

## 554. Fin type propulsive devices with varying working area of vibrating tail

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**Abstract.** The object of the study is a fin type propulsive device of robotic fish moving inside water. The aim of the study is to establish optimal control law for variation of additional area of vibrating tail, which ensures maximal positive impulse of motive forces acting on tail. The problem has been solved using the maximum principle of Pontryagin. It is demonstrated that optimal control action corresponds to the case of bound values of area limits. The proposed method makes it possible to increase effective surface area of the fin within motion cycles, when useful tractive force is formed. But within cycles, when fin motion is hindered by water resistance forces, effective surface area is decreased. Thanks to this energy losses are minimized, and operation of fin propulsive device becomes more effective. Examples on synthesis of real mechatronic systems are provided as well.

**Keywords:** robotic fish, fin, propulsive device, optimal control, vibrating tail

### Introduction

There are known different types of engineering devices and objects, which elements or units during operation interact with external air or liquid medium. In many cases such interaction results in increased damping forces, which can be the reason of additional energy losses [1]. This paper considers the possibilities to increase the efficiency of vibration devices of such type by special variation of working area of machine head interacting with external medium. This problem is solved in application to robotic fish with fin type vibration propulsive device [2].

### Dynamic model and equations of motion

Simple robotic fish model consists of two main parts – hull and tail considered as rigid bodies (Fig. 1). Tail and hull are mutually connected with pivot. Fish body moves along  $x$  axis, but tail executes angular vibrations around pivot.

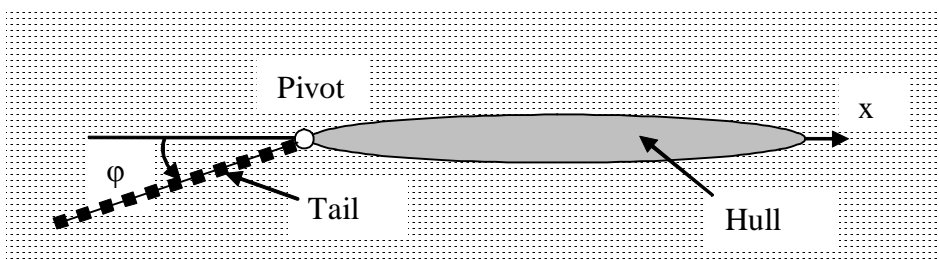


Fig. 1. Simple model of a robotic fish

In the case considered here a mass of the hull is sufficiently higher than tail mass. Besides, assuming the tail as a perfectly rigid plate, it is possible to form conditions on tractive force forming by the analysis of simplified dynamic model with one degree of freedom (Fig. 2). In accordance with this model tail is fastened to stationary base with pivot A and motion of the tail is described with one co-ordinate – angle  $\varphi$ . Elastic properties of the system are taken into account with torsion spring  $c$ .

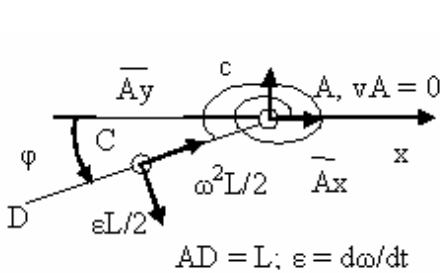


Fig. 2. Simplified dynamic model of tail motion

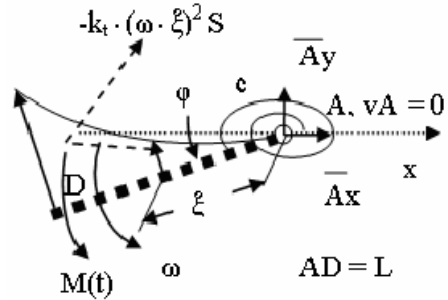


Fig. 3. Nonlinear model for tail interaction with water

Tail interaction with water is described by the hydrodynamic resistance force proportional to the square of velocity in local fin point (see Fig. 3) [3]. External excitation is simulated with moment  $M(t)$ .

Taking into account the aforementioned assumptions, the differential equation of fin angular oscillations about pivot A under external harmonic excitation  $M(t) = M_0 \cdot \sin(kt)$  can be represented as follows

$$J_A \cdot \ddot{\varphi} = M(t) - c \cdot \varphi - k_t \cdot S(t) \cdot \text{sign}(\dot{\varphi} \cdot \varphi) \cdot \int_0^L (\dot{\varphi} \cdot \xi)^2 \cdot \xi \cdot d\xi, \quad (1)$$

where  $J_A$  is fin moment of inertia about point A;  $k_t$  is coefficient of hydrodynamic resistance;  $S(t)$  is a law for variation in time of area of fin working surface;  $\xi$  is coordinate of fin local point;  $L$  is length of the fin.

In accordance with the principle of the motion of centre C of fin mass [4], the following equation in projection on  $x$  axis can be written:

$$m \left( \dot{\varphi}^2 \cdot \frac{L}{2} \cdot \cos \varphi + \ddot{\varphi} \cdot \frac{L}{2} \cdot \sin \varphi \right) = A_x - k_t S(t) \cdot \sin(\varphi) \cdot \text{sign}(\varphi \cdot \dot{\varphi}) \cdot \left( \int_0^L (\dot{\varphi} \cdot \xi)^2 \cdot d\xi \right), \quad (2)$$

where  $m$  is mass of the fin;  $A_x$  is a horizontal component of reaction at the pivot A.

### Results of mathematical analysis of motion

In order to realize an onward motion of robotic fish (motion in the direction of  $x$  axis, see Fig. 3), it is necessary to afford a negative mean value of reaction component  $A_x$  during stationary vibrations of the fin. By solving of equations (1) – (2) optimal control action  $S(t)$ , which ensures a maximal negative mean value of the reaction  $A_x$ , has been sought. And it has been taken into account, that area  $S(t)$  can change its value in the interval  $S_{\min} \leq S(t) \leq S_{\max}$ .

The search for an optimal control action  $S(t)$  has been performed using the maximum principle of Pontryagin [5, 6]. Besides, maximal negative value of impulse of reaction  $A_x$  was taken as criterion  $K$  for solving the optimization problem:

$$K = - \int_0^T A_x \cdot dt, \tag{3}$$

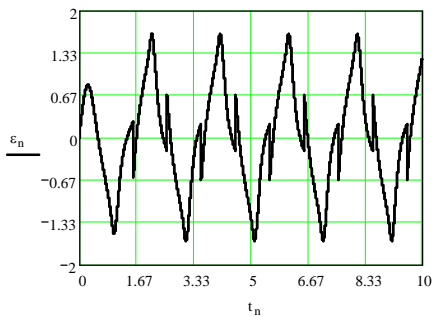
where  $T$  is a time required to move a fin from the initial stop position ( $\varphi_{\max}, \dot{\varphi} = 0$ ) to the second (end) stop position ( $\varphi_{\min}, \dot{\varphi} = 0$ ). By the use of equations (1) and (2) the optimization criterion (3) can be written in the following form:

$$K = - \int_0^T \left( m \cdot \left\{ \dot{\varphi}^2 \frac{L}{2} \cos \varphi + \frac{1}{J_A} \cdot \left[ M(t) - c\varphi - k_t \cdot S(t) \cdot \text{sign} \dot{\varphi} \cdot \dot{\varphi}^2 \cdot \frac{L^4}{4} \right] \cdot \frac{L}{2} \cdot \sin \varphi \right\} + \right. \\ \left. + k_t \cdot S(t) \cdot \sin \varphi \cdot \text{sign}(\varphi \cdot \dot{\varphi}) \cdot \dot{\varphi}^2 \cdot \frac{L^3}{3} \right) \cdot dt. \tag{4}$$

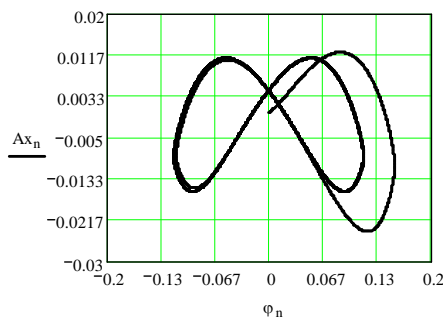
Equation (4) has been used in solution of the optimization problem [7]. It was shown that optimal control action corresponds to the case of bound values of area limits:  $S(t) = S_{\min}$  – for the case of fin motion from the neutral position ( $\varphi = 0$ ) till its momentary stop ( $\varphi_{\max}$  or  $\varphi_{\min}$ );  $S(t) = S_{\max}$  – for the case of fin motion from the momentary stop position ( $\varphi_{\max}$  or  $\varphi_{\min}$ ) till the neutral position ( $\varphi = 0$ ).

Efficiency of the proposed optimal control action  $S(t)$  is confirmed by the mathematical simulation of equations (1) and (2) using program MathCAD. Fig. 4 illustrates a temporal variation of the angular acceleration  $\varepsilon = \ddot{\varphi}$  of tail. The figure indicates that angular acceleration of tail reaches stationary cycle after one period (transition process is very short due to relatively high water resistance).

Effective operation of vibration propulsive device is dependent on the reaction component  $A_x$ . Fig. 5 shows the graph of reaction component  $A_x$  versus angle  $\varphi$ , but Fig. 6 presents a change of  $A_x$  in time  $t$ . As it may be observed, both graphs are asymmetric relative to zero level, and asymmetry is negative. As a result, mean value of the reaction  $A_x$  is also negative (see Fig.7).



**Fig. 4.** Variation in time  $t$  of the angular acceleration  $\varepsilon = \ddot{\varphi}$  of tail



**Fig. 5.** Reaction component  $A_x$  as a function of angle  $\varphi$

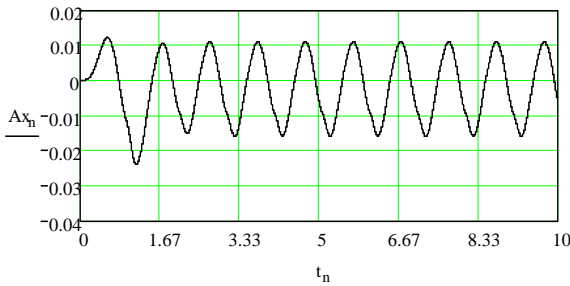


Fig. 6. Variation in time  $t$  of the reaction component  $A_x$

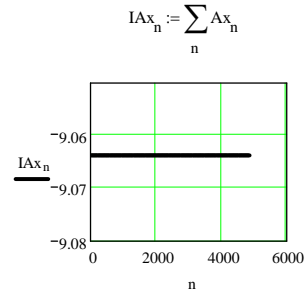


Fig. 7. Mean value of the reaction  $A_x$  in time domain

Due to  $(A_x)_m < 0$ , the mean value of tractive force will be positive (its direction will be the same as direction of  $x$  axis, see Fig. 3). And this is necessary for onward motion of robotic fish.

### Robotic fish with one-tail propulsive device

Optimal control action  $S(t)$  is realized in one-tail vibration propulsive device, which schematic diagram is shown in Fig. 8 [8]. Propulsive device is mounted in the after body 1 of robotic fish. Fin is made from two plates 2 and 3, which working surfaces are mutually parallel. Both plates have the possibility to rotate about vertical axis running through point O (turning angle  $\varphi$ ). Besides, plate 3 has the possibility to move relative to plate 2 in guides 4 (relative motion of plate 3 in the direction of axis  $x_1$ ). Rotation of plates 2 and 3 about axis O is carried out by link gear, which consists of link (it is made as an integral member with the plate 2), slide-block 5 and crank 6. Drive of the crank 6 can be executed by electric motor, which is not shown on the schematic diagram (see Fig. 8). But relative translational motion of the plate 3 is realized with the aid of mechatronic system: electromagnet 7 with rod 8, rigidly connected with the plate 3; spring 9; vibration transducer 10; block 11 making possible to form a control signal.

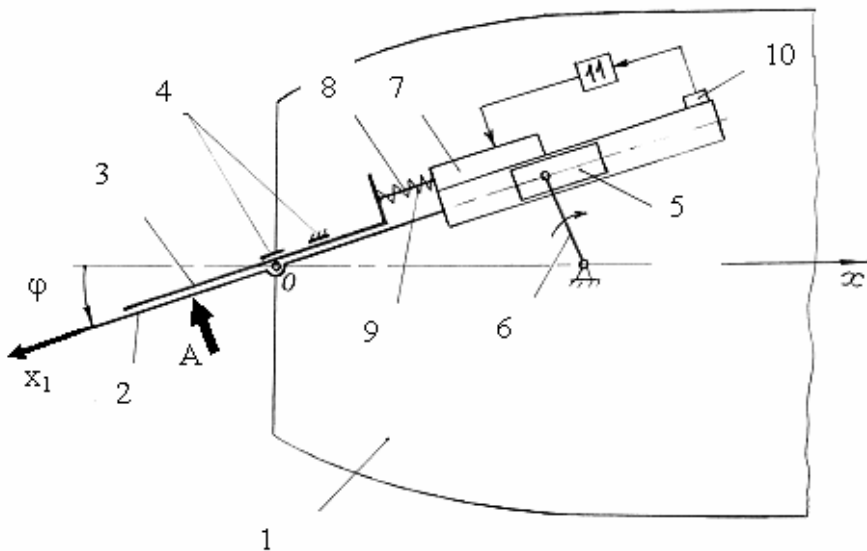


Fig. 8. Schematic diagram of the one-tail vibration propulsive device

Design variant of the outlet end of plate 2 (or 3), which interacts with water in operation regime, is shown in Fig. 9 (view A in Fig. 8). The same holes 12 are made in both plates (2 and 3), and space  $a$  between holes is equal to their width  $b$ . According to the value of relative displacement  $x_1$  of the plate 3, holes 12 in both plates (2 and 3) may be fair or unfair. If holes are fair, then effective surface area of the fin is minimal  $S_{\min}$ , but if holes are unfair, then this area is maximal  $S_{\max}$ . Therefore it is possible to change an effective surface area  $S$  of the fin within a broad range of values by simple moving of plate 3 relative to plate 2.

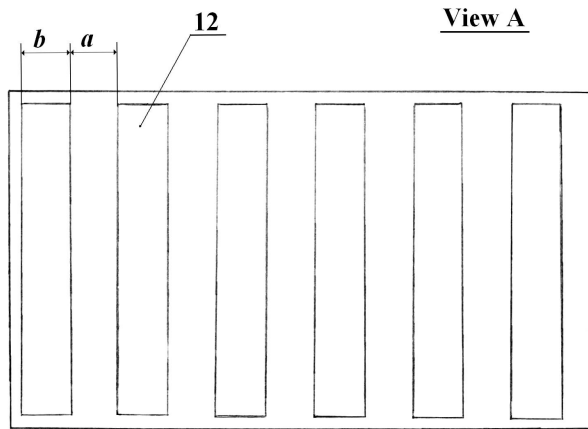


Fig. 9. Design variant of the outlet end of the fin

The proposed control system ensures a maximal value  $S_{\max}$  of the effective surface area in operation stages corresponding to fin motion from periphery till neutral position, which is fair with axis  $x$  (decreasing of turning angle  $\varphi$ , see Fig. 8). But in motion stages, when fin moves away from neutral position (increasing of turning angle  $\varphi$ , see Fig. 8), effective surface area of the fin is minimal  $S_{\min}$ .

The proposed method of tractive force forming [8] lies in making of fin from two plates 2 and 3, hinge joining of this sectional fin to robot fish body 1 and subsequent excitation of fin angular vibrations (see Fig. 8). Besides, effective surface area  $S$  of the sectional fin is adjusted in stepwise during vibrations, using mechatronic control system. In time moments, when the fin moves through neutral position  $Ox$  (turning angle  $\varphi = 0$ ), block 11 generates control signal switching-on electromagnet 7. Then electromagnet 7 retracts a rod 8 together with plate 3 connected to it rigidly, simultaneously spring 9 is compressed. As the result, plate 3 moves relative to plate 2 on specific value, holes in both plates become fair and due to this fin effective surface area takes a minimal value  $S_{\min}$ .

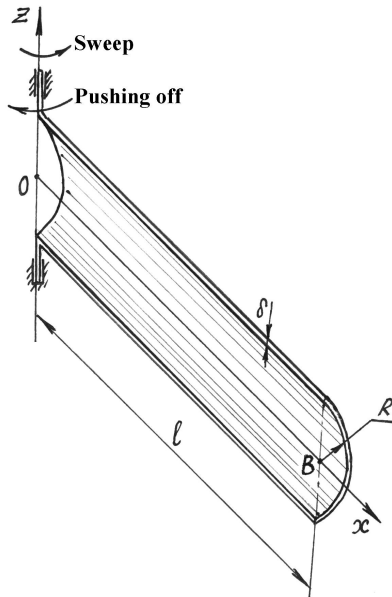
But in time moments corresponding to momentary stops of the fin (maximal values of turning angle  $\pm\varphi_{\max}$ ) block 11 generates control signal, which switches off electromagnet 7. Then spring 9 is extended, but plate 3 moves relative to plate 2 on specific value corresponding to unfair position of holes in both plates. As the result, fin effective surface area takes a maximal value  $S_{\max}$ .

Therefore the proposed method makes it possible to increase effective surface area  $S$  of the fin within motion cycles, when useful tractive force is formed (during decreasing of fin turning angle from  $\pm\varphi_{\max}$  till 0). But within cycles, when fin motion is hindered by water resistance forces (changing of turning angle from 0 till  $\pm\varphi_{\max}$ ), effective surface area  $S$  is decreased.

Thereby energy losses are minimized and operation of fin propulsive device becomes more effective.

### Robotic fish with two-tail propulsive device

The proposed idea for special variation of additional area of vibrating fin can be also used to increase efficiency of two-tail propulsive devices [2, 9]. Variation of working area of tail during operation of propulsive device can be achieved by making fin in the form of thin-walled semi-cylinder chute. Type design of such fin is illustrated in Fig. 10.



**Fig. 10.** Design variant of fin in the form of thin-walled semi-cylinder chute

Vibration propulsive device is equipped with two identical thin-walled semi-cylinder fins, which can rotate about vertical axis  $z$  (see Fig. 10). Besides, both fins are joined to propulsive device in such a manner that convex part of the chute is directed outside from the fore-and-aft axis of floating vehicle (in the direction of “sweep”). In accordance with the classification of mechanical objects [10], chute may be considered as thin-walled, if its geometrical parameters comply with conditions  $\delta \ll l$  and  $\delta \ll R$ , where  $\delta$  is fin thickness,  $l$  is fin length,  $R$  is a radius of fin medium surface.

In this case special variation of fin working surface can be achieved using effect of the loss of stability of rectilinear equilibrium form of both chutes during their motion [11]. Sequential positions of both fins during adjustment of this specific motion regime are shown in Fig. 11.

Two identical thin-walled semi-cylinder fins 2 and 3 are hinged to the back part of floating vehicle 1 (see Fig. 11, a, b). Both fins can rotate synchronously in opposite directions about fulcrum  $O$ . Hydrodynamic resistance force  $q_1(r, \omega)$ , which acts on convex surfaces of both fins during their motion from fore-and-aft axis  $y$  up to end positions (“sweep”), is as follows (see Fig. 11, a):

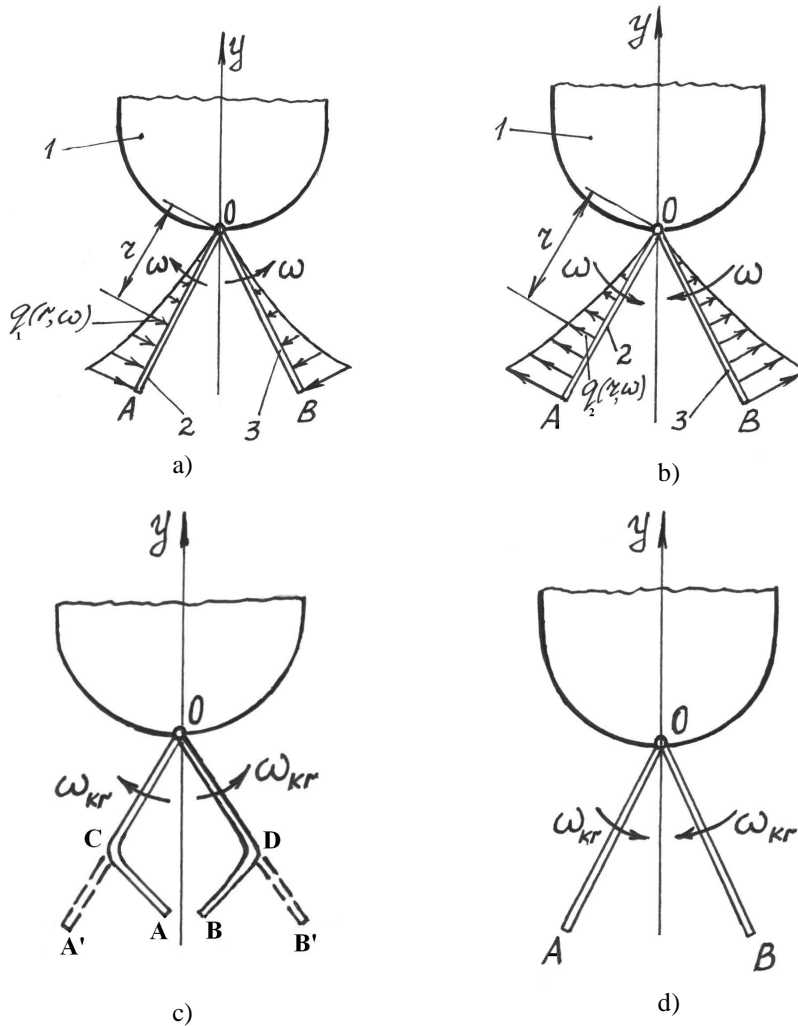
$$q_1(r, \omega) = k_1 \cdot r^2 \cdot \omega^2, \quad (5)$$

where  $k_1$  is a resistance coefficient of fluid flow going from fin convex side (“sweep”);  $r$  is fin cross-section coordinate measured from point O.

In fins’ opposite motion (“pushing off”) hydrodynamic resistance forces  $q_2(r, \omega)$  change their direction and act on concave surfaces of both chutes (see Fig. 11, b):

$$q_2(r, \omega) = k_2 \cdot r^2 \cdot \omega^2, \quad (6)$$

where  $k_2$  is a resistance coefficient of fluid flow going from fin concave side (“pushing off”).



**Fig. 11.** Sequential positions of fins during adjustment of the motion regime, which corresponds to the loss of stability of rectilinear equilibrium form of both chutes

Due to geometry of thin-walled semi-cylinder fin, coefficient  $k_1$  is always less than coefficient  $k_2$  [11]. Therefore braking force  $q_1$  in sweep is also always less than useful load  $q_2$  in pushing off.

In accordance with the equations (5) and (6), it is possible to change values of loads  $q_1$  and  $q_2$  by the variation of frequency  $\omega$  of fin angular oscillations. Besides, as it is known [11], thin-walled semi-cylinder chute can lose a stability of its initial rectilinear equilibrium form, if load  $q(r, \omega)$  exceeds certain critical value  $q_{kr}$  dependent on chute material and dimensions. As load  $q_1$  is always less than load  $q_2$ , then also  $q_{kr1} < q_{kr2}$ . Therefore by smooth variation of frequency  $\omega$  it is possible to realize a working regime with frequency  $\omega = \omega_{kr}$ , when load  $q_1$  is equal to its critical value  $q_{kr1}$  ( $q_1 = q_{kr1}$ ), but at the same time  $q_2 < q_{kr2}$ .

In Fig. 11 (a, b) fins are shown before critical condition (in the case of  $\omega < \omega_{kr}$ ), when both chutes maintain stability of rectilinear equilibrium form. Under the critical frequency  $\omega = \omega_{kr}$  both fins lose stability of its initial rectilinear equilibrium form during motion from fore-and-aft axis  $y$  of floating vehicle up to end positions (see Fig. 11, c). As the result, longitudinal axes of fins are flexed and take up positions ACO and BDO. Fluid flows around fins become almost parallel to their parts AC and BD, therefore summary braking loads on both fins in sweep are sufficiently reduced.

At the end of sweep stage both fins are straightened due to the action of internal elastic moment. During next stage ("pushing off") both fins are moved in straight form (see Fig. 11, d), because under  $\omega = \omega_{kr}$  load  $q_2$  is always less than its critical value  $q_{kr2}$ . In this case straight fins favor generation of useful tractive forces and therefore this fin position is more effective.

As the result of theoretical study, method for adjustment of operation condition of fin-type vibration propulsive device is proposed [12]. This method lies in equipping vibration propulsive device with two identical fins and making each fin in the form of thin-walled semi-cylinder chute. Fins are jointed to propulsive device in such manner that convex part of the chute is directed outside from the fore-and-aft axis of floating vehicle. Then floating vehicle is immersed into a liquid. The adjustment of operation condition of the propulsive device is carried out by gradual increasing of frequency of fin angular oscillations and by further observing equilibrium form of each fin under adjusted frequency values. Operation frequency of fins oscillations is taken for the motion regime, which corresponds to the loss of stability of rectilinear equilibrium form of both fins during their motion from fore-and-aft axis of floating vehicle up to the end positions. The proposed method ensures more effective operation of vibration propulsive device.

## Conclusions

Mathematical simulation was applied in order to determine optimal control law for variation of additional area of vibrating tail, which ensures maximal positive impulse of motive forces acting on the tail. The proposed method enables minimization of energy losses within operation cycles, when fin motion is hindered by water resistance forces. Due to such fin operation the propulsive device becomes more effective. Two examples of synthesis of propulsive devices with varying working area of vibrating tail are considered. The results of theoretical study are confirmed by experiments with robot fish models.

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