859. Analytical framework for analyzing brake squeal noise using assumed-modes approach

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(Received 15 June 2012; accepted 4 September 2012)

Abstract. Sometimes a loud noise or high pitched squeal occurs when the brakes are applied. It is generated during the braking phase and is characterized by a harmonic spectrum. Brake squeal is induced by self-excited vibrations, consequences of local nonlinearities at the contact interface. Many researchers have examined the problem with experimental, analytical, and computational techniques, but there is still no method to fully annihilate brake squeal. This paper deals with presentation of a new model to analyze the brake squeal behavior. In this paper, a lumped-continuous vibration model is presented for the braking system and nonlinear equations are obtained using the Hamilton's principle. Then, the linearization of nonlinear equations is done around the equilibrium point of system and linear stability analysis is discussed. Furthermore, the effects of different braking parameters such as friction coefficient, rotational speed, pad stiffness, calipers etc. on the brake squeal noise are investigated.

Keywords: brake squeal, noise, assumed modes method, lumped-continuous model.

Introduction

Automobiles are the main mode of transportation for people travelling from one place to another. Brakes are one of the most important safety and performance components in motor vehicles. A wide range of brake noise and vibration phenomena is described by many investigators. Squeal, groan, chatter, judder, moan, hum, and squeak are just a few of the names found in the literature [1]. Brake mechanisms are the place of complex dynamic behaviors which can lead to the emission of unpleasant sounds as squeal. No precise definition of brake squeal has gained complete acceptance, but it is generally agreed that squeal is a sustained, high-frequency (1–20 kHz) vibration of brake system components during a braking action [2]. Overall brake squeal can be annoying to the vehicle passengers, passers-by, pedestrians, etc. especially as vehicle designs become quieter. There has been significant progress in understanding the generation of brake squeal and in developing numerical methods for analyzing its characteristics [3–7]. However, as mentioned by Oberst and Lai [8], the problem of predicting and reducing brake squeal propensity remains as challenging as ever. This is because the nature of brake squeal is fugitive, transient due to its high dependency on a large number of interacting parameters, such as contact conditions, material properties and ever-changing operating conditions. Traditionally, research into brake squeal has been focused on three approaches using: (1) experimental methods to study the vibration and acoustic responses of a brake system; (2) numerical methods, predominantly finite element analysis techniques, to assess brake squeal propensity; and (3) simplified analytical models to gain insights into the mechanisms of brake squeal generation [9]. Most early works on instability analysis were performed with hand-derived equations for mass-spring models that were used to represent real structures. The analyses for more complicated mass-spring models by Jarvis and Mills [10], Earles and Lee [11], North [12], Millner [13], and many others have revealed that even when the friction coefficient is constant, the model can be unstable if the friction force couples two degrees-of-freedom together. A large-scale finite element analysis of the stability of the linearized brake system also confirms that instability arises when two modes coalesce under the influence of friction. For the first time Chan et al. [14] considered the disk and transitional disk vibrations as a plate and analytical components modes, respectively. In the work of Chowdhary et al. [15], individual brake components were modeled and solved separately for their modal characteristics. Then, these were coupled together at the contact interface and the equations of motion were derived through the Lagrangian approach. Chakraborty et al. [16] also used thin plate model for the disc. They introduced nonlinear spring for the pads. Von Wagner et al. [17] further demonstrated that the frequency of the limit-cycle vibration of the disc was very close to that of the linear unstable vibration obtained through a complex eigenvalue analysis. Lee et al. [18] expressed modal sound radiation of a brake rotor in terms of analytical solutions of a generic thick annular disk having similar geometric dimensions. They used finite element method to determine structural modes and response. Joe et al. [19] proposed a linear, lumped, and distributed parameter model to represent the floating caliper disc brake system. The complex eigenvalues were used to investigate the dynamic stability, and, in order to verify simulations which are based on the theoretical model, an experimental modal test and dynamometer test were performed.

Analytical methods cannot resolve the complex interactions between components of the braking system features to model them; however, they can provide sufficient understanding of the mechanisms causing the brake squeal phenomena. This paper deals with analytical analysis of brake squeal. First, an appropriate physical model is presented for the brake system, and then the governing equations are obtained using Hamilton's principle. The linearization of nonlinear equations is performed around the balance point and linear stability analysis is discussed. Finally, system stability is analyzed in state space and the effect of different parameters on stability of brake system is investigated.

Modeling of brake system

A lumped-continuous vibration model is used for modeling the brake system as shown in Fig. 1. This model consists of a rotating circular disk with angular velocity of Ω. Displacement in the direction normal to the disk is presented by w_d. The caliper is modeled by two bodies (m_1, m_2) above and below the disc. Displacement of the first and second mass is represented with w_{c1} and w_{c2} respectively. They are connected to a pin by two rods with length of λ. The rods are massless and are connected to each other by a torsional spring with stiffness of k_T. The lower mass is connected to the internal pad by a spring with stiffness of k_c and a damper with damping coefficient of d_c. Pads are assumed to be massless and are modeled by a spring and damper with stiffness and damping coefficient of k_p and d_p, respectively (Fig. 1).

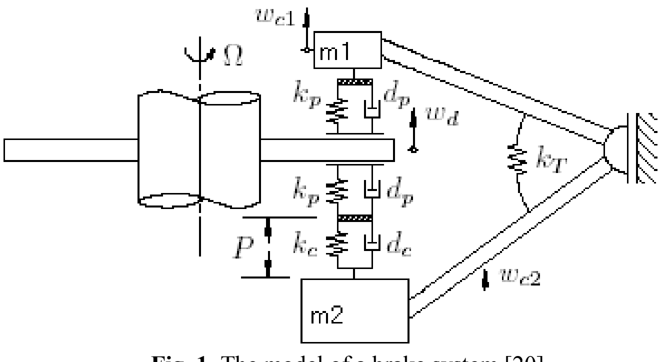

Fig. 1. The model of a brake system [20]

Analytical modeling

Hamilton's principle is expressed as [21]:

$$\int_{t_1}^{t_2} (\delta L + \delta W)dt = 0 \tag{1}$$

$$L = T - U$$

Disk kinetic energy is:

$$T_d = \frac{1}{2}\rho h \iint (\frac{\partial w_d}{\partial t} + \Omega \frac{\partial w_d}{\partial \varphi})^2 rdrd\varphi \tag{2}$$

$$\delta T_d = \rho h \iint (\frac{\partial(\delta w_d)}{\partial t} + \Omega \frac{\partial(\delta w_d)}{\partial \varphi})(\frac{\partial w_d}{\partial t} + \Omega \frac{\partial w_d}{\partial \varphi}) rdrd\varphi$$

where $\frac{\partial w_d}{\partial t}$ is the disk displacement in normal direction and $\Omega \frac{\partial w_d}{\partial t}$, is the component of radial motion of disc normal to its plane.

Disk potential energy is:

$$U_d = \frac{1}{2} D_E \iint ((\nabla^2 w_d)^2 + 2(1-\nu)((\frac{1}{r}\frac{\partial^2 w_d}{\partial r \partial \varphi} - \frac{1}{r^2}\frac{\partial w_d}{\partial \varphi})^2 - \frac{\partial^2 w_d}{\partial r^2}(\frac{1}{r^2}\frac{\partial^2 w_d}{\partial \varphi^2} - \frac{1}{r}\frac{\partial w_d}{\partial r})))rdrd\varphi \tag{3}$$

$$\delta U_d = \iint \frac{D_E}{2}[(2\nabla^2 w_d)(\delta \nabla^2 w_d) + 2(1-\nu)(2(\frac{1}{r}\frac{\partial^2 w_d}{\partial r \partial \varphi} - \frac{1}{r^2}\frac{\partial w_d}{\partial \varphi})(\delta(\frac{1}{r}\frac{\partial^2 w_d}{\partial r \partial \varphi}) - \delta(\frac{1}{r^2}\frac{\partial w_d}{\partial \varphi}))$$
$$-\frac{\partial^2 w_d}{\partial r^2}(\delta(\frac{1}{r^2}\frac{\partial^2 w_d}{\partial \varphi^2}) - \delta(\frac{1}{r}\frac{\partial w_d}{\partial r})) - \delta(\frac{\partial^2 w_d}{\partial r^2})(\frac{1}{r^2}\frac{\partial^2 w_d}{\partial \varphi^2} - \frac{1}{r}\frac{\partial w_d}{\partial r}))]rdrd\varphi$$

Potential energy of upper springs is:

$$U_{kp} = \frac{1}{2}\int_{-\varphi_0}^{\varphi_0}\int_{r_i}^{r_o} k_p (w_d - w_{c1})^2 rdrd\varphi \tag{4}$$

$$\delta U_{kp} = \int_{-\varphi_0}^{\varphi_0}\int_{r_i}^{r_o} k_p (w_d - w_{c1})(\delta w_d - \delta w_{c1})rdrd\varphi$$

where $2\phi_0$ is the hoof angle and r_0, r_1 are the inner and outer radius of disc, respectively.

Potential energy of lower springs can be obtained from Eq. (5):

$$U_{keq} = \frac{1}{2}\int_{-\varphi_0}^{\varphi_0}\int_{r_i}^{r_o} k_{eq}(w_{c2} - w_d)^2 rdrd\varphi \tag{5}$$

$$\delta U_{keq} = \int_{-\varphi_0}^{-\varphi_0}\int_{r_i}^{r_o} k_{eq}(w_{c2} - w_d)(\delta w_{c2} - \delta w_d)rdrd\varphi$$

where:

$$k_{eq} = \frac{k_p k_c}{A_L k_p k_c}$$

A_L is the area of the hoof. Potential energy of torsional spring is obtained as following:

$$U_{kt} = \frac{1}{2}\frac{k_T}{l^2}(w_{c2} - w_{c1})^2, \quad \delta U_{kt} = \frac{k_T}{l^2}(w_{c2} - w_{c1})(\delta w_{c2} - \delta w_{c1}) \tag{6}$$

In order to find the virtual work, according to Fig. 2 we can write:

$$W_f = \int_{-\varphi_0}^{\varphi_0}\int_{r_i}^{r_o} [(-f_1)(-\frac{h}{2}(\frac{\partial w_d}{r \partial \varphi})) + (-f_2)(\frac{h}{2}(\frac{\partial w_d}{r \partial \varphi}))]rdrd\varphi \tag{7}$$

where f_1, f_2 are the friction force per unit area between the discs and pads:

$$f_1 = (\frac{\mu \cos\theta}{1 + \frac{\mu \sin 2\theta}{2}})(k_p(w_d - w_{c1}) + d_p(\dot{w}_d - \dot{w}_{c1})) \tag{8}$$

$$f_2 = (\frac{\mu \cos\theta}{1 - \frac{\mu \sin 2\theta}{2}})(k_{eq}(w_{c2} - w_d) + d_p(\dot{w}_{c2} - \dot{w}_d))$$

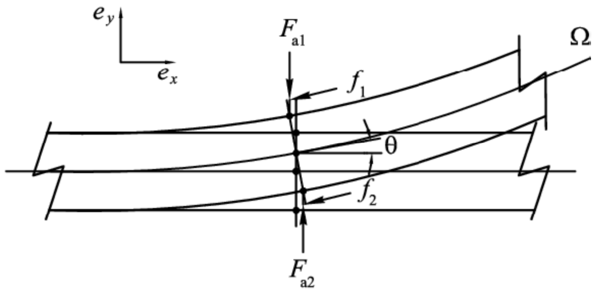

Fig. 2. Direction of friction force

Virtual work of compressive force is:

$$\delta W_p = -PA_L \delta w_{c2} + \int_{-\varphi_0}^{\varphi_0} \int_{r_i}^{r_o} p \delta w_d r dr d\varphi \tag{9}$$

Virtual work of depreciation force in upper dampers is:

$$W_{d1} = \int_{-\varphi}^{\varphi} \int_{r_i}^{r_o} d_p (\dot{w}_d - \dot{w}_{c1})(w_d - w_{c1}) r dr d\varphi \tag{10}$$

$$\delta W_{d1} = \int_{-\varphi}^{\varphi} \int_{r_i}^{r_o} d_p ((\delta \dot{w}_d - \delta \dot{w}_{c1})(w_d - w_{c1}) + (\dot{w}_d - \dot{w}_{c1})(\delta w_d - \delta w_{c1})) r dr d\varphi$$

Virtual work of depreciation force in lower dampers can be obtained as:

$$W_{d2} = \int_{-\varphi}^{\varphi} \int_{r_i}^{r_o} d_{eq} (\dot{w}_{c2} - \dot{w}_d)(w_{c2} - w_d) r dr d\varphi \tag{11}$$

$$\delta W_{d2} = \int_{-\varphi}^{\varphi} \int_{r_i}^{r_o} d_{eq} ((\delta \dot{w}_{c2} - \delta \dot{w}_d)(w_{c2} - w_d) + (\dot{w}_{c2} - \dot{w}_d)(\delta w_{c2} - \delta w_d)) r dr d\varphi$$

where:

$$d_{eq} = \frac{d_p d_c}{A_L d_p d_c}$$

It should be noted that the total amount of kinetic and potential energy of the system is summation of kinetic and potential energies of the system components:

$$U_{tot} = U_d + U_{kp} + U_{keq} + U_{kt} \tag{12}$$

$$T_{tot} = T_d + T_M + T_m$$

Based on assumed modes assumptions, we selected only one mode of the plate to obtain new lumped system against continuous system. We can obtain the natural frequencies of the plate by using the method suggested by Meirovitch [21]. The corresponding natural frequency is obtained and is shown in Table 2. Now, by neglecting higher order terms and linearization about balance point, we can obtain the final equations of the system. The final four linear equations of the system are obtained as:

$$-k_p w_{c1} A_L + k_p \left(\iint_{D0} R_{3,1}(A \cos 3\varphi + B \sin 3\varphi) r dr d\varphi\right) + \frac{k_T}{L}(w_{c1} - w_{c2}) - m_1 \ddot{w}_{c1} = 0 \tag{13}$$

$$-k_{eq} w_{c2} A_L + k_{eq} \left(\iint_{D0} R_{3,1}(A \cos 3\varphi + B \sin 3\varphi) r dr d\varphi\right) + \frac{k_T}{L}(w_{c1} - w_{c2}) - PA_L - m_2 \ddot{w}_{c2} = 0 \tag{14}$$

$$\delta A \iint \{\ddot{A}(\rho h R_{3,1}^2 \cos^2 3\varphi) + \ddot{A}(\rho h R_{3,1}^2 \cos 3\varphi \sin 3\varphi) + \dot{A}[3\Omega \rho h R_{3,1}^2 \cos 3\varphi \sin 3\varphi$$

$$-2R_{3,1}^2 \cos^2 3\varphi (d_p + d_{eq}) - 3 d_p \mu h \cos 3\varphi \sin 3\varphi \frac{R_{3,1}^2}{r}]$$

$$
\begin{aligned}
&+\dot{B}[-3\Omega\rho h R_{3,1}^2\cos^2 3\varphi-2R_{3,1}^2\cos\ 3\varphi\sin 3\varphi(d_p+d_{eq})-3d_p\mu h\cos^2 3\varphi\frac{R_{3,1}^2}{r}]\\
&+A[9\rho h\Omega^2 R_{3,1}^2\cos^2 3\varphi+R_{3,1}^2\cos^2 3\varphi(k_{eq}+k_p)+3\mu PR_{3,1}\cos\ 3\varphi\sin 3\varphi\\
&+3\mu h(k_{eq}+k_p)\cos\ 3\varphi\sin 3\varphi\frac{R_{3,1}^2}{2r}]+B[9\rho h\Omega^2 R_{3,1}^2\cos\ 3\varphi\sin 3\varphi\\
&+R_{3,1}^2\cos\ 3\varphi\sin 3\varphi(k_{eq}+k_p)-3\mu PR_{3,1}\cos^2 3\varphi-3\mu h(k_{eq}+k_p)\cos^2 3\varphi\frac{R_{3,1}^2}{2r}]\\
&-w_{c1}(k_pR_{3,1}\cos 3\varphi)-w_{c2}(k_{eq}R_{3,1}\cos 3\varphi)\}rdrd\varphi=0
\end{aligned}
\tag{15}
$$

$$
\begin{aligned}
&\delta B\iint\ \{\ddot{A}(\rho h R_{3,1}^2\cos\ 3\varphi\sin 3\varphi)+\ddot{B}(\rho h R_{3,1}^2\sin^2 3\varphi)+\dot{A}[3\Omega\rho h R_{3,1}^2\sin^2 3\varphi\\
&-2R_{3,1}^2\cos\ 3\varphi\sin 3\varphi(d_p+d_{eq})+3d_p\mu h\cos^2 3\varphi\frac{R_{3,1}^2}{r}]\\
&+\dot{B}[-3\Omega\rho h R_{3,1}^2\cos\ 3\varphi\sin 3\varphi-2R_{3,1}^2\sin^2 3\varphi(d_p+d_{eq})-3d_p\mu h\cos\ 3\varphi\sin 3\varphi\frac{R_{3,1}^2}{r}]\\
&+A[-9\rho h\Omega^2 R_{3,1}^2\cos\ 3\varphi\sin 3\varphi+R_{3,1}^2\cos\ 3\varphi\sin 3\varphi(k_{eq}+k_p)\\
&-3\mu PR_{3,1}\sin^2 3\varphi+3\mu h(k_{eq}+k_p)\sin^2 3\varphi\frac{R_{3,1}^2}{2r}]+B[9\rho h\Omega^2 R_{3,1}^2\sin^2 3\varphi\\
&+R_{3,1}^2\sin^2 3\varphi(-k_{eq}-k_p)+3\mu PR_{3,1}\cos\ 3\varphi\sin 3\varphi-3\mu h(k_{eq}+k_p)\cos\ 3\varphi\sin 3\varphi\frac{R_{3,1}^2}{2r}]\\
&+w_{c1}(k_pR_{3,1}\sin 3\varphi)+w_{c2}(k_{eq}R_{3,1}\sin 3\varphi)\}rdrd\varphi=0
\end{aligned}
\tag{16}
$$

System stability analysis

The obtained equations represent a dynamic system with four degrees of freedom. For stability analysis of the system, the above-mentioned differential equations are taken into state space and stability analysis is carried out there. For this purpose, the equations are written in a matrix form:

$$\overline{M}\ddot{\vec{q}}+\overline{C}\dot{\vec{q}}+\overline{K}\vec{q}=0 \tag{17}$$

where:

$$
q=\begin{bmatrix}A\\B\\w_{c1}\\w_{c2}\end{bmatrix},\ M=\begin{bmatrix}\rho h R_{3,1}^2\cos^2 3\varphi & 0 & 0 & 0\\ 0 & \rho h R_{3,1}^2\sin^2 3\varphi & 0 & 0\\ 0 & 0 & m_1 & 0\\ 0 & 0 & 0 & m_2\end{bmatrix},
$$

$$
C=\begin{bmatrix}0 & -3\rho h\Omega R_{3,1}^2\cos^2 3\varphi-3d_p\mu h\cos^2 3\varphi\frac{R_{3,1}^2}{r} & 0 & 0\\ -3\rho h\Omega R_{3,1}^2\cos^2 3\varphi+3d_p\mu h\cos^2 3\varphi\frac{R_{3,1}^2}{r} & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\end{bmatrix}
$$

Equations in state space will be as following:

$$\dot{\vec{X}}=\overline{F}\vec{X} \tag{18}$$

For the stability analysis in state space, the eigenvalues of matrix Φ should be obtained and stability analysis will be done according to those values. If the eigenvalues have positive real parts, the system will be unstable and break squeal phenomenon will occur.

Results and discussion

The general specifications of the brake disk are shown in Table 1. Next issue is variation of the eigenvalues with the rotational speed of the disc. In the equations, rotational speed of the disc should be considered as constant in order to obtain the results; while disc rotational speed varies during the braking process. Therefore, variation of rotational speed with eigenvalues is shown in Fig. 3. It is evident that increasing the rotational speed of the disk, results in system instability. On the other hand, it seems that the eigenvalues of the matrix Φ are proportional to pad stiffness. The variation of eigenvalues with pad stiffness is represented in Fig. 4. It is clear that by increasing pad stiffness system may become unstable and thus possibility of brake squeal occurrence increases.

Table 1. System specifications

Parameter	Value
Disk inner radius	0.08 m
Disk outer radius	0.2 m
Disk height	0.025 m
m_1	0.5 kg
m_2	1 kg
Rod length	2 m

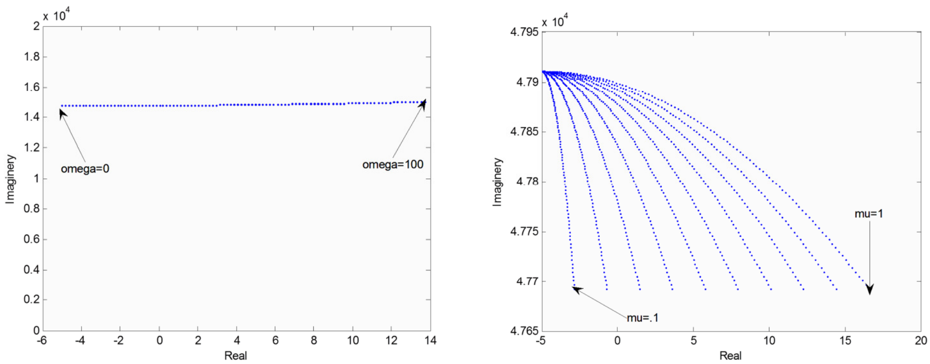

Fig. 3. Variation of eigenvalues with rotational speed

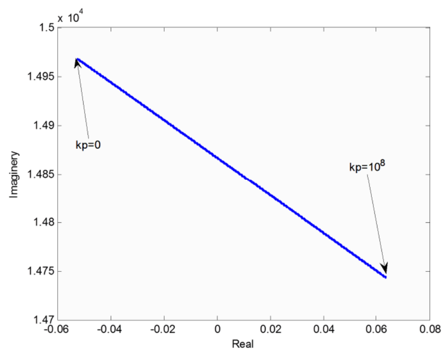

Fig. 4. Variation of eigenvalues with pad stiffness

Other system parameters such as caliper mass, length of rods and torsional stiffness of the springs have less effect on system stability. Fig. 5 is presented in order to demonstrate effect of these parameters on the eigenvalues of the matrix Φ. Variation of brake system stability, with damping coefficient is more complex. So that, at lower ranges, the damping coefficient variation does not have much effect on the system stability. But at higher ranges, it results in system instability. However, it seems that by increasing pad stiffness the brake system has more tendencies to become stable as illustrated in Fig. 6.

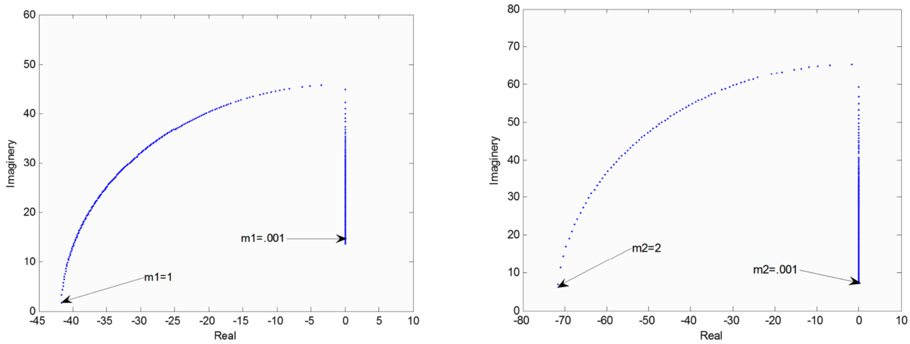

Fig. 5. Variation of eigenvalues with m_1 and m_2

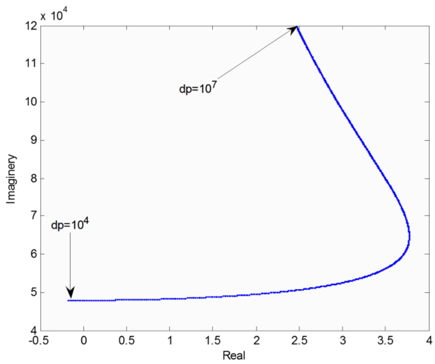

Fig. 6. Variation of eigenvalues with damping coefficient

Conclusions

In this paper, brake squeal noise in automobiles is analytically investigated by a lumped-continuous model. A new model is presented for the brake system. This model includes both lumped and continuous components. Then the governing equations of system are obtained using Hamilton's principle and are solved by the assumed-modes approach. At last, system stability is analyzed in state space and the effect of different parameters on stability of brake system is investigated. The obtained results are as follows:

- Brake system is stable for low friction coefficient, but, by increasing friction coefficient the brake system becomes unstable while other system parameters remain constant. On the other hand, since friction coefficient has a direct effect on braking torque, so, probably, there should be an optimal value for it.
- Brake system is stable for low rotational speeds, but by increasing the disc rotational speed, the system becomes unstable. It is better for braking to be an instantaneous process to prevent its occurrence.

- Increasing the mass of caliper resulted in reaching the system from oscillating mode to stable mode, that taking into account other design parameters of caliper, an optimal value can be achieved for its mass.
- Increasing the pad stiffness led to system instability, on the other hand, pad stiffness is directly proportional to the braking. The pad stiffness values are obtained through experimental data.

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