840. Investigation of eigenvalue problem of water tower construction interacting with fluid

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Abstract. The paper concerns problems, in which both the structural and fluid responses of a complex construction to mechanical actions are strongly coupled. Particularly, there are treated problems, in which the structural dynamic response to actions is significantly affected by the presence of the fluid in the structure. The work presents the evolution of the way of solving that problem of the complex solution of the generalized problem of the structure using multiphysical ANSYS program package. The formulation of fluid finite elements is discussed, considering both pressure (Euler) with/without sloshing and displacement (Lagrange) approaches. The solution is demonstrated on thin-walled steel water tower structure.

Keywords: fluid structure interaction, eigenmode, sloshing, free surface, Euler fluid, Lagrange fluid.

Introduction

A steel water tower structure alters its dynamic characteristics considerably, primarily because of the difference in mass distribution with a full or empty upper-tank. This happens during a common operation cycle. A water tower is a slender structure sensitive to dynamic loading effects. When designing a thin-walled steel structure, it is necessary to take into account stability effects as well. Furthermore, this structure is of circle section and inclined to transverse wind resonance vibration caused by vortex shedding in a wake. For this reason, a correct determination of eigen frequencies is necessary [15].

For a static analysis, a completely full upper-tank represents the most unfavorable case; fluid can be substituted by hydrostatic pressure on the bottom and walls. For a dynamic computation, the fluid substitute is further more complicated. In this case, an appropriate description of moving fluid effects plays a decisive role. This problem is shown on a water tower structure detail described below, water retained weight being 531 Mg and structure weight more than 93 Mg. The structure with more than 80 % of its weight concentrated in the upper part can be also described using a top mass concentration model. Since the hypothesis of simultaneously moving fluid is very conservative, an eigen value structure frequency is lower than it would correspond to a real situation. For very simple shapes, algorithms to determine added mass vibrating with the structure are available, although, in general, this represents a complicated task. A basic idea about the entire structure behavior can be obtained from the model of concentrated fluid mass connected to the proper water tower model (usually beam with very few degrees of freedom) using a spring-damper element. The fluid effect considered in this way can also describe the interaction of fluid volume and water tower structure vibration [13].

Present day possibilities of computational models allow a very detail description of the entire structure. For models with structure – fluid interaction, fluid influence is considered as an added value. Here, the problem of determining a moving fluid mass value must be dealt with. A fluid area model must be created and much more accurate results must be obtained. At present, Lagrange and Euler methods with two different boundary conditions are used [2].

Computation model of the fluid

The vibration of the water tower on wind induced vibration considering fluid-structure interaction should be analyzed. The motion of the computation model of the tower as a mechanical system with n degrees of freedom is described by the vector of n generalized coordinates u. The inertial, flexibility and dissipative properties of the system are described by the mass matrix **M**, stiffness matrix **K** and damping matrix **C**, derived from the expressions for the kinetic energy, resilience and dissipation of the system [6]. Assuming sufficiently small displacements of the system with respect to its appropriately selected reference configuration, elements of **M**, **K** and **C** are constants.

The external actions on the system are described by the vector of generalized forces **f** with elements f_i defined through functions of time and generalized displacements, velocities and accelerations. In the first order approach the functions f_i may be defined as the sum of a function of time f_i(t) and a function of generalized accelerations $f_i(\ddot{u}_1,..., \ddot{u}_n)$, which can be expressed as a product of the vector of generalized accelerations $\ddot{\mathbf{u}}$ and a matrix $\mathbf{M}_w$ of constant coefficients related to the water environment of system. The function f_i(t) defines e.g. the time-dependent water pressure on the wetted surface of the system. The so called "added mass" matrix $\mathbf{M}_w$ expresses the inertial effects of the mass of the water environment of the system [8]. Consequently, the motion of the system can be described by the following linear equation of motion:

$$(\mathbf{M} + \mathbf{M}_w)\ddot{\mathbf{u}} + \mathbf{C}\dot{\mathbf{u}} + \mathbf{K}\mathbf{u} = \mathbf{f}(t), \tag{1}$$

or

$$\mathbf{M}_e\ddot{\mathbf{u}} + \mathbf{C}\dot{\mathbf{u}} + \mathbf{K}\mathbf{u} = \mathbf{f}(t), \tag{2}$$

with relevant boundary and initial conditions.

In today's engineering practice, the equation of motion (1) is the basis for solution of the vibration response of the structure submerged in water to external actions. The problem is treated simply as a problem of forced vibration of the given mechanical system with its mass increased by "added mass".

The "added mass" expresses the decisive effects of the non-flowing water environment on the vibration of the system, i.e. substantially the pressure of the stagnant water on the vibrating wetted surface. Various procedures of assessing the "added mass" value are applied, both computational and experimental. However, with respect to general use of the finite element method for the solution of vibration problems, an adequate procedure is inevitable.

The commonly used program package ANSYS allows an advanced approach to the solution of the given problem [1]. This approach is based on the application of a complex computation model of the structure including besides the structure the bounded water domain. The computation model involves besides standard structural finite elements the special fluid elements FLUID30, based on the Euler's approach to describe the structure-fluid interactions using pressure as an additional variable [10].

The variable pressure component p with respect to the mean pressure in the modeled fluid domain is described by the Helmholtz acoustic wave equation:

$$\frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} = \nabla^2 p \tag{3}$$

which describes the fluid pressure distribution under the following simplifying assumptions [16]:

- the fluid is compressible (the density changes in dependence on the pressure change),
- the fluid is isotropous and homogeneous,
- the fluid is not viscous,
- the fluid is linear and ideally elastic,
- the fluid is in permanent contact with the structure,
- there is no flow,
- average density and pressure are constant in all fluid area.

Formulations of a fluid-structure interaction problem

Elasto-acoustic non-symmetric problem with fluid free surface

The structure is linear and elastic, the fluid is supposed compressible without gravity effects. The fluid-structure problem is modeled with a pressure description of the fluid problem and a displacement description of the structure problem, which is the most straightforward description of the coupled problem. Under these assumptions, the coupled fluid-structure problem is described by the following equations. The structure problem is governed by:

$$\begin{aligned}
&\rho_s \omega^2 u_i + \frac{\partial \sigma_{ij}(\mathbf{u})}{\partial x_j} && \text{in } \Omega_S \\
&u_i = 0 && \text{on } \partial\Omega_{So} \\
&\sigma_{ij}(\mathbf{u}) = n_j^s = 0 && \text{on } \partial\Omega_{S\pi} \\
&\sigma_{ij}(\mathbf{u}) = n_j^s = p n_i && \text{on } \Gamma
\end{aligned} \tag{4}$$

And the fluid problem by:

$$\begin{aligned}
&\frac{\omega^2}{c^2} p + \frac{\partial^2 p}{\partial x_i^2} = 0 && \text{in } \Omega_F \\
&p = 0 && \text{on } \partial\Omega_{Fo} \\
&\frac{\partial p}{\partial x_j} n_j^F = 0 && \text{on } \partial\Omega_{F\pi} \\
&\frac{\partial p}{\partial x_j} n_j^F = \rho_F \omega^2 u_i n_i && \text{on } \Gamma
\end{aligned} \tag{5}$$

where Ω_S is structure domain, of boundary $\partial\Omega_{So}$ (imposed displacement), $\partial\Omega_{S\sigma}$ (imposed force) and Γ (fluid-structure interface). The fluid domain is Ω_F, of boundary $\partial\Omega_{Fo}$ (free surface), $\partial\Omega_{F\pi}$ (rigid wall).

The free surface condition is expressed in the elasto-acoustic case by $p = 0$, the potential energy of the fluid free surface is then neglected. This assumption is valid in the medium and low frequency range [9].

The variational formulation of the coupled problem is obtained with the test function method with virtual field displacement $\delta\mathbf{u}$ and virtual field pressure δp:

$$-\omega^2 \int_{\Omega s} \rho_s u_i \delta u_i + \int_{\Omega s} \sigma_{ij}(\mathbf{u}) \varepsilon_{ij}(\delta\mathbf{u}) - \int_{\Gamma} p \delta u_i n_i = 0 \tag{6a}$$

$$-\omega^2 \int_{\Omega F} \frac{p\delta p}{c^2} + \int_{\Omega F} \frac{\partial p}{\partial x_i}\frac{\partial \delta p}{\partial x_i} - \rho_F \omega^2 \int_{\Gamma} u_i n_i \delta p = 0 \tag{6b}$$

The discretisation of equations (6a) and (6b) is performed with finite element method and leads to the definition of various fluid, fluid-structure interaction operators [9].

The non-symmetric eigenvalue problem is then written for the elasto-acoustic coupled system:

$$\begin{bmatrix} \mathbf{K}_s & -\mathbf{R} \\ 0 & \mathbf{K}_F \end{bmatrix} \begin{Bmatrix} \mathbf{U}(\omega) \\ \mathbf{P}(\omega) \end{Bmatrix} = \omega^2 \begin{bmatrix} \mathbf{M}_s & 0 \\ \rho_F \mathbf{R}^T & \mathbf{M}_F \end{bmatrix} \begin{Bmatrix} \mathbf{U}(\omega) \\ \mathbf{P}(\omega) \end{Bmatrix} \tag{7}$$

The fluid acoustic mode is obtained by solving the following eigenvalue problem:

$$\mathbf{K}_F \mathbf{P}(\omega) = \omega^2 \mathbf{M}_F \mathbf{P}(\omega) \tag{8}$$

Non-symmetric formulation of hydro-elastic problem with fluid free surface

A hydro-elastic problem with fluid free surface is then considered. In that case, the fluid is assumed incompressible, but the gravity effects on the fluid free surface are then considered. As in former case, the structure is supposed linear with elastic behavior.

A non-symmetric formulation of the coupled problem is obtained using the pressure-displacement approach. The structure is described by the equations (4), the fluid problem is written as:

$$\begin{aligned} &\frac{\partial^2 p}{\partial x_i^2} = 0 && \text{in } \Omega_F \\ &p = 0 && \text{on } \partial\Omega_{Fo} \\ &\frac{\partial p}{\partial x_j} n_j^F = \frac{\omega^2}{g} p && \text{on } \partial\Omega_{F\pi} \\ &\frac{\partial p}{\partial x_j} n_j^F = \rho_F \omega^2 u_i n_i && \text{on } \Gamma \end{aligned} \tag{9}$$

The free surface condition describes the evolution of the pressure field on the free surface due to gravity effects. The variational formulation of the coupled problem is then:

$$-\omega^2 \int_{\Omega s} \rho_s u_i \delta u_i + \int_{\Omega s} \sigma_{ij}(\mathbf{u}) \varepsilon_{ij}(\delta \mathbf{u}) - \int_{\Gamma} p \delta u_i n_i = 0 \tag{10a}$$

$$-\omega^2 \int_{\Omega F 0} \frac{p\delta p}{g} + \int_{\Omega F} \frac{\partial p}{\partial x_i}\frac{\partial \delta p}{\partial x_i} - \rho_F \omega^2 \int_{\Gamma} u_i n_i \delta p = 0 \tag{10b}$$

As the fluid sloshing modes involve the pressure nodes on the fluid free surface, it is possible to derive the formulation of the coupled fluid-structure problem in terms of the structure displacement $\mathbf{U}(\omega)$ and the pressure on the fluid free surface $\mathbf{P}_o(\omega)$. The eigenvalue problem is written:

$$\begin{bmatrix} \mathbf{K}_S & -\left(\mathbf{R}_o - \tilde{\mathbf{R}}\tilde{\mathbf{K}}_F^{-1}\tilde{\mathbf{K}}_F^o\right) \\ 0 & \mathbf{K}_F^o - \tilde{\mathbf{K}}_F^o\tilde{\mathbf{K}}_F^{-1}\tilde{\mathbf{K}}_F^{o\,T} \end{bmatrix} \begin{Bmatrix} \mathbf{U}(\omega) \\ \mathbf{P}_o(\omega) \end{Bmatrix} = \omega^2 \begin{bmatrix} \mathbf{M}_s + \rho_F \tilde{\mathbf{R}}\tilde{\mathbf{K}}_F^{-1}\tilde{\mathbf{R}}^T & 0 \\ \rho_F\left(\mathbf{R}_o - \tilde{\mathbf{K}}_F^o\tilde{\mathbf{K}}_F^{-1}\tilde{\mathbf{R}}\right)^T & \mathbf{M}_F^o \end{bmatrix} \begin{Bmatrix} \mathbf{U}(\omega) \\ \mathbf{P}_o(\omega) \end{Bmatrix} \tag{11}$$

where $\tilde{\mathbf{K}}_F$, $\tilde{\mathbf{K}}_F^o$, $\tilde{\mathbf{R}}$, $\tilde{\mathbf{R}}^o$, $\mathbf{M}_F^o$ arise from the static condensation algorithm of the pressure degrees of freedom on the fluid free surface.

It should be noticed that fluid sloshing modes can be obtained from the equation (11) by solving the following eigenvalue problem:

$$\left(\mathbf{K}_F^o - \tilde{\mathbf{K}}_F^o\tilde{\mathbf{K}}_F^{-1}\tilde{\mathbf{K}}_F^{o\,T}\right)\mathbf{P}_o(\omega) = \omega^2\mathbf{M}_F^o\mathbf{P}_o(\omega) \tag{12}$$

Computational model of the structure

The water tower is 47.3 m high and it can hold 530 m^3 of water. The load-carrying steel structure is welded from plates which are 4 – 50 mm thick [3]. The cylindrical water container with a conical bottom and top is firmly connected to a cylindrical shank with the outer diameter of 3.5 m and with the height of 40.2 m. It is assumed that the shank is firmly anchored into a stationary base. The water tower structure is loaded by the static effects of the structure self-weight, of the container contents, snow, etc. [14]. As for dynamic effects, the water tower structure is described under wind effects (force load) and under seismic effects (kinematic load) [12]. The response computation process is different; however, for the mode superposition method it is necessary to determine plausible eigen frequencies and eigen modes of the empty as well as full water tower structure. If the water tower is filled with fluid, the fluid changes structure frequencies considerably. As the fluid vibrates, by its movement, it loads tank walls. In practice, the computations are based on the assumption that a part of the fluid moves with the tank. For computations, this part is added to a selected part of the tank structure. The estimation of the amount of the vibrating fluid is based on experience. Nevertheless, current programs which are based on FEM make it possible to determine eigen frequencies and eigen modes in a more sophisticated manner [4, 11]. A detailed computational model of the water tower structure (including fluid) was created, see Figures 1 to 4.

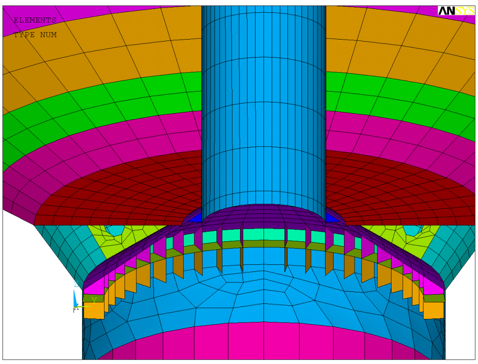


Fig. 1. Detail of mesh – tank bottom

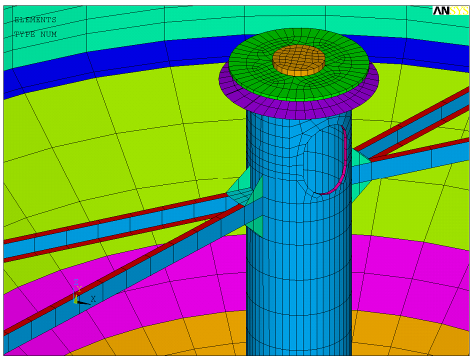


Fig. 2. Detail of mesh model – top of roof

Modal analysis

Eigen frequencies and eigen modes of the empty and filled water tower were computed. For the purpose of this study, this model was simplified; however, the basic frequency characteristics of the original model were preserved.

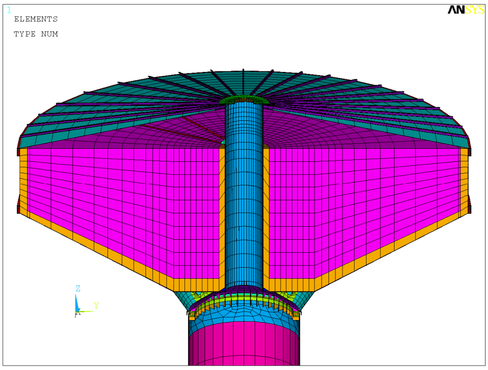


Fig. 3. Detail of mesh – tank bottom

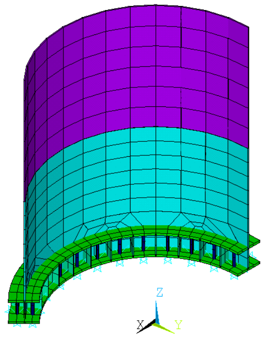


Fig. 4. Detail of mesh – bottom rims

The computational model of the load-carrying structure consists of SHELL181 four-nodal shell elements and BEAM188 two-nodal beam elements [5]. The mass of the steel part of the model is 93.160 Mg. The fluid area is modeled by FLUID30 eight-nodal 3D elements or by FLUID80 elements (mass = 531.412 Mg). Element meshing is the same in both cases. If the fluid area is modeled by FLUID30 elements, two variants of boundary conditions on the surface are analyzed. In the first case, the condition of zero pressure on the free surface is considered. This procedure was used in previous versions of the ANSYS program. In the latter case, in the computation, the possibility of the prescription of the free surface with the effects of the gravity field is taken into account. The model described corresponds to the Euler model of fluid description. The variant with FLUID80 elements corresponds to the Lagrange fluid description. The center of gravity of the empty tank is 29.630 m. The center of gravity of the full tank is 41.787 m.

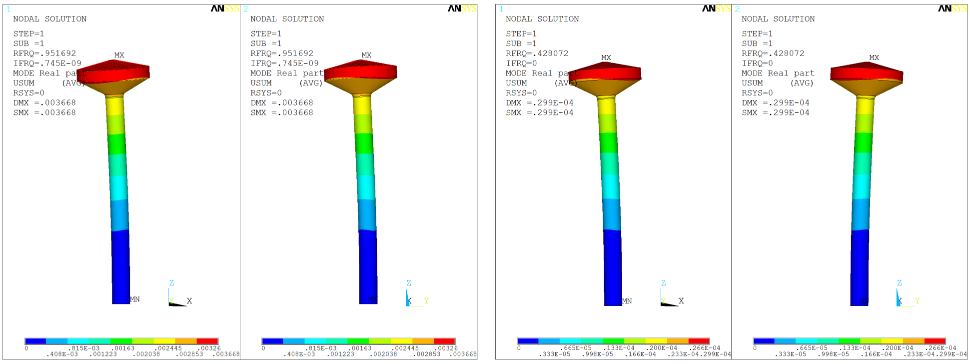
Some results of the computations are given in Table 1. The table shows the values of the frequencies corresponding to the first dominant bending modes which have the most important influence on the response under loading in the horizontal direction [7]. From the results, it is obvious that using FLUID30 elements with the boundary condition „free surface“ and FLUID80 elements makes it possible to determine the modes of the fluid with free surface movement. On the other hand, for these computations it is needed to determine hundreds or even thousands of frequencies in a required frequency spectrum. Increasing differences between the results with FLUID30FREE and FLUID80 are caused by a slower convergence of FLUID80 elements.

Table 1. Modal analysis results

Mode description	EMPTY	FLUID30	FLUID30 FREE	FLUID80	ADDMASS 30 %	ADDMASS 35 %
Convective	-	-	0.17179	0.17408	-	-
1st bending	0.95169	0.42807	0.40801	0.40133	0.47210	0.44497
			0.52407	0.51048		
2nd bending	6.81722	5.09200	5.09251	4.40036	4.27590	4.06508

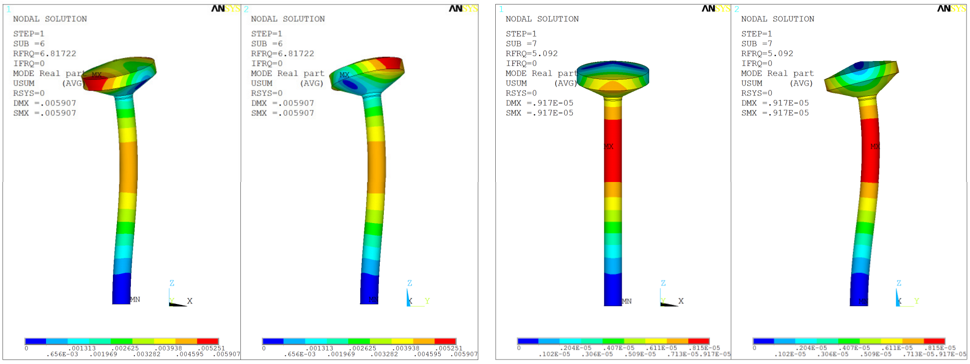
Computations in which FLUID30 elements are used can serve for computation checking. The computation with these elements is more stable. The frequencies corresponding to the first bending shape bifurcate if FLUID30FREE elements and FLUID80 elements are used. For addition, the results with an approximate substitute of the vibrating fluid which is added to the density of the material of the tank part are given in the last two columns of the Table.

For the model with FLUID80 elements, a great number of modal shapes corresponding to fluid movements with very low frequencies occur. These complicate the determination of load carrying still structure shapes. In this interval, dangerous so called slosh shapes happen; the entire fluid volume moves in one direction (from the left to the right or from the centre to the edges). In Figures 5 to 7, bending modes are given; in Figures 8 to 10, eigen modes of the fluid are shown.



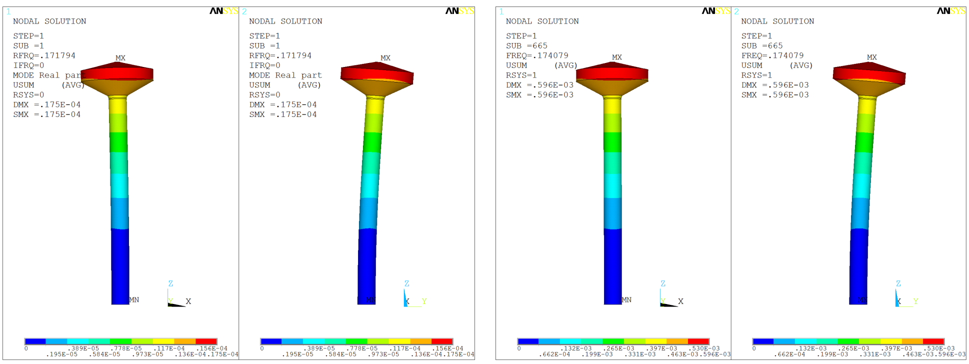
a) Empty tank b) Tank with fluid FLUID30

Fig. 5. First bending mode



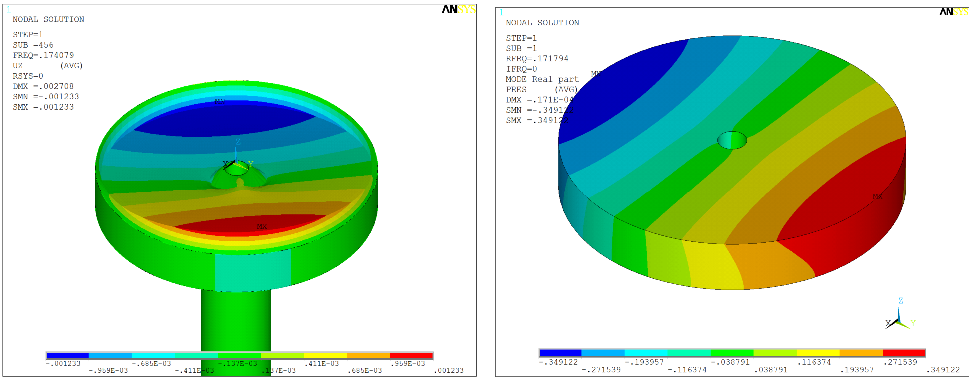
a) Empty tank b) Tank with fluid FLUID30

Fig. 6. Second bending mode



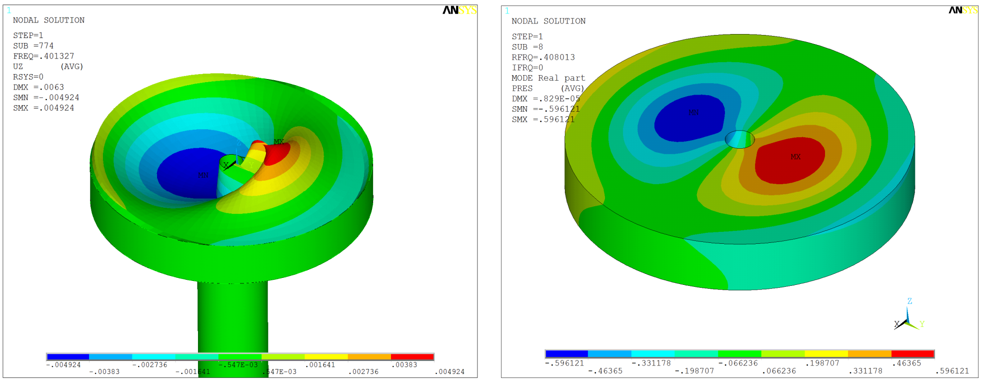
a) Tank with fluid FLUID30FREE b) Tank with fluid FLUID80

Fig. 7. Eigen mode corresponding to 1st mode of fluid movement



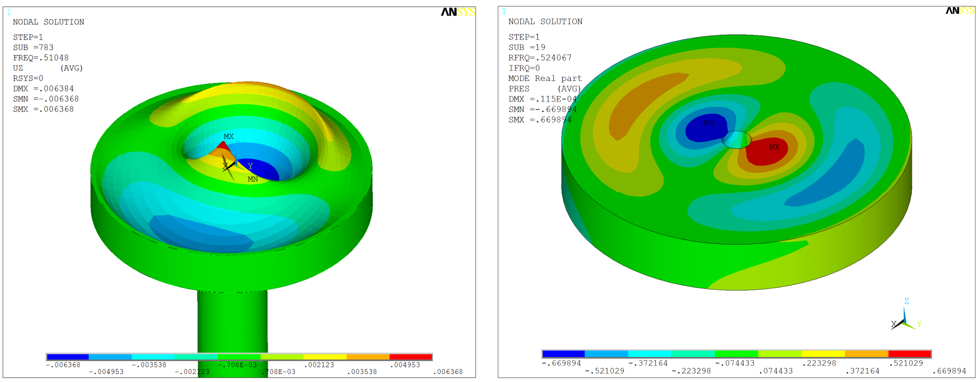
a) Fluid - FLUID80 – displacement field b) Fluid - FLUID30FREE – pres. field

Fig. 8. Eigen mode corresponding to 1st eigen mode of fluid movement



a) Fluid FLUID80 – displacement field b) Fluid FLUID30FREE – pressure field

Fig. 9. Eigen mode corresponding to 1st eigen mode of fluid movement



a) Fluid - FLUID80 – displacement field b) Fluid - FLUID30FREE – pres. field

Fig. 10. Eigen mode corresponding to 1st bending mode

Conclusion

The dynamic analysis of industrial structures taking into account fluid-structure interaction effects requires the use of coupled formulation. Non-symmetric formulations of a coupled fluid-structure problem have been studied in the present papers – the equations of the coupled problem have been recalled for a general fluid-structure problem in the case of elasto-acoustic and hydro-acoustic problem.

Several alternatives where fluid effects on eigen frequencies of a load carrying water tower structure are considered. In the results, only modes which are significant for the analysis of the response to the wind load were compared.

Acknowledgments

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