

882. A method for estimation of critical stress intensity factor for welded sheet

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Abstract. Welded structures subjected to vibration loads in modern aerospace vehicles during practices have the hazard of undergoing fatigue. Critical stress intensity factor is the key parameter in the fatigue failure criterion. Usually fracture toughness is used as an approximation of the critical stress intensity factor in fatigue crack propagation calculation, however it can be seriously influenced by welding and thickness effects when applied to sheet metal welded joints. To solve the problem, this study analyzes these effects both experimentally and theoretically. The paper considers a method for estimation of the critical stress intensity factor based on crack size at the fatigue fracture location. Fatigue tests are conducted on welded specimens made of 2219-T87 aluminum alloy and critical stress intensity factors are calculated. The relationship for critical stress intensity factor results is determined from fracture crack sizes under different loading modes. Results reveal that the estimation method that was applied to measure the factor based on the fracture crack size excludes influences of welding and thickness effects in a convenient way of measurement and calculation. The method can be adopted for welded structures in spacecrafts subjected to vibration loads for fatigue failure analysis and reference of fracture toughness in engineering practice.

Keywords: critical stress intensity factor, fatigue test, critical fatigue crack size, thickness effect, welding.

Introduction

Welded structures subjected to vibration loads in novel aerospace vehicles during practice have the possibility of fatigue. According to the damage tolerant design widely used in aeronautics and astronautics, initial defects, material properties and working conditions, safe lives of structural elements, bearing cracks can be determined by the crack propagation law [1]. Among these factors, fracture due to crack propagation is an important issue, and the critical stress intensity factor (SIF) is the key parameter to determine fracture occurring during crack growth. By analyzing test data, Forman [2] found that the maximum SIF approximates to the fracture toughness of material very much when the fatigue fracture occurs. On that ground, the maximum of the factor equals to the fracture toughness, which is chosen as a condition to determine the fracture caused by fatigue crack growth, which extended the soundness of the Paris equation [3] to the third stage of crack growth; the equation by Newman [4] improved the Forman's work by including an amend accounting for thickness effects of fracture toughness, which is called critical stress intensity factor. These failure criterions of fatigue fracture and methods of calculation for critical SIFs are still used today as proven by tests and engineering applications [5-7]. But there are some problems when considering applications to sheet metal welded construction, which is common in astronautics: although the fracture toughness of sheet made of homogeneous materials could be measured by tests [8-10] or calculated by the relationship between the fracture toughness and thickness [11-13], the coupling effects of welding and thickness cause new difficulties to the fracture toughness tests and calculation. The non-homogeneity in the welded joint area brought by welding causes inapplicability of equations based on the homogeneous materials on the one side; makes fracture toughness tests,

which need pre-cut-gaps more difficult on the other side, as only when the pre-cut gap is machined exactly at the original specimens' fatigue fractures, the accurate value of critical SIF could be achieved from the fracture toughness measured by those tests. Therefore, SIF estimation method that accounts for coupled effects between welding and thickness is necessary for fatigue life prediction of sheet metal welded joints.

After analyzing the impact of the coupled effects between welding and thickness to experimental measurements and theoretical calculations of critical SIF of sheet welding joints, the method to estimate critical SIF for sheet welding joints is proposed in this paper by applying critical dimensions of fatigue crack. We select sheet welded specimens made of 2219-T87 aluminum alloy with 4 mm thickness as research objects. The specimens are subjected to fatigue tests under different loading conditions. Critical stress intensity factor is estimated on the basis of critical fatigue cracks size in fractures of the specimens. The applicability of the results is discussed including the analysis of influence of experimental parameters on estimation results.

1. Critical stress intensity factor

1.1 Fatigue fracture criterion

At the third stage of fatigue crack growth, crack growth rate increases fast and finally causes fracture. In most cases, the number of fatigue loading cycles is only a tiny fraction of the fatigue life, however the changing patterns of crack growth rate at this stage is absolutely crucial to predict the fracture criterion for fatigue life as well as to calculate residual strength.

Fatigue test data indicates that there is a vertical asymptote in the third stage of the crack growth rate vs. Delta K curve. When Delta K closes to the boundary value, crack growth rate approaches infinity and fracture occurs. If material is in the state of plane strain, the maximum of stress intensity factor is close to or equal to the plane strain fracture toughness. Accordingly, the critical SIF failure criterion, fracture happens when the maximum stress intensity factor is equal to the plane strain fracture toughness in the state of plane strain. This theory was put forward by Forman including crack growth rate equation (1) applied to the third stage [2]:

$$\frac{da}{dN} = \frac{C(\Delta K)^m}{(1-R)K_C - \Delta K} \quad (1)$$

where: K_C – plane strain fracture toughness, R – stress ratio, ΔK – amplitude of stress intensity factor.

In the equation above, the denominator of the right side is close to zero and crack growth rate is close to infinity when the maximum SIF is close to the plane strain fracture toughness, which is in good agreement with the variation of the fatigue crack growth in the third stage. But the plane strain fracture toughness is selected as fracture criterion parameter therefore the application of the equation is limited due to the thickness effect of the fracture toughness caused by the influence of thickness to the plastic zone size of the crack tip. When the thickness is small, the material is in the state of plane stress with small restraints along the thickness direction, big plastic deformations at crack tips and high fracture toughness; with the increase of the thickness, there will be stronger restraints acting on the plastic deformations at crack tips and fracture toughness will decrease. After the thickness increases to satisfy the plane strain condition, the fracture toughness does not change with thickness anymore. As a result, corrections for the fracture toughness thickness effect are necessary when the thickness is not in the plane stress condition or plane strain condition.

According to Newman's [4] crack growth rate equation (2), the fracture toughness thickness effect is accounted by the use of the critical SIF, K_{crit} . The plane stress and plane strain fracture toughness values are used to interpolate a value for the critical SIF failure criterion, as shown in equation (3):

$$\frac{da}{dN} = C \left[\left(\frac{1-f}{1-R} \right) \Delta K \right]^n \frac{\left(1 - \frac{\Delta K_{th}}{\Delta K} \right)^p}{\left(1 - \frac{K_{max}}{K_{crit}} \right)^q} \quad (2)$$

$$\frac{K_{crit}}{K_{Ic}} = 1 + B_k e^{-\left(A_k \frac{t}{t_0} \right)^2} \quad (3)$$

where: K_{crit} – critical stress intensity, K_{Ic} – plane strain fracture toughness (Mode I), A_k – fit parameter, B_k – fit parameter, t – thickness, t_0 – reference thickness (plane strain condition).

The plane strain condition is [14]:

$$t_0 \geq 2.5(K_{Ic} / \sigma_{ys}) \quad (4)$$

1. 2 Critical stress intensity factor for sheet welded joints

Compared with the homogeneous materials discussed above, welding joints are different in critical stress intensity factor calculation as the welding process modifies local mechanical properties of the joints. These changes are embodied in the differences of the macroscopic properties between the base metal and joints, as well as the non-uniform distribution of the microstructure and mechanical properties of the welding joints. These characteristics coupled with the thickness effect, make it difficult to test or theoretically calculate critical stress intensity factor of the sheet metal welded joint.

1. 2. 1 Influence on testing

Experiment is the key approach to obtain the critical stress intensity factor. Even theoretical calculation of the factor for particular thickness required tests to obtain parameters, such as the plane strain and plane stress fracture toughness. Influences of welding onto tests are embodied in the followings factors:

The thermal cycle during the welding changes the microstructure in the welding joints, resulting in reduction of joint yield strength. This reduction requires further improvements of the standard plane strain fracture toughness test to the thickness condition. Taking 2219-T87 aluminum alloy for example, according to equation (4), the plane strain condition requires the thickness of base metal to be no less than 17.59 mm. But welding reduced the yield strength of the welding specimen and causes thicker specimen to be able to comply with the plane strain condition. In the case of the 2219-T87 aluminum alloy TIG welding joints, the yield strength measured by tests is only 49.35 % that of base metal, so according to equation (4), the thickness of the specimen must reach or exceed 85.81 mm in order to meet the plane strain requirements. As a matter of fact, welding significantly complicates specimen processing and testing.

The local thermal cycle during the welding also creates areas with different microstructures in the welding joints, which makes the joint a non-uniform body in terms of mechanical properties. Successive variations of the fracture toughness with large amplitude in a very small area increase the difficulty of critical SIF testing for welding joints. Precast notches and fatigue

cracks are necessary to be machined separately in different microstructure areas in the welding joint for existing fracture toughness tests of welding joints to get a series of measurements. So it is complex to manufacture specimen, necessarily to have a large number of specimen and carry out large scale tests. The most important problem is a greater dispersion of fatigue fracture positions, because of various elements such as weld defects, geometry and microstructure distribution of the joints. The dispersion could make it very difficult to machining the notches in the position where exactly the same position as the un-notched original specimen would fracture under fatigue load. The displacement error of the precast notches could cause appreciable error in the test results.

1. 2. 2 Influence on theoretical calculation

The plane strain fracture toughness is an important parameter for the theoretical calculation of critical SIF. Temperature field, which is the main factor affecting mechanical properties of the welding joints, is influenced by the thickness. So the microstructure distribution is different in thickness, leading to different mechanical properties between thin plates and thick plates, and the property difference increases along with the thickness difference. As a matter of fact, thin weld joint can be treated as a different material with respect to thick weld joint because of the differences in mechanical properties. Lower yield strength causes greater thickness required by the welding joint plane strain fracture toughness test. As mentioned above, large thickness difference between the original welding specimen and the plane strain fracture toughness welding specimen can cause the relationship between fracture toughness and thickness becomes much more complex than that of the base metal specimen. That difference will seriously affect the calculation accuracy of the critical stress intensity factor.

2. Method of calculation

According to the analysis presented above, we propose an estimation method for estimating critical stress intensity factor by testing welding specimen with specific thickness and calculating the parameters measured from the fatigue fracture in order to avoid the coupled effects of welding and thickness on the measurement and calculation of the factor. Aluminum alloy (2219-T87) welding joints are selected as an example for testing and calculation.

2. 1 Test specimen

The material of the specimen is 2219-T87 aluminum alloy. Chemical composition (mass fraction, %) is: Si 0.2, Fe 0.3, Cu 5.8-6.8, Mn 0.3, Zr 0.18, Al. TIG welding technology is adopted to obtain the butt joint. Welding current is 220-240 A, voltage and welding speed are respectively 30 V and 9 m/h. According to the GB/T 3075-2008 metal material fatigue test axial force control method [15], the welded plates are manufactured into specimen. The geometry and dimension are shown in Fig. 1.

The specimens are designed to small width (25 mm) and placed more than 5 months in order to reduce residual stress influence on the results of the tests.

2. 2 Test method

The welding joint specimen can be measured from 30 mm to each side off the welding center, and the width and thickness of weld foot section on both sides can be adopted from the minimum value around the cross-sectional area. While the fatigue test machine are used to exert axial alternating load on the specimen, so that different frequency and amplitude of equivalent loads as well as two level ultrasonic loads are exerted on different specimen until the specimen

split (as shown in Fig. 2). Loading frequency are 4, 6, 8 and 10 Hz. Then the shape and size of critical extension fatigue crack can be measured. The MTS fatigue test system is used in the experiment, the static precision is 0.5 % *FS* and the dynamic accuracy is 1 % *FS* (as shown in Fig. 3).

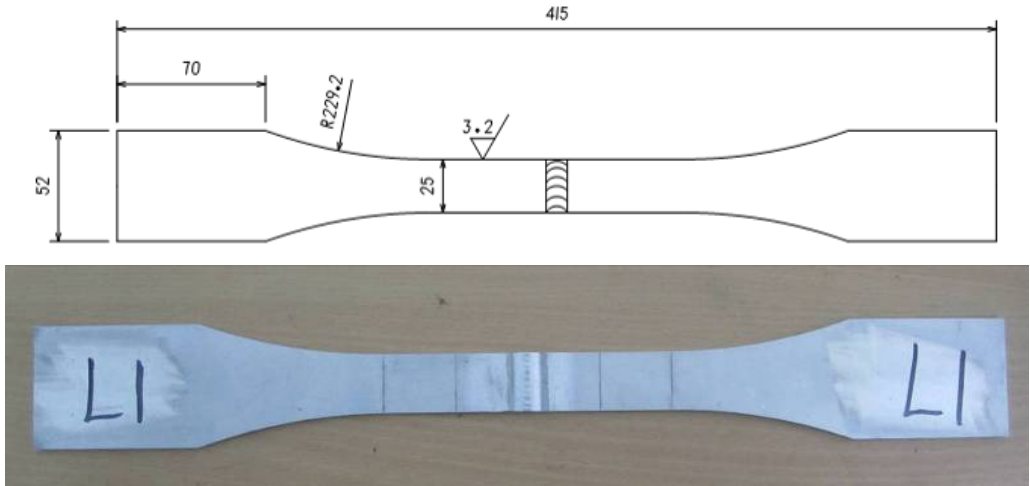


Fig. 1. Shape and dimension of the specimen

2. 3 Test results

The specimens all break along the fusion line of the welding joint. And the fatigue source area, the fatigue crack stable expansion area and the transient breaking area are visible along the fatigue fracture. While the fatigue source positions of different specimen are variable, some specimens exhibit more than one source of fatigue. As the amount and type of defects in the welding joint may be larger in comparison to the base material, the initiation positions and the initial sizes of fatigue crack have a greater range. The change range of the initial fatigue crack dimensions along the short half shaft direction is 0.7644 to 1.1734 mm, and the change is 38.20 % of the maximum amplitude. While the change range is 0.7644 to 1.1734 mm in the semi-minor axis direction, and the change is 34.86 % of the maximum amplitude. And the changes are slightly higher than the ratio of amplitude and the maximum value of the critical crack, which is 30.38 %.

Cracks initiating from different fatigue sites expand and meet, then they usually associate into one fatigue crack, finally achieving critical crack dimensions and growing into unstable fracture. Although the fatigue crack source of the welding joint may not be unique, only one piece of crack can result in fracture. Test results demonstrate that cracks from different fatigue source grow into one piece during expedition, only a few never meet but the size of main crack is much larger than the others, which is shown in Fig. 4. Therefore, the dispersion of initial defects in welding joints makes no influence in measuring the fracture toughness.

Fatigue crack stable expansion area and transiently breaking area are quite different in the macro morphology. Fatigue crack stable spreading area owns bright color and smooth fracture. While the transient breaking area is similar to the static load fracture condition, whose color is relatively close to dark grey. And it owns roughly shearing and oblique fractures, as it is shown in Fig. 4. The difference between these two areas makes it more convenient to measure the critical size. The results of critical fatigue crack measurements are listed in Table 1.

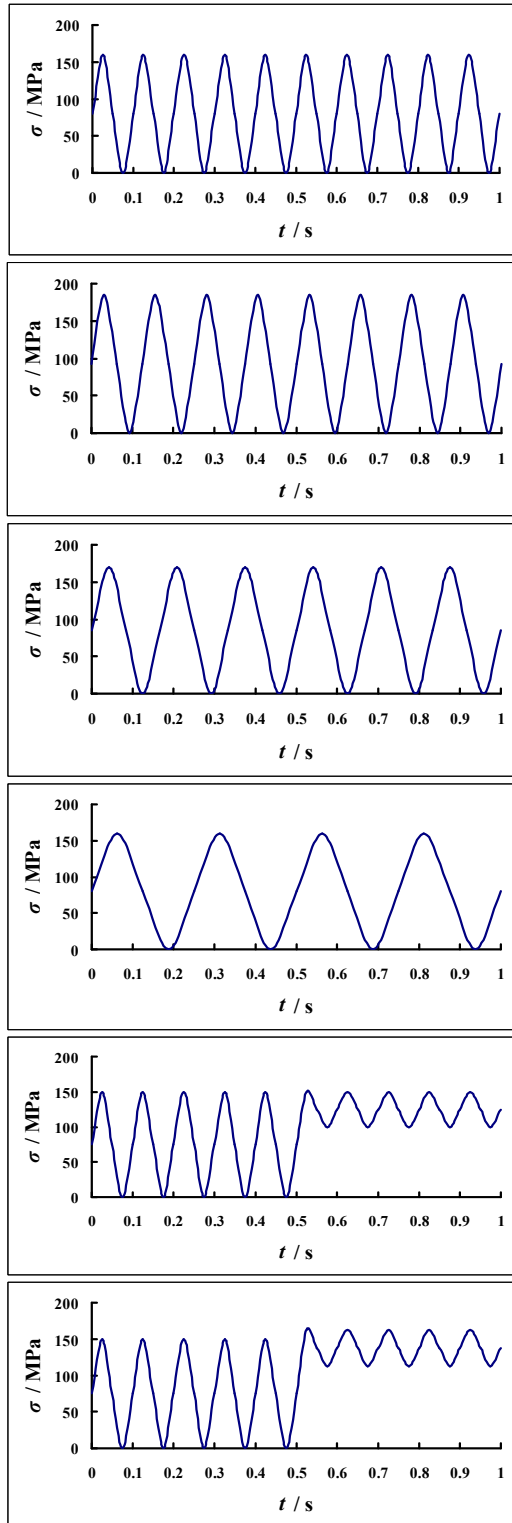


Fig. 2. Test loads



Fig. 3. Fatigue test system

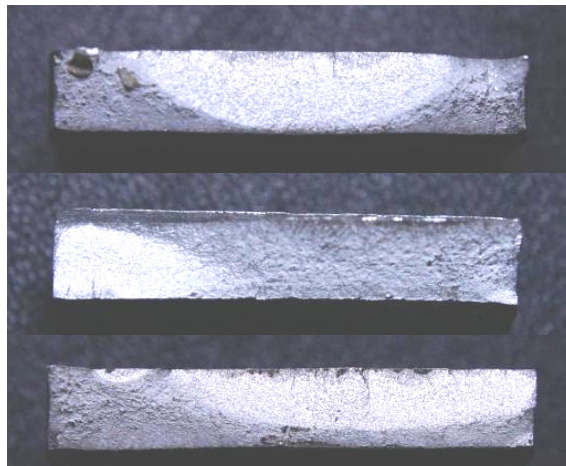


Fig. 4. Macroscopic feature of fatigue fracture

Table 1. Results of critical fatigue crack measurement

Specimen Number	Crack Type	Minor Axis /mm	Major Axis /mm
P1	Semi-elliptic Surface Flaw	3.83	9.15
P2	Edge Corner Crack	3.07	6.53
P3	Edge Corner Crack	3.24	8.11
P4	Semi-elliptic Surface Flaw	3.41	7.56
P5	Edge Corner Crack	3.35	7.62
P6	Edge Corner Crack	3.43	7.42
P7	Edge Corner Crack	3.53	7.51
P8	Semi-elliptic Surface Flaw	3.11	7.64
P9	Semi-elliptic Surface Flaw	3.47	7.53
P10	Edge Corner Crack	3.61	7.75
P11	Edge Corner Crack	3.49	7.64
P12	Edge Corner Crack	3.55	7.97
P13	Semi-elliptic Surface Flaw	3.69	7.83
P14	Edge Corner Crack	3.36	7.98
P15	Semi-elliptic Surface Flaw	3.73	8.01
P16	Semi-elliptic Surface Flaw	2.98	9.38

2. 4 Calculation

The quantitative relation among fatigue crack propagation critical crack size a_c , maximum stress σ_{max} and critical stress intensity factor K_{Crit} , is represented by equation (5):

$$K_{Crit} = K_{max} = Y\sigma_{max}\sqrt{\pi a_c} \quad (5)$$

where: K_{max} – maximum stress intensity factor of the critical crack tip, Y – form factor related to the type, size and position.

For the semi-elliptic surface flaw and edge corner crack, Y is [16]:

$$Y = \frac{\left[\sin^2 \theta + \left(\frac{a_c}{c_c} \right)^2 \cos^2 \theta \right]^{1/4}}{\phi} \quad (6)$$

$$K_{Crit} = \frac{\left[\sin^2 \theta + \left(\frac{a_c}{c_c} \right)^2 \cos^2 \theta \right]^{1/4}}{\phi} \sigma_{max} \sqrt{\pi a_c} \quad (7)$$

where: ϕ – elliptic integral of the second kind, a_c – short half shaft of critical crack, c_c – long half shaft of critical crack, θ – azimuth angle.

The critical stress intensity factor of the welding joint specimens can be acquired by inserting test results of critical fatigue crack measurement into equation (7). The calculated results are presented in Fig. 5. The dash line in Fig. 5 indicates the average value of the critical stress intensity factor $35.35 \text{ N/m}^{3/2}$.

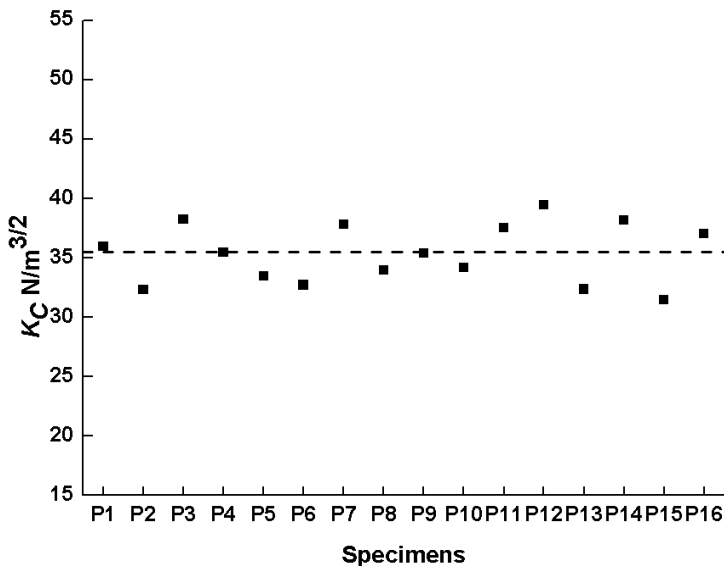


Fig. 5. Results of K_{Crit}

3. Discussion

3.1 Applicability

The coupled effects between thickness and welding make it difficult to calculate critical stress intensity factor in the case of welded sheet joint. The thickness effect makes the welding sheet joint critical SIF to be measured from the specimen with uniform thickness; or according to the relationship between the fracture toughness and thickness, calculated by the plane strain fracture toughness measured by the standard plane strain specimen. The welding brings about non-uniform mechanical properties and reduction in strength, which cause difficulties in testing welding joint fracture toughness and standard plane strain fracture toughness as well as changes the relationship between fracture toughness and thickness.

The method used in this paper has adopted specimen with uniform thickness to consider the thickness effect of critical stress intensity factor and the mechanical property changes caused by welding. It also uses the fracture crack size of the original specimen without precast notches or cracks to avoid the critical stress intensity factor error caused by the precast notch position error. Thus, to take the coupled effects between welding and thickness in to consideration, this method is applicable for welding sheet joints.

3.2 Influence of test parameters

3.2.1 Maximum loads

According to equation (5) for the critical stress intensity factor K_{Crit} , there are two important variables: maximum stress σ_{max} and critical crack length a_c . A coupled effect exists between the two variables and significantly influences the fatigue crack growth rate da/dN and fatigue life N_f .

For these specimen are under constant amplitude loads, the variation range of the maximum load is from 150 MPa to 185 MPa with the amplitude being 18.92 % of the maximum. The variation range of the critical crack length is shown in Table 1 with the amplitude being 30.38 % of the maximum. The variation range of the fatigue is in the range of 33026 - 234691 cycles with the amplitude being 85.93 % of the maximum. The variation range of the calculation results of the critical stress intensity factor constitutes 31.48 - 39.47 N/m^{3/2} with the amplitude being 20.24 % of the maximum. Taking the scatter of welding joint mechanical properties into account, the calculation results prove that the critical stress intensity factor calculated by the equation (5) are not influenced by the maximum stress or critical crack length.

The change of the maximum stress in constant amplitude loads directly influences the critical crack length, but the coupled effect between them does not influence the critical stress intensity factor.

3.2.2 Loading spectrums

When the fatigue loads change to variable amplitude loads, the fatigue crack propagation changes in a complex way because of the interaction between different load levels. There are some models to explain those changes, such as plastic zone of the crack tip model and crack closure model [17]. Critical stress intensity factor is not involved in any modifications of these models. Therefore it is reasonable to believe that critical SIF under variable amplitude loads is equal to that under constant amplitude loads.

SIF calculation results of the specimen tested under two-level variable amplitude loads (shown in Fig. 2) are congruent with the results obtained under constant amplitude loads, confirming the statement above.

But the maximum stress used to calculate the critical SIF is the maximum of the spectrum block which fracture occurred in, and that maximum is not always equal to the maximum of the whole spectrums. Take specimen P6 as an example, the maximum stress is used to calculate the critical stress intensity factor is the maximum of the fracture spectrum block 150 MPa, not the maximum of the whole spectrums 162.5 MPa.

3. 2. 3 Loading frequency

The loading frequency has no effect on the fatigue strength unless under the influence of corrosion or temperature. Therefore the fatigue crack growth rate is not influenced by loading frequency under the same conditions. A formula of critical stress intensity factor has been derived from the fatigue crack growth rate equation (shown as equation 8). According to that, there is no parameter influenced by the loading frequency. So it can be concluded that loading frequency in 5-200 Hz range has no effect on the critical stress intensity factor if it is not influenced by corrosion or temperature factors, and the critical stress intensity factor calculation results of the specimen tested under 4, 6, 8, 10 Hz fatigue loads prove that view:

$$K_{Crit} = \frac{\Delta K}{(1-R)} \cdot \left(1 + \sqrt[q]{\frac{C \left[\left(\frac{1-f}{1-R} \right) \Delta K \right]^n \cdot \left(1 - \frac{\Delta K_{th}}{\Delta K} \right)^p}{da/dN}} \right)^{-1} \quad (8)$$

4. Conclusions

1) The presented estimation method estimates the critical stress intensity factor based on the fracture crack size in a convenient way of measurement and calculation as well as considers the coupled effect between welding and thickness. It can be used for fatigue failure analysis in engineering practice.

2) The estimation results do not vary with test parameters such as loading frequency, maximum loads or loading spectrums.

3) The critical stress intensity factor measured by the fracture crack size is the local value of the welding joint fatigue fracture, could be reference to fracture toughness in the break as a complement of the welding joint fracture toughness partition measurement results and should apply to fatigue fracture analysis of welded structures subjected to vibration loads in novel aerospace vehicles.

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