# 537. Vibrations generator with a motion converter based on permanent magnet interaction

**K. Ragulskis[1], K. Kanapeckas[2], R. Jonušas[3], K. Juzėnas[4]**
Kaunas University of Technology,
Kęstučio str. 27, 44025 Kaunas, Lithuania
**E-mail:** [1] *k.ragulskis@jve.lt;* [2] *kestutis.kanapeckas@stud.ktu.lt*
[3]*remigijus.jonusas@ktu.lt;* [4]*kazimieras.juzenas@ktu.lt*

*(Received 26 December 2009; accepted 03 March 2010)*

**Abstract.** The paper deals with a new type of vibrations exciters whose operation is based on the forces developed by permanent magnets interacting between input and output members. Classification and some diagrams of vibrations exciters are presented. Mathematical models of one-dimensional vibrations exciters are set up. In cases when mean velocities of the magnets of input member and output member are equal and when velocity of an output member magnet is lower than that of an input member, dynamics is studied by means of approximate analytic methods. More complicated cases are studied by means of numerical methods. Estimated dynamic characteristics of the system, its established peculiarities and properties make it possible to apply the mentioned exciters to some technological processes.

**Keywords:** magnetic vibrations exciters, dynamic characteristics, stability.

## 1. Introduction

One of two-coordinate mechanical vibrations generators (MVG) in which the vibrations process is excited by means of permanent magnets is being dealt with.

MVG (Fig.1) consists of input member 1, output member 2, and two elastic couplings 3 and 4. The latter restrict the motion of output member 2 along $\alpha$ and $y$ coordinates. Permanent magnets fixed on input and output members at a uniform step lie on parallel surfaces. When the magnets are shifted the static equilibrium state of input member 1 is along axis OY at distance L.

Input member 1 can turn at angle $\varphi$, while an output member can do it at angle $\alpha$ and may move linearly in the direction of axis OY.

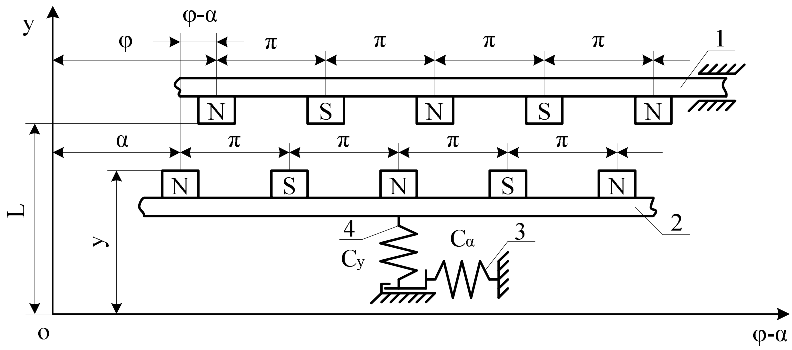



**Fig. 1.** Schematic diagram of MVG based on interaction of permanent magnets

Differential equations of the system (Fig.1) are the following:

$$I_\varphi \ddot{\varphi} - f_{\varphi-\alpha}(L-y)M_m = M_\varphi,$$
$$I_\alpha \ddot{\alpha} + f_{\varphi-\alpha}(L-y)M_m + C_\alpha \alpha + H_\alpha \dot{\alpha} = M_\alpha,$$
$$m\ddot{y} - f_y(L-y)F_m + C_y y + H_y \dot{y} = F_y, \quad (1)$$

here $I_\varphi$ and $I_\alpha$ - moments of inertia of input 1 and output 2 members, respectively;

L – distance from the origin of coordinates to the input member magnets in the OY axis direction;

y – distance from the origin of coordinates to the output member magnets in the OY axis direction;

$f_{\varphi-\alpha}(L-y)M_m$ - moment of magnetic forces generated between members 1 and 2 (Fig.2);

$f_y(L-y)F_m$ - magnetic force acting member 2 along coordinate y (Fig. 2);

$f_{\varphi-\alpha}(L-y)$, $f_y(L-y)$ - dimensionless functions describing variation of the distance between magnets (Fig. 3);

$F_y$ - external force acting member 2 along coordinate y;

α, φ, y - generalized coordinates;

m - mass of member 2;

$M_\varphi$, $M_\alpha$ - external moments acting members 1 and 2, respectively;

$C_\alpha$ and $C_y$ - factors of stiffness of elastic couplings 3 and 4, respectively;

$H_\alpha$ and $H_y$ - coefficients of external friction [1].

When $y < L$, functions $f_{\varphi-\alpha}(L-y)$ and $f_y(L-y)$ may be approximated by segments of a line:

$$f_{\varphi-\alpha}(L-y) = a_\varphi^* + b_\varphi^* y,$$
$$f_y(L-y) = a_y^* + b_y^* y, \quad (2)$$

where $a_\varphi^*, b_\varphi^*, a_y^*, b_y^*$ are constant values depending on L.

We assume approximately that:

$$M_m = M_{ma} \sin n(\varphi - \alpha),$$
$$F_m = F_{ma} \cos n(\varphi - \alpha), \quad (3)$$

here $F_{ma}$ and $M_{ma}$ - amplitudes of magnetic forces and the moments, respectively.

Having evaluated (2) and (3) we can approximate:

$$f_{\varphi-\alpha}(L-y)M_m = (a_\varphi + b_\varphi y)\sin n(\varphi - \alpha),$$
$$f_y(L-y)F_m = (a_y + b_y y)\cos n(\varphi - \alpha). \quad (4)$$

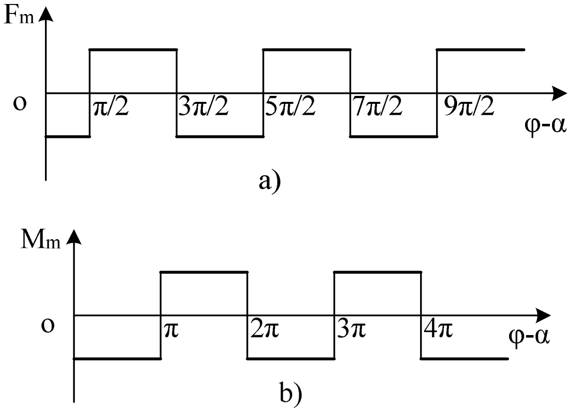



**Fig. 2.** Graphs of variation of magnetic forces (a) and the moments (b)

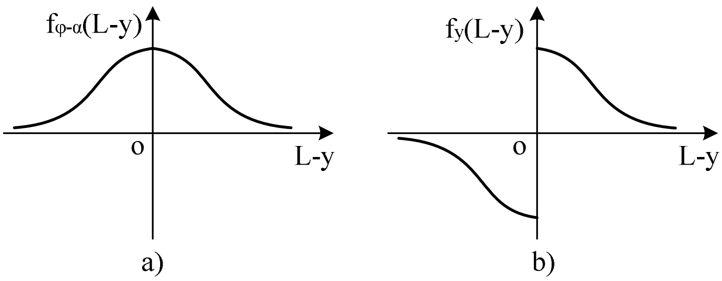



**Fig. 3.** Graphs of variation of functions $f_{\varphi-\alpha}(L-y)$ and $f_y(L-y)$ [2]

Having evaluated (4), the following form of equations (1) is obtained:

$$\ddot{\varphi} - (a_{\varphi*} + b_{\varphi*} y)\sin n(\varphi - \alpha) = M_\varphi^*,$$
$$\ddot{\alpha} + p_\alpha^2 \alpha + h_\alpha \dot{\alpha} + (a_\varphi + b_\varphi y)\sin n(\varphi - \alpha) = M_\alpha^*,$$
$$\ddot{y} + p_y^2 y + h_y \dot{y} - (a_y + b_y y)\cos n(\varphi - \alpha) = F_y^*, \tag{5}$$

$$a_{\varphi*} = \frac{a_\varphi^*}{I_\varphi} M_{ma}; \quad b_{\varphi*} = \frac{b_\varphi^*}{I_\varphi} M_{ma}; \quad a_\varphi = \frac{a_\varphi^*}{I_\alpha} M_{ma};$$
$$b_\varphi = \frac{b_\varphi^*}{I_\alpha} M_{ma}; \quad a_y = \frac{a_y^*}{m}\left|F_m\right|; \quad b_y = \frac{b_y^*}{m}\left|F_m\right|,$$
$$h_\alpha = \frac{H_\alpha}{I_\alpha}; \quad h_y = \frac{H_y}{m}; \quad P_\alpha^2 = \frac{C_\alpha}{I_\alpha}; \quad P_y^2 = \frac{C_y}{m},$$
$$M_\varphi^* = \frac{M_\varphi}{I_\varphi}, \quad M_\alpha^* = \frac{M_\alpha}{I_\alpha}, \quad F_y^* = \frac{F_y}{m} \tag{6}$$

where a, b, n, p, h are constants.

**Case 1:**

$$\varphi = \omega t\,,\ \ y = 0\,, \tag{7}$$

and a differential equation of motion is:

$$\ddot{\alpha} + p_\alpha^2 \alpha + h_\alpha \dot{\alpha} + a_\varphi \sin n(\omega t - \alpha) = M_\alpha^*\,. \tag{8}$$

The steady regime is sought approximately by an analytic method [3].

When damping is high and amplitudes of the steady statevibrations is low, then:

$$\alpha \approx \overline{\alpha} + \tilde{\alpha}\,, \tag{9}$$

where $\overline{\alpha}$ and $\tilde{\alpha}$ are slowly and rapidly varying quantities, respectively.

From (8, 9) a linear differential equation is set up for rapidly varying $\tilde{\alpha}$ , assuming that $\overline{\alpha}$ is a constant:

$$\ddot{\tilde{\alpha}} + p_\alpha^2 \tilde{\alpha} + h_\alpha \dot{\tilde{\alpha}} + a_\varphi \sin n(\omega t - \overline{\alpha}) = 0\,. \tag{10}$$

Equation (10) yields:

$$\tilde{\alpha} = \frac{a_\varphi}{[p_\alpha^2 - (n\omega)^2]^2 + (n\omega h_\alpha)^2}[-(p_\alpha^2 - n^2\omega^2)^2 \sin n(\omega t - \overline{\alpha}) + n\omega h_\alpha \cos n(\omega t - \overline{\alpha})]\cdot \tag{11}$$

Referring to (8, 9) a differential equation of a slow motion is the following:

$$\ddot{\overline{\alpha}} + p_\alpha^2 \overline{\alpha} + h_\alpha \dot{\overline{\alpha}} + a_\varphi \overline{\sin n(\omega t - \overline{\alpha} - \tilde{\alpha})} = \overline{M}_\alpha^*\,, \tag{12}$$

where a bar over a sinusoidal represents the averaging according to *t*.

Restricting to the straight-line, expanding $\sin n(\omega t - \overline{\alpha} - \tilde{\alpha})$ by a power series it is obtained:

$$\ddot{\overline{\alpha}} + p_\alpha^2 \overline{\alpha} + h_\alpha \dot{\overline{\alpha}} - \frac{a_\varphi^2 n\omega h_\alpha}{2\{[p_\alpha^2 - (n\omega)^2]^2 + (n\omega h)^2\}} = \overline{M}_\alpha^*\,, \tag{13}$$

where under the steady regime:

$$\overline{\alpha} = \frac{1}{2}\left(\frac{a_\varphi}{p_\alpha}\right)\frac{n\omega h_\alpha}{[p_\alpha^2 - (n\omega)^2]^2 + (n\omega h)^2} + \frac{\overline{M}_\alpha^*}{p_\alpha^2}\,. \tag{14}$$

It implies that (14) is shifted approximately along angle $\alpha$ along axis O$\alpha$ in a positive direction by an average constant $\overline{\alpha} > 0$, i.e. along the sense of rotation of angle $\varphi$ and it vibrates harmonically $\tilde{\alpha}$ around that state.

Let us examine how the analyzed vibrating systems change, how the state of (8) and (15) is coordinated in the phase plane when:

$$h_\alpha = 0.1;\ p_\alpha^2 = 0.5;\ a_\varphi = 0.5.$$

$$\ddot{\alpha} + p_\alpha^2 \alpha + h_\alpha \dot{\alpha} = a_\varphi \sin(\omega t - \alpha) \tag{15}$$

Fig. 4 presents typical phase trajectories of the motion of system (15) at different values of $\omega$.

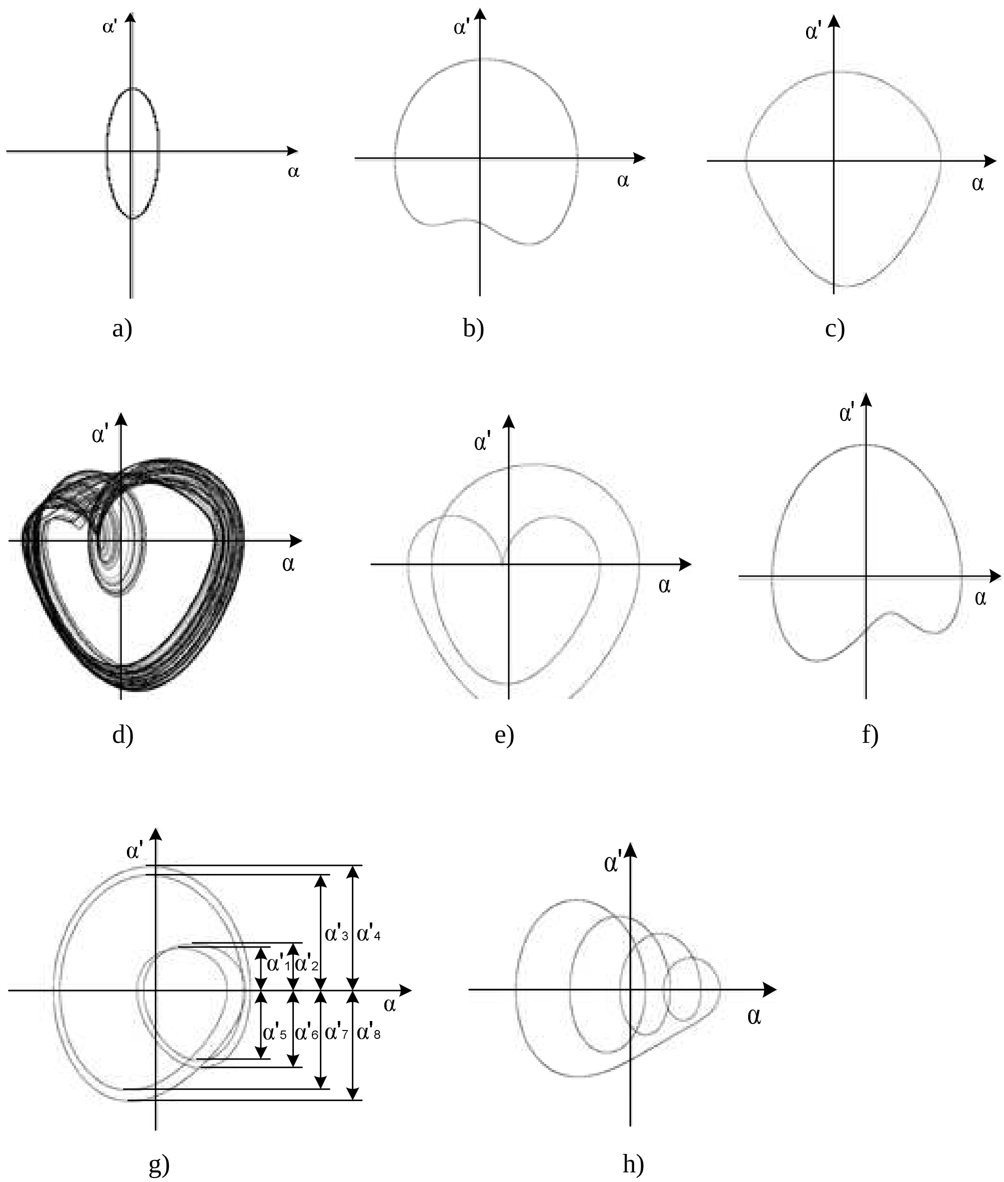



**Fig. 4.** Phase trajectories of the motion of system (15): a) $\omega = 2$; b) $\omega = 1.6$; c) $\omega = 1.2$; d) $\omega = 1$; e) $\omega = 0.8$; f) $\omega = 0.6$; g) $\omega = 0.4$; h) $\omega = 0.2$

It is evident that changing $\omega$ single-harmonic (Fig. 4 a), poly-harmonic (Fig. 4 b, c, e, f. g, h) and chaotic Fig. 4 d) regimes settle down.

Puankare diagram (Fig. 5) illustrates a general state of system (15).

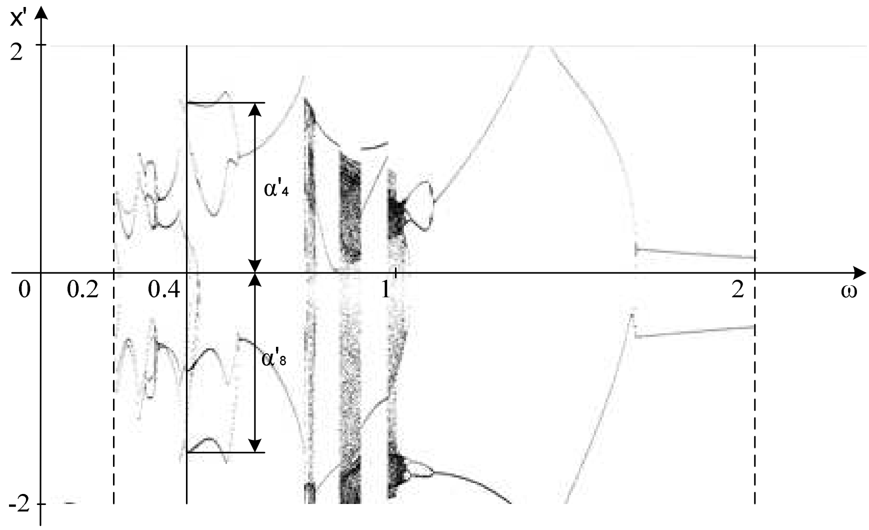



**Fig. 5.** Puankare diagram where $\alpha_4'$ and $\alpha_8'$ correspond to system's frequency $\omega = 0.4$ (Fig. 4 (g))

**Case 2:**

$$\varphi = \omega t\,,\ y \neq 0 \tag{16}$$

When damping is high and amplitudes of the steady statevibrations is low, then equations (5, 6) are examined jointly.

Analogically, (9) is assumed as:

$$\alpha = \overline{\alpha} + \tilde{\alpha}\,,\ y = \overline{y} + \tilde{y}\,. \tag{17}$$

Having evaluated (17), equations (5, 6) acquire the following forms:

$$\begin{aligned} &\ddot{\tilde{\alpha}} + p_\alpha^2 \tilde{\alpha} + h_\alpha \dot{\tilde{\alpha}} + (a_\varphi + b_\varphi \overline{y}) \sin n(\omega t - \overline{\alpha}) = 0\,, \\ &\ddot{\tilde{y}} + p_y^2 \tilde{y} + h_y \dot{\tilde{y}} - (a_y + b_y y) \cos n(\omega t - \overline{\alpha}) = 0\,, \end{aligned} \tag{18}$$

$$\begin{aligned} &\ddot{\overline{\alpha}} + p_\alpha^2 \overline{\alpha} + h_\alpha \dot{\overline{\alpha}} + \overline{[a_\varphi + b_\varphi(\overline{y} + \tilde{y})] \sin n(\omega t - \overline{\alpha} - \tilde{\alpha})} = \overline{M}_\alpha^*\,, \\ &\ddot{\overline{y}} + p_y^2 \overline{y} + h_y \dot{\overline{y}} - [a_y + b_y(\overline{y} + \tilde{y})] \cos n(\omega t - \overline{\alpha} - \tilde{\alpha}) = \overline{F}_y^*\,. \end{aligned} \tag{19}$$

In the case of a rapid motion equations (18) yield:

$$\tilde{\alpha} = \frac{a_\varphi + b_\varphi \bar{y}}{D_\alpha}[n\omega h_\alpha \cos n(\omega t - \bar{\alpha}) - \Delta_\alpha \sin n(\omega t - \bar{\alpha})],$$
$$\tilde{y} = \frac{a_y + b_y \bar{y}}{D_y}[\Delta_y \cos n(\omega t - \bar{\alpha}) + n\omega h_y \sin n(\omega t - \bar{\alpha})], \quad (20)$$

where $D_\alpha = (p_\alpha^2 - n^2\omega^2)^2 + (n\omega h_\alpha)^2$, $D_y = (p_y^2 - n^2\omega^2)^2 + (n\omega h_y)^2$,
$$\Delta_\alpha = p_\alpha^2 - n^2\omega^2, \ \Delta_y = p_y^2 - n^2\omega^2. \quad (21)$$

In equations (19) confining to the linear terms of a power series according to $\tilde{\alpha}$ expansion, after averaging it is obtained:

$$\bar{\alpha} = \frac{1}{2p_\alpha^2}[-\frac{(a_\varphi + b_\varphi \bar{y})^2 n^2 \omega h_\alpha}{D_\alpha} + \frac{b_y(a_y + b_y \bar{y})\Delta_\alpha}{D_y}],$$
$$\bar{y} = \frac{1}{2p_y^2}[-(a_\varphi + b_\varphi \bar{y})(a_y + b_y \bar{y})\frac{n\Delta_\alpha}{D_\alpha} + b_y(a_y + b_y \bar{y})\frac{n\omega h_\alpha}{D_y}]. \quad (22)$$

The obtained expressions (20) and (22) indicate that along the coordinates α and y motions consist of constant $\bar{\alpha}, \bar{y}$ and varying $\tilde{\alpha}, \tilde{y}$ parts.

## 2. Experimental research

Experiments have been made on the stand (Fig. 1) whose input member 1 and output member 2 carry 8 permanent magnets each. Magnet poles N and S are laid alternately, i.e. uniform poles are in 4 ($n = 4$). Coercive force of magnets is $P = 37.3N$ [4]. The moment of inertia of an input link is $I_\alpha = 0.0016kgm^2$, that of the output link $I_\varphi = 0.0058kgm^2$, output link mass $m = 2.2kg$. The space between the magnets fixed on the input and output links is $\Delta = 2mm$. Stiffness coefficients of elastic couplings are $C_y = 451\frac{kN}{m}$; $C_\alpha = 4.7\frac{kN}{m}$.

Experimental research has been carried out by varying the position between input and output links along their common rotating axis, i.e. varying the overlapping degree of input and output members and varying rotation frequency $\omega$ of an input member [5].

Some experimental research results are given in Fig. 6.

It is quite evident that harmonic is prevailing, its frequency is $\frac{n\omega}{2}$ ($n = 4$ – number of uniform magnet poles) i.e. its frequency is twofold lower than that of the excitation. It indicates that parametric vibrations take place in the system. These vibrations are described by equation of Mathieu which may be derived from (8) analyzing its homogeneous expression and underestimating external damping [6]:

$$\alpha'' + (u - 2q\cos 2\tau)\alpha = 0, \quad (23)$$

here $\alpha' = \frac{d\alpha}{d\tau}$; $\tau = \frac{n\omega t}{2}$; u, q – constant parameters of the vibrating system.

$$u = \left(\frac{2p_\alpha}{n\omega}\right)^2; \; q = \frac{2a_\varphi}{n\omega^2}.$$

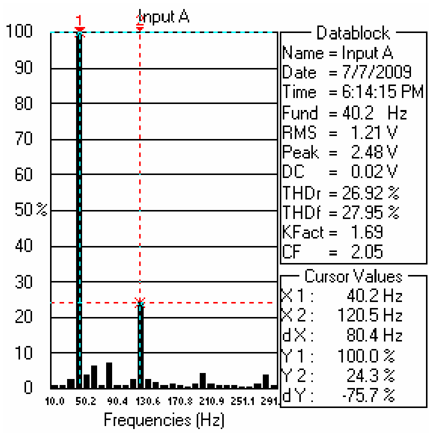
Dissemination of the linerized part of eq. (8) by the gradual series according to $\alpha$ , gives:

$$\ddot{\alpha} + p_\alpha^2 \alpha + h_\alpha \dot{\alpha} + a_\varphi \sin n\omega t - a_\varphi n\alpha \cos n\omega t = M_\alpha^* \tag{24}$$

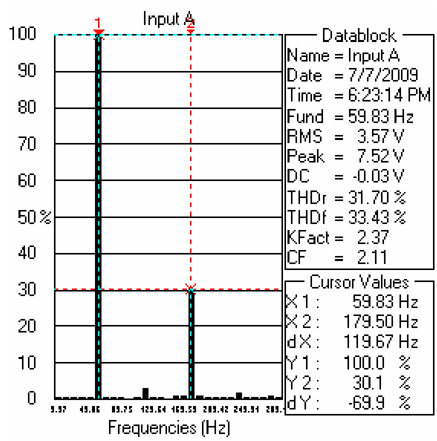
The main homogeneous equation of the steady state:

$$\ddot{\alpha} + (p_\alpha^2 + a_\varphi n \cos n\omega t)\alpha = 0 \tag{25}$$

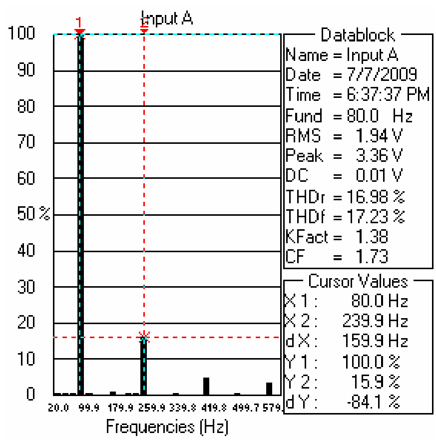
here $\ddot{\alpha} = \left(\frac{2}{n\omega}\right)^2 t\alpha''$.




a)




b)




c)

**Fig. 6.** Frequency spectrum of output member rotating vibrations when a) $\omega = 20Hz$ ; b) $\omega = 30Hz$ ; c) $\omega = 40Hz$

Then stability of such a system depends on the values of parameters $u$ and $q$ and it is expressed by Ince-Streett diagram. When the values of parameter $q$ are moderate, stability conditions may be expressed in the following way:

$$1+q<b<1-q \tag{26}$$

With an increase in $\omega$ both parameters $u$ and $q$ decline. Ratio of both parameters $k=\frac{q}{u}=\frac{na_\varphi}{2p_\alpha^2}$ remains constant. The state of the system (either stable or unstable) depends on the position of the point on line $q=ku$ because with an increase in $\omega$ the stable and unstable zones replace each other in series [7].

## 3. Conclusions

This research has indicated that mechanical vibrations exciters based on the interaction of permanent magnets can be applied to excitation of various regimes, such as:

- harmonic,
- poly-harmonic,
- chaotic.

In the system of constant parameters the vibrations regimes are changed by changing the system excitation frequency $\omega$.

When damping is high and amplitudes of the steady statevibrations is low, equation (14) is shifted approximately along angle $\alpha$ along axis O$\alpha$ in a positive direction by an average constant $\bar{\alpha}>0$, i.e. along the sense of rotation of angle $\varphi$ and it vibrates harmonically $\tilde{\alpha}$ around that state.

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