888. A Lamb wave signal processing method for damage diagnosis in the plate structure

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Abstract. In this work, a damage location method for plate-like structures is developed based on Lamb wave signal processing using matching pursuit method, which is employed to decompose Lamb wave signals into a linear expansion of several chirplet atoms using a fast realization algorithm. The relationship between Lamb wave dispersion and the chirplet chirp rate is established, which can be used to identify the modes of Lamb waves. Then a method for damage location is developed based on the difference between the baseline and the damaged signals. The effectiveness and accuracy of the proposed method in identifying the modes and in locating defects are demonstrated through experimental tests on the isotropic plate structure and honeycomb sandwich composite structure.

Keywords: Lamb waves, signal processing, chirplet atom, damage detection.

1. Introduction

Structural health monitoring (SHM) is an emerging technology that combines advanced sensor technology with advanced signal processing technology to determine the health conditions of structures in real time. Therefore, it can improve the reliability and safety of structures, enhance their operating performance and reduce their lifecycle costs. Lamb waves can propagate over a long distance in beam-, plate-, and shell-like structures, with damage monitoring method based on Lamb waves considered a promising and active SHM method [1, 2]. Lamb wave has two fundamental modes, the symmetrical mode (S mode) and the antisymmetrical mode (A mode). It also has the dispersion characteristic, which means that the propagation velocity varies with the frequency. Due to this dispersion nature, the pulse, which is a narrow bandwidth of Lamb wave, can spread in time, and its amplitude diminishes, making the signal-to-noise and time-frequency resolutions worse, especially during propagation over a long distance. In addition, mode conversion, which occurs when Lamb wave interacts with damage or boundary conditions, makes the identification of the damage location difficult.

Scattering signals are generated when Lamb wave interacts with defects, which can be achieved from the difference between healthy and damaged signals. However, the amplitude of scattering signals is very small, and extraction of information on the damage from the measured signals in the presence of strong noise is difficult. The development and study of advanced signal processing methods are necessary to process the measured signals. Recently, signal processing methods, such as short-time Fourier transform (SHFT), wavelet transform, and Hilbert-Huang transform (HHT), have been used to process Lamb wave signals successfully [3-5], but the time-frequency resolution obtained by these methods is not optimal. Further, they do not take into account the dispersion characteristic. Some researchers have used matching pursuit method to process monitoring signals. Ruiz-Reyes used matching pursuit method to improve ultrasonic flaw detection in nondestructive testing [6]. Hong employed the same method to analyze the guided wave signals measured in rod structures [7]. Ajay also utilized matching pursuit method using chirplet dictionary to process Lamb wave signals in a plate structure. However, the method does not establish the relationship between the dispersion and the chirp rate of the chirplet atom, and the results are not accurate [8].

Matching pursuit method with chirplet atom is applied to handle Lamb wave in a plate-like structure in the present study. When Lamb wave interacts with boundary or damages, some Lamb wave modes are generated due to mode conversion. Distinguishing these Lamb modes is necessary to determine the velocities of the different Lamb wave modes. The relationship between dispersion and chirp rate of the chirplet atom is established to distinguish Lamb wave modes, such as the A_0 and S_0 modes.

The current research is organized as follows. In Section 2, matching pursuit method is briefly reviewed, and a fast implementation method is introduced to decompose Lamb wave signals. And the feasibility of chirplet atom matching with dispersion pulse is studied. In Sections 3, the effectiveness and accuracy of the proposed method are verified by a simulation and experiment on isotropic plate and honeycomb sandwich composite structures.

2. Matching Pursuit Method

Matching pursuit method is an adaptive signal processing method proposed by Mallat and Zhang in 1993 [9]. Around the same time, Qian and Chen independently developed a similar algorithm [10]. The method projects the signal onto a large and redundant dictionary of waveforms and chooses a waveform that is best adapted to approximate part of the waveform from the dictionary. Compared with other time-frequency signal processing methods, matching pursuit method possesses some advantages, such as its high time-frequency resolution, robustness to noise, lack of interference, and high computational efficiency. Assuming signal $f(t)$ belongs to Hilbert space $L^2(R)$, matching pursuit method can decompose signal $f(t)$ into a linear combination of several time-frequency atoms (with $R^0 f = f$) with the following steps:
(a) The best atom g_r is chosen from dictionary D:

$$g_{\gamma_m} = \arg\max_{g_\gamma \in D} \left| \left\langle R^{m-1} f, g_\gamma \right\rangle \right| \tag{1}$$

(b) After step (a), the residual signal can be obtained by:

$$R^m f = R^{m-1} f - \left\langle R^{m-1} f, g_{\gamma_m} \right\rangle g_{\gamma_m} \tag{2}$$

The symbol <·> is defined as $\langle f_1, f_2 \rangle = \int_{-\infty}^{\infty} f_1^*(t) f_2(t) dt.$ The search and subtraction of the residual signals are repeated until a fixed number of iterations or a threshold of the residual signals is reached; signal $f(t)$ can then be decomposed into:

$$f = \sum_{m=0}^{M-1} \langle f, g_{rm} \rangle g_{rm}(t) + R^M f \tag{3}$$

where M is the number of iterations. During decomposition, $R^m f$ and g_{rm} are orthogonal, and the decomposition follows the energy conservation law.

In the current work, the dictionary of chirplet atoms $g_r(t)$ is used in matching pursuit method:

$$g_\gamma(t) = \frac{1}{\sqrt{s}} g\left(\frac{t-u}{s}\right) * \exp\left[i\left(\omega_0 (t-u) + \frac{c}{2}(t-u)^2 \right) \right] \tag{4}$$

where $g(t) = 2^{1/4} e^{-\pi t^2}$, $\|g_\gamma(t)\| = 1$, index $\gamma = (s, u, \omega_0, c)$; s is the dilation parameter; u is the translation parameter; ω_0 is the center of the angular frequency; and c is the chirp rate. The angular frequency $\omega(t) = \omega_0 + c(t - u)$ and varies with time. The chirplet atom and its Wigner-Ville distribution (WVD) are shown in Fig. 1.

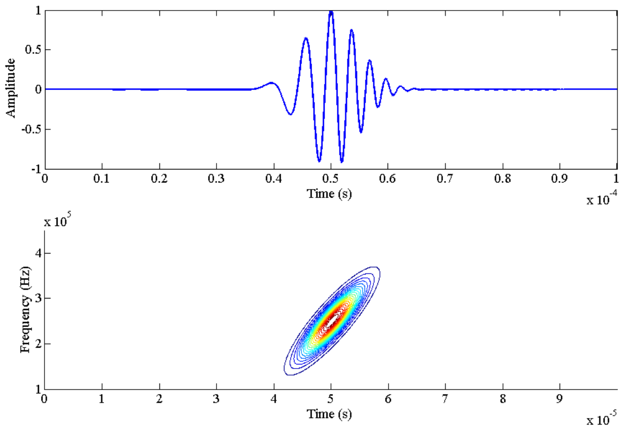


Fig. 1. Chirplet atom and its WVD

A narrow bandwidth signal is chosen as excitation signal, and scale s and angular frequency ω_0 of the excitation signal are determined beforehand. Therefore, matching pursuit method can be simplified to process the monitoring signals. Index γ has four parameters, s, u, ω_0, and c, where s and ω_0 are already determined; only parameters u and c need to be defined. The method can be repeatedly simplified using a two-phase algorithm.

(a) The chirp rate $c_0 = 0$ is defined in chirplet atom $g_\gamma(t)$. With the chirplet atom equivalent to the Gabor atom, the best atom is chosen using matching pursuit method, and the initial values are obtained, such as time u_0, amplitude A_0, and phase ϕ_0. The inner product of the signals and the chirplet atoms can be efficiently calculated in the frequency domain for all translation $\left\langle f(t),\, g_{(s,\,u,\,\omega_0,\,c)}(t)\right\rangle = f(u) * g_{(s,\,0,\,\omega_0,\,c)}(u)$, where $*$ denotes the convolution which can be implemented using FFT.

(b) The nonlinear least square algorithm [11] is used to determine the optimal values of u_1 and c_1, using u_0 and c_0 as the initial values:

$$(u_1, c_1) = \arg \min_{\substack{u_1 \in [0, N-1] \\ c_1 \in [-\pi, \pi]}} \left| R^{m-1} f - A_0 g_{\gamma_0 m} \right| \tag{5}$$

Once the index is determined, the exact value of amplitude A_1 and phase ϕ_1 can be determined from the inner product. The algorithm is repeated until a fixed number of iterations or a threshold of the residual signals is reached. When decomposition of the signal is completed, information, such as the time of flight of the reflected waves and the dispersion values, can be obtained. Monitoring signals can be reconstructed using index γ_1, amplitude A_1, and phase ϕ_1.

The dispersion characteristic makes the elastic wave pulse distorted when it propagates, the amplitude small, and the waveform of the signal spread in the time domain. The excitation signal, which is a narrow bandwidth signal modulated by a Gaussian window, is expressed as:

$$g_{(s,\omega_0)}(t) = 2^{1/4} e^{-\pi (t/s)^2} e^{j\omega_0 t} \tag{6}$$

where $\omega_0 = 2\pi f_0$, and f_0 is the center frequency of the excitation signal. In the frequency domain, the excitation signal can be expressed as:

$$G_{(s,\omega_0)}(\omega) = 2^{1/4} s \cdot \exp\left(-\frac{\left[s(\omega - \omega_0) \right]^2}{4\pi} \right) \tag{7}$$

When the excitation signal travels along a waveguide for time duration u, its Fourier transform can be expressed as:

$$G_{(u,s,\omega_0)}(\omega) = G_{(s,\omega_0)}(\omega)\cdot e^{-ju\omega} \tag{8}$$

where u is the pulse location in the time domain. If the signal is dispersive, the Fourier transform of $G_{(u,s,\omega_0,D(\omega))}(\omega)$ can be expressed as:

$$G_{(u,s,\omega_0,D(\omega))}(\omega) = G_{(u,s,\omega_0)}(\omega)\cdot e^{-jD(\omega)} = G_{(s,\omega_0)}(\omega)\cdot e^{j\Phi(\omega)} \tag{9}$$

where $D(\omega)$ represents the dispersion effect, and phase $\Phi(\omega) = -(D(\omega) + u\omega)$. The signal in the time domain can be obtained by inverse FFT.

The group delay $\tau(\omega)$ can be obtained by the derivative of the phase:

$$\tau(\omega) = -d\Phi(\omega)/d\omega = dD(\omega)/d\omega + u \tag{10}$$

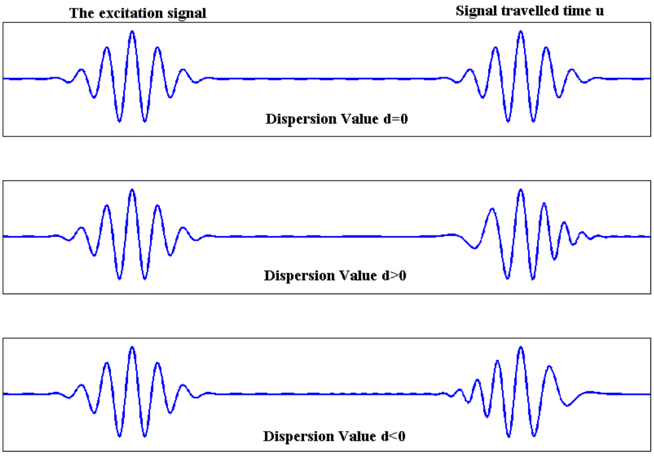


Fig. 2. Influence of dispersion value on the impulse waveform

A good estimation of $D(\omega)$ is very important for the accurate representation of $\tau(\omega)$. The excitation signal is a narrow bandwidth pulse. The relationship between group velocity and frequency can be approximated as a linear relationship in a frequency range of narrow bandwidth (Fig. 2). The group delay $\tau(\omega)$ can be approximated as:

$$\tau(\omega) \cong d(\omega-\omega_0)+u \tag{11}$$

where d is the dispersion value. The influence of dispersion value d on the pulse waveform is shown in Fig. 3. Compared with Eq. (10), function $D(\omega)$ can be approximated as:

$$dD(\omega)/d\omega \cong d(\omega-\omega_0) \tag{12}$$

The change in phase between $G_{(s,\omega_0)}(\omega)$ and $G_{(s,u,\omega,c)}(\omega)$, which is the Fourier transform of $g_\gamma(t)$, is:

$$\Phi(\omega) = -\frac{cs^4(\omega-\omega_0)^2}{(8\pi^2+2c^2s^4)} - u\omega \tag{13}$$

The group delay $\tau(\omega)$ can be expressed as:

$$\tau(\omega) = -d\Phi(\omega)/d\omega = \frac{cs^4}{(4\pi^2+c^2s^4)}(\omega-\omega_0)+u \tag{14}$$

The dispersion value can be expressed as:

$$d_{mp} = cs^4 / \left(4\pi^2 + c^2 s^4\right) \tag{15}$$

Once d is determined, $D(\omega)$ can be found. The influence of dispersion on the waveform of the signals is obtained. Therefore, the chirplet atom can be used to approximate the distorted pulse.

3. Experimental Study

3. 1. Damage Location for Isotropic Structure

The proposed method is verified in this section using an experimental study. An experimental setup containing an NI-5412 arbitrary waveform generator and an NI-6115 multifunction data acquisition card is established for signal generation and acquisition. The experiment is controlled using a program written by LabVIEW. The specimen is an aluminum plate whose dimensions are 100 cm × 100 cm × 0.1 cm. A pair of piezoelectric patches whose diameters are 12 mm is bonded on the plate for excitation and sensing. A photograph of the experimental setup is shown in Fig. 3(a), and the schematic diagram of the experiment is depicted in Fig. 3(b). The center frequency of the excitation signal is 200 kHz, the frequency at which the actuator and sensor will mainly excite and sense the S_0-mode wave. Two bolts whose diameters are 1 cm are bonded on the plate to model the defects, and the distances between the defects and the sensor are 17 and 20 cm, respectively.

Damage information can be obtained from the difference between the baseline and damaged signals. After the baseline signals are recorded under a no-defect condition, the bolts are attached to the structure to model the defects, and the damaged signals under the condition of two defects are measured. The differences between the baseline and the damaged signals are shown in Fig. 3(c); only the signals after the excitation are shown, and the pulse contains two scattering signals reflected from the two defects. The reconstructed signals completed using matching pursuit method are shown in Fig. 3(c); the residual signals whose amplitudes are very small are shown in Fig. 3(c). The time–frequency energy distribution obtained by matching pursuits is shown in Fig. 3(d), especially the two overlapped scattering signals that can be resolved by matching pursuit method. Other signal processing methods, such as wavelet transform, cannot distinguish the overlapped signals (as demonstrated in Fig. 3(e)). The results of the analysis are listed in Table 1. The dispersion values of the two pulses are positive, and their modes are the S_0 mode. The errors of location are 0.95 % and 4.65 %, respectively.

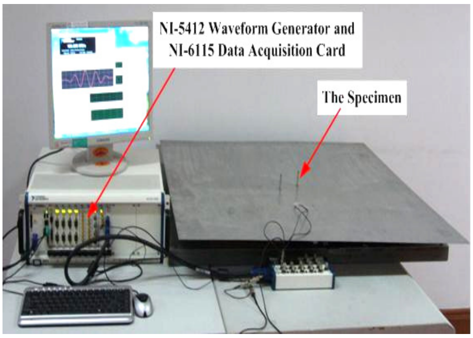
Table 1. Lists of experimental results

Atom	t_0 μs	c Hz·s^{-1} (E-10)	d (E-11)	Energy (E-7)	Mode	Actual distance (cm)	Distance from MP (cm)	Error %
1	75.6	1.66	0.24	0.61	S_0	20	20.19	0.95
2	66.6	2.74	0.26	0.19	S_0	17	17.79	4.65

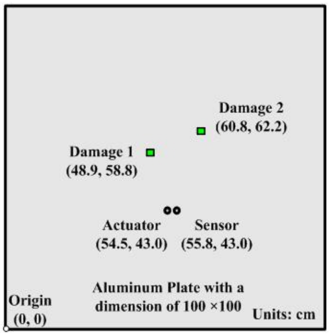
The method can determine the radial distance of the defect. Using three pairs of transducers to pinpoint the location of the defect in a plate-like structure, this defect is found at the intersection of three circles.

3. 2. Location of Impact Damage in a Honeycomb Sandwich Composite Structure

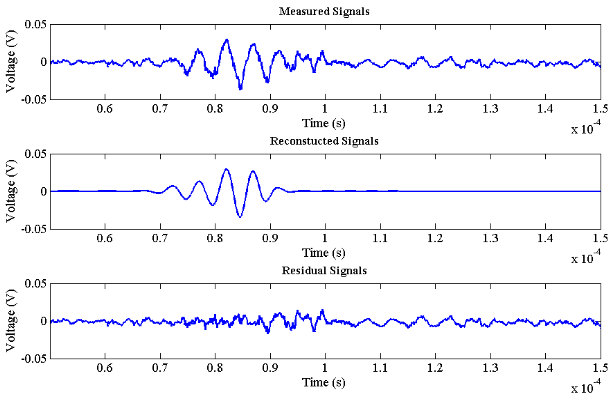
A honeycomb sandwich composite structure, widely used in aircrafts, is an anisotropic structure characterized by light weight and high strength. However, it is generally poor at resisting impact damage. In this section, a damage monitoring method based on Lamb wave and matching pursuit method is presented to detect the impact damage in a honeycomb sandwich composite structure. The specimen is an airplane fin control surface containing two honeycomb sandwich composite plates and an aluminum chuck. The honeycomb sandwich composite is constructed from two thin carbon composite sheets bonded to a paper honeycomb core. The specimen and the experimental setup are shown in Fig. 4(a). In the experiment, a drop hammer with impact energy of 10 J is used to strike the structure. A piezoelectric patch array with dimensions of 16 cm × 20 cm, diameter of 1.2 cm, and thickness of 0.1 cm is bonded to the specimen to actuate and sense Lamb wave. A rectangular coordinate system is established with point *O* as origin, path 1-2 as the *x*-axis, and path 1-4 as the *y*-axis (Fig. 4(b)).



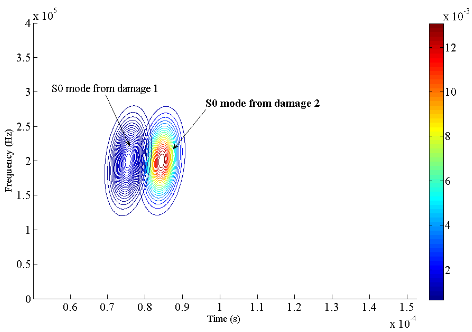
(a) The specimen and measurement instrument



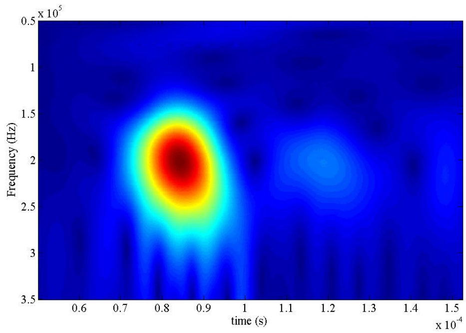
(b) The schematic diagram of the specimen



(c) The measured signals and the reconstructed signals

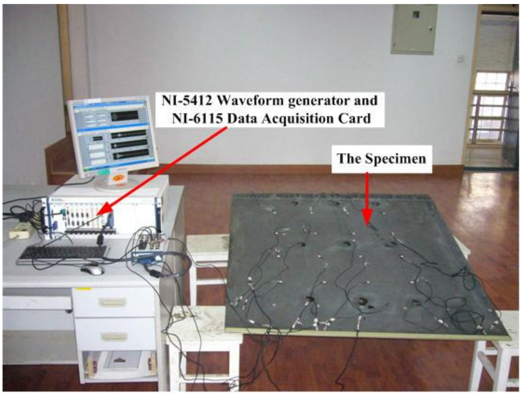


(d) The time-frequency energy distribution obtained by matching pursuits method

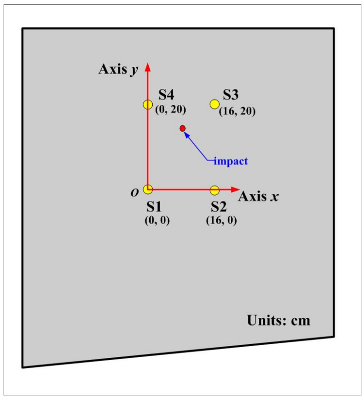


(e) The wavelet scalogram of the experimental result

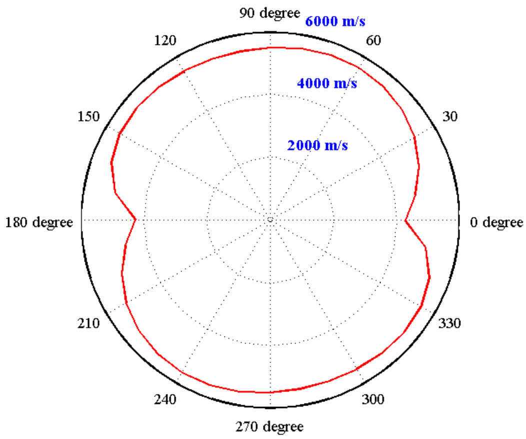
Fig. 3. Experimental setup and the results



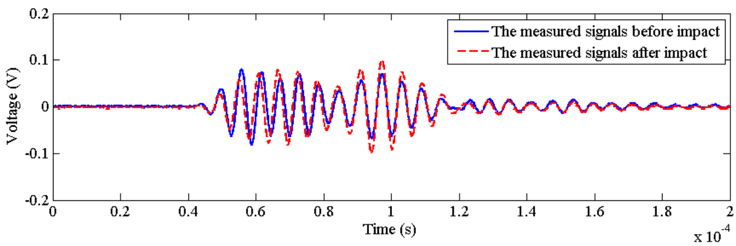
(a) Photograph of the experimental setup



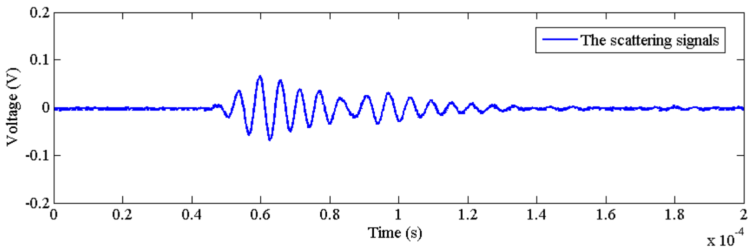
(b) Sketch of the specimen



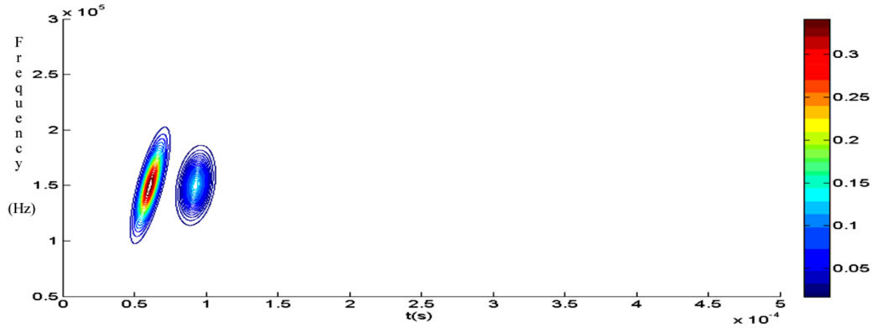
(c) Group velocity curves at a frequency of 150kHz



(d) Signals sensed by PZT S_3 before and after impact



(e) Difference between the signals sensed by PZT S3 before and after impact



(f) The time-frequency energy distribution obtained by the MP method

Fig. 4. Experimental results of the honeycomb sandwich composite structure

The development of a model to determine analytically the velocity of Lamb wave is difficult for the honeycomb sandwich composite structure. In the current research, the velocity of Lamb wave in several directions is measured using the experimental method, and the group velocity of Lamb wave as it varies with the direction is shown in Fig. 4(c). The average velocity in all directions is 5400 m/s at a frequency of 150 kHz. A scattering wave, which is the signal difference between undamaged and damaged signals, is generated when Lamb wave interacts with the impact damage. The flight time of the scattering signals can be extracted using matching pursuit method. The location of the impact damage can be determined by ellipse location method [14], and the damage is found at the intersection point of two ellipses, with the sensor and actuator as its foci.

In the experiment, piezoelectric patch S4 is used as the actuator, whereas the other patches are used as sensors. The signals sensed by patch S3 before and after the impact are shown in Fig. 4(d), and the scattering signals are shown in Fig. 4(e). The flight time of the signals is 37.2 μs in path 4-3 and 43.0 μs in path 4-1. With the velocity measured by the experiment, two ellipses can be obtained. The intersection of the two ellipses is at point (7.2, 14.2) cm (Fig. 5), and the distance to the real damage is 0.5 cm.

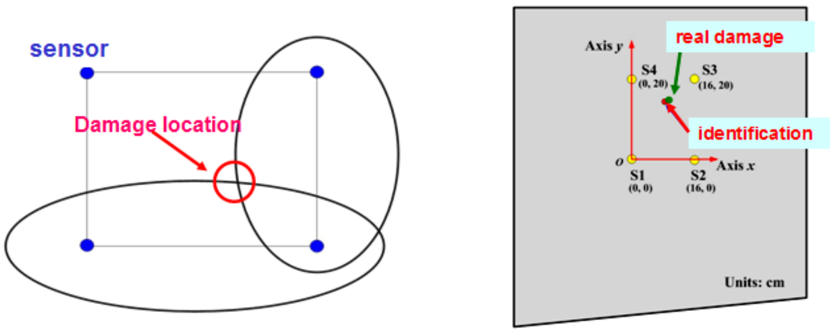


Fig. 5. Ellipse location method

4. Conclusions

Current study has presented a damage location method for plate-like structures based on Lamb wave and matching pursuit method with chirplet dictionary. The proposed implementation method of matching pursuit can quickly decompose the monitoring signals into a linear expansion of several chirplet atoms and can extract useful data, such as the flight time of the scattering signals, the dispersion values, and the Lamb wave mode. Compared with other signal processing methods, such as wavelet transform and HHT, the considered method can identify the mode and resolve the overlapped pulse reflected from several damages. This result has been verified by the experiment.

(1) The experimental results from the isotropic aluminum plate demonstrate that the method can also identify Lamb wave mode and distinguish overlapped pulses.

(2) The experimental results from the honeycomb sandwich composite structure demonstrate that matching pursuit method coupled with ellipse location method can accurately locate the impact damage.

Acknowledgments

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