539. Investigation of vibrations of variable cross-section linking elements

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Abstract. The applicability of analytical models for vibrations of variable cross-section two-layered cylindrical structural elements is investigated. Natural frequencies of longitudinal and lateral vibrations are calculated and validated experimentally. Natural frequency $\omega_0^{(n)}$ dependence on material properties (ρ, E) and geometric parameters (l, R, r, S) of structural element provide means for optimization of vibration amplitude characteristics.

Keywords: vibrations, mechanical elements, natural frequencies.

Introduction

In a mechanical system (element, their components), resilient vibrations excited by a force $F(t)$ with an amplitude spread with velocity $v(t)$ to other parts of the structure [1]. Performing Fourier transformation of $F(t)$ and $v(t)$ and expressing $F(j\omega)/v(j\omega)= Z(j\omega)$ by a mechanical (see [2]) impedance we could mathematically render a relation in this mechanical system between the sound frequency vibrations and sound spreading intensity.

Based on the relationship between mechanical system impedance $Z(j\omega)$ and complex frequency characteristic function $H(j\omega)$, system transfer function is defined. Specifying [1, 2] vibro-acoustic characteristics of this system by parameter $\psi(\omega) = v_2(\omega)/v(\omega)$ and knowing the complex acoustic impedance $Z_a(\omega)$, the sound power of this mechanical system is:

$$P(\omega) = Z_a \cdot \psi(\omega) \cdot F(\omega)/Z_F(\omega). \qquad (1)$$

The prediction of spreading of mechanical vibrations through system links is viable when a possibility exists to foresee $\psi(\omega)$ transfer function (frequency characteristic of complex $H(\omega)$ or transient mechanical impedance, [2]). The relationship between $H(\omega)$ and $Z(\omega)$ can be expressed [5] as follows:

$$H(\omega) = \frac{1}{j\omega} \cdot \frac{1}{Z(\omega)}, \qquad (2)$$

where M is the system mass, $\omega=2\pi f$, f is the vibrations frequency, $\omega_0=\sqrt{K/M}$ is the natural mechanical system frequency, K is the resilience, $1/Q=C/\sqrt{KM}$ is the qualitative vibro-acoustic damping index, C is the vibro-acoustic damping (active resistance); $Z_F(\omega)$ is the complex (total mechanical resistance (impedance)) spectrum.

The spreading of mechanical vibrations from the source to outlet points of vibro-acoustic signals $\psi(\omega)$, $Z_F(\omega)$, structural elements damping C, mass – resilience $Z=j\omega M-k/j\omega$ were studied in practice using analytical and experimental methods. The advantage of the latter (two-channel high-speed Fourier transformation capabilities [6]) is more prominent in determining amplitude frequency characteristics of realistic mechanical systems or of structural elements [1, 3, 4, 5]. While assessing amplitude-frequency characteristics of structural elements to estimate and control spreading of vibrations even during design stage or structure upgrade, mathematical modeling is applied to the process of mechanical vibrations.

The research objective is development of mathematical apparatus for determination of natural frequencies of longitudinal and lateral vibrations of linear two-layer and variable cross-section elements using mathematical model for resilient oscillations of tubular structural elements.

Methodology and research results

In many cases, profiled structural elements (pipes, rods, etc.) used in complex mechanical system links can be ascribed to longitudinal elements (shafts, half-axles, cylinders, etc.). Process of longitudinal and transverse oscillations of such mechanical elements with resilience ρ_i, length l_i, solid or two-layer cross-section S was studied. Cases when one end of the longitudinal element is free or is under force were considered. In metals, vibro-acoustic damping is small. This is why changes of transmission function frequency characteristics are mostly influenced by the ratio of mass and resilient elements.

Fourier method is used for estimation of longitudinal natural frequencies of mechanical oscillations [4, 5]. Since axial stresses of the structural elements do not depend on time (t), thus at resonance frequencies (ω_0), when t=0, resilient mechanical oscillation amplitude $w(z)$ can be written as:

$$w(z,t)=w(z)=A\cos(k_0^n l)+B\sin(k_0^n l), \qquad (3)$$

where A and B are the Fourier transformation coefficients [4], k is the wave number [1, 4], k_0^n is the resonance wave number.

Due to forces affecting vibroconductors (P_F) the displacement amplitude of resilient layers according to [5] $w(z)=P/K=ES/l$ and the fundamental frequency of linear natural mechanical oscillations $\omega_0^{(n)}$ is calculated as follows:

$$\omega_0^{(1)}=\sqrt{ES/m}/l=\left(\frac{\pi}{l}\right)\sqrt{\frac{ES}{m}} \qquad (4)$$

The process of the transverse vibrations was studied also applying the laws of the elastic theory of mechanical bodies. In this case shifts of spots of an elastic body can be described by the Lam equations. Theoretical study was conducted at the same value initial conditions as the above mentioned exceptional vibration process. The composed cylindrical element transverse shift (deflection) amplitude equations, when $t=0$, were solved applying Fourier transformation and Bubnov-Galerkin methods where N_i are the shape functions of the finite element.

$$u_n(x,t) = \sum_{n=1}^{\infty} C_n \left(1 - cos \frac{\pi(2n+1)x}{2l} \right) sin(\omega_0^n t + \varphi_n), \quad (5a)$$

Where $\omega_0^n = \frac{\pi^2(2n+1)^2}{4l^2} \sqrt{\frac{EJ}{\rho S}} \sqrt{\frac{\pi(2n+1) - 4(-1)^n}{3\pi(2n+1) - 8(-1)^n}}$ are natural oscillation frequencies of a pipe, from which the fundamental frequency with <0,3 % error could be calculated as follows:

$$\omega_0^1 = \frac{9\pi^2 r}{8l^2} \sqrt{\frac{E}{\rho}} \sqrt{\frac{3\pi+4}{9\pi+8}}, \quad (5b)$$

To calculate transverse and longitudinal vibrations of variable cross-section parts or structure elements the basic equations of elasticity theory can be used for the mathematical model. Partial differential equations were developed to describe the process of vibration of mechanical elements with variable diameter [5]. Applying an approximate solution method to these equations, simple and applicable in practice formulas were obtained to calculate the natural frequencies of such parts.

The equation to calculate the fundamental natural oscillation frequency of a cone tube $\omega_0^{(1)}$ is as follows:

$$\omega_0^{(1)} = \frac{3\pi}{2l^2} \sqrt{\frac{E}{\rho A}} \times \sqrt{\frac{9\pi^2}{16} \left(\begin{array}{l} r_2^2 B - \frac{8k_1 r_1 l}{9\pi^2} - \\ - \frac{16 k_1 l^2}{27\pi^3} + \frac{k_1 l(r_1 + 2r_2)}{6} + \\ + \frac{k_1 l(r_1 + r_2)}{9\pi^2} \end{array} \right) + \\ + k_1 l (r_2 + k_1 l A) - 3B k_1^2 l^2}, \quad (6)$$

where $A = \frac{3}{2} + \frac{4}{3\pi}, B = \frac{1}{2} + \frac{2}{3\pi}, k_1 = \frac{r_1 - r_2}{l}$.

When $k_1 = 0$, then the conical pipe is replaced with a cylindrical pipe: $r_1 = r_2 = r$. Then (5b) equations are suitable to calculate $\omega_0^{(1)}$.

The obtained $\omega_0^{(1)}$ equations are applied while solving vibration spreading and $Z_1(j\omega)$ specified mechanical system (rod, pin, etc.) into $Z_2(j\omega)$ characterized by structural elements (e.g. intermediary mass – chock, etc.). The processes of resilient vibrations in a linkage of such mechanical elements are described by electromechanical analogy methods [4].

$$F_1 = F_2 \cos kl + jZ_e V_2 \sin kl, \quad (7a)$$

$$V_1 = F_2 / Z_c \sin kl + V_2 \cos kl, \quad (7b)$$

where $k=\omega/c$ is a wave propagation factor, m^{-1}.

In this case, $Zc = cm$ is the natural mechanical impedance or damping capacity of a mechanical element. At $\omega = \omega_0$, $Z_\varepsilon = QC$, where Q is the vibro-acoustic damping index. $c = \sqrt{ES/m}$ is the velocity of spreading of vibrations, where E, m and S are respectively, elastic modulus, mass per unit length and cross-section area of vibroconductive material.

Controlling each process of vibrations in the mechanical link, in which $\omega_0^{(n)}$ is predicted according to equation (5), the process of vibrations is determined by their characteristic parameters $Z_{c,i}$. The process of vibrations $\omega_0^{(n)}$ in the rod – mass system was studied with respect to control possibilities. The obtained equation of this system $tgkl = \frac{\rho l S}{M} \cdot \frac{l}{kl}$ is solved using a graphical method, as presented in Fig. 1 with respect to kl_0, projection of points by $tgkl$ and kl curves to the coordinate axis.

Frequency characteristics of amplitude of lateral vibrations is analyzed using equations (1, 7) for applicability of $\rho_1 \neq \rho_2$ of two layers to calculate $\omega_0^{(n)}$.

The requirement of lateral vibrations of a thick two-wall tube is that both layers are connected so that during deformation tangential stresses would not break the common structure. The equation of vibration of such a tube is analogous to equations (5), (7); where $A_1 = \sqrt{\frac{E_1 I_1 + E_2 I_2}{\rho_1 S_1 + \rho_2 S_2}}$, E_1, I_1, ρ_1, S_1 are parameters of the inner tube layer and E_2, I_2, ρ_2, S_2 are parameters of the outer layer.

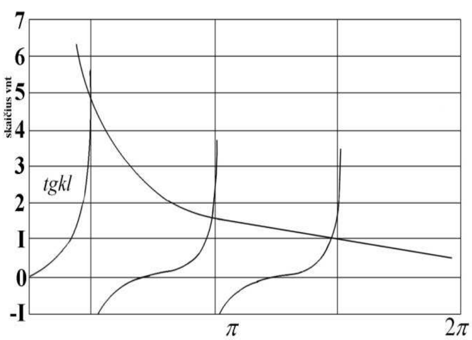


Fig. 1. Natural vibration frequencies ($k_0^{(1)}$) of the rod – mass system

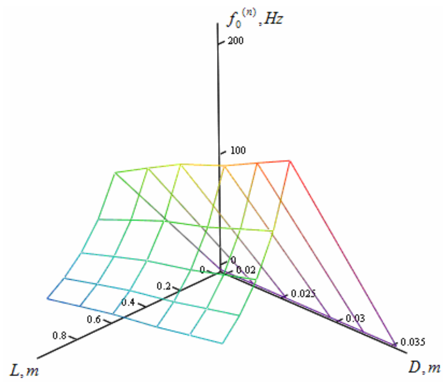
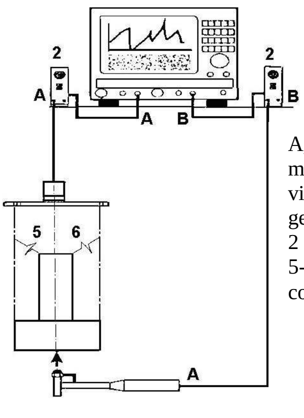


Fig. 2. Chart of natural frequency of two-wall metal – rubber tube

Using equation (6) and the corresponding values of coefficient A from equation (7), a matrix to calculate frequencies of lateral natural oscillations (Ω_0^n) of a two-wall tube element (a hollow rod) has been obtained and the results are presented in Figure 4. Comparative assessments of the results of $\omega_0^{(1)}$ for one-layer and two-layer tubes have been performed. For the two-layer longitudinal element, depending on the D, l values, the reduction of $\omega_0^{(1)}$ is significant. Natural frequency of lateral oscillation Ω_0^1 decreases. Furthermore, with respect to vibro-acoustic damping ($(Q >> 1)$, see equation (2)), this system has a better vibro-acoustic damping characteristics. The presented results demonstrate the advantage of conjugate smoothing over the procedure of conjugate approximation and the relationship of the required smoothing with the relative error norms is evident. This enables to propose various algorithms for the adaptive control of the smoothing parameter.

Results of calculation of natural oscillation frequencies of the mechanical system (see Fig. 2) were based on the experimental results of the analyzed mechanical systems transfer functions $j\omega \cdot Z(\omega)$ and vibro-mobility and transient mechanical impedance $Z_{12}(j\omega)$.

Complete information on the mechanical system and amplitude frequency characteristics of the elements can be obtained from experiments for determining the active and reactive components, of the mechanical impedance module $|Z|$ and phase angle (φ) between the vectors of force (F) and velocity (V). The designated or transient mechanical impedance module $|Z_i|$ and the phase angle φ_i values depend on the changes of the excitation frequency f. The measurements of these parameters were obtained using special instrumentation, where the vibro-acoustic signals were analyzed by the First Fourier Transform (FFT), see Fig. 3. The mechanical system is excited either by impulse or periodic force and response vibrations are measured by accelerometer (impedance hammer type 8202 and the vibration sensors type 8200 from Brüel&Kjær were used). The signal analysis was carried out using two-channel spectrum analyzer type 2034. Experimental results are given in Fig. 4.



A, B are the electro-mechanical links of vibration velocity and generation force;
2 is the load amplifier;
5-6 are the elastic construction hanger

Fig. 3. Experimental set-up for measurement of mechanical impedance

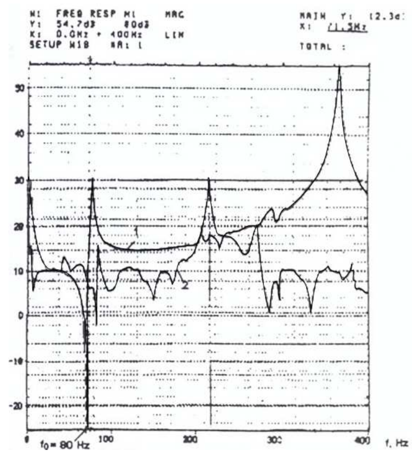


Fig. 4. Frequency characteristic of cross mechanical permeability of the single-ply steel tube (ply thickness h=2 mm, length l=0.8, thickness D-3/4): 1 – straight tube, 2 – tube with curvature radius R=0.8 m

Transient mechanical impedance is significantly reduced when f=74,56 Hz. This is the frequency of the first harmonic $f_0^{(1)}$. The difference between calculated and measured values is 7,24 %.

Conclusions

Mathematical model of vibrations of cylindrical elements has been developed. The dependence of natural frequencies of mechanical links on geometric parameters and material properties has been demonstrated. It can be used to optimize vibro-acoustic characteristics of machine parts with respect to excitation forces.

Obtained equations for computation of natural frequencies of longitudinal and lateral vibrations of cylindrical hollow rods are less complicated than the transcendental equations, which are used in classical methods for calculation of natural frequencies of rods and tubes. Therefore they could have wider possibilities for practical application.

Using equations and matrixes for calculation of natural vibration frequencies, dependencies $f_0^n = F(\varsigma;\rho;h;h_2;l;D_2)$, have been established, which are beneficial in design and adjustment work.

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