515. Determination of eigenfunction and frequency response function of constrained dynamic system

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Abstract. Structural modification needs to prescribe some structural frequencies and mode shapes. This study derives the eigenfuction and frequency response function matrix of constrained dynamic system based on measured test data. The modified eigenfunction is derived by utilizing the measured modal data of the actual system as constraints to govern a part of the behavior of modified system and minimizing cost functions of the difference between analytical and corrected parameter matrices with them. It is shown that the modified eigenfunction incorporates the modified parameter matrices. The frequency response function matrix modified by measured constraints is also derived by minimizing a cost function of the dynamic strain energy to be expressed by dynamic stiffness matrix and the difference between analytical and measured modal displacements. The validity of the proposed methods is demonstrated in applications.

Keywords: dynamic strain energy, constraint, eigenfunction, frequency response function, modal parameter

1. Introduction

Modal data are extremely useful information that can assist in the design of almost any structure. The visualization of mode shapes is invaluable in the design process and the development of a modal model is useful for simulation and design studies.

Structural modification is a procedure aimed at identifying the changes required in a structural system to modify its dynamic behaviour (natural frequencies, structural modes, frequency response) in the desired direction. Each step in structural design process requires the analysis of a modified structure that is often only slightly different from a structure previously analyzed. This complete reanalysis of the structure may be an expensive and time-consuming task, and make the detailed refinement of the proposed structure difficult.

Enhancement of the structural response is one of the common goals of structural modification processes. Vibration is becoming increasingly important in the design of mechanical and structural systems. The change of the structural behavior to alleviate vibration problems gives rise to the structural modification problem. The problem of determining the structural modification needed to prescribe some natural frequencies and mode shapes is considered. The structural dynamic modification techniques attempt to reduce dynamic design time and can be implemented beginning with spatial models of structures, dynamic test data or updated models. The mathematical models are extracted from dynamic test data viz. frequency response function (FRF).

The structural modification is usually the direct problem and the inverse problem. The direct problem consists in determining the effect of already established modifications. This is a verification problem aimed at establishing the efficiency of given changes on the dynamic behaviour of the considered system. The solution of the inverse problem can not be unique. The inverse problem tries to identify the most appropriate changes required to obtain the desired dynamic behaviour. System identification and damage detection methods belong to this category.

Baldwin and Hutton [1] provided a detailed review of structural dynamic modification techniques. Sesteri [2] considered the direct problem of determining the new response of a system after some modifications are introduced into the system based on the modal database and the FRF database. Kundra [3] discussed the structural modification methods for getting desired dynamic characteristics by using modifiers namely mass, beams and tuned absorbers. Ram [4] discussed two methods for determining the damped natural frequencies of a viscously damped system, which is changed by structural modification based on transfer function, and eigenvalues and mode shapes. Braun and Ram [5] discussed the effect of modal truncation on structural and modal modification, and the impossibility of obtaining an exact solution for structural modification when using an incomplete set of eigenvectors.

Minimizing a residual matrix norm based on the Rayleigh-Ritz approximation, Ram et. al [6] proposed an analytical method for the approximation of a modified structure eigensystem to have only an incomplete set of modes and frequencies of the original model, and the amount of modification in the mass and stiffness matrices.

This study presents the analytical methods to determine eigenfunction and FRF matrix of modified dynamic systems without carrying out complete reanalysis based on the measured test data as constraints. The modal parameters and FRF matrix are derived by minimizing the cost functions mentioned by the past researchers. It is shown that the modified eigenfunction is expressed by the modified parameter matrices such as mass or stiffness matrix. The validity of the proposed methods is illustrated in applications.

2. Modal Parameters of Locally Modified System

Measured and analytical data are unlikely to be equal due to measurement noise and model inadequacies, and damages. If updated model exactly reproduces inaccurate measurements any subsequent analysis may be flawed. Assuming that experimental data are accurate, this involves comparing the experimental data and the model predictions. Boundary conditions, geometry, material properties, and local damages are the parameters that can have a large effect on the responses predicted by finite element model. These parameters are subjected to uncertainties, which lead to errors in the model.

Modal parameters should be evaluated due to localized mass and stiffness changes, and be corrected to obtain exact dynamic characteristics deviated by measurement noise, model inadequacies, and local damages. The dynamic behaviour of a structure which is assumed to be linear and approximately discretized for n degrees of freedom can be described by the equations of motion

$$\mathbf{M}_a\ddot{\mathbf{u}} + \mathbf{C}\dot{\mathbf{u}} + \mathbf{K}_a\mathbf{u} = \hat{\mathbf{F}}(t) \tag{1}$$

where $\mathbf{M}_a$ and $\mathbf{K}_a$ denote the $n \times n$ analytical mass and stiffness matrices, $\mathbf{u} = [u_1 \quad u_2 \quad \cdots \quad u_n]^T$, and $\mathbf{C} \in R^{n\times n}$ is the damping matrix. And $\hat{\mathbf{F}}(t)$ is the $n \times 1$ load excitation vector. Without loss of generality, Rayleigh damping is adopted as

$$\mathbf{C} = \alpha\mathbf{M}_a + \beta\mathbf{K}_a \tag{2}$$

where α and β are the two proportionality constants which can be related to the damping ratios of the first and second natural modes. Assuming the system is lightly damped and the free vibration, the dynamic equation of Eqn. (1) becomes

$$\mathbf{M}_a\ddot{\mathbf{u}}+\mathbf{K}_a\mathbf{u}=\mathbf{0} \tag{3}$$

Assume the displacement in exponential form as

$$\mathbf{u}=\hat{\mathbf{U}}e^{j\omega t} \tag{4}$$

where $\hat{\mathbf{U}}$ denotes the modal coordinate vector, ω is the natural frequency, and $j=\sqrt{-1}$. Substitution of Eqn. (4) into Eqn. (3) with $e^{j\omega t}\neq 0$ yields the equation

$$\left(\mathbf{K}_a-\omega_i^2\mathbf{M}_a\right)\hat{\mathbf{U}}=\mathbf{0}, \qquad i=1,\quad 2,\quad \cdots,\quad n \tag{5}$$

The desired modal data are required to be modified to fulfill the eigenvalue equation using modal test data. In order to establish the relation between the analytical and corrected mass matrices under the assumption that the mass is not a function of time *t*, this study utilized the cost function of Berman and Nagy [7] by

$$J=\frac{1}{2}\left\|\mathbf{M}_a^{-1/2}\left(\mathbf{M}-\mathbf{M}_a\right)\mathbf{M}_a^{-1/2}\right\| \tag{6}$$

where **M** is an $n\times n$ corrected mass matrix. Assuming that the modal data of the dynamic system are measured at several degrees of freedom, the measured modal data become constraints to describe a part of full modal data of the system

$$\mathbf{AU}=\mathbf{0} \tag{7}$$

where **A** is an $m\times n$ $(m<n)$ matrix and $\mathbf{U}$ denotes an $n\times 1$ actual modal coordinate vector. The corrected eigenfunction is obtained by minimizing the cost function of Eqn. (6) subjected to the constraints of Eqn. (7).

In order to insert Eqn. (7) into Eqn. (6), the constraint equations of Eqn. (7) are modified as

$$\mathbf{SM}_a^{1/2}\mathbf{U}=\mathbf{0} \tag{8}$$

where $\mathbf{S}=\mathbf{AM}_a^{-1/2}$. Solving Eqn. (8) with respect to $\mathbf{M}_a^{1/2}\mathbf{U}$, it is derived as

$$\mathbf{M}_a^{1/2}\mathbf{U}=\left[\mathbf{I}-\mathbf{S}^{+}\mathbf{S}\right]\mathbf{d} \tag{9}$$

where **d** is an arbitrary vector and '+' denotes the Moore-Penrose inverse.

The analytical mass matrix $\mathbf{M}_a$, the natural frequency ω_i, and the corresponding mode shape vector $\hat{\mathbf{U}}$ in Eqn. (5) for updated dynamic system are replaced by the corrected mass matrix $\mathbf{M}$, natural frequency ω_i^*, and mode shape vector $\mathbf{U}$ due to the measurement and modeling errors, and local damages, respectively.

$$\left(\mathbf{K}_a-\omega^{*2}_i\mathbf{M}\right)\mathbf{U}=\mathbf{0} \tag{10}$$

Equation (10) can be modified as

$$\mathbf{M}^{1/2}\mathbf{U} = \omega_i^{*-2}\mathbf{M}^{-1/2}\mathbf{K}_a\mathbf{U} \tag{11}$$

Applying the condition to minimize the cost function of Eqn. (6), we obtain from Eqns. (9) and (11) that

$$\left[\mathbf{I} - \mathbf{S}^+\mathbf{S}\right]\mathbf{d} = \omega_i^{*-2}\mathbf{M}_a^{-1/2}\mathbf{K}_a\mathbf{U} \tag{12}$$

Solving Eqn. (12) with respect to the arbitrary vector **d**, it follows that

$$\mathbf{d} = \left[\mathbf{I} - \mathbf{S}^+\mathbf{S}\right]\omega_i^{*-2}\mathbf{M}_a^{-1/2}\mathbf{K}_a\mathbf{U} + \mathbf{S}^+\mathbf{S}\mathbf{v} \tag{13}$$

where **v** denotes an arbitrary vector. Substituting Eqn. (13) into Eqn. (9) and arranging the result, the constrained eigenfunction can be written as

$$\left[\mathbf{K}^* - \omega_i^{*2}\mathbf{M}_a\right]\mathbf{U} = \mathbf{0} \tag{14}$$

where $\mathbf{K}^* = \left[\mathbf{I} - \mathbf{M}_a^{1/2}\left(\mathbf{A}\mathbf{M}_a^{-1/2}\right)^+\mathbf{A}\mathbf{M}_a^{-1}\right]\mathbf{K}_a$. Equation (14) represents the eigenfunction to be constrained by the measured modal displacements due to uncertainties or local damages. From Eqn. (14), it is observed that the deviation of the dynamic responses of the intact system due to the uncertainties or the local damages can be incorporated in the corrected stiffness matrix.

By the similar approach, the eigenfunction modified by constraints of measured test data can be obtained by replacing the cost function of Eqn. (6) by a different cost function [8] of

$$J = \frac{1}{2}\left\|\mathbf{K}_a^{-1/2}\left(\mathbf{K} - \mathbf{K}_a\right)\mathbf{K}_a^{-1/2}\right\| \tag{15}$$

where **K** denotes the corrected stiffness matrix. Assuming that the motion of the system of Eqn. (3) is constrained by Eqn. (7), the constraint equation of Eqn. (7) is modified as

$$\mathbf{R}\mathbf{K}_a^{1/2}\mathbf{U} = \mathbf{0} \tag{16}$$

where $\mathbf{R} = \mathbf{A}\mathbf{K}_a^{-1/2}$. Solving Eqn. (16) with respect to $\mathbf{K}_a^{1/2}\mathbf{U}$, it follows that

$$\mathbf{K}_a^{1/2}\mathbf{U} = \left[\mathbf{I} - \mathbf{R}^+\mathbf{R}\right]\mathbf{v} \tag{17}$$

where **v** is an arbitrary vector.

Utilizing the corrected stiffness matrix **K,** natural frequency ω_i^* and mode shape vector $\hat{\mathbf{U}}$ due to the measurement and modeling errors, and local damages into the eigenfunction of Eqn. (5), it is written as

$$\left(\mathbf{K} - \omega_i^{*2}\mathbf{M}_a\right)\mathbf{U} = \mathbf{0} \tag{18}$$

Expressing Eqn. (18) with respect to $\mathbf{K}^{1/2}\mathbf{U}$, it is written by

$$\mathbf{K}^{1/2}\mathbf{U} = \omega_i^{*2}\mathbf{K}^{-1/2}\mathbf{M}_a\mathbf{U} \tag{19}$$

Utilizing Eqns. (17) and (19), and giving the condition to minimize the cost function of Eqn. (15) into the result, we obtain the relation

$$\left[\mathbf{I} - \mathbf{R}^+\mathbf{R}\right]\mathbf{v} = \omega_i^{*2}\mathbf{K}_a^{-1/2}\mathbf{M}_a\mathbf{U} \tag{20}$$

Solving Eqn.(20) with respect to the arbitrary vector **v**, it follows

$$\mathbf{v} = \left[\mathbf{I} - \mathbf{R}^{+}\mathbf{R}\right]\omega_i^{*2}\mathbf{K}_a^{-1/2}\mathbf{M}_a\mathbf{U} + \mathbf{R}^{+}\mathbf{R}\mathbf{x} \tag{21}$$

where **x** is an arbitrary vector. Substituting Eqn. (21) into Eqn. (17) and arranging the result, the corrected eigenfunction is written as

$$\left(\mathbf{K}_a - \omega_i^{*2}\mathbf{M}^{*}\right)\mathbf{U} = \mathbf{0} \tag{22}$$

where $\mathbf{M}^{*} = \left[\mathbf{I} - \mathbf{K}_a^{1/2}\left(\mathbf{A}\mathbf{K}_a^{-1/2}\right)^{+}\mathbf{A}\mathbf{K}_a^{-1}\right]\mathbf{M}_a$. The modal parameters of the updated system are obtained by solving the eigenfunction of Eqn. (22). It is shown that the effects of the uncertainties or damages of dynamic system can be incorporated in the corrected mass matrix.

The natural frequencies and their corresponding mode shapes of modified dynamic system are obtained by solving the eigenfunction of Eqn. (14) of corrected stiffness matrix or Eqn. (22) of corrected mass matrix with the constraints of the measured modal data.

3. Frequency response function

It is important to establish the relationships between FRF and modal parameters for successful modal testing. Inserting $\mathbf{u} = \hat{\mathbf{U}}e^{j\Omega t}$ and $\hat{\mathbf{F}} = \mathbf{F}e^{j\Omega t}$ into Eqn. (1) and expressing it as the form of frequency domain, it follows that

$$\left(\mathbf{K} - \Omega^2\mathbf{M} + j\Omega\mathbf{C}\right)\hat{\mathbf{U}}(\Omega) = \mathbf{F}(\Omega) \tag{23}$$

where Ω denotes the excitation frequency and $j = \sqrt{-1}$. Using the FRF matrix, the response of the original structure, described by $\hat{\mathbf{U}}(\Omega)$, to an external excitation, described by $\mathbf{F}(\Omega)$, is given by

$$\hat{\mathbf{U}}(\Omega) = \mathbf{H}_0(\Omega)\mathbf{F}(\Omega) \tag{24}$$

where $\mathbf{H}_0(\Omega) = \left(\mathbf{K} - \Omega^2\mathbf{M}_a + j\Omega\mathbf{C}_a\right)^{-1}$ is the FRF matrix of the original structure, whose elements can be receptances. Using the impedance-type matrix of the structure $\mathbf{B}_0(\Omega) = \mathbf{H}_0^{-1}(\Omega)$, the equation of motion for the initial structure in the frequency domain is expressed by

$$\mathbf{F}(\Omega) = \mathbf{B}_0(\Omega)\hat{\mathbf{U}}(\Omega) \tag{25}$$

If the system is undamped or only lightly damped, the characteristic features of the system such as Eqn. (1) are the natural frequencies ω_i and the corresponding normal modes $\boldsymbol{\varphi}_i$, which can be calculated from the eigenvalue problem

$$\left(\mathbf{K}_a - \omega_i^2\mathbf{M}_a\right)\boldsymbol{\varphi}_i = \mathbf{0} \tag{26}$$

Substituting $\mathbf{u} = \boldsymbol{\varphi}\mathbf{q}$ into Eqn. (1), premultiplying the result by $\boldsymbol{\varphi}^T$ and normalizing the mode shapes to unit modal mass $m_i\left(i = 1, \quad 2, \quad \cdots \quad n\right)$, it follows:

$$\ddot{\mathbf{q}} + \boldsymbol{\Gamma}\dot{\mathbf{q}} + \boldsymbol{\Lambda}\mathbf{q} = \mathbf{F} \tag{27}$$

where $\boldsymbol{\varphi}_i^T\mathbf{M}\boldsymbol{\varphi}_i = 1$, $\boldsymbol{\varphi}_i^T\mathbf{M}\boldsymbol{\varphi}_k = 0$ ($i \neq k$), $\mathbf{F} = \boldsymbol{\varphi}^T\mathbf{F}(t)$

$$\mathbf{\Gamma} = \begin{bmatrix} 2\omega_1\xi_1 & 0 & \cdots & 0 \\ 0 & 2\omega_2\xi_2 & \vdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 2\omega_n\xi_n \end{bmatrix}, \quad \mathbf{\Lambda} = \begin{bmatrix} \omega_1^2 & 0 & \cdots & 0 \\ 0 & \omega_2^2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \omega_n^2 \end{bmatrix}, \tag{28}$$

and $\mathbf{F}$ is modal excitation, $\boldsymbol{\varphi}$ and $\mathbf{q}$ are the modal matrix and mode coordinate vector, and the superscript 'T' indicates the transpose of matrix.

Transforming Eqn. (27) into the frequency domain leads to

$$\left(-\Omega^2 + 2j\xi_i\omega_i\Omega + \omega_i^2\right)\mathbf{Q}(\Omega) = \mathbf{F}(\Omega) \tag{29}$$

Modal transformation using the real eigenvalues and eigenvectors leads to the representation of the FRF matrix for an excitation frequency Ω :

$$\mathbf{H}_0(\Omega) = \sum_{i=1}^{n} \frac{\boldsymbol{\varphi}_i\boldsymbol{\varphi}_i^T}{\omega_i^2 - \Omega^2 + 2j\xi_i\omega_i\Omega} \tag{30}$$

with $\hat{\mathbf{U}}(\Omega) = \mathbf{H}_0(\Omega)\mathbf{F}(\Omega)$ and $\mathbf{B}_0(\Omega) = \mathbf{H}_0^{-1}(\Omega)$.

The structural dynamic features can be changed by the unexpected environmental change or damages of the system and should be found based on the measurement data. Under the consideration of the excitation frequency, let us assume that the modal coordinates in the frequency domain are measured as

$$\mathbf{A}\,\mathbf{U}(\Omega) = \mathbf{0} \tag{31}$$

which $\mathbf{A}$ denotes a Boolean matrix to define measurement points and Eqn. (31) represents constraints to locally govern the dynamic responses. The dynamic equation in the frequency domain of Eqn. (29) should be modified due to the existence of the measured test data of Eqn. (31).

Let us consider the variation of the dynamic strain energy in the frequency domain expressed as the dynamic stiffness matrix $\mathbf{B}_0(\Omega)$ and the displacement difference between constrained and unconstrained dynamic systems.

$$S = \frac{1}{2}\left(\mathbf{U} - \hat{\mathbf{U}}\right)^T \mathbf{B}_0\left(\mathbf{U} - \hat{\mathbf{U}}\right) \tag{32}$$

where $\hat{\mathbf{U}}(\Omega) = \mathbf{H}_0(\Omega)\mathbf{F}(\Omega)$. Let us modify the constraint equation as

$$\mathbf{A}\mathbf{B}_0^{-1/2}\mathbf{B}_0^{1/2}\mathbf{U}(\Omega) = \mathbf{0} \tag{33}$$

Because the matrix $\mathbf{A}$ is a rectangular matrix, the Moore-Penrose inverse must be utilized. Utilizing $\mathbf{Z} = \mathbf{A}\mathbf{B}_0^{-1/2}$ into Eqn. (33) and solving the equation with respect to $\mathbf{B}_0^{1/2}\mathbf{U}$, it follows

$$\mathbf{B}_0^{1/2}\mathbf{U} = \left[\mathbf{I} - (\mathbf{Z})^+\mathbf{Z}\right]\mathbf{y} \tag{34}$$

where $\mathbf{y}$ is an arbitrary vector.

Minimizing the variation in the dynamic strain energy of the dynamic system of Eqn. (32)

with Eqn. (34) is to satisfy the following equation:

$$\left[\mathbf{I} - (\mathbf{Z})^{+}\mathbf{Z}\right]\mathbf{y} = \mathbf{B}_0^{1/2}\hat{\mathbf{U}} \tag{35}$$

Utilizing the properties of the Moore-Penrose inverse of $\left[\mathbf{I} - \mathbf{Z}^{+}\mathbf{Z}\right]^{+} = \mathbf{I} - \mathbf{Z}^{+}\mathbf{Z}$ and $\mathbf{Z}^{+}\mathbf{Z}\mathbf{Z}^{+} = \mathbf{Z}^{+}$, the solution with respect to the arbitrary vector $\mathbf{y}$ of Eqn. (35) is obtained as

$$\mathbf{y} = \left[\mathbf{I} - \mathbf{Z}^{+}\mathbf{Z}\right]\mathbf{B}_0^{1/2}\hat{\mathbf{U}} + \mathbf{Z}^{+}\mathbf{Z}\mathbf{w} \tag{36}$$

where $\mathbf{w}$ is another arbitrary vector. Substituting Eqn. (36) into Eqn. (34) and pre-multiplying both sides of the result by $\mathbf{B}_0^{-1/2}$ leads to the constrained displacement.

$$\mathbf{U} = \left(\mathbf{I} - \mathbf{B}_0^{-1/2}\mathbf{Z}^{+}\mathbf{Z}\mathbf{B}_0^{1/2}\right)\hat{\mathbf{U}} = \left[\mathbf{I} - \mathbf{B}_0^{-1/2}\left(\mathbf{A}\mathbf{B}_0^{-1/2}\right)^{+}\mathbf{A}\right]\mathbf{B}_0^{-1}\mathbf{F} \tag{37}$$

The final form of the FRF matrix of the constrained dynamic system can be written as

$$\mathbf{H}_{cons} = \left[\mathbf{I} - \mathbf{B}_0^{-1/2}\left(\mathbf{A}\mathbf{B}_0^{-1/2}\right)^{+}\mathbf{A}\right]\mathbf{B}_0^{-1} \tag{38}$$

where $\mathbf{B}_0(\Omega) = \mathbf{H}_0^{-1}(\Omega) = \sum_{i=1}^{n}\frac{\omega_i^2 - \Omega^2 + 2j\xi_i\omega_i\Omega}{\boldsymbol{\varphi}_i\boldsymbol{\varphi}_i^T}$. Using Eq. (38) into Eq. (37), it leads to the modal displacements of constrained dynamic system of full degrees of freedom.

Equation (38) represents the FRF matrix of the system subjected to constraints of measured modal displacements of Eqn. (31) and gives the relationship between modal parameters of dynamic system modified by measurement errors or local damages.

4. Applications

As an application, let us consider a six DOF mass-spring system in Fig. 1. Describing the displacements by $\mathbf{u} = [u_1 \quad u_2 \quad u_3 \quad u_4 \quad u_5 \quad u_6]^T$, the dynamic equations of motion for the system can be written by

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{K}\mathbf{u} = \mathbf{F}(t) \tag{39}$$

where $\mathbf{M} = diag[m_1 \quad m_2 \quad m_3 \quad m_4 \quad m_5 \quad m_6]$ and

$$\mathbf{K} = \begin{bmatrix} k_1 + k_2 + k_6 & -k_2 & 0 & 0 & -k_6 & 0 \\ -k_2 & k_2 + k_3 + k_7 & -k_3 & 0 & -k_7 & 0 \\ 0 & -k_3 & k_3 + k_4 + k_9 & -k_4 & 0 & -k_9 \\ 0 & 0 & -k_4 & k_4 + k_5 & 0 & 0 \\ -k_6 & -k_7 & 0 & 0 & k_6 + k_7 + k_8 & -k_8 \\ 0 & 0 & -k_9 & 0 & -k_8 & k_8 + k_9 \end{bmatrix} \tag{40}$$

Each of masses weighs 10kg, and the springs have stiffness of 100MN/m except for spring k_1 whose stiffness is 300MN/m. The modal properties of the mass-spring system of Eqn. (39) are listed in Table 1.

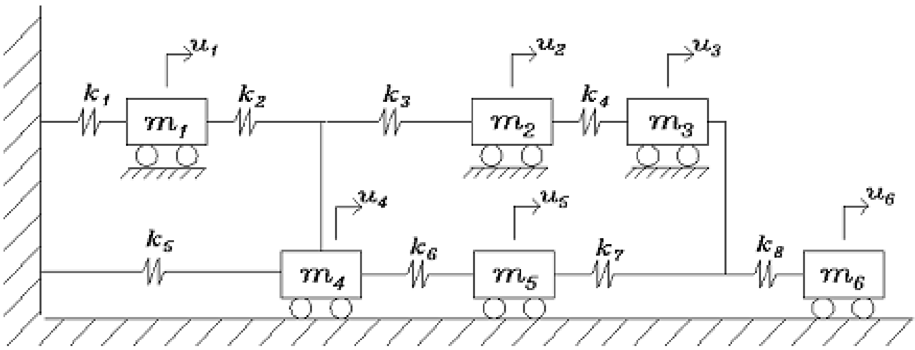

Fig. 1. A structural dynamic system

Table 1. Modal parameters of the unconstrained mass-spring system

	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5	Mode 6
Natural frequency(Hz)	30.046	63.913	75.209	91.235	111.531	119.112
Mode shapes	0.1920	0.1638	0.1381	0.1110	-0.1194	0.9597
	0.4384	0.1724	0.6038	-0.2766	-0.2006	-0.1867
	0.4541	-0.4440	0.0380	-0.2907	0.7812	0.1304
	0.2642	-0.7672	0.1803	0.5892	-0.3475	-0.0479
	0.4603	0.3826	-0.1937	0.5176	0.1791	-0.1575
	0.5320	-0.1062	-0.7381	-0.4599	-0.4271	0.0100

Table 2. Modal parameters of the constrained mass-spring system

	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5	Mode 6
Natural frequency(Hz)	31.52	66.78	84.97	100.7	119.1	-
Mode shapes	0.1834	0.2598	0.244	0.0171	0.9158	
	0.435	0.4144	0.2254	0.7146	-0.2784	
	0.541	-0.1735	-0.5096	-0.0372	0.0778	
	0.3365	-0.7258	0.5995	0.0186	-0.022	
	0.4101	0.4275	0.2998	-0.6974	-0.2702	
	0.4507	-0.1452	-0.4251	-0.0314	0.065	

Let us assume the relationship of the measured mode shapes at nodes 3 and 6 as

$$U_3 = 1.2U_6 \qquad (41)$$

Substituting Eqns. (39) and (41) into Eqns. (14) or (22), and solving the eigenfunction, the modal parameters are calculated as Table 2. Although the eigenfunctions of Eqns. (14) or (22) are derived based on the corrected mass and analytical stiffness matrices, and the analytical mass and corrected stiffness matrices, respectively, it is shown that the numerical results are the same and the mode shapes at nodes three and six satisfy the measured relation of Eqn. (41). Though the system is six degree-of-freedom system, the constrained dynamic system becomes five degree-of-freedom system due to a constraint of a measured mode shape relation and exhibits five mode shapes. From the application, it is observed that the modal parameters of modified dynamic system can exactly explain the eigenfunction without any complete reanalysis process.

As another application, let us consider a vibrating system with the forcing frequency 2rad./sec. and the damping ratio 2% on all modes as shown in Fig. 2. Expressing the dynamic equations of the system in terms of modal parameters, the FRF matrix of the initial dynamic system is obtained as

$$\mathbf{H}_0 = \begin{bmatrix} 0.072-0.006j & -0.042-0.008j & -0.122-0.005j & -0.055-0.001j \\ -0.042-0.008j & -0.092-0.019j & -0.268-0.014j & -0.122-0.005j \\ -0.122-0.005j & -0.268-0.014j & -0.2-0.02j & -0.091-0.008j \\ -0.055-0.001j & -0.122-0.005j & -0.091-0.008j & 0.05-0.006j \end{bmatrix}$$

If the dynamic mode shapes of the system is constrained by the mode relation of $U_3 = U_4$

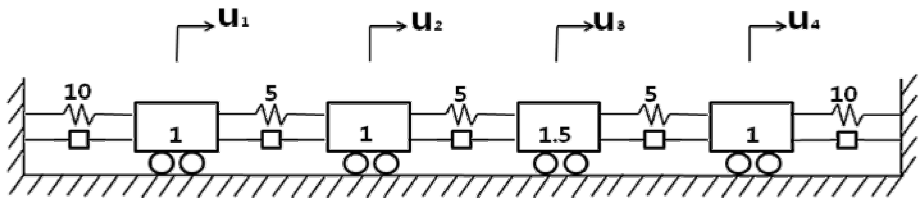

Fig. 2. A damped dynamic system

the FRF matrix of the constrained systems based on Eqn. (38) is derived as

$$\mathbf{H}_{cons} = \begin{bmatrix} 0.082-0.005j & -0.02-0.006j & -0.106-0.002j & -0.035-0.0005j \\ -0.009-0.006j & -0.019-0.013j & -0.214-0.007j & -0.051-0.003j \\ -0.061-0.002j & -0.134-0.005j & -0.1-0.009j & 0.038-0.006j \\ -0.061-0.002j & -0.134-0.005j & -0.1-0.009j & 0.038-0.006j \end{bmatrix}$$

Comparing the mode shapes at nodes three and four, it is shown that the FRF matrix satisfies the constraint. From the applications, it is found that the proposed method can easily and explicitly determine the FRF with the physical information of the original structure and the constraints of modal coordinates only without any numerical scheme and other mechanical properties.

5. Conclusions

This study presented the reanalysis method to calculate the modal parameters and FRF matrix of modified dynamic system subjected to constraints such as measured modal data. The reanalysis methods to determine the modal parameters and FRF matrix were derived by minimizing the cost functions in the satisfaction of the constraints. It was observed that the modal parameters of modified structure were determined based on the measured natural frequency and modal data. It was shown that the proposed methods have an advantage to be able to determine the modal parameters and FRF of modified system without any complete reanalysis process. The validity of the methods was illustrated in applications.

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