760. Damage localization by wavelet analysis of uniform load surface

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Abstract. This paper presents a new technique for identifying damages in beam-type structures based on wavelet analysis of uniform load surface (ULS). Having come into focus in the field of damage detection, wavelet analysis revealed itself as a practical and state-of-the-art signal processing method for discerning abnormalities of mode shapes of structures, where the irregularities are considered as the indications of cracks. ULS is beneficial in terms of participating lower mode shapes and better immunity to noise. Continuous wavelet transform (CWT) is employed to analyze the ULS in order to find the damage site. The proposed method does not require any prior knowledge of the structure and combines the synergistic advantages of ULS technique and CWT to provide more accurate results in the face of high signal to noise ratio. A numerical study is conducted and three kinds of wavelets are utilized to evaluate the technique, namely *symmetrical 4*, *Gaussian 4* and *bior 6.8.* The procedure is also experimentally investigated using a free-free beam structure.

Keywords: damage identification, continuous wavelet transform (CWT), uniform load surface (ULS).

1. Introduction

Stemming from environmental effects or accidental loads, defects and flaws in some structures such as railways and road bridges or aircraft bodies may result in disastrous consequences. Due to their crucial importance, several investigations with the purpose of introducing new effective approaches to localize damages in structures have been performed over the past century.

Nondestructive damage detection techniques include two groups: local and global methods. As global approaches, vibration-based techniques have been developed owing to their inherent advantages over the local methods like the ultrasonic and X-ray, where preliminary knowledge of the position is needed to exactly find the location of a crack. These techniques have been classified according to their characteristics in different ways such as non-baseline model and baseline model methods, linear and nonlinear techniques, categorized based on features extracted, time vs. frequency methods, model-based approaches or non-model-based approaches and etc. [1-6].

The idea of using vibration-based methods is to identify damage in any system due to the fact that it causes changes in the dynamic behavior of structure. Altered by damages, modal parameters (mode shapes, damping ratios, natural frequencies) can be used as useful indicators of damages. When a crack occurs in a structure, stiffness and damping reduces and increases respectively, which results in reduction of natural frequencies. Due to this interesting fact, vibration-based damage detection techniques have been developed during the past decades. Among all these techniques, since time domain data, which are directly acquired from vibration measurements are hard to study for damage identification, modal domain data obtained from time data has gained more attention.

Wavelet transform has achieved a considerable reputation as an effective method for localizing damages in structures due to its intrinsic properties to recognize small irregularities of signals. Significant efforts have been devoted to use wavelet transform as a practical tool for analyzing the signals obtained from vibration measurements. Taha et al [7] provided a useful and intensive review of using wavelet transform in structural health monitoring. Damage detection based on continuous wavelet transform (CWT) of time signal was first proposed by Surace and Ruotolo [8], where they analyzed signals acquired from a numerical simulation of a beam. Spatial discrete wavelet transform was firstly used by Liew and Wang [9] with the aim of finding cracks in a numerical model of a simply supported beam. Their results showed that wavelet transform is a robust method over traditional eigenvalue techniques. Although their method used spatial wavelet transform, the technique needed an oscillating excitation along the length of the beam. To overcome this shortcoming, Wang and Deng [10] suggested a new approach based on spatial Haar discrete wavelet transform with the purpose of directly studying structural response signals, for example displacement, strain or acceleration measurements.

A study on sensitivity to various boundary conditions, geometries of damages and different kinds of wavelets was performed by Quek et al [11]. This investigation concentrated on profiles of beams which were subjected to static and impact loads. CWT was applied on mode shapes by Hong et al [12]. In their study, Mexican hat wavelet transform was used to evaluate Lipschitz exponent of a damaged beam. Results of their investigations for detecting damages in beam-type structures showed that the value of Lipschitz exponent can play an important role in probing damages. Gentile and Messina [13] provided a useful study on applying continuous wavelet transform (Gaussian family) on mode shapes of a beam with open crack. They compared derivatives with CWTs and deduced that a CWT with m vanishing moment and small scale s is equivalent to m-th derivatives, aside from points near the boundary conditions. Thus number of vanishing monuments should be at least 2. Effects of different boundary conditions and noisy data were numerically examined.

Chang and Chen [14] were the first authors who used spatial wavelet transform to estimate the depths and locations of damages in a multiple cracked beam. They also carried out studies into damage detection in a rectangular plate with multiple cracks [15]. Application of complex Gaussian wavelet transform with the aim of identifying damages in structures was primary proposed by Poundel et al [16]. Complex Gaussian wavelet was applied on differential mode shapes of intact and damaged beam. A comparative study of Gaussian wavelet, complex Gaussian wavelet and Morlet wavelet was conducted and results obtained from experimental test on simply support beam were presented. Bayissa et al [17] put operation energy content of the wavelet coefficients into the space domain and interrogated damages in plate-like structure. Method compared with mean square value (MSV), modal flexibility (MF) and uniform load surface (ULS) and it showed its superiority over them.

A new approach based on wavelet transform, called ‘integrated wavelet transform’ proposed by Qia and Cao [18] where they employed stationary wavelet transform in conjunction with CWT. The procedure encompasses two steps, first, mode shapes obtained from test or simulation is upgraded by SWT-based multi-resolution analysis then abnormalities in refined mode shapes are magnified by CWT-based multi-scale analysis. Fan and Qiao [19] numerically and experimentally conducted an investigation of damages in a plate with the help of 2-D CWT (experimental test was performed on a FRP composite plate using smart piezoelectric actuators and sensors). This approach was compared with 2-D gapped smoothing method (GSM) and 2-D strain energy method (SEM). Although 2-D SEM method was found less sensitive to number of measurement points, it needs an analytical or numerical model of intact structure while two other methodologies are non-model-based algorithms. Recently, Bagheri et al [20] employed curvelet as a new multi-scale representation method with the purpose of finding cracks in a plate-structure, where they claimed that discrete curvelet transform provides better representation than wavelet transform.

Damage detection technique based on changes in flexibility of structures developed due to the fact that although stiffness matrix is changed by cracks, this matrix is dominated by higher mode shapes, which are difficult to measure in practice. In conflict with stiffness, flexibility matrix is dominated by lower mode shapes and then it is easier to form. This method was first brought into play by Pandey and Biswas [21]. Using absolute difference between flexibility matrix of intact and damaged beam estimated from a few of lower frequency modes, they not only detected availability of a crack in beam but also localized the accurate location of the crack. The idea was numerically and experimentally evaluated.

Bernal and Gunes [22] put forward a technique on the basis of out-put only data. Realizing that it is impossible to form flexibility matrix from out-put only data, they used alternative matrix which did not cause any problem to employ damage locating vector (DLV) procedure. This matrix differed from flexibility just by a scalar multiplier. Based on the method, after subtracting flexibility matrix of intact structure from flexibility matrix of damaged structure, null space is considered as loads on structure, and then zero stress fields can be regarded as potentially damaged elements. Duan et al [23] proposed another proportional flexibility matrix (PFM), which was utilized as a replacement for original flexibility matrix in DLV method. Two numerical simulations were conducted with the aim of evaluating the accuracy of methodology.

Developing FE model based approach, Jaishi and Ren [24] performed an optimization algorithm to minimize an objective function, which was comprised of flexibility residual and its gradient, then damage detection techniques were numerically and experimentally carried out. Yang and Liu [25] used eigenparameter decomposition of flexibility change to localize damages. By verifying the number of damaged elements based on non-zero eigenvalues, they identified and recognized cracks. Yang [26] introduced a new approach for the purpose of finding damages in structures by using ambient modal data. Tomaszewska [27] not only provided a statistical investigation of error in flexibility and mode shape curvature methods but also put forward an Absolute Damage Index (ADI) in order to distinguish between the fake and genuine outcomes of damage detection approaches. He suggested that in order to avoid obtaining false results from contaminated data, two damage indicators can be used simultaneously. Reynders and Roeck [28] came up with a new idea based on flexibility method called local flexibility (LF). Output-only data used to form quasi-static flexibility matrix and changes in local stiffness utilized to identify damages.

Zhang and Aktan [29] suggested utilizing uniform load surface (ULS) instead of flexibility matrix. They not only introduced and extended ULS approach and explained its robustness over traditional flexibility method but also conducted numerical and experimental tests on a highway bridge to analyze the truncation error. Wu and Law [30] developed ULS curvature for 2-D plate and they used Chebyshev polynomial approximation as an alternative way to localize damage. Their methodology needed both mode shapes of intact and damaged structure like Zhang and Aktan method. Different boundary conditions, noise levels, mode truncation and sensor spacing were numerically investigated.

Generalized Fractal Dimension, Simplified Gapped-smoothing, central difference and Chebyshev polynomial in association with ULS have been used as damage identification methodologies [23, 30-32]. The objective of this paper is to propose a new approach based on CWT in conjunction with ULS technique. A brief review of two concepts is introduced, and then a numerical simulation is conducted with the aim of demonstrating procedure. In order to probe the sensitivity of method to size of the cracks and its immunity to noisy data, different size of damages and noise levels are studied. Finally, for the purpose of showing the practicability of this technique, an experimental test is performed.

2. Theory of Wavelet

Slight damages do not induce serious changes in dynamic behavior of structures and therefore signal processing methods should be used as an instrument to diagnose these small changes in response signals acquired from vibration tests. One of the state-of-the-art signal processing methods which has recently been come into focus in damage detection area is wavelet transform owing to its advantages over traditional methods to analyze signals based upon windowing and variable sized regions process. Continuous wavelet transform (CWT) of a space-domain signal $f(x)$ is presented as:

$$W(\tau,s)=\int_{-\infty}^{+\infty} f(x)\psi_{\tau,s}^{*}(x)dx=\frac{1}{\sqrt{s}}\int_{-\infty}^{+\infty} f(x)\psi^{*}\left(\frac{x-\tau}{s}\right)dx \tag{1}$$

where $s>0$ and τ are scaling (dilation) and position (shift) parameters of the wavelet function respectively. Position parameter denotes moving window site and dilation points to the size of this window. Selecting different complex functions $\psi_{\tau,s}^{*}(x)$ called *mother wavelet* provides us with various families of wavelets. Although mother wavelet can be chosen any function, it should satisfy the following condition:

$$\int_{-\infty}^{+\infty} \frac{|\hat{\psi}(\omega)|^2}{|\omega|} d\omega < \infty \tag{2}$$

where $\hat{\psi}(\omega)$ is Fourier transform of $\psi(x)$. A wavelet has *n* vanishing moments if:

$$\int_{-\infty}^{+\infty} x^n \psi(x)dx = 0 \tag{3}$$

Number of vanishing moments is important when it comes to finding abnormalities of signals. In order to determine the location of damage from mode shape based data, real wavelets is used in this work. For any wavelet with *n* vanishing moments there is a function $\theta(x)$ which is defined as [33]:

$$\psi(x)=(-1)^n \frac{d^n\theta(x)}{dx^n}, \int_{-\infty}^{+\infty}\theta(x)dx \neq 0 \tag{4}$$

Gentile and Messina [13] indicated that a CWT with *n* vanishing moments and small scale *s* is equivalent to *n*th derivatives except for intervals near the boundary conditions, where there are discrepancies in two approaches. As the number of points increases, accuracy of CWT in boundaries upsurges. Although wavelets holding greater number of vanishing moments lead to better performance, we should yearn for reconciliation between number of vanishing moments and number of data points, because higher vanishing moments need longer support.

Granted, number of vanishing moments plays a prominent role in damage detection, but there is another important parameter and it is a mother wavelet. Up to now, several kinds of mother wavelets have been used to analyze mode shapes [13, 14, 36-40]. In this study three kinds of CWTs are considered in order to find abnormalities of ULS including: *Gaussian* with four vanishing moments, *symmetrical 4* and *bior 6.8*.

3. Theory of ULS Method

Definition of modal flexibility matrix for a system with *m* lower measured modes is given by [21]:

$$f_{i,j}=\sum_{k=1}^{m}\frac{\phi_k^i\phi_k^j}{\omega_k^2} \tag{5}$$

where ϕ_k^i and ϕ_k^j are i^{th} and j^{th} elements of k^{th} mass normalized mode shape respectively. ω_k denotes k^{th} natural frequency and $f_{i,j}$ is the modal flexibility at point *i* due to the unit load at the point *j*. For a linear system, modal deflection at point *i* due to the uniform load all over the structure is obtained as [23]:

$$u(i)=\sum_{j=1}^{n} f_{i,j}=\sum_{j=1}^{n}\left(\sum_{k=1}^{m}\frac{\phi_k^i\phi_k^j}{\omega_k^2}\right)=\sum_{k=1}^{m}\frac{\phi_k^i\sum_{j=1}^{n}\phi_k^j}{\omega_k^2} \quad (6)$$

where *n* indicates the number of degrees of freedom. Having its advantages over modal flexibility, ULS is privileged in terms of lower contribution of higher order modes, which leads to more convergence with the lower modes and makes it a valuable and stable damage indicator. Moreover, it is less vulnerable to noise due to the fact that summation of all the modal coefficients of a specific mode averages out the random error at every measurement point [30].

Objective of this section is met in two steps. First, a numerical investigation of the proposed damage detection methodology on a cantilever beam is carried out and its sensitivity to noisy data and severity of crack is analyzed. Then a laboratory test is performed with the aim of examining the method.

4. Numerical Study

In order to interrogate damages by suggested technique, mode shapes obtained from damaged beam is utilized to form the ULS based on eq. (6) and then CWT is used to investigate abnormality as damage site. To explain how wavelet transforms in association with ULS method can be used as a damage detection indicator, a cantilever beam is considered. Geometry of the beam and its material properties are as follows: Young's modulus E = 200 GPa, density $\rho = 7850\ \mathrm{kg}/\mathrm{m}^3$, length L = 750 mm, cross-section area $A = w\times h$ = 50 mm×10 mm. Total number of elements of the FE model is 50. Damage is considered as a reduction in stiffness matrix while mass matrix is unchanged and it is simulated by the method proposed by Gentile and Messina [40].

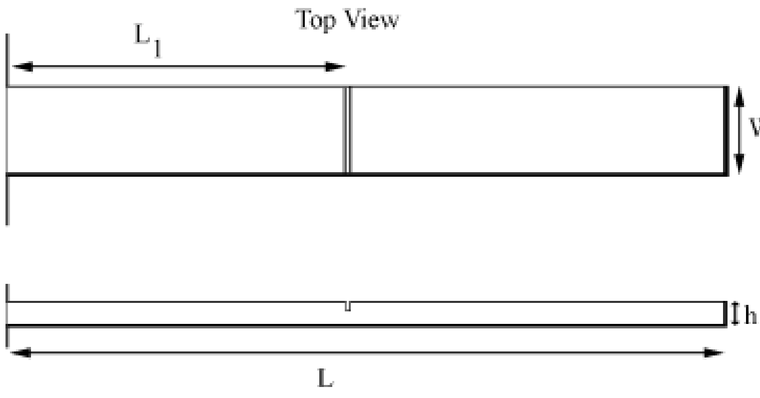

Fig. 1. Layout of cantilever beam with a crack

4.1 Noise Free Mode Shapes

Three damage cases are considered, and they are induced at $L_1 = 300$ mm with 30%, 40% and 50% severity respectively. In Fig. 2 frequency response functions (FRFs) of intact and damaged beam for each case at a typical measurement point is presented. As expected, damages result in shifts in natural frequencies. Fig. 3 shows that the first three mode shapes of the beam before and after inducing 30% damage are not noticeably different.

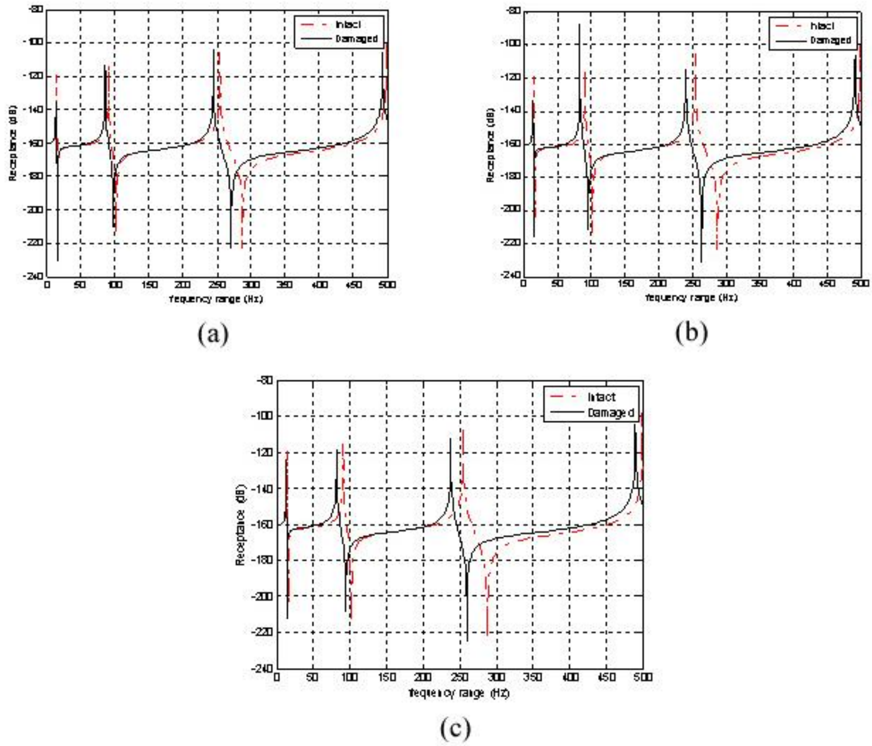

Fig. 2. FRF of intact and damaged beam at a typical measurement point: (a) 30% damage, (b) 40% damage, (c) 50% damage

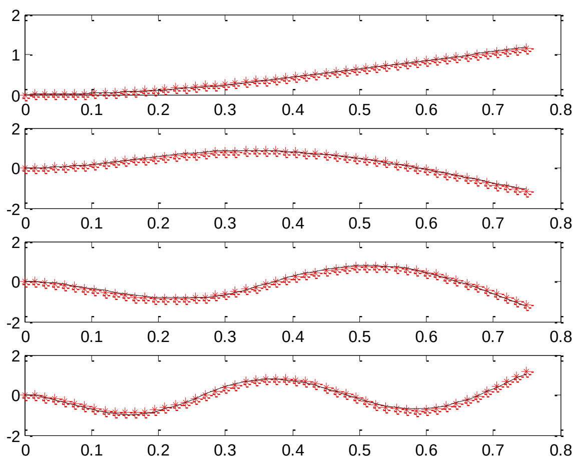

Fig. 3. First three mode shapes of intact (*) and damaged (-) beam for 30% damage

In order to perform the method, ULS for three damage cases calculated by eq. (6) and then the spline interpolation and extrapolation were utilized to obtain refined grid points between measurement points and at boundary condition points respectively. Refined ULSs was analyzed through three aforementioned CWTs. Figs. 4-6 show the results. Although coefficients are nearly zero at the locations far from crack site, they considerably surge when they meet the crack.

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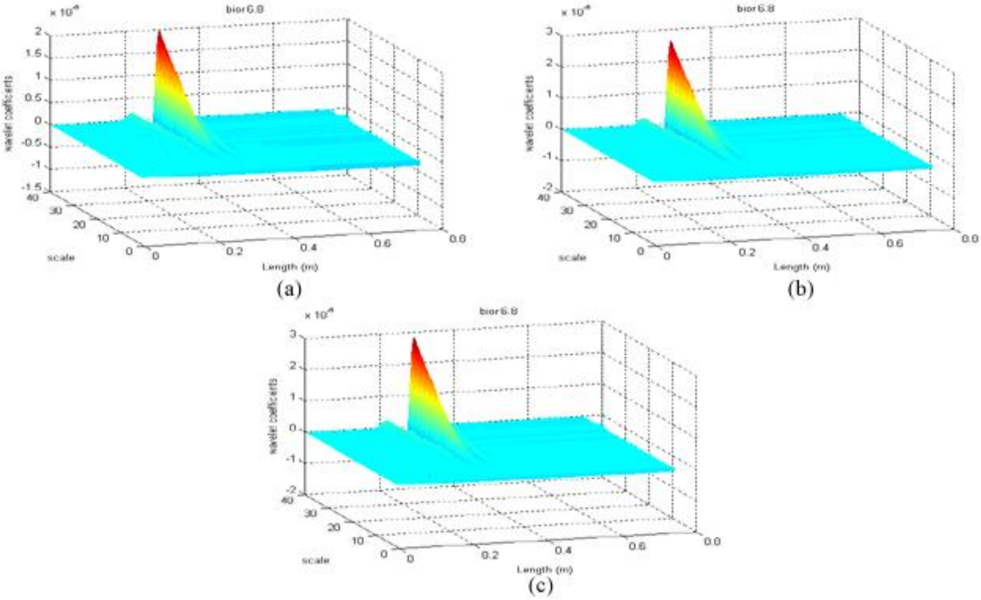

Fig. 4. Wavelet coefficients of *bior 6.8* for (a) %30 damage, (b) %40 damage, (c) %50 damage

In Fig. 4, wavelet coefficients of *bior 6.8* for three damages are presented. Wavelet coefficients at the location of damage are sensitive to damage severity and as the depth of crack increases, wavelet coefficients also enlarge.

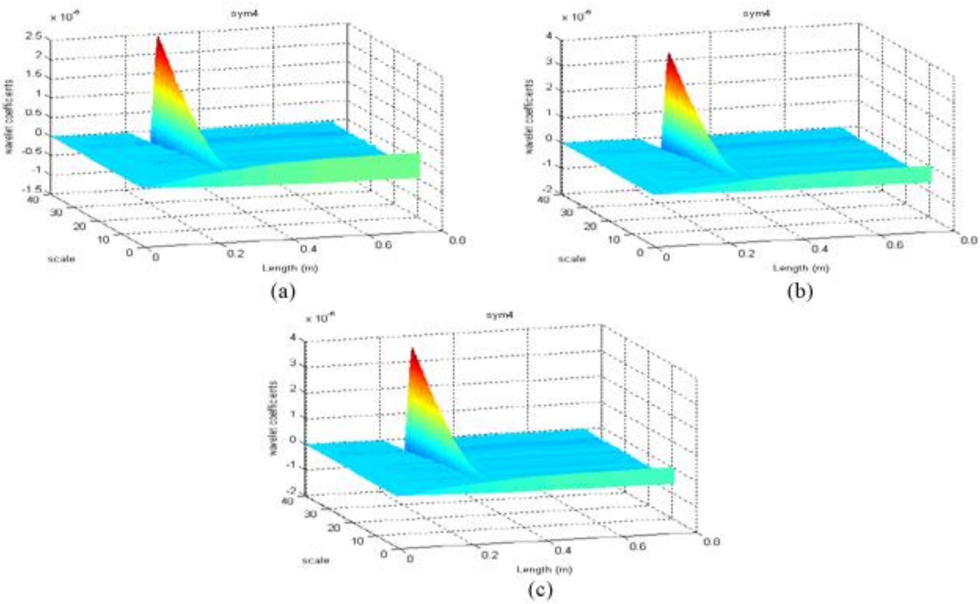

Fig. 5. Wavelet coefficients of *symmetrical 4* for (a) %30 damage (b) %40 damage (c) %50 damage

The same procedure was followed and *symmetrical 4* was used to find abnormalities in ULS. As indicated in Fig. 5, although this wavelet like *bior 6.8* is a useful instrument for localizing damage, there are irregularities in some scales. These irregularities tend towards less conspicuousness when the damage is relatively considerable. In comparison with *bior 6.8*, maximum values of coefficients are slightly greater at the crack site.

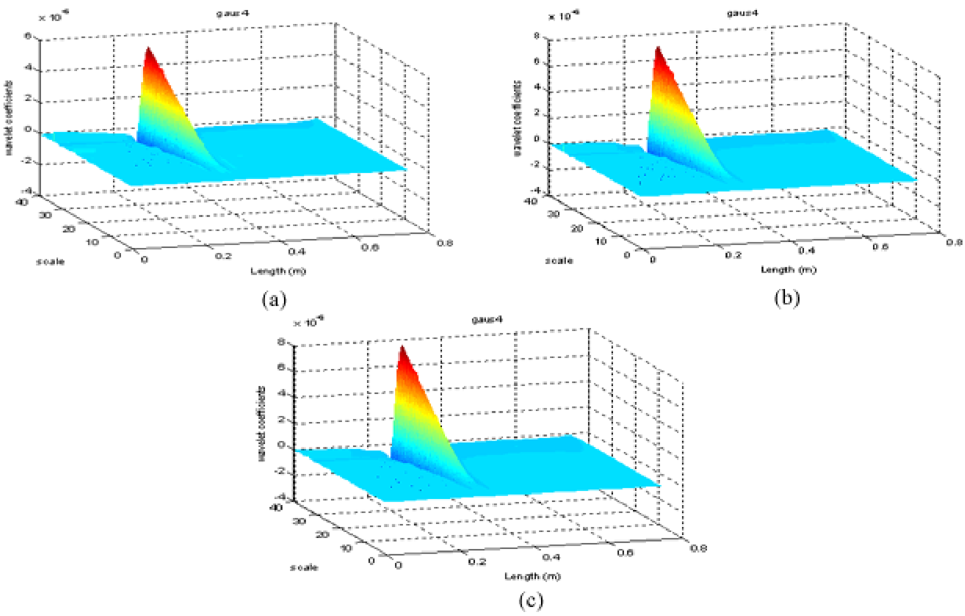

Fig. 6. Wavelet coefficients of *Gaussian 4* for (a) %30 damage (b) %40 damage (c) %50 damage

In Fig. 6, results of applying Gaussian wavelet with four vanishing moments for detecting abnormalities in ULSs obtained from three damage cases are presented. Maximum values of wavelet coefficients are greater than two other wavelets in the face of damages, but irregularities are noticeable in some scales and it may lead to minor mistakes in damage localization of structures, especially in high scales.

4.2 Contaminated Mode Shapes

To cope with noises which are inexorable in experimental tests, effects of noise in damage detection methods should be evaluated. In this section with the aim of investigating the ability of the proposed method to deal with noise, contaminated mode shapes are utilized to form the ULS and outcome has been used to draw a comparison between earlier ULS method that employs finite difference and the introduced approach, which applies wavelet transform to magnify irregularities. Simulated mode shapes of three damage cases are contaminated with different noise levels. Function that is used to aim to this goal is as follow [1]:

$$\varphi'_{ij} = \phi_{ij} + r_{ij}\rho\phi_{rms,j} \tag{7}$$

where ρ is the level of random noise, ϕ_{ij} and φ'_{ij} are noise free mode shapes and contaminated mode shapes respectively; r stands for normally distributed random variables with a zero mean and variance equals to 1 and $\phi_{rms,j}$ is the root of mean square of j^{th} mode shape. Simulated mode shapes contaminated by several levels of noise for three damage case were considered and three aforementioned wavelets have been used to interrogate crack location. Results pertaining to 30%, 40% and 50% induced damage tainted with $\rho = 0.0001$ noise level are provided in Figs. 7-9.

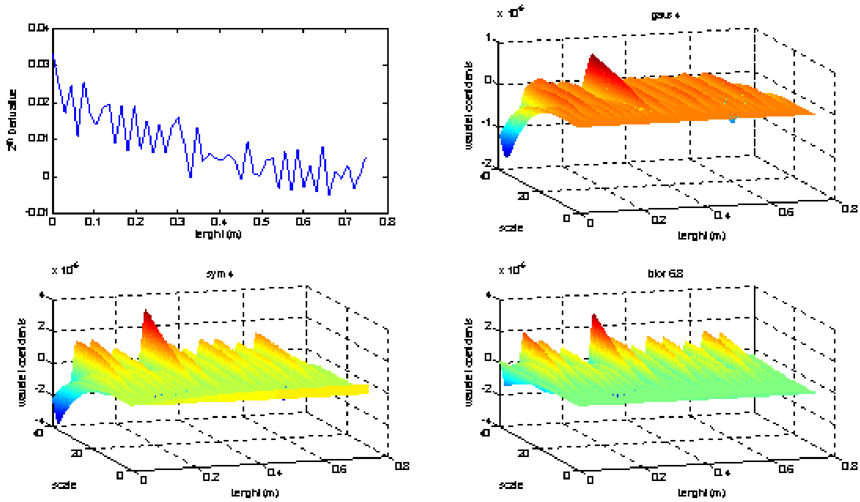

Fig. 7. Wavelet coefficients for 30% damage with ρ = 0.0001 (Contaminated mode shapes)

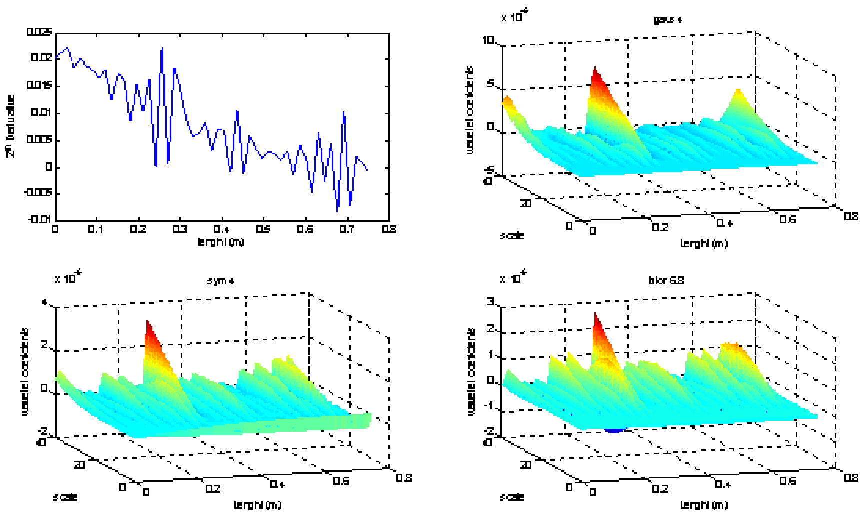

Fig. 8. Wavelet coefficients for 40% damage with ρ = 0.0001 (Contaminated mode shapes)

As it can be noticed, all three types of wavelets have been able to recognize the damage while the second derivative is unable when the damage is 30% and it is feeble in the face of 40% and 50% damage cases.

5. Experimental Study

A free-free beam was used as an experimental case to investigate the practicability of the proposed method. The dimensions of the beam are 940x50x10 mm and a 35% damage was located at $l_1 = 510$ mm. The beam was excited using a roving hammer at 48 points while an accelerometer was placed at 480 mm from one end.

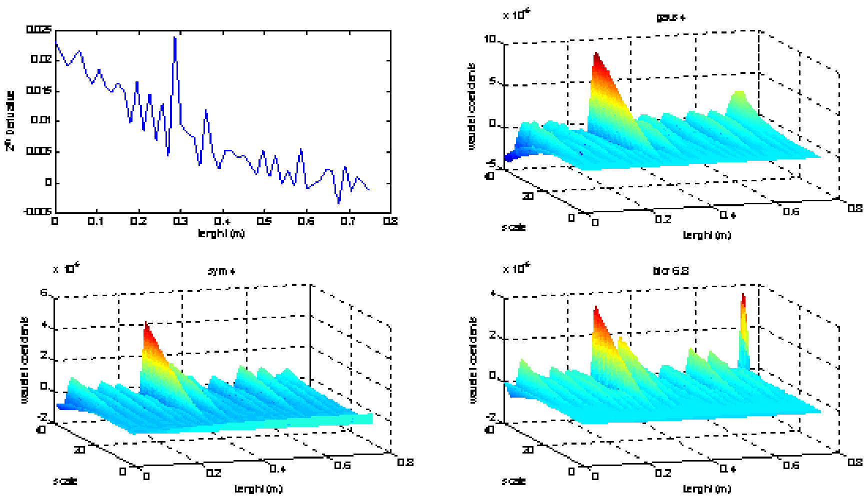

Fig. 9. Wavelet coefficients for 50% damage with ρ = 0.0001 (Contaminated mode shapes)

Fig. 10. Experimental test setup and its instruments

Fig. 11 indicates the first three mode shapes of the beam and as it shows there is no sign of the location of the induced damage. ULS is formed using eq. (6) and three aforementioned wavelets are applied to detect the location of the damage like the procedure that was followed for the numerical case.

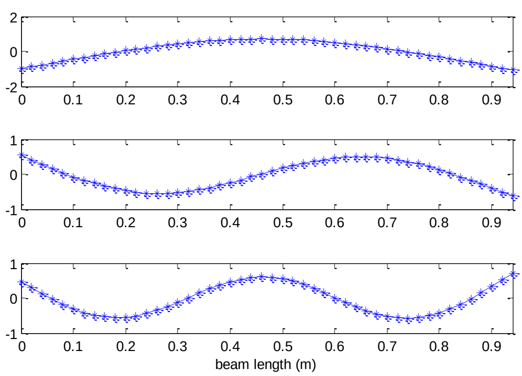

Fig. 11. First three experimental mode shapes

As results in Fig. 12 reveal, although there are some distortions at the boundary condition points, especially in *Gaussian 4*, three wavelets manifest their maximum values at the crack site.

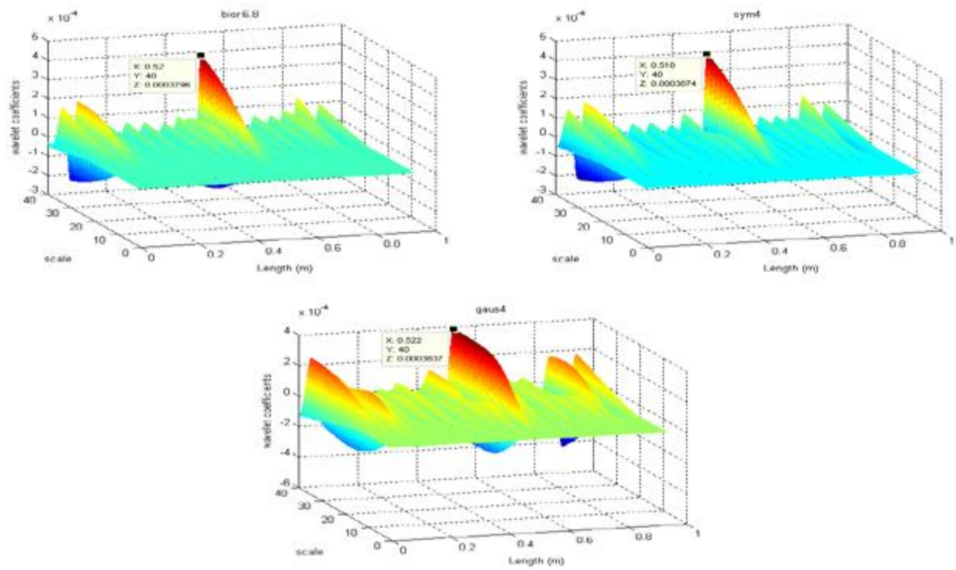

Fig. 12. Wavelet coefficients for experimental test

6. Conclusions and Remarks

New approach has been espoused for detecting damages of structures using uniform load surface method in conjunction with wavelet transforms, where the wavelets have been applied to scrutinize the ULS, obtained from the damaged beam, in preference to classic finite difference approach. Advised procedure compared with classic method in the face of both noise free mode shapes and the contaminated mode shapes and the new approach exhibited its capabilities in the face of noise. A beam-type structure has been used to verify the procedure as an experimental case. Although *bior 6.8* and *symmetrical 4* wavelets were capable of discerning damages, *Gaussian* wavelet could not act as a practicable damage indicator in the performed test. The main reason for this incapacity stems from irregularities at the boundary condition points and changing the extrapolation function at these points may result in better conclusions for this wavelet.

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