425. Calculation of stresses in the coating of a vibrating beam

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Abstract. The model for the analysis of stresses in the coating of a straight beam performing transverse vibrations is presented. The developed numerical procedure is based on the technique of conjugate approximation with smoothing adapted for this specific problem. Relative error norms are calculated and their relationship with the required smoothing is determined.

Keywords: beam vibrations, eigenmode, conjugate approximation, conjugate smoothing, finite elements.

Introduction

The numerical calculation and analysis of stresses in photo-elastic coatings is important in hybrid experimental – numerical procedures used in experimental mechanics [1, 2]. One of the simplest and important models in this field is the analysis of transverse vibrations of a straight beam. It is assumed that the coating is thin and has no effect on the vibrations of the beam.

The model for the analysis of stresses in the coating of a straight beam performing transverse vibrations is presented. It is based on the model for the analysis of beam bending described in [3].

The developed numerical procedure is based on the technique of conjugate approximation [4, 5] with smoothing [6] adapted for this specific problem.

Relative error norms [7, 8] are calculated and their relationship with the required smoothing is determined.

The calculation of the stress field is important in hybrid experimental – numerical procedures, which were analyzed in the previous papers [9, 10]. One dimensional model which is analyzed in this paper enables to represent the results graphically and thus provides better understanding of the behavior of approximation as well as necessity of smoothing of the stress field.

Numerical procedure

Further x, y and z denote the axes of the orthogonal Cartesian system of coordinates.

The beam bending element has two nodal degrees of freedom: the displacement w in the direction of the z axis and the rotation Θ_y about the y axis. The displacement u in the direction of the x axis is expressed as $u=z\Theta_y$.

The mass matrix has the form:

$$[M] = \int [N]^T \begin{bmatrix} \rho h & 0 \\ 0 & \rho \dfrac{h^3}{12} \end{bmatrix} [N] dx, \tag{1}$$

where ρ is the density of the material of the beam, h is the thickness of the beam and:

$$[N] = \begin{bmatrix} N_1 & 0 & \dots \\ 0 & N_1 & \dots \end{bmatrix}, \tag{2}$$

where N_i are the shape functions of the finite element.

The stiffness matrix has the form:

$$[K] = \int \begin{pmatrix} [B]^T \left[\dfrac{E}{1-\nu^2} \dfrac{h^3}{12} \right] [B] + \\ + [\overline{B}]^T \left[\dfrac{E}{2(1+\nu)1.2} h \right] [\overline{B}] \end{pmatrix} dx, \tag{3}$$

where:

$$[B] = \begin{bmatrix} 0 & \dfrac{dN_1}{dx} & \dots \end{bmatrix}, \tag{4}$$

$$[\overline{B}] = \begin{bmatrix} \dfrac{dN_1}{dx} & N_1 & \dots \end{bmatrix}, \tag{5}$$

where E is the modulus of elasticity of the beam, ν is the Poisson's ratio of the beam.

The i-th eigenfrequency ω_i and the corresponding eigenmode $\{\delta_i\}$ are determined from:

$$([K] - \omega_i^2 [M])\{\delta_i\} = \{0\}. \tag{6}$$

The nodal values of the stress in the coating are determined from:

$$\int \left([\hat{N}]^T [\hat{N}] + [\hat{B}]^T \lambda [\hat{B}] \right) dx \{\delta_c\} = \int [\hat{N}]^T \sigma_c dx, \tag{7}$$

where:

$$[\hat{N}] = [N_1 \quad \dots], \tag{8}$$

$$[\hat{B}] = \begin{bmatrix} \dfrac{dN_1}{dx} & \dots \end{bmatrix}, \tag{9}$$

where λ is the smoothing parameter, $\{\delta_c\}$ is the vector of nodal values of the stress in the coating, σ_c is the stress in the coating determined from:

$$\sigma_c = \left[\dfrac{E_c}{1-\nu_c^2} \right] \dfrac{h}{2} [B] \{\delta\}, \tag{10}$$

where E_c is the modulus of elasticity of the coating, ν_c is the Poisson's ratio of the coating, $\{\delta\}$ is the vector of generalized displacements of the analyzed eigenmode.

The relative error norms for the i-th finite element are calculated as:

$$\psi_i = \frac{\int_e \left[\frac{E_c}{1-\nu_c^2}\right]\left(\varepsilon_c - \frac{h}{2}[B]\{\delta\}\right)^2 dx}{\int_e \left[\frac{E_c}{1-\nu_c^2}\right]\varepsilon_c^2 dx}, \tag{11}$$

where the integrals are over the analyzed finite element and the values of ε_c are calculated from:

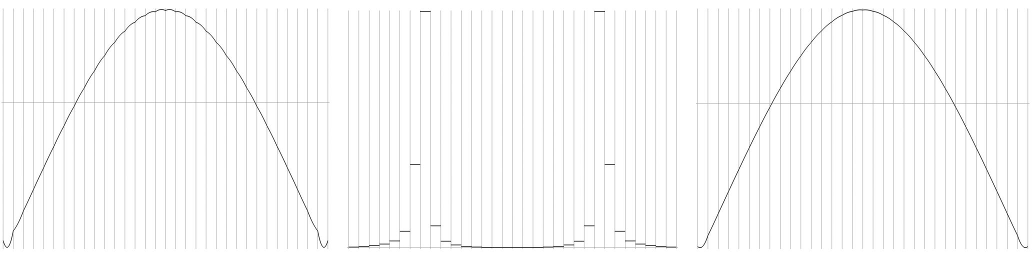
$$\varepsilon_c = \frac{\sigma_c}{\left[\frac{E_c}{1-\nu_c^2}\right]}, \tag{12}$$

where the stress in the coating is calculated by using the shape functions of the finite element from the vector $\{\delta_c\}$ obtained by using the procedure of conjugate approximation (λ=0).

Results of analysis

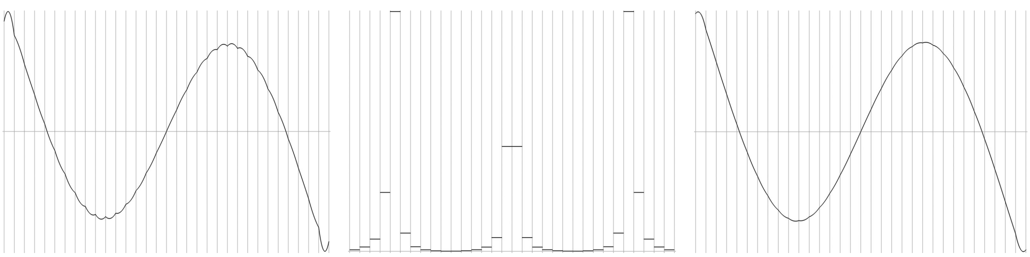
At both ends of the beam both generalized displacements are assumed equal to zero.

The stress field for the first eigenmode obtained by using the procedure of conjugate approximation is presented in Fig. 1a, the histogram of the relative error norms is presented in Fig. 1b, and the stress field obtained by using the procedure of conjugate smoothing is presented in Fig. 1c. The corresponding results for the second eigenmode are presented in Fig. 2, …, for the sixth eigenmode - in Fig. 6.



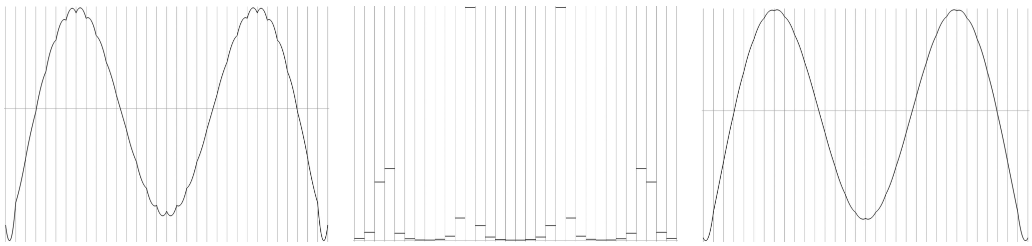
a) b) c)

Fig. 1. The first eigenmode: a) stress field obtained by using the procedure of conjugate approximation, b) the histogram of the relative error norms, c) stress field obtained by using the procedure of conjugate smoothing



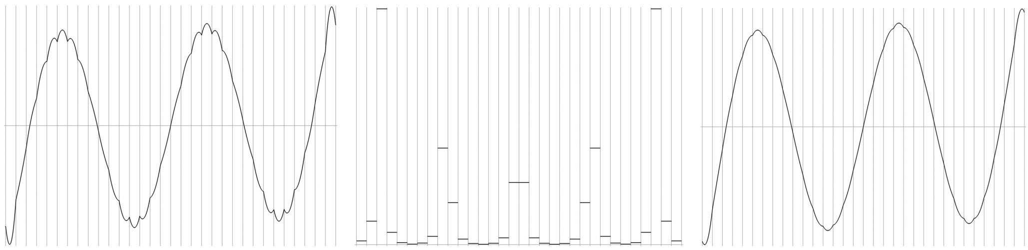
a) b) c)

Fig. 2. The second eigenmode: a) stress field obtained by using the procedure of conjugate approximation, b) the histogram of the relative error norms, c) stress field obtained by using the procedure of conjugate smoothing



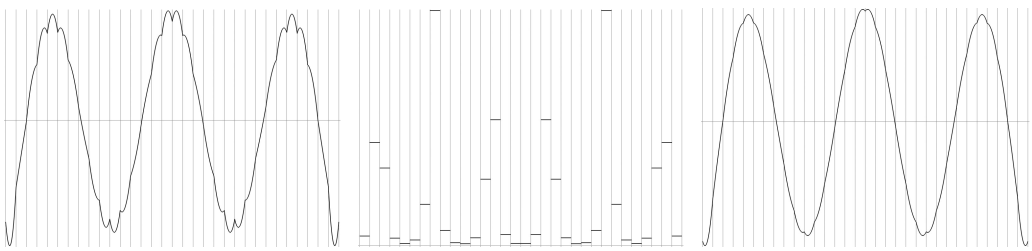
a) **b)** **c)**

Fig. 3. The third eigenmode: a) stress field obtained by using the procedure of conjugate approximation, b) the histogram of the relative error norms, c) stress field obtained by using the procedure of conjugate smoothing



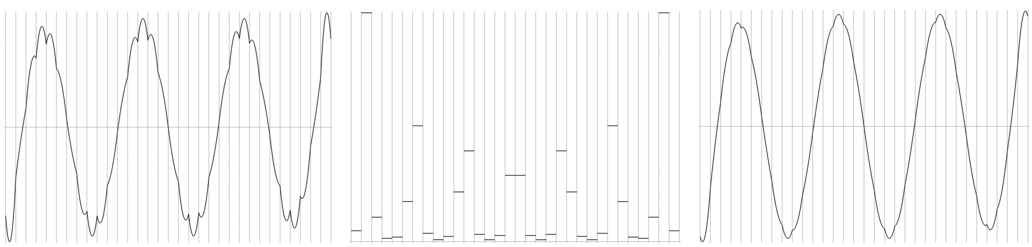
a) **b)** **c)**

Fig. 4. The fourth eigenmode: a) stress field obtained by using the procedure of conjugate approximation, b) the histogram of the relative error norms, c) stress field obtained by using the procedure of conjugate smoothing



a) **b)** **c)**

Fig. 5. The fifth eigenmode: a) stress field obtained by using the procedure of conjugate approximation, b) the histogram of the relative error norms, c) stress field obtained by using the procedure of conjugate smoothing



a) **b)** **c)**

Fig. 6. The sixth eigenmode: a) stress field obtained by using the procedure of conjugate approximation, b) the histogram of the relative error norms, c) stress field obtained by using the procedure of conjugate smoothing

From the presented results the advantage of conjugate smoothing over the procedure of conjugate approximation and the relationship of the required smoothing with the relative error norms are evident. This enables to propose various algorithms for the adaptive control of the smoothing parameter.

Conclusions

The model for the analysis of stresses in the coating of a straight beam performing transverse vibrations is presented. The developed numerical procedure is based on the technique of conjugate approximation with smoothing adapted for this specific problem. Relative error norms are calculated and their relationship with the required smoothing is determined.

From the presented results the advantage of conjugate smoothing and the relationship of the required smoothing with the relative error norms are evident. This enables to propose various algorithms for the adaptive control of the smoothing parameter.

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