417. Inversion of the generalized abel transform in the analysis of fluid vibration in a tube

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(Received:02 November; accepted: 02 December)

Abstract. Fluid vibrations in axi-symmetric geometry according to the first harmonic in the circumferential direction are analyzed. This problem has a practical application in the analysis of transverse vibrations of fluid in an axi-symmetric pipe. In order to obtain volumetric strains from the holographic image of the vibrating fluid the inversion of the generalized Abel transform is to be performed. A numerical procedure for the solution of this problem is proposed.

Keywords: generalized Abel transform, vibration of fluid, axi-symmetric problem, volumetric strain, numerical integration.

Introduction

Interpretation of experimental results, especially when such experimental techniques as laser holography is applied for investigation of high frequency vibrations of fluid as described in references [1, 2, 3, 4], is an important engineering problem. The fluid vibrations according to the first harmonic in the circumferential direction of the cylindrical system of coordinates are investigated in a tube. Generalized Abel transform [4] must be exploited for the interpretation of interference fringes due to the fact that the laser beam travels different lengths through the fluid at different positions of the laser rays penetrating through the surface of the tube.

In order to interpret the obtained interference pattern the field of volumetric strain is to be calculated. For this purpose the inversion of the generalized Abel transform for each scan line of the hologram is performed on the basis of the proposed numerical procedure.

Generalized Abel Transform

Further x, y and z denote the axes of the orthogonal Cartesian system of coordinates. The penetration of the laser beam into the liquid (in the direction of the z-axis) is related to the radial coordinate. The incorporation of the viewing direction is performed by assuming that the pipe is turned by an angle φ and thus the angular coordinate Θ is understood as indicated in the figure. The angles can be determined from the following relationship:

$$\tan(\Theta+\varphi)=\frac{z}{x}=\frac{\pm\sqrt{r^2-x^2}}{x}, \qquad (1)$$

where r is the radial coordinate. The two solutions are denoted as:

$$(\Theta+\varphi)_1=\tan^{-1}\frac{\sqrt{r^2-x^2}}{x},$$
$$(\Theta+\varphi)_2=\tan^{-1}\frac{-\sqrt{r^2-x^2}}{x}. \qquad (2)$$

Then the generalized Abel transform for the analyzed problem of tube vibration has the following form:

$$\Phi(x,y)= = \int_x^{\infty}\varepsilon_{v\cos}(r,y)\begin{bmatrix}\cos((\Theta+\varphi)_1-\varphi)+ \\ +\cos((\Theta+\varphi)_2-\varphi)\end{bmatrix}\frac{rdr}{\sqrt{r^2-x^2}}, \qquad (3)$$

where $\Phi(x, y)$ is the phase of the laser beam, $\varepsilon_{v\cos}(r, y)$ are the amplitudes of circumferential variation of the volumetric strains.

When viewing direction is $\pi / 2$ no interference fringes are observed and the viewing area in the hologram is white. This follows directly from the expression of the generalized Abel transform and the properties of the trigonometric functions.

The holograms obtained from different viewing directions are used in order to determine the direction of vibrations. For further analysis the hologram with the highest possible number of fringes is chosen and used to determine the volumetric strain.

The minimum number of observation directions in order to get a good estimate of volumetric strain depends on our knowledge of the direction of vibrations. When the direction of vibrations is known then a single observation direction is sufficient, but when the direction of micro-vibrations is not evident a number of observation directions are to be used.

Thus for the hologram with the highest number of fringes which is used for the interpretation of volumetric strains it follows that:

$$\cos\Theta = \frac{x}{r}, \tag{4}$$

and the generalized Abel transform takes the form:

$$\Phi(x, y) = 2x\int_x^\infty \varepsilon_{v\cos}(r, y)\frac{dr}{\sqrt{r^2 - x^2}}. \tag{5}$$

For each scan line of the hologram by using the right rectangle numerical integration rule the generalized Abel transform is represented in the following way:

$$\Phi_j = \sum_{i=j+1}^{n} 2\varepsilon_{v\cos i}\frac{(x_{\min} + j\Delta)\Delta}{\sqrt{(x_{\min} + i\Delta)^2 - (x_{\min} + j\Delta)^2}}, \tag{6}$$

where Φ_j is the phase of the laser beam at the pixel j of the scan line, $\varepsilon_{v\cos i}$ is the volumetric strain at the pixel i of the scan line, $x_{\min}$ is the minimum value of the radial coordinate in the hologram, Δ is the radial distance between the adjacent pixels.

Inversion of the Generalized Abel Transform

The computational scheme of the computational process for the formation of the digital fluid hologram is presented in Fig. 1.

For each scan line of the hologram the phase of the laser beam for each pixel of the scan line is obtained. Schematic representation of the implementation of this procedure is illustrated in Fig. 2.

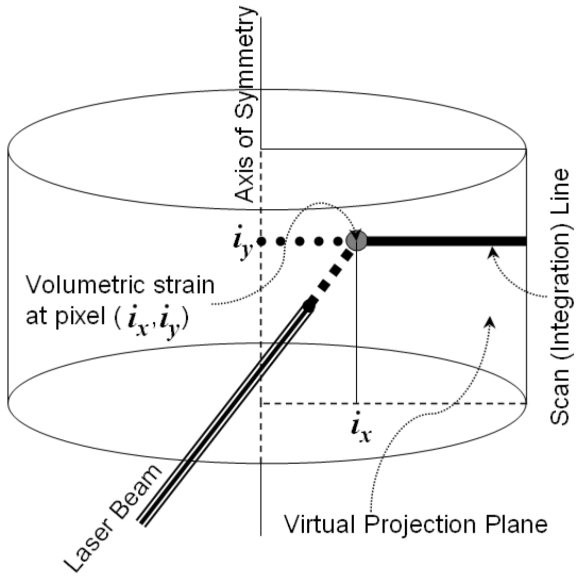

Fig. 1. The geometrical scheme of the computational process

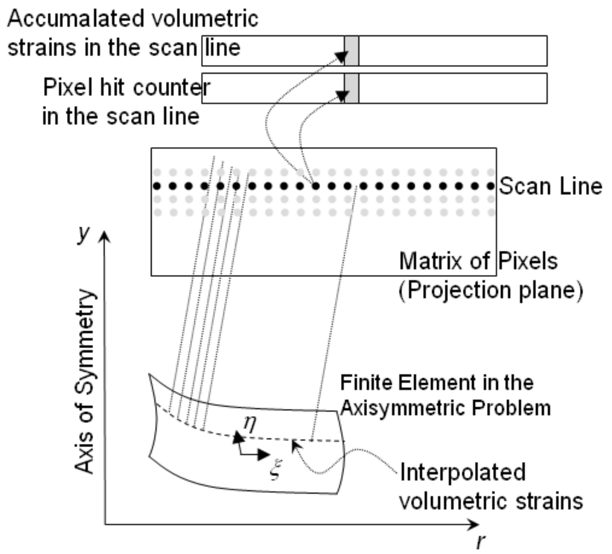

Fig. 2. Implementation of the determination of the phase of the laser beam for a scan line of the hologram

In experimental procedures determination of volumetric strains from the phase of the laser beam is an important problem. On the basis of equation (6) the inversion of the generalized Abel transform is performed in the following way:

$$\varepsilon_{v\cos n} = \frac{\Phi_{n-1}}{2\dfrac{(x_{\min} + (n-1)\Delta)\Delta}{\sqrt{(x_{\min} + n\Delta)^2 - (x_{\min} + (n-1)\Delta)^2}}}, \tag{7}$$

$$\varepsilon_{v\cos n-1} = \frac{\Phi_{n-2} - 2\varepsilon_{v\cos n}\dfrac{(x_{\min}+(n-2)\Delta)\Delta}{\sqrt{(x_{\min}+n\Delta)^2 - (x_{\min}+(n-2)\Delta)^2}}}{2\dfrac{(x_{\min}+(n-2)\Delta)\Delta}{\sqrt{(x_{\min}+(n-1)\Delta)^2-(x_{\min}+(n-2)\Delta)^2}}}, \quad (8)$$

$$\varepsilon_{v\cos n-2} = \frac{\Phi_{n-3} - 2\varepsilon_{v\cos n-1}\dfrac{(x_{\min}+(n-3)\Delta)\Delta}{\sqrt{(x_{\min}+(n-1)\Delta)^2 - (x_{\min}+(n-3)\Delta)^2}} - 2\varepsilon_{v\cos n}\dfrac{(x_{\min}+(n-3)\Delta)\Delta}{\sqrt{(x_{\min}+n\Delta)^2 - (x_{\min}+(n-3)\Delta)^2}}}{2\dfrac{(x_{\min}+(n-3)\Delta)\Delta}{\sqrt{(x_{\min}+(n-2)\Delta)^2-(x_{\min}+(n-3)\Delta)^2}}}, \quad (9)$$

and so on until finally:

$$\varepsilon_{v\cos 1} = \frac{\Phi_0 - \sum_{i=2}^{n} 2\varepsilon_{v\cos i}\dfrac{x_{\min}\Delta}{\sqrt{(x_{\min}+i\Delta)^2 - x_{\min}^{\ 2}}}}{2\dfrac{x_{\min}\Delta}{\sqrt{(x_{\min}+\Delta)^2 - x_{\min}^{\ 2}}}}. \quad (10)$$

The previous equations can be represented in the form:

$$\varepsilon_{v\cos j+1} = \frac{\Phi_j - \sum_{i=j+2}^{n} 2\varepsilon_{v\cos i}\dfrac{(x_{\min}+j\Delta)\Delta}{\sqrt{(x_{\min}+i\Delta)^2 - (x_{\min}+j\Delta)^2}}}{2\dfrac{(x_{\min}+j\Delta)\Delta}{\sqrt{(x_{\min}+(j+1)\Delta)^2-(x_{\min}+j\Delta)^2}}}, \quad (11)$$

where $j=n-1, n-2, \ldots, 0$.

The variation of the phase of the laser beam in a scan line of the hologram is shown in Fig. 3. The obtained variation of volumetric strain is presented in Fig. 4.

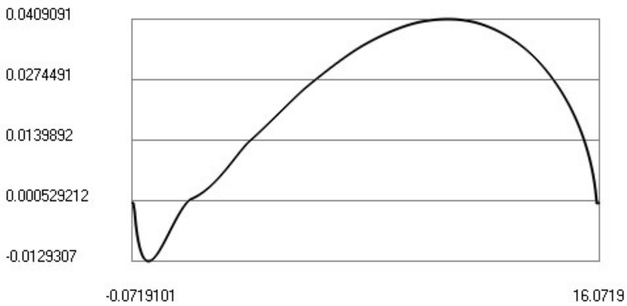

Fig. 3. Phase of the laser beam in a scan line of the hologram

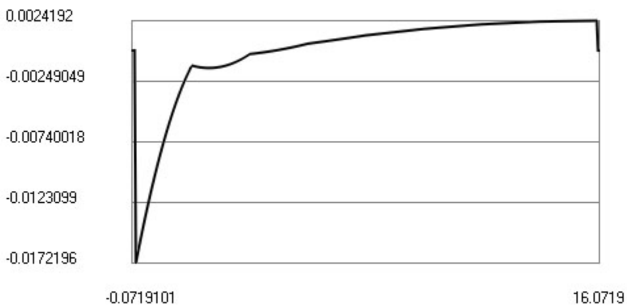

Fig. 4. Volumetric strain in a scan line of the hologram.

Conclusions

The fluid vibrations according to the first harmonic in the circumferential direction in axi-symmetric geometry are analyzed. The procedure for the calculation of the amplitudes of the circumferential variation of the volumetric strain is proposed. Thus the holographic images are used for the determination of volumetric strains in vibrating tubes.

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