565. Design of elastomeric shock absorbers with variable stiffness

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Abstract. Over the past 20-30 years, usage of specific properties of rubber (high elasticity, resistance to environmental influences, good dynamic performance, low compressibility, nearly linear stress-strain relationship for strains of up to 15% ÷ 20%) enabled development of shock absorbers with nonlinear characteristics "force - settlement". The paper proposes a method of calculation of a cylindrical rubber shock absorber cords with hard side stops by considering weak compressibility of rubber. The solution is obtained by applying Ritz method and using the principle of the minimum complete potential energy of deformation. Obtained solutions can be used to find the dependence of „force - settlement" for cylindrical shock absorbers, as well as in design of such shock absorbers. This is required when developing a shock absorber with a given non-linear stiffness characteristics.

Keywords: rubber, shock-absorber, weak compressibility, side stops, stiffnes.

Introduction

Over the past 20-30 years, usage of specific properties of rubber (high elasticity, resistance to environmental influences, good dynamic performance, low compressibility, almost linear relationship between stress and strain at strains up to 15% ÷ 20%) enabled development of shock absorbers with nonlinear characteristics "force - settlement". They can also be used in different designs engineering and construction as elastic ties for the implementation of the given linear and nonlinear elastic characteristics, effectively replacing hydro- pneumatic - spring compensating device. An example may serve shock absorbers with „self-tuning" stiffness when subjected to wide variation of external forces (including the weight of equipment).

Suppose we want to design a shock absorber loaded with axial compressive force P with a nonlinear or piecewise nonlinear characteristic P = P(Δ) "force - settlement", providing a definite correlation between the stiffness of the shock absorber and level of loading, i.e. at any point characteristic P (Δ) should comply

$$\frac{dP(\Delta)}{d\Delta}\cdot\frac{1}{P(\Delta)}=f(P(\Delta)) \qquad (1)$$

If **f(P(Δ)) = const = C**, then from equation (1) to describe P(Δ) we obtain the solution:

$$P(\Delta)=A\cdot\exp(C\cdot\Delta) \qquad (2)$$

The constant of integration A is determined from additional conditions. For example, the condition of minimum weight (the minimum amount of rubber layer) of the shock absorber, for which the shock absorber performs its functions (for example, provides an initial draft of Δ_H shock absorber). In this case, constant **A** we get:

$$A = P(\Delta_H) \cdot \exp(-C \cdot \Delta_H) \quad (3)$$

From (1) - (3) that the shock absorber stiffness characteristics:

$$P(\Delta) = P(\Delta_H) \cdot \exp\left[C(\Delta - \Delta_H)\right] \quad (4)$$

will provide the desired balance between rigidity and the level of shock absorber loading. Qualitative characteristics (4) shown in Figure 1.

In particular, if $C = \rho^2/g$ (where: ρ - the natural frequency of equipment that is equipped with the projected shock absorber; g - acceleration of gravity) and Δ_H - sludge from the minimum weight of the shock absorber, the execution of the design of the shock absorber of the equation (4) provides equifrequency absorber .Value (4) can be implemented using special characteristics of P (Δ), which provides a choice of a particular geometry of the shock absorber. Commonly, shock absorbers with absolutely rigid side stops are used. In the calculation of shock weak compressibility of rubber is unrecorded [1]. But in this paper, we design a damper on the example of a cylindrical solid absorber with absolutely rigid vertical side stops (Fig. 2), which should provide a measure of stiffness properties of P (Δ) of type (4). A qualitative graph is presented in Figure 2. Stages of the shock absorber circuit are provided in Figure 3.

The first stage (Fig. 3a). No contact of the rubber layer side with a side stop, force P is less than force P_k, in which they come into contact with the side stop. The dependence of the "force - settlement" P(Δ) in the light of weak compressibility of rubber is determined by using the principle of minimum total potential energy of deformation $J(u_1,w_1)$ [1,2].

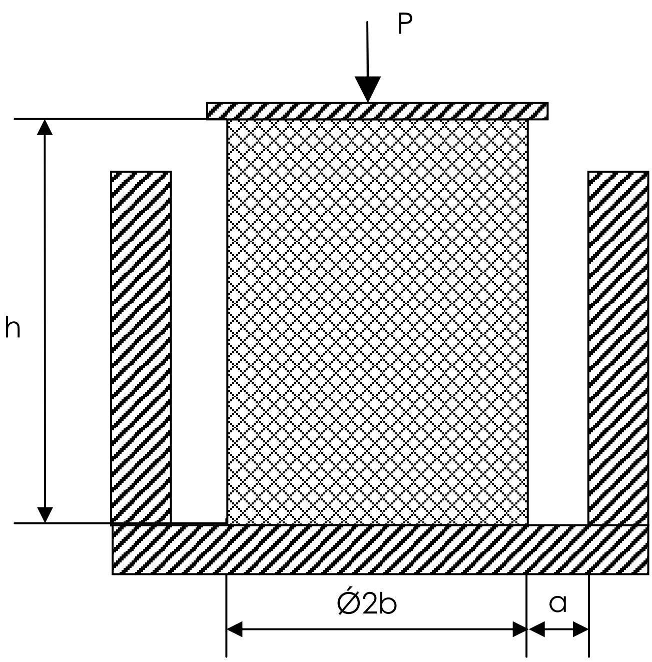

Fig. 1. Cylindrical solid absorber with absolutely rigid vertical side stops

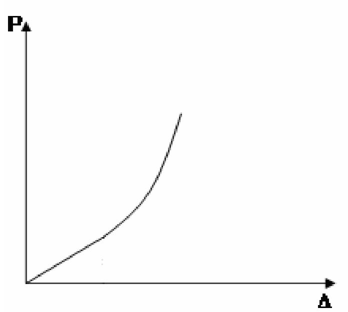

Fig. 2. Stiffness properties of P(Δ)

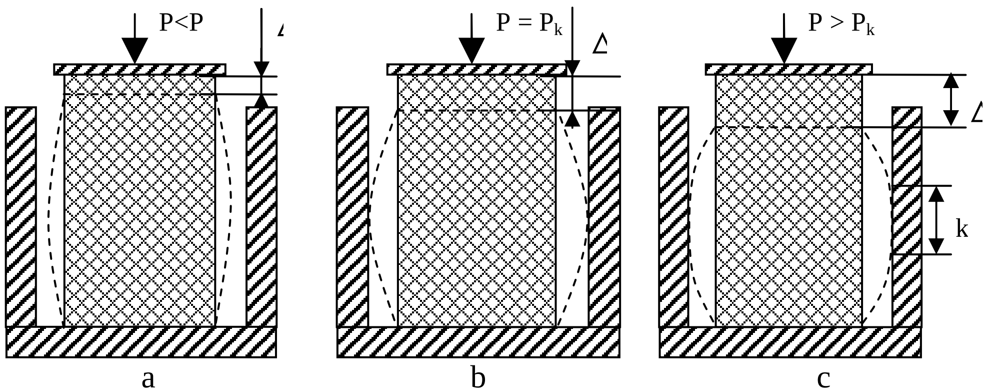

Fig. 3. Principle of operation of cylindrical solid absorber with absolutely rigid vertical side stops

$$J = 2\pi G \int_{-\frac{h}{2}}^{\frac{h}{2}} \int_{0}^{b} \left[\begin{array}{l} (\frac{\partial u}{\partial r})^2 + \left(\frac{u}{r}\right)^2 + (\frac{\partial w}{\partial r})^2 + \frac{1}{2}\left((\frac{\partial u}{\partial z}) + (\frac{\partial w}{\partial r})\right)^2 + \\ + \frac{3\mu}{1+\mu} s \left((\frac{\partial u}{\partial r}) + \frac{u}{r} + (\frac{\partial w}{\partial z})\right) - \frac{9\mu(1-2\mu)}{4(1+\mu)^2} s^2 \end{array} \right] r\, dr\, dz - P\Delta_1 \quad (5)$$

$$u_1 = A_1 r \left(1 - \frac{4z^2}{h^2}\right), \quad w_1 = -\frac{3\Delta_1}{2h}\left(z - \frac{4z^3}{3h^2}\right) \quad (6)$$

Choosing the displacement functions u_1 (r, z), w_1 (r, z), respectively, along the axes r and z, that satisfy the geometric boundary conditions:

$$u_1(0, z) = 0, \quad u_1(r, \pm\frac{h}{2}) = 0, \quad u_1(r = b, z = 0) \leq a \quad , \quad (7)$$

$$w_1(r, 0) = 0 \qquad w_1(r, \pm\frac{h}{2}) = \mp\frac{\Delta_1}{2}$$

From the principle of minimizing the functional (A_1, B_1) the dependence of the force - for the first phase of sediment loading (up to contact the rubber layer with a lateral focus), with the weak compressibility of rubber, has the form (for $0 \le P \le P_k$):

$$\Delta_1 = \frac{Ph}{\pi b^2 G}\left[1{,}8 + \frac{1{,}2 + 1{,}5\alpha^2}{\left(1 + 3\frac{1-2\mu}{2\mu}\alpha^2\right)}\right]^{-1}, \quad \alpha = b/h \tag{8}$$

For further calculations the dependence (8) is conveniently represented in the form:

$$\Delta_1 = \Delta_{1f} + \Delta_{1\theta} \tag{9}$$

Where: Δ_{1f} - settlement first stage at the expense of forming a rubber layer.

$$\Delta_{1f} = \frac{Ph}{\pi b^2 G} \cdot \frac{1}{3 + 1.5\alpha^2} \tag{10}$$

$\Delta_{a\theta}$ - settlement first stage due to bulk deformation of rubber layer

$$\Delta_{1\theta} = \frac{Ph}{2\pi b^2 G} \cdot \frac{3(1-2\mu)\alpha^2}{3 + 1.5\alpha^2} \tag{11}$$

The second stage (fig. 3.b). When the force $P = P_k$ is point contact rubber layer with a side stop. Meaning of P_k is determined from the point of contact with the side stop:

$$u_1(r = b, z = 0) = 0{,}75\,\Delta_1\, b/h = a \tag{12}$$

where: a - the size of the gap between the lateral surface of the rubber layer and the side stops, from (8) and (12), taking into account only the deformation of shape change:

$$P_k = \frac{4}{3}\pi b\ G\, a\left[3{,}0 + 1{,}5\alpha^2\right] \tag{13}$$

The third stage (fig. 3.c). At this stage, the force $P > P_k$ and the process of deformation of the rubber layer will substantially depend on the width of its contact with the lateral k emphasis (fig. 3.c), which value is not known in advance and must be determined in the solution process. For the third stage, fulfilling the condition of zero volumetric strain:

$$\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{\partial z} = 0 \tag{14}$$

Displacement functions for the rubber layer can be chosen as:

- for $0 \leq r \leq b$; $0 \leq z \leq 0{,}5k$:

$$u_{3,1} = \frac{ar}{b}, \quad w_{3,2} = -\frac{2a}{b} z \tag{15}$$

- for $0 \leq r \leq b$; $0{,}5\ к < z \leq 0{,}5h$:

$$\begin{gathered} u_{32} = \frac{ar}{b(1-\chi)^2}\left[1 - 2\chi + \frac{4kz}{h^2} - \frac{4z^2}{h^2}\right] \\ w_{3,2} = -\frac{2a}{b(1-\chi)^2}\left[(1-2\chi)z + \frac{2kz^2}{h^2} - \frac{4z^3}{3h^2}\right] + \frac{ak}{b(1-\chi)^2}(0.5 - 2\chi + \frac{2}{3}\chi^2) \end{gathered} \tag{16}$$

that satisfy the geometric boundary conditions:

$$\begin{gathered} u_3(0, z) = 0, \quad u_3(r, \pm\frac{h}{2}) = 0, \quad u_{31}(r = b, z \leq 0{,}5k) = a, \\ w_{32}(r, \pm\frac{h}{2}) = \mp\frac{\Delta_3}{2} \end{gathered} \tag{17}$$

From analysis of the shock absorber, it follows that the first stage for the load $P \leq P_k$ force characteristics "force - settlement" is known. To determine the characteristics of the "force - settlement" at $P > P_k$, it is necessary to take into account that the transition from the second phase of the third on an already strained state realized at $P = P_k$ is imposed an additional strain condition u_2 (r, z) and w_2 (r, z) to the force $P_c = P - P_k$. In view of (6), (15) and (16) deformed state, which is superimposed on the deformation of the first stage, described by the displacements u_2 (r, z) and w_2 (r, z)

$$\begin{gathered} \left.\begin{aligned} u_{21} &= u_{31} - u_1 \\ w_{21} &= w_{31} - w_1 \end{aligned}\right\} \quad 0 \leq r \leq b,\ 0 \leq z \leq 0.5k \\ \left.\begin{aligned} u_{22} &= u_{32} - u_1 \\ w_{22} &= w_{32} - w_1 \end{aligned}\right\} \quad 0 \leq r \leq b,\ 0.5k \leq z \leq 0.5h \end{gathered} \tag{18}$$

From the condition $w_{22}(r, z = 0.5h) = -\frac{\Delta_2}{2}$ have

$$\Delta_2 = \frac{2axh}{3b(1-\chi)^2}\left(\frac{4}{3} - \frac{1}{3}\chi + \frac{1}{5}\chi^2\right) \tag{19}$$

Where $\chi = \frac{k}{h}$

The width of the contact zone of the rubber layer with a side stop is an unknown parameter and the challenge of its definition is of focal character. In general, when determining the width of the contact zone k, one must take into account the actual configuration of the rubber layer during its contact with the side stop. This greatly complicates the calculations, because it will be necessary to integrate along a curved side surface of the rubber layer. In [3] it is demonstrated that the deformation of rubber and 40% of small increments of stress and strain imposed on the already loaded body are subject to a linear law. In this case, taking into account the incompressibility condition (14), in determining the integral force characteristics (type "force - settlement"), the integration, the transition from the coordinate system and displacement functions of the first stage, a coordinate system, coordinates and the displacement of the third stage can be carried out on the initial undeformed configuration of the rubber layer of the first stage of deformation.

Selected displacement functions (6), (15) - (19), fulfilling the conditions of continuity of displacement throughout the volume of the rubber layer does not meet the conditions of continuity of stresses on the surface of $0 \le r \le b$, $z = 0{,}5$ k. In [1,4] it is demonstrated that using an approximate solution for the minimum principle to the functional (5), and if the conditions of continuity for the displacements, the stress continuity conditions at the boundary of partitions can not perform because they are natural boundary conditions for functional (5). Decision (15) - (19) unknown quantity is the only parameter k (the width of the contact zone of the rubber layer with aside stop), we find the value which minimizes the functional:

$$J_2 = 4\pi\left[\int^{k}\int_{0}^{b} J_{21} r dr\, dz + \int_{0.5k}^{0.5h}\int_{0}^{b}\right]$$

$$J_2 = 4\pi\left[\int_{0.5k}^{0.5h}\int_{0}^{b} J_{21} r dr\, dz + \int_{0.5k}^{0.5h}\int_{0}^{b} J_{22} r dr\, dz\right] - P_c \Delta_2 \tag{20}$$

Where $J_{21}(u_{21}, w_{2,1})$, $J_{22}(u_{22}, w_{22})$

Parameter k is found from the equation:

$$\frac{\partial J_2}{\partial k} = 0 \tag{21}$$

Since the parameter k is included in the limits of integration, then, in accordance with [5], equation (21) is written as:

$$\frac{\partial J_2}{\partial k} = 4\pi\int_0^b\left[\int_0^{0.5k}\frac{\partial J_{21}}{\partial k}dz + 0.5J_{21}\Big|_{z=0.5k} + \int_{0.5k}^{0.5h}\frac{\partial J_{22}}{\partial k}dz + 0.5J_{22}\Big|_{z=0.5k}\right]rdr - \\ - P_c\frac{\partial \Delta_2}{\partial k} = 0 \tag{22}$$

All the solutions obtained by methods of linear theory of elasticity, so the deformation of up to 10% ÷ 15% in real shock absorbers width of contact will vary in the range 0 ≤ k ≤ 0,5 h. Therefore, the parameter æ = k / h can be regarded as a small parameter. In this case, we neglect the terms of higher-order of (20) - (22) to obtain the expression of force Pc.

$$P_c = \frac{2\pi Gab\chi}{(1-0.8\chi+0.5\chi^2)(1-\chi)^2}\left[\left(0.8-0.5\chi+0.2\chi^2\right)+\alpha_1^2(1.2-0.4\chi+0.15\chi^2)\right] \tag{23}$$

Where $\alpha_1 = \dfrac{2b}{h(1-\chi)}$

The width of the contact of the rubber layer is calculated by means of (19), if you set the movement Δ_2 or formula (24), if you set the force Pc. Formulas (9), (19), (20) and (24) can be used (with known geometric parameters of the harness and the level of loading) to calculate the nonlinear characteristic of "force - settlement" for a solid cylindrical damper with side stops.
In this case:
- the total settlement absorber, in accordance with formulas (10), (11) and (19), is equal to:

$$\Delta_\Sigma = \Delta_1 + \Delta_2 + \Delta_{\Sigma\Theta} \tag{24}$$

- Maximum compressive force, in accordance with formulas (13) and (24), is:

$$P_\Sigma = P_1 + P_2 \tag{25}$$

For example consider a real cylindrical shock absorber with side stops and obtain for him depending "force - settlement".

Parameters of the shock absorber:
b = 6cm, h = 6 cm, a = 0.2, μ = 0.495, G = 7 kg/cm^2, b/h = 4

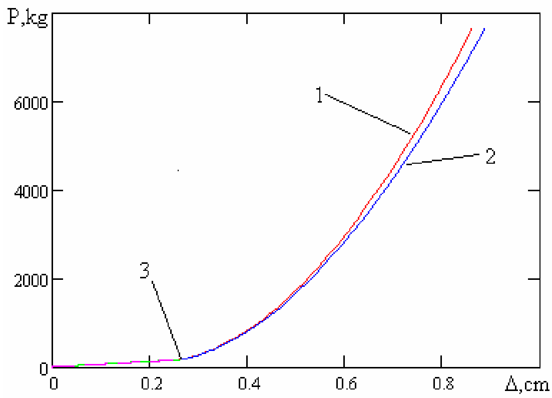

Fig. 4. dependence „force - settlement" for shock absorber, b/h = 1

„1" line - dependence „force - settlement" for shock absorber, b/h = 1 without taking into account the weak compressibility of rubber;
„2" line - dependence „force - settlement" for shock absorber, b/h = 1 with taking into account the weak compressibility of rubber;
„3" point - point of contact of the rubber with side boards

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Parameters of the shock absorber:
b = 6cm, h = 2 cm, a = 0.2, μ = 0.495, G = 7 kg/cm^2, b/h = 1

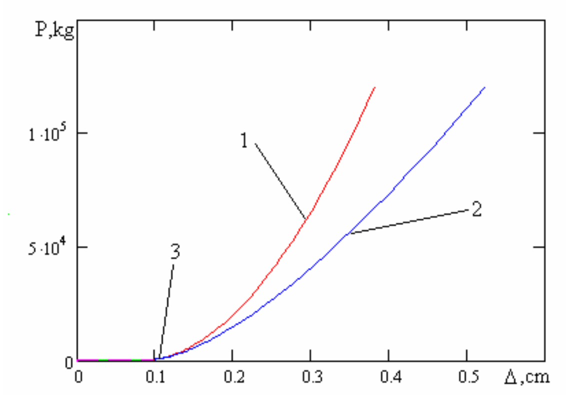

Fig. 4. dependence „force - settlement" for shock absorber, b/h = 3

„1" line - dependence „force - settlement" for shock absorber, b/h = 3 without taking into account the weak compressibility of rubber;
„2" line - dependence „force - settlement" for shock absorber, b/h = 3 with taking into account the weak compressibility of rubber;
„3" point - point of contact of the rubber with side boards

Conclusion

A method was proposed for determination of rigidity dependence "force - settlement" for shock-absorbing elements with absolutely rigid vertical side stops being under pressure, and it allows to take into account low compressibility of material of rubber layers. Obtained solutions can be used to determine the dependence „force - settlement" for cylindrical shock absorbers, as well as applied during design of such shock absorbers when you need to design a shock absorber with a given non-linear stiffness characteristics. Presented examples indicate that the omission of the weak compressibility of the rubber in the calculation according „force - settlement" for cylindrical shock absorbers with side stops when b/h = 1 does not lead to significant errors. When b/h = 3 neglecting weak compressibility of rubber results in significant error.

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