860. Criteria of evaluating initial model for effective dynamic model updating

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Abstract. Finite element model updating is an important research field in structural dynamics. Though a variety of updating methods have been proposed in the past decades, all the methods could be effective only on the assumption that the initial finite element model is updatable. The assumption has led to the fact that many researchers study on how to update the model while little attention is paid to studies on whether the model is updatable. This has become inevitable obstacle between research and engineering applications because the assumption is not a tenable hypothesis in practice. To circumvent this problem, the evaluation of model updatability is studied in this paper. Firstly, two conditional statements about mapping are proved as a theoretical basis. Then, two criteria for evaluation of initial models are deduced. A beam is employed in the numerical simulations. Two different initial models for the beam are constructed with different boundary conditions. The models are evaluated using the proposed criteria. The results indicate that the criteria are able to distinguish the model updatability.

Keywords: finite element model, dynamic model updating, initial model, evaluation criteria, numerical simulation.

Introduction

In modern engineering, finite element model (FEM) plays a key role in structural design. However, there are always errors in finite element modeling of a structure due to various uncertainties and assumptions. According to Mottershead and Friswell [1, 2], there are three commonly encountered forms of errors which give rise to inaccuracy: (1) model structure error; (2) model parameter error; (3) model order error. Hence, model updating must be performed to minimize the deviation between finite element analysis results and experimental analysis results so that the FEM could be applied with confidence.

In the past decades, varieties of model updating methods have been proposed. Although in recent years, the concept of model updating has recently been extended to multiple-scale problem [3], nonlinear problem [4, 5], stochastic problem [6] or static problem [7], most of the current methods and applications deal with single-scale, linear and dynamic problems [8-12]. These methods could be classified into two categories depending on how updating objects are defined: matrix updating and design parameter updating. For the former, elements of mass matrix and stiffness matrix are taken as updating objects. For the latter, design parameters such as inertia moment, Young's modulus, density, cross-section area, length of rigid element, etc., are taken as updating objects. It's obvious that, for the latter, the updated value can be easily interpreted by the engineers because of its obvious physical meaning.

As it is known to all, initial finite element model (initial model for the simplification) plays a key role in finite element model updating. Not only does it provide the initial values for optimization problem, which model updating used to be concluded as, but also it gives the model configurations that may or may not contain the structure error or order error. Initial values are values for updating objects such as density, inertia moment, section area, etc. For optimization problem, the initial values should be as close as possible to the real solutions. The values of the initial models will definitely have great influence on the speed and accuracy of updating convergence. Comparing with the initial values, the configurations have much more significant influence. If there are erroneous configurations, the model may contain structure error or order error which are hard to be corrected by model updating. And, the updated model can be nothing but a partially equivalent model. Therefore, the initial model has a pronounced influence on the success of model updating.

In the past years, few researchers had focused on the initial models, or more specifically, on their updatability. Although all the proposed updating methods had been proved to be able to improve the initial models in the studies, there is one assumption that had been used by all the researchers but never been clearly stated, that is, the initial model is updatable. In other words, initial models had always been assumed to be good enough for updating.

However, that is not a tenable hypothesis in practice. In fact, the quality of initial models has to be subjected to the experience and skill of the engineer. The model might in all possibility not be updatable. Hence, before updating, each initial model must be evaluated to determine whether it worth updating or whether it is updatable.

There are hardly any studies on the initial model evaluation. Research works worthy of regarding are mainly concerned with correlation analysis. Modal assurance criterion (MAC), proposed by Allemang [13], has been widely used as an indicator for the correlation analysis between the analytical mode shapes and the identified experimental mode shapes. MAC helps the researchers to form the first impression on whether and how close the computational results and experimental results are. Ewins [14] proposed another method to perform correlation analysis between two sets of vibration data. Experimental and computational modal frequencies are marked in rectangular coordinate as *X*-axis and *Y*-axis. If all the points scatter on the same side of the diagonal, there might be errors on material properties. The motivations of these two studies are actually find the methods for correlation analysis. Neither of the methods is able to give the definite assessment whether the initial model is updatable or not. This has become inevitable obstacle between research and engineering applications.

To circumvent the problem, initial model evaluation criteria for effective dynamic model updating and the theoretical basis are considered in this paper. This study aims to provide simple but feasible criteria for initial model evaluation. The paper is organized as follows: in Section 2, two conditional statements about mapping are proved. In Section 3, initial model evaluation criteria are deduced based on the two condition statements in Section 2 as theoretical basis. In Section 4, model evaluations using numerical simulation are presented.

1. Two conditional statements about mapping

Supposing there are n independent variables:

$$x_i \in \left[x_i^l \quad x_i^u \right],\ i = 1,\ 2,\ \cdots,\ n \tag{1}$$

where x_i^u and x_i^l are upper and lower bound of definitional domain for x_i respectively.

In addition, supposing there are m dependent variables:

$$y_j = y_j\left(x_1, x_2, \cdots, x_n\right) \in \left[y_j^l \quad y_j^u \right],\ j = 1,\ 2,\ \cdots,\ m \tag{2}$$

where y_j is defined to be the function with respect to x_1, x_2, $\cdots$, x_n, y_j^u and y_j^l are upper and lower bound of value range for y_j respectively.

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Obviously, independent variables and dependent variables meet the requirements of mapping, i. e. uniqueness and ergodicity.

For a given $Y^e = \left\{y_1^e, y_2^e, \cdots, y_m^e\right\}$, the following two conditional statements could be proved.

(1) If $\exists y_j^e \notin \left[y_j^l \quad y_j^u\right], j = 1,\ 2,\ \cdots,\ m;$ then there doesn't exist $X^e = \left\{x_1^e,\ x_2^e,\ \cdots,\ x_n^e\right\}$, satisfying $x_i^e \in \left[x_i^l \quad x_i^u\right],\ i = 1,\ 2,\ \cdots,\ n$.

Verification:

Supposing there exists $X^e = \left\{x_1^e,\ x_2^e,\ \cdots,\ x_n^e\right\}$, which satisfies:

$$x_i^e \in \left[x_i^l \quad x_i^u\right],\ i = 1,\ 2,\ \cdots,\ n \tag{3}$$

According to the uniqueness, there exists unique $y_j^e,\ j = 1,\ 2,\ \cdots,\ m,$ which satisfies:

$$y_j^e \in \left[y_j^l \quad y_j^u\right],\ j = 1,\ 2,\ \cdots,\ m \tag{4}$$

This conflicts with the hypothesis:

$$\exists y_j^e \notin \left[y_j^l \quad y_j^u\right],\ j = 1,\ 2,\ \cdots,\ m \tag{5}$$

Therefore, the assumption is not true.

(2) If $\forall y_j^e \in \left[y_j^l \quad y_j^u\right], j = 1,\ 2,\ \cdots,\ m;$ then there exists $X^e = \left\{x_1^e,\ x_2^e,\ \cdots,\ x_n^e\right\}$, satisfying $x_i^e \in \left[x_i^l \quad x_i^u\right],\ i = 1,\ 2,\ \cdots,\ n$.

Verification:

According to properties of subjective mapping, for each $y_j^e \in \left[y_j^l \quad y_j^u\right], j = 1,\ 2,\ \cdots,\ m,$ there must exist:

$$X^e = \left\{x_1^e,\ x_2^e,\ \cdots,\ x_n^e\right\} \tag{6}$$

where x_i^e satisfies:

$$x_i^e \in \left[x_i^l \quad x_i^u\right],\ i = 1,\ 2,\ \cdots,\ n \tag{7}$$

2. Criteria for initial model evaluation

Considering an initial finite element model, the model obviously represents the implicit functional relationships of modal frequencies with respect to structural parameters. This gives the motivation to employ the above conditional statements in the initial model evaluation.

Supposing there are n structural parameters for the initial model:

$$p_i \in \left[p_i^l \quad p_i^u\right],\ i = 1,\ 2,\ \cdots,\ n \tag{8}$$

where p_i is i^{th} parameter, p_i^u and p_i^l are upper and lower bound of definitional domain for p_i respectively.

Supposing there are m modal frequencies of concerned modes for the initial model:

$$f_j = f_j\left(x_1, x_2, \cdots, x_n\right) \in \left[f_j^l \quad f_j^u\right], j = 1,\ 2,\ \cdots,\ m \tag{9}$$

where f_j is the modal frequency of j^{th} mode, f_j^u and f_j^l are upper and lower bound of value range for f_j respectively. Also f_j is the function defined by the initial model.

Since the function between structural parameters and modal frequencies meet the requirements of mapping, the conditional statements proved in section 2 are applicable here. Based on the two conditional statements, two similar conditional statements and criteria about initial model evaluation could be derived as follows.

$F^e = \left\{f_1^e, f_2^e, \cdots, f_m^e\right\}$ is a group of experimental modal frequencies to be used in the dynamic finite element model updating, of which f_j^e is the experimental modal frequency of j^{th} mode.

(1) If $\exists f_j^e \notin \left[f_j^l \quad f_j^u\right], j = 1,\ 2,\ \cdots,\ m;$ then there doesn't exist $P^e = \left\{p_1^e,\ p_2^e,\ \cdots,\ p_n^e\right\}$, satisfying $p_i^e \in \left[p_i^l \quad p_i^u\right],\ i = 1,\ 2,\ \cdots,\ n$.

Criterion 1: If any of the experimental modal frequency i. e. f_j^e, is not contained in the corresponding range $\left[f_j^l \quad f_j^u\right]$, the initial model is not updatable with the given definition domain of structural parameters.

(2) If $\forall f_j^e \in \left[f_j^l \quad f_j^u\right], j = 1,\ 2,\ \cdots,\ m;$ then there exists $P^e = \left\{p_1^e,\ p_2^e,\ \cdots,\ p_n^e\right\}$, satisfying $p_i^e \in \left[p_i^l \quad p_i^u\right],\ i = 1,\ 2,\ \cdots,\ n$.

Criterion 2: If each of the experimental modal frequency i. e. f_j^e, is contained in the corresponding range $\left[f_j^l \quad f_j^u\right]$, the initial model is updatable with the given definition domain of structural parameters.

3. Numerical simulation: model evaluation

3. 1. Elastic boundary beam

In the numerical simulation, a beam with elastic boundary condition is used [2]. Fig. 1 illustrates configuration of the beam. The length of the beam is 0.7 m. The values of parameters are considered as the real structural parameter values. The mass density is $7850\,\mathrm{kg/m^3}$ and the area of the cross section is $4.2654\times10^{-4}\,\mathrm{m^2}$. The EI of the beam is $4560\,\mathrm{Nm^2}$ in which E is the Young's Modulus and I is the moment of section inertia. The translation and rotation stiffness of the boundary are $K_t = 4.0\times10^7\,\mathrm{Nm^{-1}}$ and $K_r = 1.0\times10^5\,\mathrm{Nm/rad}$ respectively.

The beam is used to compute the simulated experimental modal frequencies and mode shapes. Simulated experimental modal frequencies of the first six modes are listed in Table 1.

Table 1. Simulated experimental modal frequencies of the first six modes

Mode order	1	2	3	4	5	6
Experimental modal frequencies (Hz)	37.5	236.8	657.6	1256.3	1996.6	2889.7

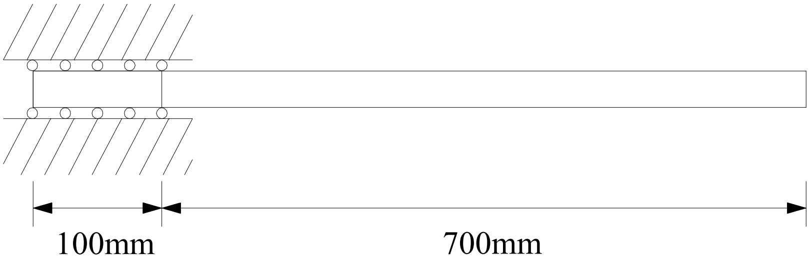

Fig. 1. Beam with elastic boundary condition

3. 2. Evaluation of initial model with erroneous boundary condition

In the modeling of the beam, the boundary should be modeled as elastic end. But, in this case, the boundary condition is intentionally modeled incorrectly. The beam is modeled to be a fixed-end beam as shown in Fig. 2.

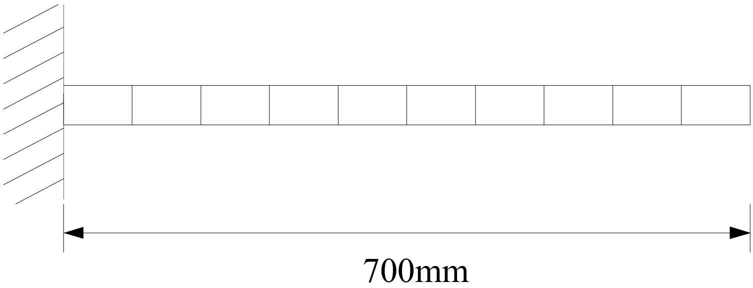

Fig. 2. Incorrect initial model with fixed-end boundary condition

The initial model consists of ten beam elements. The value of EI is set to be $4500\ \text{Nm}^2$. In fact, whatever the parameters are, the initial model is naturally to be a wrong model due to the incorrect configuration of boundary condition. For the purpose of optimization constraints, the definition domain for EI is set to be $\left[3420\ \text{Nm}^2 \quad 5700\ \text{Nm}^2\right]$, which are 0.75 and 1.25 times of those 'real' values given in 3.1. It is worth noting that the definition domain for EI may be determined according to engineering experience. And, the updatability could be affected by the definition domain. However, in this case where erroneous boundary condition exists in initial model, the correctness of the criterions proposed in Section 3 will not be influenced.

Modal frequencies of the initial model are listed in the fifth column of Table 2. The lower and upper bound for the modal frequencies are listed in the third and forth column of Table 2.

Table 2. Lower and upper bounds for modal frequencies: incorrect initial model

Mode order	Experimental modal frequency (Hz)	Lower bound (Hz)	Upper bound (Hz)	Initial modal frequency (Hz)	Modal frequency ($EI = 4560\ \text{Nm}^2$)
1	37.5	36.5	47.1	41.9	42.1
2	236.8	228.7	295.3	262.4	264.1
3	657.6	640.6	827.0	734.8	739.7
4	1256.3	1256.2	1621.8	1441.0	1450.6
5	1996.6	2079.9	2685.1	2385.8	2401.6
6	2889.7	3115.9	4022.6	3574.2	3597.9

From the second column to the forth column of Table 2 it is observed that the fifth and the sixth experimental modal frequencies are not within the bounds. This implies that it is impossible to find certain groups of parameter values in the given closed interval $\left[3420\ \text{Nm}^2 \quad 5700\ \text{Nm}^2\right]$ that could minimize the deviations between the experimental and the computational modal frequencies of these two modes using the given model configuration. According to criterion 1, with this given definition domain for the selected parameter, the model is not updatable.

For another point of view, modal frequencies of the beam computed using fixed-end configuration and correct parameter value ($4560\ \text{Nm}^2$) are listed in the sixth column. It is obvious that the computed modal frequencies significantly differ from the experimental data even if there is no parameter error.

3. 3. Evaluation of initial model with correct boundary condition

In this case, the beam is modeled correctly using the spring elements to model the elastic boundary condition. Fig. 3 illustrates the initial model of elastic-end beam.

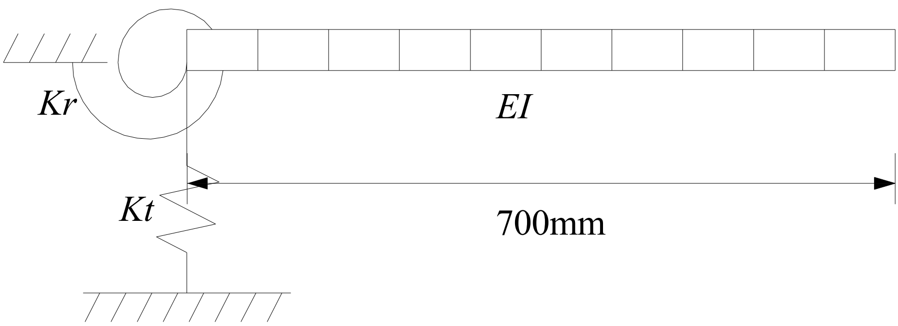

Fig. 3. Correct initial model with elastic-end boundary condition

The initial model also consists of ten beam elements. The value of EI is set to be $4500\ \text{Nm}^2$. The values of spring stiffness are set to be $K_t = 2.0\times10^7\ \text{Nm}^{-1}$ and $K_r = 5.0\times10^4\ \text{Nm/rad}$. And the definition domain for EI is set to be $\left[3420\ \text{Nm}^2 \quad 5700\ \text{Nm}^2\right]$. The definition domains of spring stiffness are set to be 0.3 and 2 times those of the 'real' values given in 3.1, they are $\left[1.2\times10^7\ \text{Nm}^{-1} \quad 8\times10^7\ \text{Nm}^{-1}\right]$ and $\left[3\times10^4\ \text{Nm}^{-1} \quad 2\times10^5\ \text{Nm}^{-1}\right]$ respectively. The range of definition domain of spring stiffness exceeds the range of EI because the uncertainties in boundary conditions are generally larger than the uncertainty in EI.

Modal frequencies of the initial model are listed in the fifth column of Table 3. The lower and upper bounds for the modal frequencies are listed in the third and forth columns of Table 3.

From the second column to the forth column of Table 3, it is observed that all of the six experimental modal frequencies are within the lower bound and the upper bound. It means that the initial model could be updated to minimize the deviations between the experimental and the computational modal frequencies given the model configuration. According to criterion 2, with these given ranges for the selected parameters, this model is updatable.

Table 3. Lower and upper bounds for modal frequencies: correct initial model

Mode order	Experimental modal frequency (Hz)	Lower bound (Hz)	Upper bound (Hz)	Initial modal frequency (Hz)
1	37.5	28.3	43.7	33.9
2	236.8	187.5	273.9	220.6
3	657.6	522.1	760.9	613.9
4	1256.3	974.9	1464.2	1155.0
5	1996.6	1538.2	2350.5	1819.8
6	2889.7	2297.4	3397.8	2687.1

Conclusions

Each initial finite element model must be evaluated to determine whether it is updatable before conducting the updating procedure. However, initial model evaluation is an issue that has been neglected over the years. This paper presents a study on the criteria of initial model evaluation for effective dynamic model updating. Two conditional statements about mapping are proved as theoretical basis. Then, two criteria are deduced as follows:

Criterion 1: If any of the experimental modal frequency i. e. f_j^e , is not contained in the corresponding range, the initial model is not updatable with the given definition domain of structural parameters.

Criterion 2: If each of the experimental modal frequency i. e. f_j^e , is contained in the corresponding range, the initial model is updatable with the given definition domain of structural parameters.

To demonstrate the effectiveness of the criteria, two initial models are constructed for a beam with one including erroneous boundary condition and the other with correct boundary condition. Results confirm that the criteria are able to distinguish the model updatability.

The authors realize that the proposed method is just the preliminary step in the research of initial model evaluation. The method need to be further investigated on many factors that may affect the success of model evaluation, e. g. noise-contamination on modal frequencies. This research work is now in progress and the results will be reported in the near future.

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