \frac{\Omega f_1}{2\omega_x} a_1 \sin\theta_1 = 0, \tag{34}$$

$$\sigma_1 - \frac{3\gamma_2}{4\omega_x} a_1^2 - \frac{\eta_2}{2\omega_x} a_2^2 - \frac{\Omega f_1}{\omega_x} \cos\theta_1 = 0, \tag{35}$$

$$-c_y a_2 - \frac{1}{2} G_2 a_2 + \frac{\Omega Q}{2\omega_y} \sin\theta_2 = 0, \tag{36}$$

$$a_2 \sigma_2 - \frac{3\gamma_3}{8\omega_y} a_2^3 - \frac{\eta_3}{4\omega_y} a_2 a_1^2 + \frac{\Omega Q}{2\omega_y} \cos\theta_2 = 0. \tag{37}$$

From Eqs. (34)-(37) we have the following cases:

(i) $a_1 \neq 0, a_2 = 0$, (ii) $a_2 \neq 0, a_1 = 0$ and (iii) $a_1 \neq 0, a_2 \neq 0$ (practical case).

For the practical case ($a_1 \neq 0, a_2 \neq 0$), Squaring Eqs. (34) and (35), then adding the squared results together, similarly to Eqs. (36) and (37) gives the following frequency response equations:

$$\sigma_1^2 + \left[-\frac{3\gamma_2}{2\omega_x} a_1^2 - \frac{1}{\omega_x} \eta_2 a_2^2 \right] \sigma_1 + \left[4c_x^2 + G_1^2 + 4c_x G_1 + \frac{9}{16\omega_x^2} \gamma_2^2 a_1^4 + \frac{1}{4\omega_x^2} \eta_2^2 a_2^4 + \frac{3}{4\omega_x^2} \eta_2 \gamma_2 a_1^2 a_2^2 - \frac{\Omega^2 f_1^2}{\omega_x^2} \right] = 0, \tag{38}$$

$$\sigma_2^2 + \left[-\frac{3\gamma_3}{2\omega_y} a_1^2 - \frac{1}{\omega_y} \eta_3 a_1^2 \right] \sigma_2 + \left[4c_y^2 + G_2^2 + 4c_y G_2 + \frac{9}{16\omega_y^2} \gamma_3^2 a_2^4 + \frac{1}{4\omega_y^2} \eta_3^2 a_1^4 + \frac{3}{4\omega_y^2} \eta_3 \gamma_3 a_1^2 a_2^2 - \frac{\Omega^2 Q^2}{\omega_y^2 a_2^2} \right] = 0. \tag{39}$$

On the way to determine the stability of the fixed point solutions of Eqs. (34)-(37), we introduce the following forms:

$$A_0 = \left(\frac{p_1 - iq_1}{2} \right) e^{i\sigma_1 T_1}, \quad B_0 = \left(\frac{p_2 - iq_2}{2} \right) e^{i\sigma_2 T_1}, \tag{40}$$

where p_1, p_1, q_1, q_2 are real coefficients. Substitution from Eq. (40) into the linear form of Eqs. (27) and (28), that is:

$$-2i\omega_x A'_0 - 2i\omega_x c_x A_0 - i\omega_x G_1 A_0 = 0, \tag{41}$$

$$-2i\omega_y B'_0 - 2i\omega_y c_y B_0 - i\omega_y G_2 B_0 = 0. \tag{42}$$

Substituting from Eq. (40) into Eqs. (41) and (42) and equating the imaginary and real parts of Eq. (41) and Eq. (42) we contain:

$$p'_1 + \left(c_x + \frac{G_1}{2} \right) p_1 + (\sigma_1) q_1 = 0, \tag{43}$$

$$q'_1 + (-\sigma_1) p_1 + \left(c_x + \frac{G_1}{2} \right) q_1 = 0, \tag{44}$$

$$p'_2 + \left(c_y + \frac{G_2}{2} \right) p_2 + (\sigma_2) q_2 = 0, \tag{45}$$

$$q'_2 + (-\sigma_2) p_2 + \left(c_y + \frac{G_2}{2} \right) q_2 = 0. \tag{46}$$

The stability of a particular fixed point with respect to a proportional to $e^{\lambda T_1}$ (where λ is an

eigenvalue) is determined by zeros of the characteristic equation:

$$\begin{vmatrix} \lambda + c_x + \frac{G_1}{2} & \sigma_1 & 0 & 0 \\ -\sigma_1 & \lambda + c_x + \frac{G_1}{2} & 0 & 0 \\ 0 & 0 & \lambda + c_y + \frac{G_2}{2} & \sigma_2 \\ 0 & 0 & -\sigma_2 & \lambda + c_y + \frac{G_2}{2} \end{vmatrix} = 0. \quad (47)$$

To analyze the stability of the non-trivial solution, one uses Eq. (47) to obtain:

$$\lambda^4 + r_1\lambda^3 + r_2\lambda^2 + r_3\lambda + r_4 = 0, \quad (48)$$

where:

$$\begin{aligned} r_1 &= 2(c_x + c_y) + (G_1 + G_2), \\ r_2 &= \sigma_1^2 + \sigma_2^2 + c_x G_1 + c_y G_2 + c_x^2 + c_y^2 + \frac{1}{4}(G_1^2 + G_2^2) + G_1 G_2 + 2(c_x G_2 + c_y G_1) + 4c_x c_y, \\ r_3 &= \sigma_1^2 G_2 + \sigma_2^2 G_1 + 2(\sigma_1^2 c_y + \sigma_2^2 c_x) + 2(c_x c_y^2 + c_y c_x^2) + (c_x^2 G_2 + c_y^2 G_1) \\ &\quad + \frac{1}{2}(c_y G_1^2 + c_x G_2^2) + \frac{1}{4}(G_1^2 G_2 + G_2^2 G_1) + 2c_x c_y (G_1 + G_2) + G_1 G_2 (c_x + c_y), \\ r_4 &= \sigma_1^2 \sigma_2^2 + \sigma_1^2 c_y^2 + \sigma_2^2 c_x^2 + \frac{1}{4}(\sigma_1^2 G_2^2 + \sigma_2^2 G_1^2) + \sigma_1^2 c_y G_2 + \sigma_2^2 c_x G_1 + c_x^2 c_y^2 \\ &\quad + \frac{1}{4}(c_x^2 G_2^2 + c_y^2 G_1^2) + \frac{1}{4}(c_x G_2^2 G_1 + c_y G_1^2 G_2) + \frac{1}{16} G_1^2 G_2^2 + c_x^2 c_y G_2 + c_y^2 c_x G_1 + c_x c_y G_1 G_2, \end{aligned}$$

are constants. According to the Routh-Hurwitz criterion, the necessary and sufficient conditions for all the roots of Eq. (48) to possess negative real parts are: $r_1 > 0$, $r_1 r_2 - r_3 > 0$, $r_3(r_1 r_2 - r_3) - r_1^2 r_4 > 0$ and $r_4 > 0$.

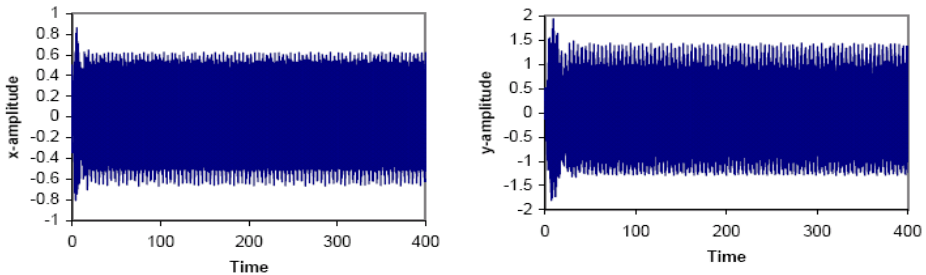


Fig. 2. Non-resonant time response solution at selected values: $c_x = 0.2$, $\alpha_2 = 0.3$, $\beta_2 = 0.2$, $\gamma_2 = 0.3$, $\eta_2 = 0.4$, $c_y = 0.1$, $\alpha_3 = 0.1$, $\gamma_3 = 0.1$, $\eta_3 = 0.1$, $\omega_x = 7.5$, $\omega_y = 7.8$, $\Omega = 1.4$, $\Omega_1 = 5.3$, $\Omega_2 = 4.75$, $\Omega_3 = 3.35$, $f_1 = 4.0$, $f_2 = 3.0$, $F = 2.0$, $Q = 4.0$

5. Numerical results

5.1. Active control effect

In order to verify the analytic results, the equations of motion Eqs. (11) and (12) is numerically integrated using a fourth order Runge-Kutta algorithm. Fig. 2 shows the non-resonant system behavior, where the maximum steady state amplitudes x and y are about 30 % and 70 % of the external excitation amplitude F respectively, this case can be regarded as a basic case. The worst

resonance case of the system is the simultaneous sub-harmonic and combined resonance case $\Omega_1 \cong 2\omega_x, \Omega_2 + \Omega_3 \cong \omega_y$, where the steady-state amplitudes are increased to about 250 % and 200 %. We added a negative linear velocity feedback to this worst resonance case as shown in Fig. 3. We found that the vibration amplitudes of the system are decreased. Best effectiveness of the system is obtained when the linear velocity feedback controller added to the system. The effectiveness of the controller is determined from the relation ($E_a =$ steady state amplitude of the system without controller/steady state amplitude of the system with controller), where ($E_a = 2000$ % for the amplitude x and $E_a = 800$ % for the amplitude y) at the worst resonance case which is the simultaneous sub- harmonic and combined resonance case.

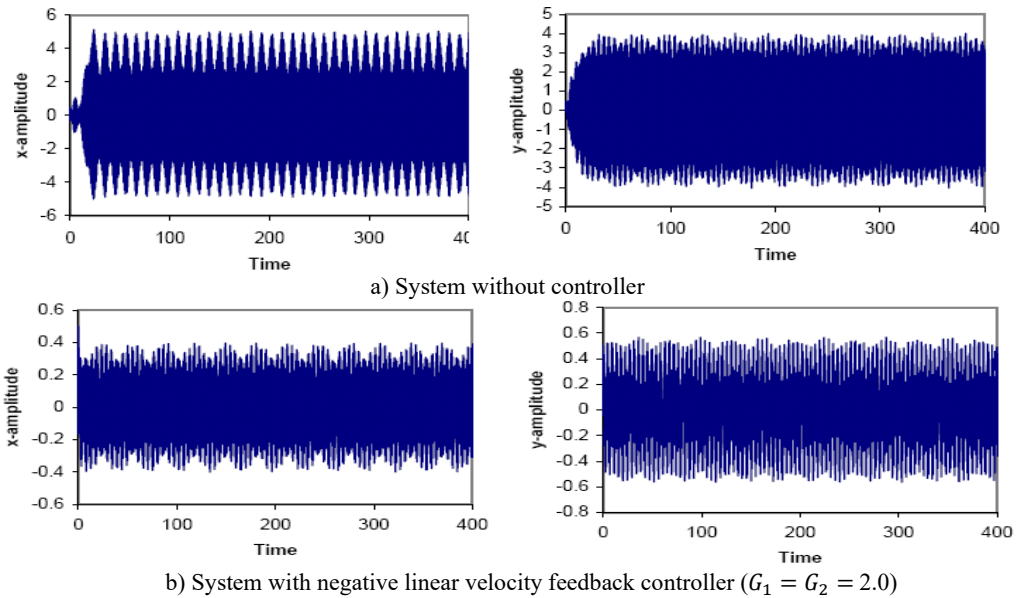


Fig. 3. Simultaneous sub-harmonic and combined resonance case ($\Omega_1 \cong 2\omega_x, \Omega_2 + \Omega_3 \cong \omega_y$)

Table 1. Numerical solutions of frequency response equations

Parameters	Effect	Fig. 4(a) (a_1, σ_1)	Parameters	Effect	Fig. 5(a) (a_2, σ_2)
Excitation force f_1	M.I.	Fig. 4(b)	Excitation force Q	M.I.	Fig. 5(b)
Natural frequency ω_x	M.I. with R.D.	Fig. 4(c)	Natural frequency ω_y	M.D.	Fig. 5(c)
Damping coefficient c_x	M.D.	Fig. 4(d)	Damping coefficient c_y	M.D.	Fig. 5(d)
Non-Linear parameter γ_2	H&S	Fig. 4(e)	Non-Linear parameter γ_3	H&S	Fig. 5(e)
Non-Linear parameter η_2	S.A.	Fig. 4(f)	Non-Linear parameter η_3	SA.	Fig. 5(f)
Gain G_1	M.D.	Fig. 4(g)	Gain G_2	M.D.	Fig. 5(g)
Ω	M.I.	Fig. 4(h)	Ω	M.I.	Fig. 5(h)

M.I. denotes that the amplitude is monotonic increasing function in the parameter;
 M.D. denotes that the amplitude is monotonic decreasing function in the parameter;
 R.D. denotes that the region is decreased;
 S.A. denotes that the amplitude is Saturation;
 H&S means that the parameter has hardening and softening non-linearity effects.

5.2. Response curves and effects of different parameters

The frequency response Eqs. (38) and (39) are nonlinear algebraic equations of a_1 against σ_1 and a_2 against σ_2 respectively. These equations are solved numerically as shown in Figs. 4, 5. Some figures possess hard and soft effects. This bending leads to multi-valued solutions and jump phenomenon. There are stable (solid line) and unstable (dashed lines) solutions. The Table 1

shows all effects of the different parameters of the considered system.

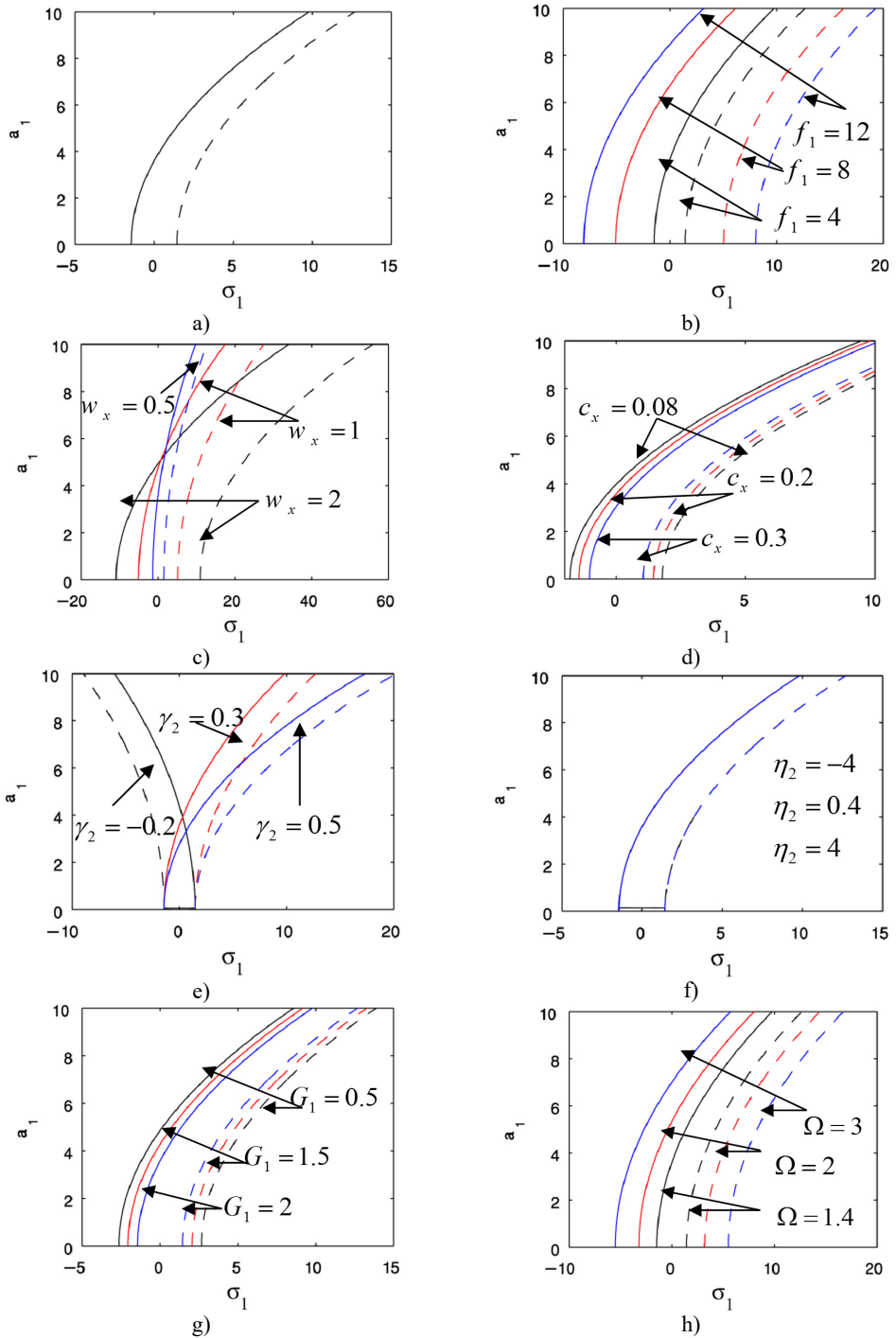


Fig. 4. Theoretical frequency response curves:
 $c_x = 0.2, \gamma_2 = 0.3, \eta_2 = 0.4, \omega_x = 2, \Omega = 1.4, f_1 = 4, G_1 = 2$

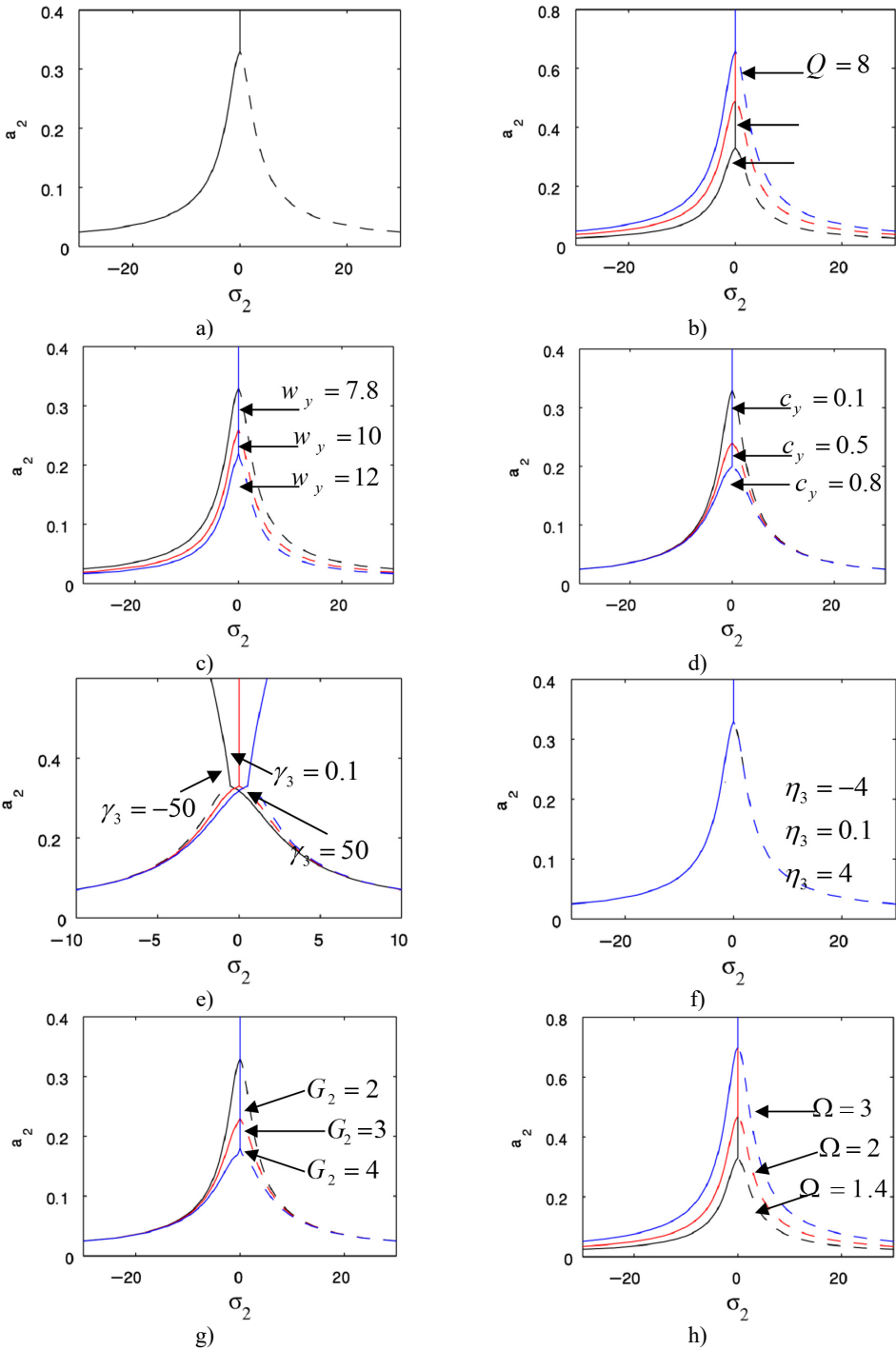


Fig. 5. Theoretical frequency response curves:
 $c_y = 0.1, \gamma_3 = 0.1, \eta_3 = 0.1, \omega_y = 7.8, \Omega = 1.4, Q = 4, G_2 = 2$

6. Conclusions

The vibrations of second order nonlinear differential equations of a suspended cable system

subjected to external, parametric and tuned excitation forces can be controlled via a negative linear velocity feedback controller. Multiple time scale perturbation technique is useful to determine approximate solutions for the differential equations describing the system up to second order approximation. To study the stability of the system, the frequency response equations are applied. The effects of the different parameters of the system are studied numerically.

From the above study the following may be fulfilled:

- 1) The worst resonance case of the system is the simultaneous sub-harmonic and combined resonance case $\Omega_1 \cong 2\omega_x$, $\Omega_2 + \Omega_3 \cong \omega_y$.
- 2) Negative linear velocity feedback active controller is the best one for the reported worst resonance case as it reduces the vibration dramatically.
- 3) The effectiveness of the best controller at the reported worst resonance case are about $E_a = 2000$ for x and $E_a = 800$ for y , respectively.
- 4) The steady-state amplitudes of both modes are monotonic increasing functions in the parameters f_1 , Q , ω_x and Ω while decreasing functions in the coefficients c_x , c_y , ω_y , G_1 and G_2 .
- 5) Also, for the nonlinear parameters γ_2 , γ_3 the steady-state amplitudes of both modes are making hardening and softening effects.
- 6) The nonlinear parameters η_2 , η_3 have no significant effects on the steady state amplitudes denoting the occurrence of saturation phenomenon.

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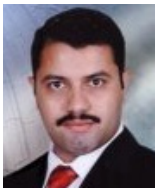
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