377. INFORMATION ENTROPY DETERMINATION IN SCALES MEASUREMENT INCLUDING MECHATRONIC APPROACH

M. Rybokas[1,a], R. Bansevicius[2,b], V. Giniotis[1,a]

[1] Vilnius Gediminas Technical University, Sauletekio al. 11, LT-10223 Vilnius-40, Lithuania
[2] Kaunas University of Technology, Kestucio 27, LT-44025 Kaunas, Lithuania
E-mail: [a] MRybokas@gama.vtu.lt ;[b] bansevicius@cr.ktu.lt ; [c] vg@ai.vgtu.lt

Abstract. Analysis of information entropy of mechatronic measuring systems, computer modelling; research of the accuracy of multiparameter systems and the solutions for determination of the deviations of those parameters for the purpose of application it to the adaptive correction and for assurance of mechatronic control of the system are presented in the paper. The mechatronic means for correction of systematic errors of information-measuring systems including measurement of linear and circular scales are presented. An application of active materials for smart correction by piezoactuators is given.

Keywords: mechatronics, measurement, information entropy, scales

Introduction

Production equipment and the communication systems of high technology feature by tremendous volumes of information data and their input/output complexity. It is especially important having in mind that modern measuring systems consist of "smart" transducers that can perform some logic functions and "smart" sensors and actuators [1], complicated control systems with special programming languages able to control the functions of a measuring system by digital and alphanumeric information [2]. The amount of data in such systems sometimes exceeds the gigabytes rather in average complexity of the system. Features and data of mechatronic information systems are discussed in this work. Information about the quantitive and qualitative state of the object is selected using mathematic statistic methods and means. The information selected and processed is to be used to determine a statistical evaluate of the mechatronic object that is required for essential it apprehension and for relevant impact (for example, error correction [3]) making. This all is under consideration including all kind of information about the technical parameters of the object to be controlled, its displacement with the accuracy parameters and the control of the position and micro/nano displacement. It is especially important with the tasks for the position accuracy control in nano displacement systems, information – measuring systems, such as raster and coded scales of high accuracy, transducers, rotary encoders, etc.

Computer simulation system is developed including of visualization of the object and mathematic equations for an assessment of mutual information on the mechatronic object as well. The circular raster scale as a real example to be simulated is foreseen in this area.

Information processing in scales measurement

The angle measuring methods using a rotary encoder, autocollimator and polygon, circular scale and microscopes are used for measurements (calibration) of raster, coded and ordinary scales [4]. During the calibration it is possible to determine only a restricted number of values, dependant mainly on number of the points of the reference measure (for example, polygon). An analysis of the results shows that during the accuracy calibration some larger values of the error can be omitted including significant ones. Therefore, it is important to determine the information quantity on an object that has been evaluated providing a more thorough result assessment during the calibration processes. The problem exists due to the great amount of information that is gained in the calibration of scales, information – measuring systems of the numerically-controlled machines, such as coordinate measuring machines (CMM) in their total volume. It is technically difficult and economically demanding to calibrate the enormous number of points available, e.g. the 324,000 steps of the rotary table of such machine in the measuring volume arising from six rotary axes or millions parts of the meter for a laser interferometer. The measurement interval of the length no fewer than 20 positions should be measured as it is indicated in CMM documentation. These requirements stated by written standards show the same problem – information inside the interval of measurement remains unknown. This problem is analyzed and

293

information on measurement is supplemented by evaluating the information joining it with the general expression of the measurement result, i.e., expressing the systematic part of the result, the uncertainty of the assessment, and adding to it the quantity of information entropy that shows the indeterminacy of the result [5]. This is performed by adding an information entropy parameter to the expression of the measurement result X. The full information entropy according to [2] can be expressed as:

$$H_0 = -\sum_{\Delta} \log_a \frac{1}{m} = \log_a m \; ;. \qquad (1)$$

The information received after the calibration of the scale, it is, the determination of the accuracy of part of the strokes b of the scale will be:

$$H_1 = \log_a b, \qquad (2)$$

where $b = \dfrac{m}{k}$ is the number of calibrated strokes in the scale having m number of strokes and k is the pitch of the calibration. These strokes were measured c times each for the statistical evaluation. Then the reduction in the information uncertainty (indeterminacy) due to the information received will be:

$$I = H_0 - H_1 = \log_a m - \log_a b \; ;$$
and
$$\log_a b = \log_a m - I \; ;$$
and
$$b = a^{(\log_a m - I)} = m \times a^{-I} \; . \qquad (3)$$

Since the total number of measurements is $n = bc,$ (each calibration measurement is performed c times), the expression for the measurement result at a given probability and the reduction in information indeterminacy becomes:

$$X = \overline{X} \pm \frac{t \cdot S}{\sqrt{a^{-I} mc}} \; , P, I(H_0, H_1) \cdot \qquad (4)$$

It means that the measurement result is determined expressing the measurand with the uncertainty assessed by probability level P and with the *indeterminacy* assessed by the entropy $I(H_1, H_0)$ by evaluation of that part of all the data in question. It gives a better assessment of the results of measurement allowing one to know which part of all the information was assessed during this process.

Information entropy assessment

Information entropy [2, 5] permits the presentation to the user that part of the information that is available from the measurement data that is determined with appropriate statistical evaluation. An additional value to this approach

would give an indication of the sampling value in the result of the measurement. The entropy is the uncertainty of a single random variable [10, 11]. The reduction I in the uncertainty due to the information assessed (in our case the information received after the calibration) is $I = H_0 - H_1$, where H_0 is the entropy before receiving the information and H_1 is the entropy after receiving it. As it follows from the multiplication theorem of probabilities [2]:

$$P(H_j \mid A) = \frac{P(H_j \cap A)}{P(A)} = \frac{P(H_j)P(A \mid H_j)}{P(A)}. \qquad (5)$$

Where $P(H_j \mid A)$ - a full probability of events H_j under an *a posteriori* circumstances of already happened event A. A probability $P(A)$ of a set of discrete values can be considered as having a uniform law of distribution, and a part of values that are assessed according to the normal law of distribution. In case of a centered variable that is distributed according to the normal distribution this expression becomes

$$H^*(X) = \log \sqrt{2\pi e}\, \sigma_X \qquad (6)$$

The relative entropy of independent variables X and Y in terms of probabilities is [2]:

$$H(Y / X) = -\sum_i \sum_j p(X_i, Y_j) \log p(Y_j / X_i)$$

and

$$H(X / Y) = -\sum_i \sum_j p(X_i, Y_j) \log p(X_i / Y_j) \qquad (7)$$

The differential entropy in case of continuous functions is

$$H^*(Y / X) = -\int_{-\infty}^{+\infty} \int f(x) f(y/x) \log f(y/x) dx dy$$

$$(8)$$

These assumptions can help in determination of mechatronic impact on the information – measuring system for the elimination of systematic errors. A case of scale errors correction is presented in [6-8]. Piezoplates and simplified bodies from piezomaterial were applied for micrometric displacement for correction of the systematic errors of the raster scale. Correction of errors is accomplished by the control of a potential energy of the piezoplate. In technical point of view it can be done by changing a voltage supplied to the electrode electromechanically coupled with the raster scale, changing the width of the electrode coating of the

294

piezoplate. According to the principle of minimum of potential energy $\frac{\delta U}{\delta \varepsilon_1} = 0$, and taking into account, that longitudinal displacements are related to deformations by relationship $\varepsilon_1(x) = \frac{du}{dx}$, it is possible to obtain the equation of equilibrium of the piezoplate considered and to control an accuracy of linear raster scale. Two-dimensional or three-dimensional measuring systems distributions of errors can be expressed by two functions

$u(x, y) = -k_1 \delta_x(x, y)$ and

$v(x, y) = -k_2 \delta_y(x, y)$, where u, v - components of displacements at the zone controlled into the direction of appropriate coordinates. k_1, k_2 - coefficients of electromechanical coupling. For more exact assessment of error distribution and possible its correction the relative entropy parameters must be evaluated.

The relative entropy helps to assume the distribution of the random value as q when true distribution of this variable is p. This parameter is assumed as a measure of the distance between two distributions of one variable. Assuming the explanations presented above, the mutual information appears as the most acceptable method to assess the multi-coordinate measuring systems. The mutual information model of 3D measurement is investigated below. x, y and z axes are subdivided into k, l and m steps (divisions), respectively, and Δ is the length of a single step of the scale of the information-measuring system. Symbols b and c indicate the number of calibrated steps for axes x and y, respectively, as a result of performing calibration at pitches of d_1 and d_2. Therefore, the intervals of measurement values extend to

$$
\begin{aligned}
&0 \le x \le k, \\
&0 \le y \le l \text{ and} \\
&0 \le z \le m.
\end{aligned}
\tag{9}
$$

If all dimensions are independent of each other no information is gained about any of the variables by fixing the value in one dimension. For the sake of economizing space only the example of fixing dimension z is provided. The instances of fixing two remaining dimensions produce analogous results using previous equations [4].

$$
I(X;Y \mid Z) = \sum p(x, y, z) \log \frac{p(X, Y \mid Z)}{p(X \mid Z) p(Y \mid Z)} =
\tag{10}
$$

$$
\sum \frac{1}{klm} \log \frac{1/kl}{1/k \cdot 1/l} = \sum \frac{1}{klm} \log 1 = 0
$$

For the continuous version we need to combine the expressions for trivariate normal density and multivariate conditional normal distributions. These expressions are provided and simplified below.

The general expression for a bivariate normal distribution is given by the formula below:

$$
p(x, y) = \frac{1}{2\pi \sigma_x \sigma_y \sqrt{1 - r_{xy}^2}} \times e^{-\zeta},
\tag{11}
$$

where

$$
\zeta = \frac{1}{2(1 - r_{xy}^2)} \left[\frac{(x - m_x)^2}{\sigma_x^2} - \frac{2 r_{xy}(x - m_x)(y - m_y)}{\sigma_x \sigma_y} + \frac{(x - m_y)^2}{\sigma_y^2} \right]
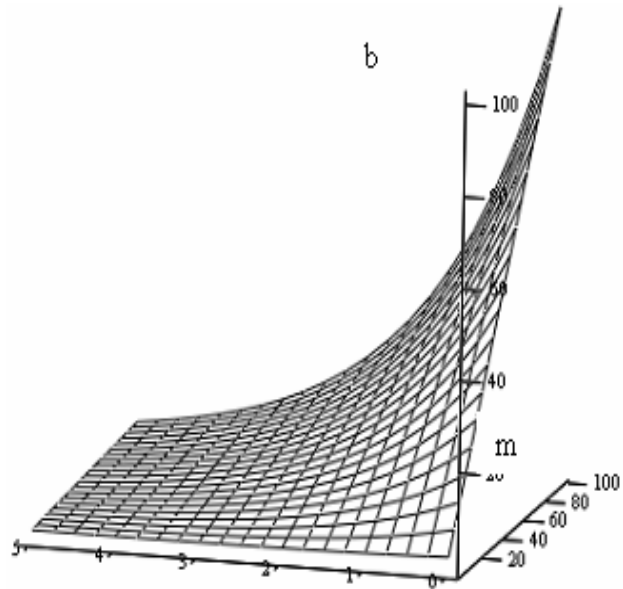$$

Fig. 1. The relationship between the total and assessed number of strokes with the information quantity of the scale, where m is the number of strokes on the scale, b is the number of the assessed strokes on the scale and I is the quantity of information

The expressions for bivariate conditional normal distributions to be placed in the denominator of equation (12) are provided below

$$
p(y \mid z) = \frac{1}{\sigma_y \sqrt{1 - r_{yz}^2} \sqrt{2\pi}} e^{-\frac{1}{2(1 - r_{yz}^2)} \left(\frac{y - m_y}{\sigma_y} - r_{yz} \frac{z - m_z}{\sigma_z} \right)^2},
\tag{12}
$$

$$
p(x \mid z) = \frac{1}{\sigma_x \sqrt{1 - r_{xz}^2} \sqrt{2\pi}} e^{-\frac{1}{2(1 - r_{xz}^2)} \left(\frac{x - m_x}{\sigma_x} - r_{xz} \frac{z - m_z}{\sigma_z} \right)^2},
\tag{13}
$$

where r's are correlation coefficients between corresponding random variables, while m's and σ's are means and standard deviations of random variables indicated by the indexes, respectively.

In Fig. 1. the relationship between the total number of strokes on the scale m, its information I and the number of the strokes position already assessed on the scale b is displayed. It is assumed that the base of logarithm is 2, and this base is used in Formula (3). The general expression for a trivariate normal distribution [see mathworld.worlfram.com] is given by the formula below:

295

$$p(x,y,z) = \frac{e^{-w/[2(r_{xy}^2 + r_{xz}^2 + r_{yz}^2 - 2r_{xy}r_{xz}r_{yz} - 1)]}}{2\sqrt{2}\pi^{3/2}\sqrt{1 - (r_{xy}^2 + r_{xz}^2 + r_{yz}^2) + 2r_{xy}r_{xz}r_{yz}}},$$

(14)

where

$$w = x^2(r_{yz}^2 - 1) + y^2(r_{xz}^2 - 1) + z^2(r_{xy}^2 - 1) + $$
$$2[xy(r_{xy} - r_{xz}r_{yz}) + xz(r_{xz} - r_{xy}r_{yz}) + yz(r_{yz} - r_{xy}r_{xz})].$$

15)

More complicated interactions of normally distributed variables must be discussed separately.

Computer simulation of the scale measurement

The simulation of angular scale measurement starts at the *Measurement* control group at the upper left corner or the program window (Fig. 2). The number of strokes to simulate can be chosen virtually. However, to be able to create a static array for calculations in the computer program the upper limit of 720 strokes has been chosen having been noted that this number represents a sensible value from a measurement standpoint (each stroke divides the circle into half a degree), and at this number of strokes individual strokes still remain visible. The visualization can also be adjusted by choosing a line width for strokes from the Line width list on the right of the control group.
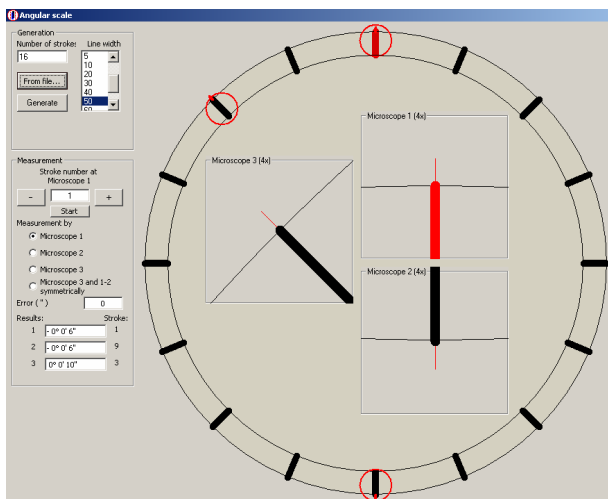

Fig. 2. The program screen after generating results for 16 strokes. The width of stroke lines has been increased to 50 for a better visual representation

Nevertheless, no matter how many strokes are entered the program still computes the results displaying them in the Measurement group at the centre left of the screen at the three strokes near the strokes' position reading microscopes as well as generating a results file with the results at every stroke.

Conclusions

- Mathematic dependencies for theoretic assessment of multiparameter mechatronic systems are proposed for the investigation of their accuracy and for the development of correction of micro- and nano- displacement control means.
- Information entropy assessment is given allowing to evaluate the calibration of the circular scales or rotary encoders supplementing it by an information which part of it all was assessed during this process.
- Computer simulation means are developed for the measuring information evaluation of the circular raster scales with the possibility to select wide range of parameters of the scale.

Acknowledgements

References

[1] **Ash R. B.** Information Theory, Dover Publications, New York, 1990. 352 p.
[2] **Bansevicius R., Giniotis V.** *Mechatronic means for machine accuracy improvement,* Mechatronics, 12, 2002, p. 1133-1143
[3] **Jablonski R., Shimokohbe A. and Nagai A.** Calibration system for precision angle standards. *Bull. P.M.E. (T.I.T.)*, No 45, 1980, p. 17-24.
[4] **Rybokas M.** The information analysis and the research on entropy for measurement data: Summary of Doctoral Dissertation, Vilnius: Technika, 2006, 25 p.
[5] **Bansevičius R., Barauskas R., Giniotis V. and Ragulskis K**. *Control of pitch accuracy of measuring grids (rasters)* (Vibration Engineering, Vol. 1, No 1, 1987), p. 251-262
[6] **Bansevičius R., Knight J. A. G.** *Intelligent Mechanisms with Piezoactive Links: State of the Art, Problems, Future Developments* (Proc. of 10th World Congress on the Theory of Machines and Mechanisms Oulu, Finland, 1999), p.2008-2013
[7] **Giniotis V., Bansevicius R. and Knight J. A. G.** *Complex accuracy assessment of multicoordinate machines* (Proceedings of the 6th ISMQC IMEKO Symposium Vienna, Austria, 1998), p. 195-199.
[8] **Giniotis V.** Position and Displacement Measurement. Vilnius: Technika, 2005, p. 215 (Padėties ir poslinkių matavimas) (in Lithuanian).

296