855. 3D holographic visualization of vibrations of cylindrical piezoceramic transducers

K. Ragulskis[1], R. Vasiliauskas[2], L. Patašienė[3], A. Fedaravičius[4]
[1, 3]Kaunas University of Technology, Kęstučio str. 27, LT-44312 Kaunas, Lithuania
[2]Mykolas Romeris University, Faculty of Public Security
V. Putvinskio 70, LT-4421 Kaunas, Lithuania
[4]Kaunas University of Technology, Institute of Defence Technologies
Kęstučio str. 27, LT-44312 Kaunas, Lithuania
E-mail: [1]*k.ragulskis@jve.lt*, [2]*r.vasiliauskas@mruni.eu*, [3]*laima.patasiene@ktu.lt*, [4]*alfedar@ktu.lt*

(Received 10 June 2012; accepted 4 September 2012)

Abstract. The piezoelectric material used in cylindrical transducers requiring high-precision displacements indicates that accuracy depends on design and technological factors. The analyzed criteria have enabled selection of piezoelectric material for optimal mechatronic systems having maximum displacement. Experimental investigation of precision dynamic systems by means of 3D holographic visualization enabled to collect appreciably more information about the vibrating surface in comparison with traditional methods. The developed methodology of analysis of experimental data derived from 3D holographic visualization using holographic measurement stand allowed to obtain results that are indispensable for optimization of design of mechatronic systems or its constituent elements.

Keywords: piezomaterial, mechatronics systems, 3D holographic visualization, cylindrical transducers vibrations.

Introduction

Piezoceramics can be significantly more sensitive to electric and mechanical effects compared to natural crystals. In addition, it is mechanically strong, chemically inert and resistant to atmospheric effects. Piezoelectric cells can be made of various sizes and parameters which enable them to be used in any manufactured structures with 90 % efficiency. Theoretical investigation of vibrations of cylindrical transducers and dynamic analysis of their components have indicated that increase in the loading force and initial tension decrease the harmonic components of fluctuations (Figs.1-2).

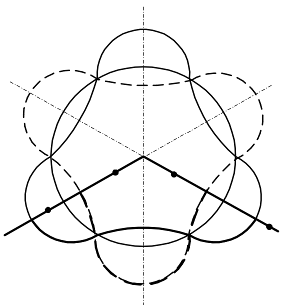


Fig. 1. Mode of vibration in a cylindrical transducer

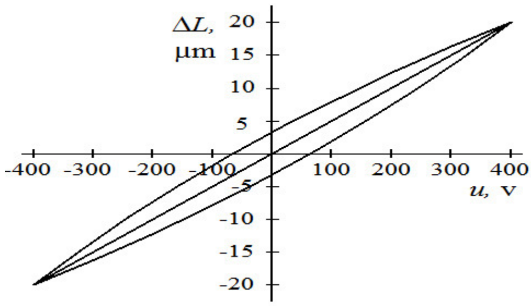


Fig. 2. Hysteresis loops corrected in cylindrical transducers vs. the feedback value

Fig. 3 shows an interferogram when a control signal is sent only to one active control cylindrical transducer of the mechatronic system. In this case, even deformation of the working part surface in the operation area of the active cylindrical transducer is observed. Slight surface deformation in the operation areas of other active piezostack emerges due to conditions of their fixing onto the surface of the working part of the mechatronic system.

This study employs the double-exposure holographic visualization technique for the quality assessment of surface deformation. The essence of this technique is the recording of holograms of two objects (being in different conditions, initial and deformed, for instance, before and after increase of voltage) on the same layer of a light-sensitive photographic plate. Upon illumination of the hologram after two exposures with a copy of the cylindrical transducers, both of them reflected by the object surface before and after the deformation, are restored at the same time [1, 6].

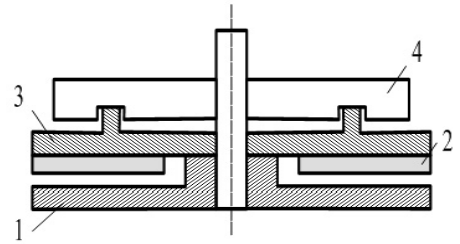


Fig. 3. Construction of the transducer: 1 - holder, 2 - piezomaterial, 3 - stator, 4 - rotor

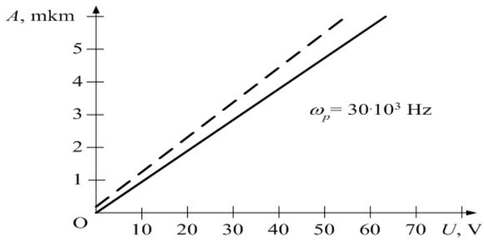


Fig. 4. Dashed line: experimental investigations, continuous line: theoretical investigations

The results of their interference – the system of interferential bands is observed against the background of the surface of the object image, which provides information about changes in the object status that occurred in the period between expositions. 3D holographic visualization of vibrations of cylindrical transducers used in the experimental work has strengthened the expressions of differential equations and was used for drawing final conclusions of the investigation. The ineffective electrical energy is stored as electrostatic energy in the piezoceramic material and reverts it to the power supply in the final process of an operating cycle. The analyzed criteria have made it possible to choose the piezomaterial for an optimal construction having a maximum displacement [7, 8].

Experimental investigation of precision vibrosystems by means of holographic interferometry enables one to obtain appreciably more information about the vibrating surface in comparison with traditional methods. The paper deals with the consideration of methods for determination of the vibrational characteristics of precision mechanical systems from the holographic interferograms of linked analysis of these characteristics received by using numerical techniques based on the theories of mechanical vibrations and holographic interferometry [4, 5, 9].

Method of holographic visualization of vibrations of cylindrical transducers

When wave properties are defined by a standing wave, quantitative analysis of interferograms shall be performed in the following method.

Think about point i located on a surface of piezoceramic cylinder (Fig. 5) whose spatial vibration vector may be defined by equation:

$$\overline{R}_i(\varphi,t) = U(\varphi,t)\hat{i} + V(\varphi,t)\hat{j} + W(\varphi,t)\hat{k} \tag{1}$$

where φ is an angular coordinate of the analyzed point on piezoceramic cylinder; t – time; $\hat{i}$, $\hat{j}$, $\hat{k}$ – unit vectors of corresponding coordinate axis z, t, or r; W – normal constituent of spatial vibration at point i; U and V – tangential constituents.

If an observation unit vector of point i is named $\hat{K}_0$, and vector opposite to an illumination unit vector of point i is named $\hat{K}_i$, the sensitivity vector of point i will be defined by the following equation:

$$\overline{K}^i = K_z^i\hat{i} + K_t^i\hat{j} + K_r^i\hat{k} \tag{2}$$

where $\hat{K}_z^i$, K_t^i, K_r^i shall be calculated from the following system of equations:

$$\begin{aligned} K_z^i &= \cos\varphi_2^i + \cos\varphi_1^i \\ K_t^i &= \sin\theta_1^i \sin\phi_1^i - \sin\theta_2^i \sin\phi_2^i \\ K_t^i &= \cos\theta_1^i \sin\phi_1^i + \cos\theta_2^i \sin\phi_2^i , \end{aligned} \tag{3}$$

where θ_1^i, θ_2^i are angles formed by the observation and illumination vectors of point *i* in relation to coordinate axis *r*; φ_1^i, φ_2^i are the corresponding angles formed by the observation and an illumination vectors in relation to coordinate axis *z*.

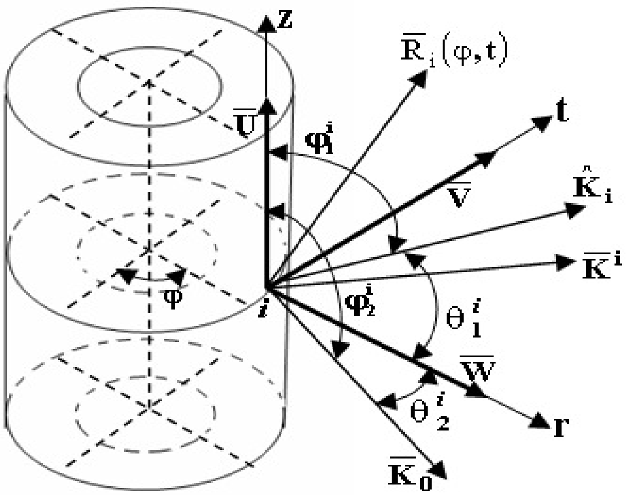


Fig. 5. Interpretation of vibrations of a standing wave in a piezoceramic transducer

A change of a light wave phase that appears due to surface vibration of a piezoceramic cylinder, when the light travels from the source to the analyzed point *i* located on the surface of piezoceramic cylinder and then proceeds to holographic interferogram, is defined as [3]:

$$\Omega_i = 2\pi \overline{R}_i(\varphi, t) \frac{\overline{K}^i}{\lambda} \tag{4}$$

where λ is a wavelength of a laser light used for fixing the interferogram; $\overline{R}_i(\varphi, t)$, and $\overline{K}^i$ are defined by equations (1) and (2).

Scalar product of spatial vibration and sensitivity vectors is defined as:

$$\overline{R}\overline{K} = U(\varphi, t)K_z^i + V(\varphi, t)K_t^i + W(\varphi, t)K_r^i \tag{5}$$

It we insert this equation into the expression of the change of a light wave phase (4), we get:

$$\Omega_i = \frac{2\pi}{\lambda}\left[U(\varphi, t)K_z^i + V(\varphi, t)K_t^i + W(\varphi, t)K_r^i\right] \tag{6}$$

where tangential constituents *U* and *V* of spatial vibration vector $\overline{R}_i(\varphi, t)$ and normal vibration constituent *W* at a surface point *i* are expressed as:

$$\begin{aligned} U(\varphi, t) &= U_0^i(\varphi)\cos(\omega t + \alpha_i), \\ V(\varphi, t) &= V_0^i(\varphi)\cos(\omega t + \beta_i), \\ W(\varphi, t) &= W_0^i(\varphi)\cos(\omega t + \gamma_i), \end{aligned} \tag{7}$$

where $U_0^i(\varphi)$, $V_0^i(\varphi)$, $W_0^i(\varphi)$ are values of amplitude tangential and normal vibration constituents of spatial vibration vector $\overline{R}_i(\varphi,t)$ that are expressed as:

$$U_0^i(\varphi)=\sum_1^n A_j^u F_{ij}^u,\ V_0^i(\varphi)=\sum_1^n A_j^v F_{ij}^v,\ W_0^i(\varphi)=\sum_1^n A_j^w F_{ij}^w, \tag{8}$$

where i – number of point i located on a surface of cylinder; j – number of a type of self-excited vibration of a piezoceramic cylinder; F_{ij} – value of the amplitude of a number of self-excited vibration type j in point i; A_j – coefficient of influence of self-excited vibration type j; n – number of self-excited vibrations types.

Characteristic function of the distribution of interference bands on a surface of piezoceramic cylinder, when holographic interferogram is fixed at time-average and harmonic vibration is present, is defined as [2]:

$$M_x(\Omega_i)=\frac{1}{T}\int_0^T \exp(i\Omega_i)dt \tag{9}$$

Inserting expressions (7) into the equation (6) we get the value of Ω_i that should be inserted into characteristic function of band distribution (9). We obtain the following:

$$M_x(\Omega_i)=\frac{1}{T}\int_0^T \exp\left[i\frac{2\pi}{\lambda}(\Omega_1\cos\omega t-\Omega_2\sin\omega t)\right]dt, \tag{10}$$

where:

$$\Omega_1=U_0^i(\varphi)K_z^i\cos\alpha_i+V_0^i(\varphi)K_t^i\cos\beta_i+W_0^i(\varphi)K_r^i\cos\gamma_i \tag{11}$$

$$\Omega_2=U_0^i(\varphi)K_z^i\sin\alpha_i+V_0^i(\varphi)K_t^i\sin\beta_i+W_0^i(\varphi)K_r^i\sin\gamma_i \tag{12}$$

After the use of the equation [1] the equation (10) will look like:

$$M_x(\Omega_i)=J_0\left[n\left(\Omega_1^2+\Omega_2^2\right)^{\frac{1}{2}}\right], \tag{13}$$

where J_0 – Bessel function of the first kind of order zero.

Using (7), (11), (12) and (13) values of point i located on the surface of piezoceramic cylinder, we will obtain the following equation of the distribution of the interference bands on the surface of vibrating piezoceramic cylinder:

$$\begin{aligned}\frac{\Omega_i\lambda^2}{4\pi}=&\left[\left(\sum_1^n A_j^w F_{ij}^w\right)K_r^i\cos\gamma_i+\left(\sum_1^n A_j^v F_{ij}^v\right)K_t^i\cos\beta_i+\left(\sum_1^n A_j^u F_{ij}^u\right)K_z^i\cos\alpha_i\right]^2+\\&\left[\left(\sum_1^n A_j^w F_{ij}^w\right)K_r^i\sin\gamma_i+\left(\sum_1^n A_j^v F_{ij}^v\right)K_t^i\sin\beta_i+\left(\sum_1^n A_j^u F_{ij}^u\right)K_z^i\sin\alpha_i\right]^2\end{aligned} \tag{14}$$

The equation (14) is commonly used for identification of spatial vibration coordinates, when the object is analyzed in a method of time-average. In the equation, we know the parameter Ω_i, which is obtained for the centre points of dark interference bands of interferograms with the help of the following equation (1):

$$\Omega_p=(p-0.25)\pi+\frac{0.125}{(p-0.25)\pi}, \tag{15}$$

where p is the order number of an interference band (in a holographic interferogram), in the centre of which point i is located; the order number is calculated from the brightest nodular band.

Coefficients K_r^i, K_t^i, K_z^i of the equation (14) are obtained from the equation (3), using optical scheme parameters obtained during the experiment of fixing holographic interferogram. Coefficients F_{ij}^w, F_{ij}^v, F_{ij}^u are calculated using analytical expressions of self-excited vibration types related to geometrical shapes of transducers and conditions of their fixing in various structures.

In an attempt to calculate coefficients A_j^w, A_j^v, A_j^u and angles α_i, β_i, γ_i we need to minimize the equation formed on a basis of the equation (14) in a method described [2]:

$$F_i = \left[\left(\sum_1^n A_j^w F_{ij}^w\right)K_r^i \cos\gamma_i + \left(\sum_1^n A_j^v F_{ij}^v\right)K_t^i \cos\beta_i + \left(\sum_1^n A_j^u F_{ij}^u\right)K_z^i \cos\alpha_i\right]^2 - \left[\left(\sum_1^n A_j^w F_{ij}^w\right)K_r^i \sin\gamma_i + \left(\sum_1^n A_j^v F_{ij}^v\right)K_t^i \sin\beta_i + \left(\sum_1^n A_j^u F_{ij}^u\right)K_z^i \sin\alpha_i\right]^2 - \left[\frac{\Omega_i \lambda^2}{4\pi}\right]^2 \tag{16}$$

Function F_i shall be differentiated with respect to each unknown component of vibration. In respect of A_j^w we obtain the following:

$$G_j^{(i)} = \frac{\partial F_i}{\partial A_j^w} = 2\left[\left(\sum_1^n A_j^w F_{ij}^w\right)K_r^i \cos\gamma_i + \left(\sum_1^n A_j^v F_{ij}^v\right)K_t^i \cos\beta_i + \left(\sum_1^n A_j^u F_{ij}^u\right)K_z^i \cos\alpha_i\right]F_{ij}^w K_r^i \cos\gamma_i - 2\times \left[\left(\sum_1^n A_j^w F_{ij}^w\right)K_r^i \sin\gamma_i + \left(\sum_1^n A_j^v F_{ij}^v\right)K_t^i \sin\beta_i + \left(\sum_1^n A_j^u F_{ij}^u\right)K_z^i \sin\alpha_i\right]F_{ij}^w K_r^i \sin\gamma_i \tag{17}$$

Analogous expressions will be obtained in respect to other unknown components of vibration, i.e. A_j^v, A_j^u.

In respect of the unknown γ_i:

$$G_{1+3n}^{(i)} = \frac{\partial F_i}{\partial \gamma_i} = 2\left[\left(\sum_1^n A_j^w F_{ij}^w\right)K_r^i \cos\gamma_i + \left(\sum_1^n A_j^v F_{ij}^v\right)K_t^i \cos\beta_i + \left(\sum_1^n A_j^u F_{ij}^u\right)K_z^i \cos\alpha_i\right]\left(\sum_1^n A_j^w F_{ij}^w\right)K_r^i(-\sin\gamma_i) + \left[\left(\sum_1^n A_j^w F_{ij}^w\right)K_r^i \sin\gamma_i + \left(\sum_1^n A_j^v F_{ij}^v\right)K_t^i \sin\beta_i + \left(\sum_1^n A_j^u F_{ij}^u\right)K_z^i \sin\alpha_i\right]\left(\sum_1^n A_j^w F_{ij}^w\right)K_r^i \cos\gamma_i \tag{18}$$

Analogous expressions will be obtained in respect to the other unknowns α_i and β_i.

In such a way q number of points evenly located on a deformed (due to vibration) surface of piezoceramic cylinder is analyzed. According to the method described above, the calculation of coefficients K_r^i, K_t^i, K_z^i and F_{ij}^w, F_{ij}^v, F_{ij}^u of the equation (14) is followed by the formation of the matrix $\left|G^{(i)}\right|$. If q is the number of points analyzed and n is the number of types of self-excited vibration of piezoceramic cylinder obtained in result of the resolution of vibration at point i on a surface of piezoceramic cylinder in respect to corresponding coordination axis r, t and z, then matrix $\left|G^{(i)}\right|$ will feature a dimension $q\times 3(2n+3)$.

Solutions of a non-linear equation (14) will be obtained in a way of iteration. We will have to find out the following vector of unknowns:

$$N_V = \left[A_1^U, A_2^U, \ldots, A_n^U, A_1^V, A_2^V, \ldots A_n^V, A_1^W, A_2^W, \ldots A_n^W, \alpha, \beta, \gamma\right], \tag{19}$$

that will ensure minimum value of correction vector defined from the following expression:

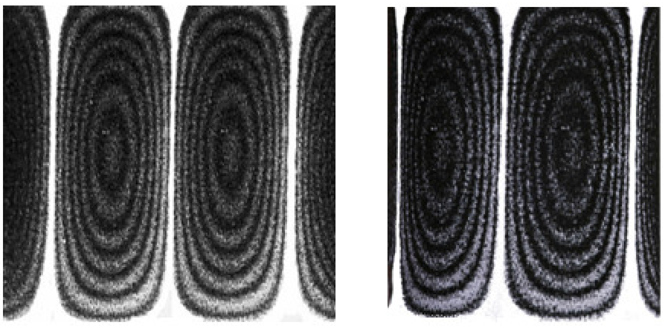
$$\{R\} = [T]^{-1}\{P\}, \tag{20}$$

where $[T]$ and $\{P\}$ are obtained from:

$$T_{1j} = \sum_{i=1}^{q} G_1^{(i)} G_j^{(i)}, \text{ where } j = 1, 2, \ldots, (3+3n) \tag{21}$$

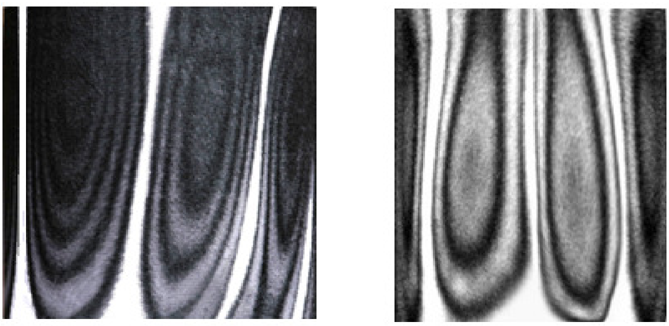
$$P_j = -\sum_{i=1}^{q} F_i G_j^{(i)}, \text{ where } j = 1, 2, \ldots, (3+3n) \tag{22}$$

Thus, performing the analysis of holographic interferograms in transducers with excited vibrations of a standing wave by means of the described approach, we will obtain several holographic interferograms from various illumination and observation angles. We select q number of points evenly located in the centers of interference bands on the surface of transducers. Sensitivity vector projections $\hat{K}_z^i$, K_t^i, K_r^i are defined for each point i $(i = 1,\ 2,\ \ldots,\ q)$ selected in a way of an experiment. F_{ij}^w, F_{ij}^v, F_{ij}^u of each point i selected are calculated using analytical expressions of the first n self-excited vibrations related to geometrical shape and fixing conditions of transducers. Ω_i is defined for each selected point i located in the centre of dark interference bands. These values shall be inserted into the equation (14). For the calculation of coefficients A_j^w, A_j^v, A_j^u and angles α_i, β_i, γ_i equation (16) shall be transformed to vector $|G^{(i)}|$ and, using (20) and (21), minimum value correction vector is formed.



a) b)

Fig. 6. Waves formed on holographic interferogram of a surface of piezoceramic cylinder when parameters of excitation and claims are optimal: a) angle of light for fixed hologram 80^0; b) angle of light for fixed hologram 40^0



a) b)

Fig. 7. Waves formed on holographic interferogram of a surface of piezoceramic cylinder when parameters of excitation and claims are not optimal: a) angle of light for fixed hologram 80^0; b) angle of light for fixed hologram 40^0

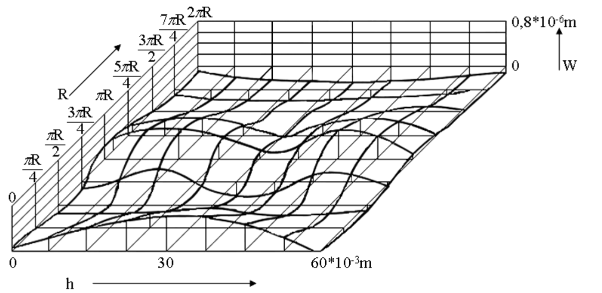


Fig. 8. Theoretical investigation of the radial oscillation amplitude of a surface of piezoceramic cylinder with application of interferograms in Figs. 6-7

Conclusion

The analyzed criteria enabled selection of the piezomaterial for optimal construction exhibiting maximum displacement. The automatic control has been determined to affect the correction of the hysteresis loop thus leading to reduction of displacement error up to 0.2 %. Performing the analysis of holographic interferograms in transducers with excited standing wave vibrations by means of the reported method allowed formation of several holographic interferograms from various illumination and observation angles. Experimental investigation of cylindrical transducers makes it possible to determine optimal initial tension force, the dependence of displacement of free cylindrical transducers in terms of constructional and technological parameters. The experimental results are easy to access and they are applied for development of tools for loosening rigid tightenings and eliminating corrosion impurities in machining and tool adjustment, medical devices, optical systems as well as items used in criminology for adjustment of various elements.

References

[1] **G. Baurienė, A. Fedaravičius, L. Patašienė, K. Ragulskis** Investigation of combined piezostacks and their application in optical systems. Proceedings of the International Conference “Vibroengineering – 2004”, Kaunas, 2004, p. 60-63.

[2] **Tonin R. F., Bies D. A.** General theory of time-average holography for the study of three-dimensional vibrations at a single frequency. J. Optical Soc. of America, 1978, Vol. 68, p. 924-931.

[3] **Ch. West** Holographic Interferometry. Moscow: Mir, 1982, 504 p.

[4] **Palevičius A.,Vasiliauskas R.** Application of laser interferometry and holographic methods for analysis and design of vibrational systems. Vibroengineering, 1998, p. 358-381.

[5] **K. Ragulskis, A. Palevičius, R. Vasiliauskas** Calculation of projections of the sensitivity vector on the basis of the photos of holographic interferograms in the process of decoding of spatial vibrations. Vibroengineering, Vol. 3(56), 1986, p. 133-141.

[6] **Patašienė Laima, Vasiliauskas Raimondas, Fedaravičius Algimantas** Holographic interferometry method for determination of layer piezostack parameters. Ultrasound, Kaunas: Technologija, 2007, Vol. 1(62), p. 23-25.

[7] **Baurienė Genovaitė, Fedaravičius Algimantas, Patašienė Laima** Dynamics of piezomaterials used in the optical systems. Journal of Electroceramics, Dordrecht: Springer, 2008, Vol. 20, No. 3-4, p. 313-316.

[8] **Patašienė Laima, Ragulskis Kazimieras, Fedaravičius Algimantas** Dynamics processes in exciting vibration of piezoactuators. Rare Attractors and Rare Phenomena in Nonlinear Dynamics: Proceedings of the 2nd International Symposium RA'11 held in Riga-Jūrmala, Latvia, 16-20 May 2011, Riga: RTU, 2011, p. 142-145.

[9] **R. Vasiliauskas, A. Bubulis, L. Patašienė, K. Ragulskis** Investigation of wave processes in circular transformers of motion by using the method of holographic interferometry. Herald of Machine Building, Moscow, 1981, No. 1, p. 16-18.