790. FRF-based model updating using SMURF technique

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Abstract. Finite element (FE) method is a well-known technique in structural dynamic analysis. However, FE models may be inaccurate or even incorrect due to erroneous modeling, geometrical over-simplification or uncertainties in the element and joint properties. In contrast, modal models are generally considered to be correct or at least closely matching the actual dynamic behavior of a structure. Therefore, a model updating procedure should be introduced for adjusting the analytical model in order to reconcile theoretical and experimental results. In this paper, a new FRF-based model updating method is proposed based on the Structural modification using experimental frequency response functions (SMURF) method. It is demonstrated that the proposed method updates the parameters accurately using just a few frequency response functions from the mis-modeled regions. A 12-DOF mass-spring system is considered as a test case in a simulated experiment. The convergence of the method and its capacity to improve the accuracy of the FE model are evaluated. Moreover, the paper considers the effect of the number of modes, the frequency range of interest used in the calculations as well as the coordinate incompleteness and noise on the quality of the updated model. The updated models are compared in terms of the predicted natural frequencies, mode shapes and frequency response functions.

Keywords: model updating, FRF, SMURF, finite element, frequency domain.

Introduction

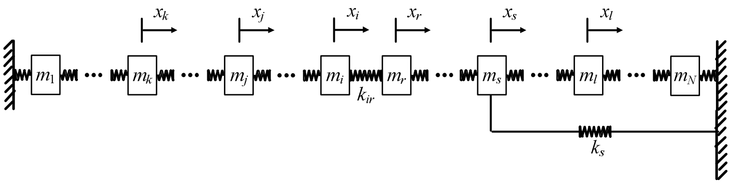
Due to a lack of confidence in the analytical models, modal testing of structures has become a classical procedure to validate the existing analytical models, especially FE models. However, the test results are not usually in perfect agreement with FE results. Therefore, the FE and modal databases need to be reconciled for further analysis. Neither of these two methods can be assumed to be perfect, but if they are combined a more accurate description of the dynamic behavior of structure can be obtained. Basically, it is believed that the experimental modal models are more reliable than the FE models. Therefore, model updating methods have been developed to improve the FE models using the modal test results.

A number of model updating methods have been proposed in recent years as shown in the surveys by Imregun and Visser [1], Mottershead and Friswell [2] and Natke [3]. Model updating methods can be classified in two groups as far as the data used for updating is concerned. One group includes methods which use modal data for updating, and the other group contains methods which use direct frequency response data. There have been attempts to use directly the measured frequency response function (FRF) data for model updating of FE models. A technique named Response Function Method (RFM) has been developed by Lin and Ewins [4], which uses FRFs to update FE model. Imregun et al. [5, 6] devised several methods using simulated and experimental data to show the effectiveness of this technique. Recently, the FRF data based methods have acquired increasing attention of the researchers due to the flexibility which these methods offer in the choice of updating parameters. Arora et al. [7], Lin and Zhu [8, 9] proposed FRF-based methods to deal with complex FRFs. Asma and Bouazzouni proposed a model updating using FRF measurements based on least square approximation [10]. They modified and extended the previously-developed nonlinear least squares method to update the FE model using separation of mass and stiffness parameters [11].

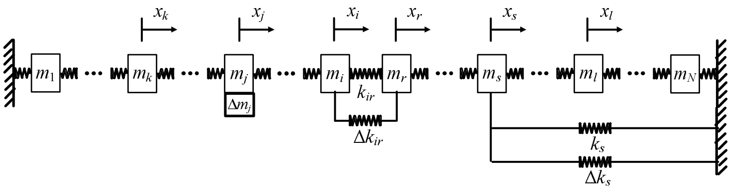
In this paper, a new FRF-based model updating method is proposed based on the Structural modification using experimental frequency response functions (SMURF) method [12]. This research work is a detailed study of the aforementioned model updating method which uses FRFs to update a finite element model of a twelve DOF mass-spring system. The convergence of the method and the accuracy with which it predicts the corrections required in a finite element model is investigated. The effect of the number of modes on the quality of the updated model is also studied. Moreover, the effects of complete, incomplete and noisy experimental data on the updated model are considered. The updated models are compared on the basis of some error indices in terms of the predicted natural frequencies, mode shapes and response functions.

Theory

An N-DOF mass-spring system is shown in Fig. 1a. It is assumed that the j^{th} mass (m_j), the spring located between the i^{th} and r^{th} DOFs (k_{ir}) and the spring between the s^{th} DOF and the ground (k_s) have been underestimated by Δm_j, Δk_{ir} and Δk_s values respectively (Figure 1b).



(a)



(b)

Fig. 1. An N-DOF system: (a) inaccurate, (b) accurate

Fig. 2 provides the free body diagram of system in which the action and reaction forces between the system and Δm_j, Δk_s, Δk_{ir} elements are presented. If the system is excited at k^{th} DOF and the response is measured at l^{th} DOF, the governing equation for the experimental system can be given by:

$$x_l = \alpha_{lk} F_k + \alpha_{lj} R_j + \alpha_{ls} R_s + \alpha_{li} R_i + \alpha_{lr} R_r \tag{1}$$

where x_l is the displacement of l^{th} DOF of the system shown in Fig. 1b. α_{lk}, α_{lj}, α_{ls} and α_{lr} are the receptances of the system shown in Fig. 1a. F_k is the excitation force and R_j, R_s, R_i and R_r are the reaction forces of the added components at j^{th}, s^{th}, i^{th} and r^{th} DOFs respectively. According to Newton's third law:

$$R_i = -R_r \tag{2}$$

Defining $R_{ir} = R_i = -R_r$ and substituting R_i and R_r by R_{ir} and $-R_{ir}$ respectively according to the Eq. (2) in Eq. (1), we have:

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$$x_l = \alpha_{lk}F_k + \alpha_{lj}R_j + \alpha_{ls}R_s + (\alpha_{li} - \alpha_{lr})R_{ir} \tag{3}$$

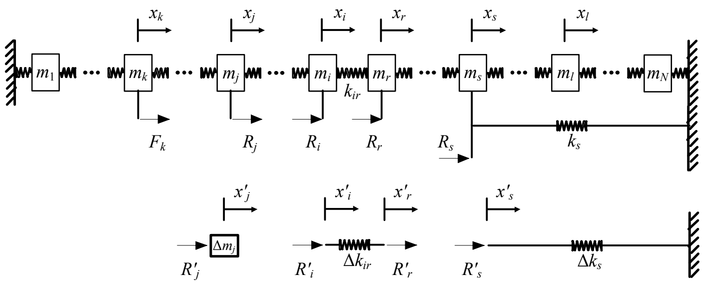


Fig. 2. Free body diagram of the *N*-DOF system

The governing equations for the added components are:

$$\begin{cases} X'_j = \alpha'_{jj}R'_j \\ X'_s = \alpha'_{ss}R'_s \\ X'_i = \alpha'_{ii}R'_i \\ X'_r = \alpha'_{rr}R'_r \end{cases} \tag{4}$$

where α's are the receptances of the added components.

The constraint equations are:

$$\begin{cases} X'_j = x_j \\ X'_s = x_s \\ X'_i = x_i \\ X'_r = x_r \end{cases}, \quad \begin{cases} R'_j + R_j = 0 \\ R'_s + R_s = 0 \\ R'_i + R_i = 0 \\ R'_r + R_r = 0 \end{cases} \tag{5}$$

Therefore:

$$\begin{aligned} x_j &= -\alpha'_{jj}R_j \\ x_i - x_r &= -\alpha'_{ii}R_i + \alpha'_{rr}R_r = -(\alpha'_{ii} - \alpha'_{rr})R_{ir} \\ x_s &= -\alpha'_{ss}R_s \end{aligned} \tag{6}$$

Substituting $\alpha'_{ii} - \alpha'_{rr}$, α'_{ss} and $-\alpha'_{jj}$ by $\frac{1}{\Delta k_{ir}}$, $\frac{1}{\Delta k_s}$ and $\frac{1}{-\Delta m_j \omega^2}$ in Eq. (6) respectively and by using Eq. (5), the following relations for the displacements are obtained:

$$\begin{aligned} x_j &= \frac{1}{\Delta m_j \omega^2} R_j \\ x_i - x_r &= \frac{-1}{\Delta k_{ir}} R_i \\ x_s &= \frac{-1}{\Delta k_s} R_s \end{aligned} \tag{7}$$

The reaction forces of the added components are:

$$R_j = \Delta m_j \omega^2 x_j$$
$$R_i = -\Delta k_{ir}(x_i - x_r) \tag{8}$$
$$R_s = -\Delta k_s x_s$$

Introducing Eq. (8) into Eq. (3) and considering that $R_i = R_{ir}$ from Eq. (2), the displacement at DOF i is obtained as:

$$x_l = \alpha_{lk} F_k + \alpha_{lj} \Delta m_j \omega^2 x_j - \alpha_{ls} \Delta k_s x_s - (\alpha_{li} - \alpha_{lr}) \Delta k_{ir} (x_i - x_r) \tag{9}$$

Dividing both sides of Eq. (9) by F_k , we have:

$$\alpha^*_{lk} = \alpha_{lk} + \alpha_{lj} \Delta m_j \omega^2 \alpha^*_{jk} - \alpha_{ls} \Delta k_s \alpha^*_{sk} - (\alpha_{li} - \alpha_{lr}) \Delta k_{ir} (\alpha^*_{ik} - \alpha^*_{rk}) \tag{10}$$

where α^* 's are the receptances of the experimental system.

If instead of adding one mass, n masses (m_1, m_2, ..., m_n) are added at DOFs j = (j_1, j_2, ..., j_n), instead of adding one spring, p springs are added between the DOFs (i, r) = (i_1, r_1), (i_2, r_2), …, (i_p, r_p) and instead of one spring, q springs are added between the DOFs s = (s_1, s_2, ..., s_q) and the ground, Eq. (10) is modified as:

$$\alpha^*_{lk} = \alpha_{lk} + \sum_{j=j_1}^{j_n} \alpha_{lj} \Delta m_j \omega^2 \alpha^*_{jk} - \sum_{s=s_1}^{s_q} \alpha_{ls} \Delta k_s \alpha^*_{sk} - \sum_{ir=i_1 r_1}^{i_p r_p} (\alpha_{li} - \alpha_{lr})(\alpha^*_{ik} - \alpha^*_{rk}) \Delta k_{ir} \tag{11}$$

$\Delta\alpha_{lk}$ is defined as the difference between the experimental and the analytical receptances:

$$\Delta\alpha_{lk} = \alpha^*_{lk} - \alpha_{lk} \tag{12}$$

Introducing Eq. (12) into Eq. (11), we have:

$$\Delta\alpha_{lk} = \sum_{j=j_1}^{j_n} \alpha_{lj} \Delta m_j \omega^2 \alpha^*_{jk} - \sum_{s=s_1}^{s_q} \alpha_{ls} \Delta k_s \alpha^*_{sk} - \sum_{ir=i_1 r_1}^{i_p r_p} (\alpha_{li} - \alpha_{lr})(\alpha^*_{ik} - \alpha^*_{rk}) \Delta k_{ir} \tag{13}$$

The mass and stiffness errors can be defined as:

$$\Delta m_j = a_j m_j$$
$$\Delta k_{ir} = b_{ir} k_{ir} \tag{14}$$
$$\Delta k_s = c_s k_s$$

where a, b and c are the correction coefficients.

Substituting Eq. (14) into Eq. (13), results in:

$$\Delta\alpha_{lk} = \sum_{j=j_1}^{j_n} \alpha_{lj} a_j m_j \omega^2 \alpha^*_{jk} - \sum_{s=s_1}^{s_q} \alpha_{ls} c_s k_s \alpha^*_{sk} - \sum_{ir=i_1 r_1}^{i_p r_p} (\alpha_{li} - \alpha_{lr})(\alpha^*_{ik} - \alpha^*_{rk}) b_{ir} k_{ir}. \tag{15}$$

Eq. (15) has $n+p+q$ unknowns. Therefore this equation is required to be written more than $n+p+q$ times for different frequency points and different DOFs to become over-determined and can be solved then using SVD techniques. This leads to the matrix expression:

$$\begin{bmatrix} \Delta\alpha_{lk}(\omega_1) \\ \Delta\alpha_{lk}(\omega_2) \\ \vdots \\ \Delta\alpha_{lk}(\omega_{fps}) \end{bmatrix} = \begin{bmatrix} A & \vdots & B & \vdots & C \end{bmatrix} \begin{bmatrix} a_{j_1} \\ a_{j_2} \\ \vdots \\ a_{j_p} \\ b_{i_1r_1} \\ b_{i_2r_2} \\ \vdots \\ b_{i_pr_p} \\ c_{s_1} \\ c_{s_2} \\ \vdots \\ c_{s_q} \end{bmatrix} \tag{16}$$

where A, B and C are:

$$A = \begin{bmatrix} \alpha_{lj_1}\omega_1^2 m_{j_1}\alpha^*_{j_1k} & \alpha_{lj_2}\omega_1^2 m_{j_2}\alpha^*_{j_2k} & \cdots & \alpha_{lj_n}\omega_1^2 m_{j_n}\alpha^*_{j_nk} \\ \alpha_{lj_1}\omega_2^2 m_{j_1}\alpha^*_{j_1k} & \alpha_{lj_2}\omega_2^2 m_{j_2}\alpha^*_{j_2k} & \cdots & \alpha_{lj_n}\omega_2^2 m_{j_n}\alpha^*_{j_nk} \\ \vdots & \vdots & \vdots & \vdots \\ \alpha_{lj_1}\omega_{nfps}^2 m_{j_1}\alpha^*_{j_1k} & \alpha_{lj_2}\omega_{nfps}^2 m_{j_2}\alpha^*_{j_2k} & \cdots & \alpha_{lj_n}\omega_{nfps}^2 m_{j_n}\alpha^*_{j_nk} \end{bmatrix} \begin{matrix} \leftarrow \omega_1 \\ \leftarrow \omega_2 \\ \vdots \\ \leftarrow \omega_{nfps} \end{matrix}$$

$$B = \begin{bmatrix} (\alpha_{li_1}-\alpha_{lr_1})(\alpha^*_{i_1k}-\alpha^*_{r_1k})k_{i_1r_1} & (\alpha_{li_2}-\alpha_{lr_2})(\alpha^*_{i_2k}-\alpha^*_{r_2k})k_{i_2r_2} & \cdots & (\alpha_{li_p}-\alpha_{lr_p})(\alpha^*_{i_pk}-\alpha^*_{r_pk})k_{i_pr_p} \\ (\alpha_{li_1}-\alpha_{lr_1})(\alpha^*_{i_1k}-\alpha^*_{r_1k})k_{i_1r_1} & (\alpha_{li_2}-\alpha_{lr_2})(\alpha^*_{i_2k}-\alpha^*_{r_2k})k_{i_2r_2} & \cdots & (\alpha_{li_p}-\alpha_{lr_p})(\alpha^*_{i_pk}-\alpha^*_{r_pk})k_{i_pr_p} \\ \vdots & \vdots & \vdots & \vdots \\ (\alpha_{li_1}-\alpha_{lr_1})(\alpha^*_{i_1k}-\alpha^*_{r_1k})k_{i_1r_1} & (\alpha_{li_2}-\alpha_{lr_2})(\alpha^*_{i_2k}-\alpha^*_{r_2k})k_{i_2r_2} & \cdots & (\alpha_{li_p}-\alpha_{lr_p})(\alpha^*_{i_pk}-\alpha^*_{r_pk})k_{i_pr_p} \end{bmatrix} \begin{matrix} \leftarrow \omega_1 \\ \leftarrow \omega_2 \\ \vdots \\ \leftarrow \omega_{nfps} \end{matrix}$$

$$C = \begin{bmatrix} -\alpha_{ls_1}k_{s_1}\alpha^*_{s_1k} & -\alpha_{ls_2}k_{s_2}\alpha^*_{s_2k} & \cdots & -\alpha_{ls_q}k_{s_q}\alpha^*_{s_qk} \\ -\alpha_{ls_1}k_{s_1}\alpha^*_{s_1k} & -\alpha_{ls_2}k_{s_2}\alpha^*_{s_2k} & \cdots & -\alpha_{ls_q}k_{s_q}\alpha^*_{s_qk} \\ \vdots & \vdots & \vdots & \vdots \\ -\alpha_{ls_1}k_{s_1}\alpha^*_{s_1k} & -\alpha_{ls_2}k_{s_2}\alpha^*_{s_2k} & \cdots & -\alpha_{ls_q}k_{s_q}\alpha^*_{s_qk} \end{bmatrix} \begin{matrix} \leftarrow \omega_1 \\ \leftarrow \omega_2 \\ \vdots \\ \leftarrow \omega_{nfps} \end{matrix}$$

The frequency points should be selected randomly, but the selected frequency points must remain constant during the iterations [13].

Solving Eq. (16) and finding the correction coefficient matrix, Δk_s, Δk_{ir} and Δm_j are obtained using Eq. (14). It can be seen from Eq. (14), that the experimental receptances at the modified DOFs are required, in the updating procedure. However, some DOFs of the system such as the DOFs inside the body and the rotational DOFs cannot be measured in practice [13]. In these cases, the analytical counterparts need to be used instead and the result is obtained after some iterations.

Numerical case study

A twelve DOF mass-spring system is considered here as the numerical case study (see Fig. 3). Certain known discrepancies are introduced in the mass and stiffness element values as the FE model errors. The mass and stiffness values of the model and the introduced discrepancies are given in Table 1. The frequency range of the simulated experiment is 0-95.5 Hz covering all the twelve modes of the system. The convergence criterion is based on the norm of the difference between two successive vectors of cumulative fractional correction factors. The norm value of 0.001 is used as a convergence criterion for this study. The cumulative fractional correction factors are defined by:

$$\overline{P}_i^j = \left(1+\boldsymbol{p}_i^1\right)\left(1+\boldsymbol{p}_i^2\right)\ldots\left(1+\boldsymbol{p}_i^j\right)-1 \quad (17)$$

where $\boldsymbol{p}_i^j$ is i^{th} element of the vector of fractional correction factors $\{p\}$ in j^{th} iteration and $\overline{P}_i^j$ is the cumulative value of i^{th} element of $\{p\}$ in j^{th} iteration.

To increase the possibility of convergence, a move limit value of 0.05 is imposed on the p values in each iteration. This will ensure that the optimum p values will eventually be reached within a tolerance of the move limit [14].

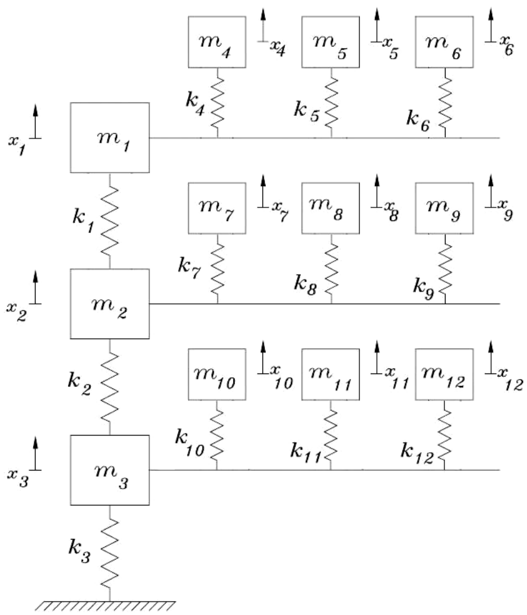


Fig. 3. A twelve DOF mass-spring system

Three test cases have been considered. In the first case, it is assumed that the measurements are available at all DOFs of the FE model. This case has been referred to as the case of complete data. Thus, one complete column of the FRF matrix and all the eigenvalues and eigenvectors falling in this measurement frequency range are known as the case of complete data. In practice, it is not possible to measure all DOFs at the points on the structure, either due to the physical inaccessibility or the difficulties encountered in the measurement of rotational DOFs. The second case is referred to as the case of incomplete data, where it is assumed that the measurements are not available at all DOFs of the FE model. The incompleteness is considered by assuming that the measurements are available at only six nodes (nodes 2, 3, 5, 6, 8 and 12). In practice, the measured FRFs are contaminated by noise and consequently the extracted modal parameters are affected. In the third case the FRFs are supposed to be contaminated by 2 % random noise. Besides, the effect of the quantity of the measured data on the quality of an updated model is studied. This is done by varying the updating frequency range to cover 1 to 6 modes.

Table 1. Mass and stiffness values and discrepancies between the FE and experimental model

DOF	Mass (kg)	Mass error (%)	Stiffness (N/m)	Stiffness error (%)
1	1	0	62169	0
2	2	0	71050	0
3	3	0	44406	0
4	1	+10	97693	0
5	2	0	8437	-40
6	3	0	62169	-10
7	1	0	88812	0
8	2	0	7105	0
9	3	0	53287	0
10	1	0	106575	0
11	2	0	5329	0
12	3	0	79931	0

To assess the progress of the iterations, certain model quality indices have been used [15]. Percentage Average Error in Natural Frequencies (AENF), percentage Average Error in Mode Shape (AEMS) and percentage Average Error in FRFs (AEFRF) are calculated using the following expressions:

$$AENF = \frac{100}{m}\sum_{i=1}^{m} abs\left(\frac{f_A - f_X}{f_X}\right)_i \tag{18}$$

$$AEMS = \frac{100}{m \times n}\sum_{j=1}^{m}\sum_{i=1}^{n} abs\left(\frac{\{\varphi_i\}_A^j - \{\varphi_j\}_X^i}{\{\varphi_i\}_X^j}\right) \tag{19}$$

$$AEFRF = \frac{100}{nf \times n}\sum_{j=1}^{nf}\sum_{i=1}^{n} abs\left(\frac{\left([\alpha(f_j)]_{ik}\right)_A - \left([\alpha(f_j)]_{ik}\right)_X}{\left([\alpha(f_j)]_{ik}\right)_X}\right) \tag{20}$$

where f_A and f_X are the natural frequencies, $\{\varphi\}_A$ and $\{\varphi\}_X$ are the mode shapes while $[\alpha]_A$ and $[\alpha]_X$ are the receptance FRF matrices corresponding to the analytical and experimental model respectively.

Results and discussion

For the case of complete data the error indices are calculated before and after updating. Table 2 shows that the error has been eliminated completely irrespective to the number of modes covered inside the updating frequency range. Fig. 4 provides the convergence of the cumulative fractional correction factors for one mode coverage. It should be noted that the results are identical when the updating range covers one to six modes. The method converges in twelve iterations and predicts the unknown fractional correction factors to the elements of mass and stiffness matrices exactly irrespective to the measurement frequency range and the number of modes. For different numbers of modes included inside the updating frequency range, the final values of the fractional correction factors are obtained. As the results are exactly the same for different numbers of modes, the values related to one mode coverage are shown in Fig. 5. It can be observed that the discrepancies have been exactly predicted by the method, irrespective to the updating frequency range and selected frequency points. The first value in figure shows the correction factor related to m_4, the second and third ones are related to k_5 and k_6. Due to the completeness of simulated measured data, the convergence is quite rapid and stable, that is, without any excessive or oscillatory variation during the iterations. It should be noted that the convergence happens more rapidly by increasing the value of move limit on the p values.

Table 2. Error indices for the case of complete experimental data

No. of modes inside the updating frequency range	Error calculated over the entire measurement range					
	AENF		AEMS		AEFRF	
	Before updating	After updating	Before updating	After updating	Before updating	After updating
1	2.249	0.000	7.561	0.000	281.506	0.000
2	2.834	0.000	3.780	0.000	281.506	0.000
3	4.883	0.000	2.520	0.000	281.506	0.000
4	5.796	0.000	1.890	0.000	281.506	0.000
5	5.120	0.000	1.512	0.000	281.506	0.000
6	4.306	0.000	1.260	0.000	281.506	0.000

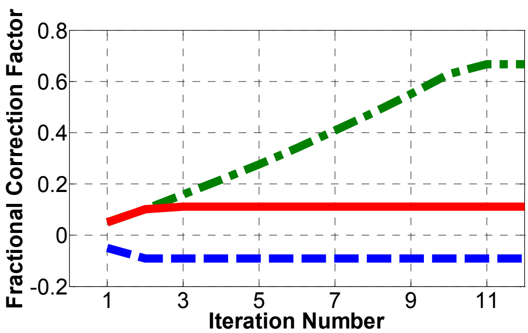


Fig. 4. Convergence of the fractional correction factors of m_4 (--), k_5 (—·) and k_6 (—) for the case of complete data

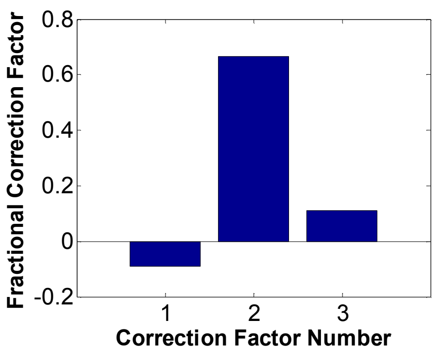


Fig. 5. The final fractional correction factors for the case of complete data

In the case of incomplete data, it is expected that the error level increases due to the reduction in the number of measured coordinates, but as can be observed in Table 3 the error has been eliminated for different updating frequency ranges. Fig. 6 shows that the method convergences in different number of iterations. The convergence speed increases as the number of modes increase in general, but the convergence speed depends also on the number and values of the selected frequency points, so the number of iterations and the changes in the cumulative fractional correction factors during iterations can be different in every run of the program due to the random selection of frequency points. Fig. 7 indicates that the method predicts the unknown fractional correction factors to the elements of mass and stiffness matrices exactly, irrespective to the updating frequency range and selected frequency points.

Table 3. Error indices for the case of incomplete experimental data

No. of modes inside the updating frequency range	Error calculated over the entire measurement range					
	AENF		AEMS		AEFRF	
	Before updating	After updating	Before updating	After updating	Before updating	After updating
1	2.249	0.000	7.561	0.000	281.506	0.000
2	2.834	0.000	3.780	0.000	281.506	0.000
3	4.883	0.000	2.520	0.000	281.506	0.000
4	5.796	0.000	1.890	0.000	281.506	0.000
5	5.120	0.000	1.512	0.000	281.506	0.000
6	4.306	0.000	1.260	0.000	281.506	0.000

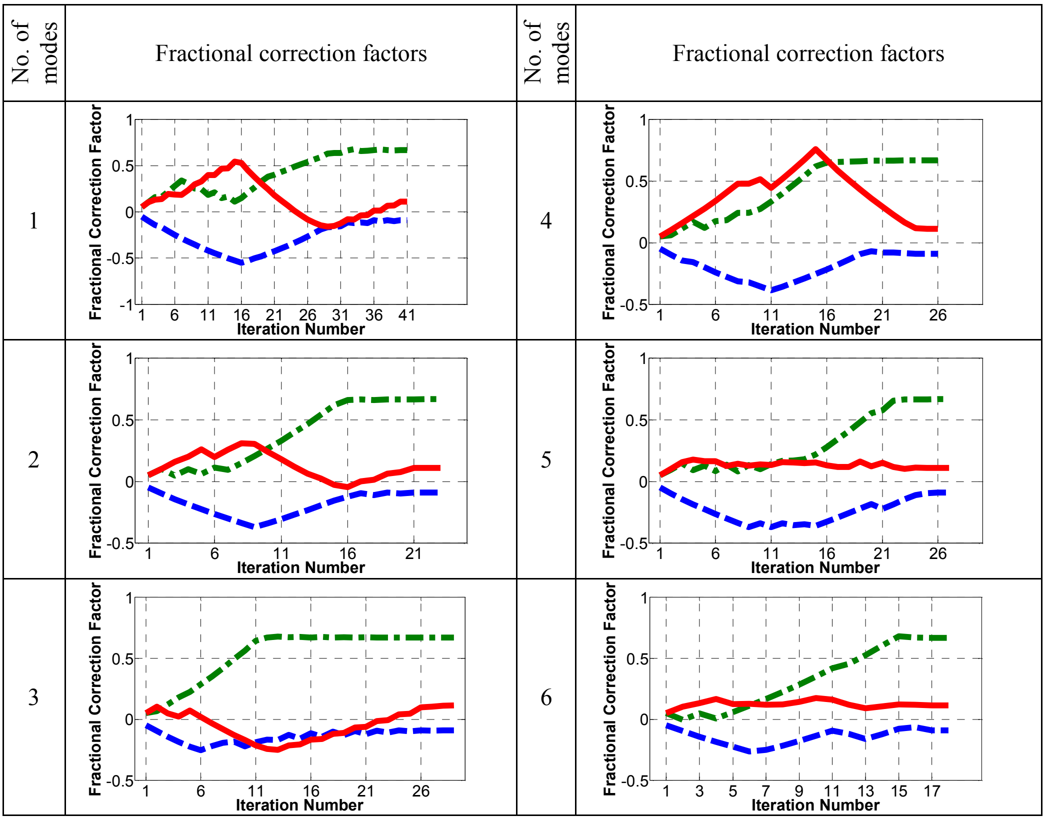


Fig. 6. Convergence of the fractional correction factors of m_4 (--), k_5 (—·) and k_6 (—) for the case of incomplete data

Table 4 and Figs. 8-9 provide the results for the case of incomplete data with 2 % noise. It can be observed that the convergence process remains stable even with the reduction in the number of modes included in the updating procedure which is evident in Fig. 8. It is experienced that the method requires almost the same number of iterations to find an acceptable solution compared to the data without noise, except for the case of 1 and 6 modes coverage. This difference can be due to the noise in data, number and values of selected frequency points. Table 4 reveals that the Average Error in Mode Shape (AEMS) and the Average Error in FRFs (AEFRF) have generally decreased as updating range is extended to encompass a greater number of modes (see Figures 10 and 11). In contrast, the Average Error in Natural Frequency (AENF) generally increases with inclusion of greater number of modes in the updating frequency range (see Fig. 12). The final values of the fractional correction factors for different number of modes included inside the updating range are shown in Fig. 9. It can be observed that the method predicts the fractional correction factors exactly when the updating frequency range covers more than three modes. For less than four modes the correction factor of the k_6 is not accurate.

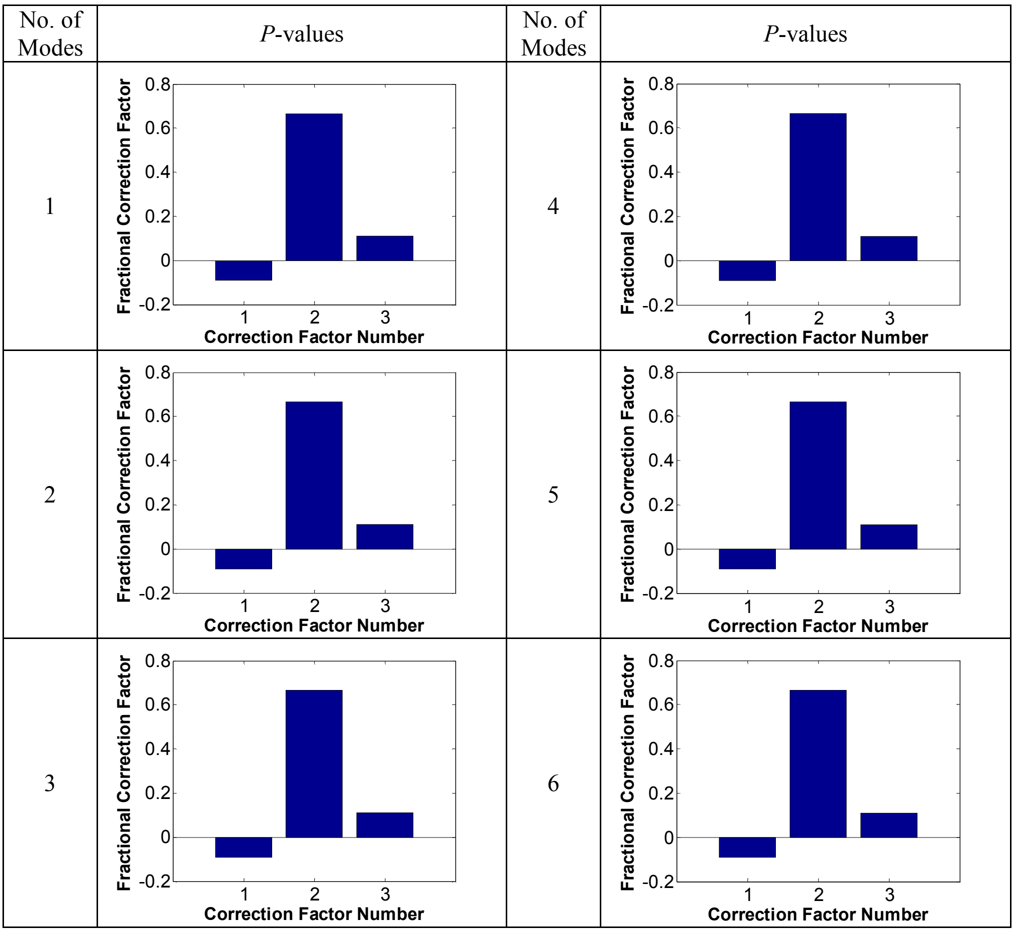


Fig. 7. The final fractional correction factors for the case of incomplete data

Fig. 13 shows the FRFs of the finite element, experimental and updated models, when updating range covers only one mode. The selected frequency points are shown by circles.

Table 4. Error indices for the case of incomplete experimental data with 2 % noise

No. of modes inside the updating frequency range	Error calculated over the entire measurement range					
	AENF		AEMS		AEFRF	
	Before updating	After updating	Before updating	After updating	Before updating	After updating
1	2.249	0.001	7.561	0.046	281.506	18.445
2	2.834	0.001	3.780	0.023	281.506	24.170
3	4.883	0.000	2.520	0.003	281.506	3.660
4	5.796	0.003	1.890	0.002	281.506	4.840
5	5.120	0.005	1.512	0.003	281.506	3.995
6	4.306	0.002	1.260	0.001	281.506	1.796

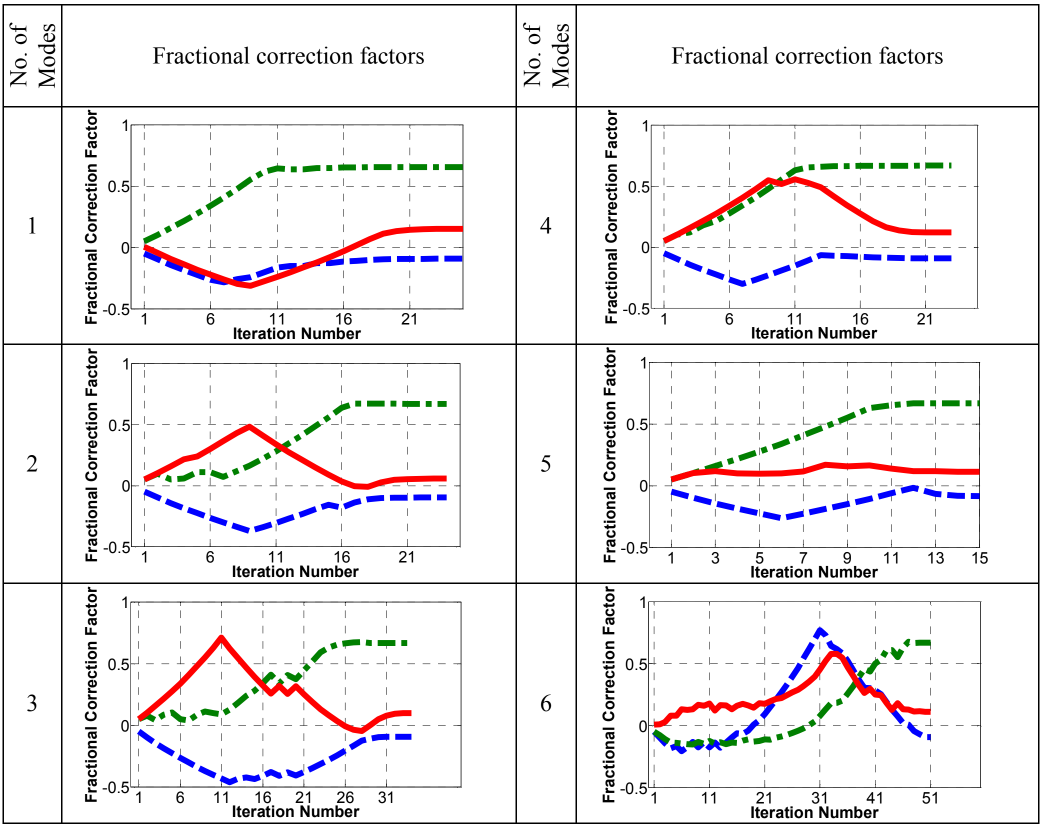
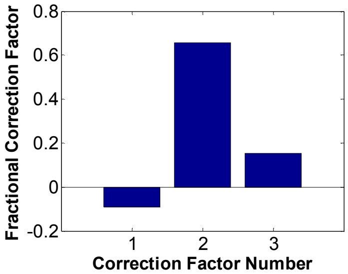
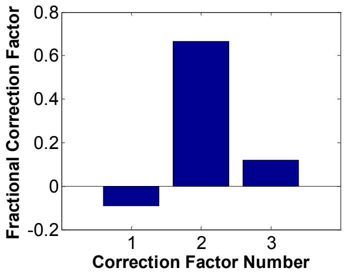


Fig. 8. Convergence of the fractional correction factors of m_4 (--), k_5 (—·) and k_6 (—) for the case of incomplete data with 2 % noise

Conclusions

In this paper a new method of model updating based on the Structural modification using experimental frequency response functions (SMURF) method is proposed. 12-DOF mass-spring system is considered as a test case in a simulated experiment. The numerical case study demonstrates that the method can predict the error in the FE model exactly for the cases of complete and incomplete data irrespective to the number of modes covered in the frequency range and with a reasonable accuracy for the case of incomplete noisy data.

<table>
<tr><th>No. of Modes</th><th>P-values</th><th>No. of Modes</th><th>P-values</th></tr>
<tr><td>1</td><td>
</td><td>4</td><td>
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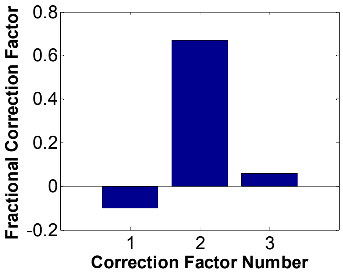
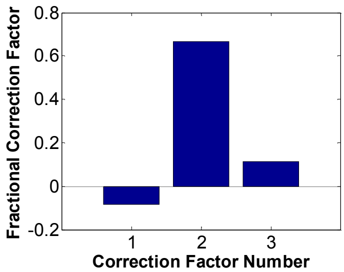
<tr><td>2</td><td>
</td><td>5</td><td>
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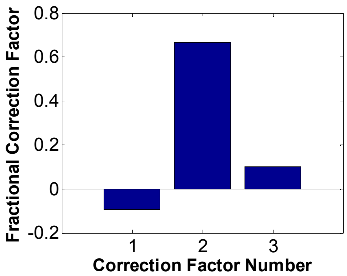
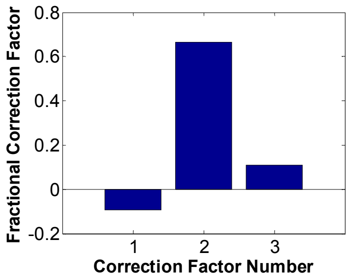
<tr><td>3</td><td>
</td><td>6</td><td>
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Fig. 9. The final fractional correction factors for the case of incomplete data with 2 % noise

The numerical study indicates that the method is able to eliminate the three error indices, AENF, AEMS and AEFRF for the cases of complete and incomplete data. In the noisy environment it is able to eliminate AENF. The technique is also able to decrease the AEMS and AEFRF, and even to eliminate the AEMS when more than three modes cover in the updating frequency range. It is experienced that the error levels decrease as updating range is extended to encompass a greater number of modes.

Moreover, the method exhibits good stability and predicts the discrepancies accurately even in the noisy environment, irrespective to the number of modes covered in the frequency range of up-dating process, but sometimes for less modes it requires more iterations to find an acceptable solution compared to the data without noise.

This approach is beneficial, because it is able to determine the exact value of discrepancies even in the case of incomplete data. Also the method requires data only from few coordinates and it works irrespective to the number of modes covered in the frequency range of updating.

Acknowledgements

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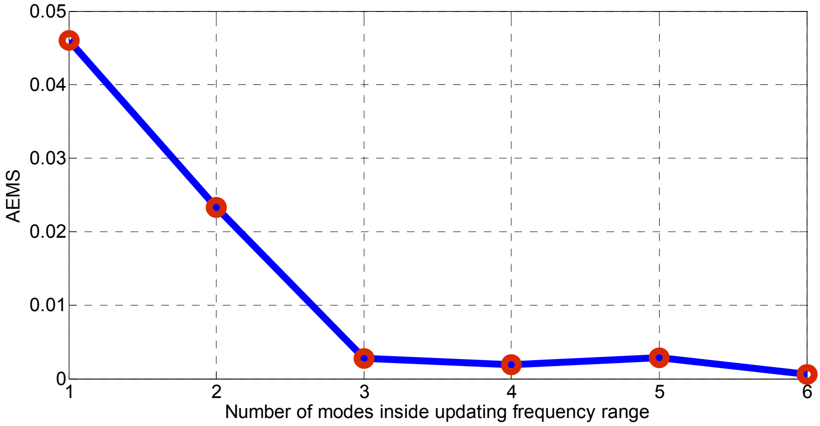

Fig. 10. Average Error in Mode Shape (AEMS) after updating process

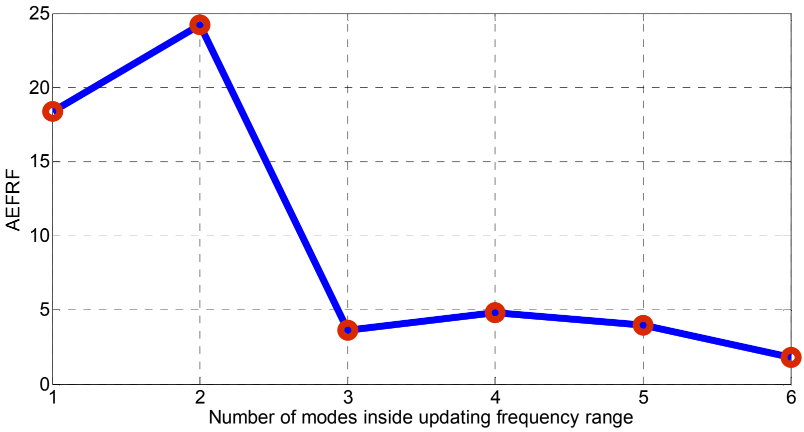

Fig. 11. Average Error in FRF (AEFRF) after updating process

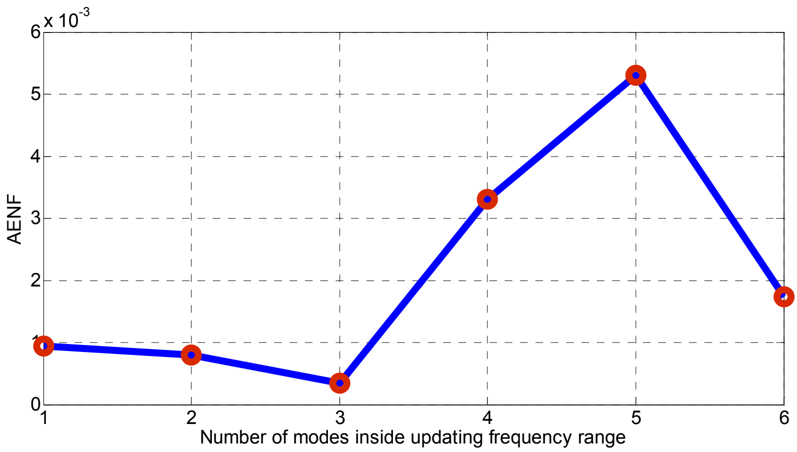

Fig. 12. Average Error in Natural Frequency (AENF) after updating process

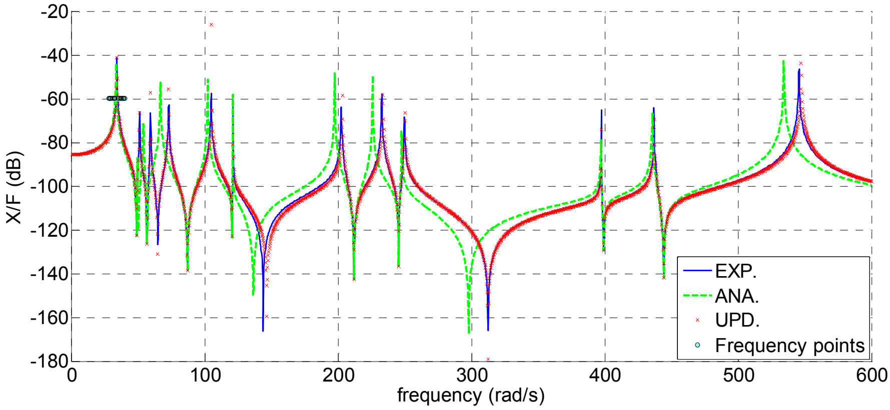

Fig. 13. FRFs of the experimental (─), analytical (--) and updated (×) models and the selected frequency points (o) for one mode coverage

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