359. Parametrization-based shape optimization of shell structures in the case of free vibrations

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Abstract: A finite-element-based shape optimization methodology has been developed for three-dimensional shell structures and shape optimization of shell structures has been performed. The shape optimization program is implemented by a job control language and commercial finite element analysis software ANSYS is used for structural analysis. Principles of structural analysis and automatic mesh generation are applied for achieving shape optimization. The objective is to minimize the weight of the shell structure under frequency constrains and the move limit for each design variable. In this paper several optimization examples are provided.

Keywords: Shape optimization, shell structures, finite element analysis, parametrization.

Introduction

Mathematical methods for structural optimization and shape optimization have been developed in the last 30 years [1,5]. In recent years, shape optimization of shell structures has attracted more attention. Through the increase in capacity and speed of modern computers and the theoretical foundations from before, together with modern optimization algorithms, significant progress has been made in the development of finite element-based shape optimization [7,8]. The design of shell structures under dynamic loads is a common problem in engineering practice. In order to obtain an optimal design of these structures one generally seeks to keep their weight or volume as low as possible (i.e. to minimize their cost), while constraining their structural response in terms of displacements, accelerations, frequencies or stress resultants. Alternatively one can minimize the displacement or acceleration at some point of the structure or its global displacement while keeping its volume constant. In the case of free vibrations the objective is to maximize the frequency, corresponding to the vibration mode that one seeks to stiffen, keeping the shell volume constant. In special cases of shells with multiple eigenvalues, the intention is to keep their volume as low as possible considering frequency constrains in order to avoid clusters. This paper presents a finite element-based shape optimization program that has been developed to perform automatic shape optimization of three-dimensional shell structures. An example of shape optimization of shell structures is provided as well. The objective is to minimize the weight of shell structures with constraints that are the ranges of natural frequencies and the move limit for each design variable. Example is provided to demonstrate the capabilities of this shape optimization program The implementation is performed based on the ANSYS code.

Modeling approach

In shape optimization process, the shape of a structure is changed in each iteration step. In this case, a fixed finite element mesh is no longer appropriate. The finite element mesh should be updated in each iteration for the new shape, loading conditions and any possible distortion of elements. It is obvious that the design variables that control the optimization model should also control the finite element mesh. In the finite element displacement approach, the modal analysis consists of solving the following equilibrium equation:

$$K\varphi = \omega^2 M\varphi \, . \tag{1}$$

where K is the structural stiffness matrix, formed by assembling all element stiffness matrices, M is the structural mass matrix, φ is the unknown eigenshape matrix and ω is the natural frequency vector.

A shape optimization problem is to find $b \in Z^n$ and $\omega \in Z^m$, minimizing the objective function $\Phi_0(b)$. Here b are the optimization parameters, and ω are the state variables, Z^n and Z^m are the *n*- and *m*-dimensional real number spaces correspondingly. The state equation for vibratory process can be written in the following form:

$$K(b)\varphi = \omega^2 M(b)\varphi \, , \tag{2}$$

subject to constrains:

$$\psi(b,\omega) \geq 0 . \tag{3}$$

The corresponding quantities may be developed into the following form:

$$\Phi_0(b) = \Phi_0(b_1, b_2, ..., b_n), \tag{4}$$

$$\psi(b,\omega) = [\psi_1(b,\omega), \psi_2(b,\omega), ..., \psi_m(b,\omega)]^T, \tag{5}$$

$$K(b) = \left[K_{ij}(b)_{ij}\right], i, j = \overline{1,l} . \tag{6}$$

Eq. (6) is the stiffness matrix of the considered structure, while the mass matrix has the following form:

$$M(b) = \left[M_{ij}(b)_{ij}\right], i, j = \overline{1,l} . \tag{7}$$

where l is the number of degrees of freedom of the structure.

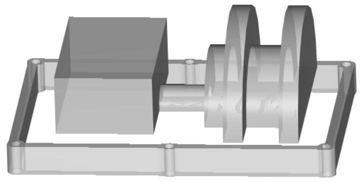
Geometrical size limitations for the shell might be present as well – for example, the height of the corner. In the beginning a rough evaluation of the shell geometric limits must be made. This is followed by the concept refinement phase with the initial model taken into consideration, aiming to minimize the shell mass by means of its geometric shape modification. The basic algorithm for structural shape optimization can be summarized in the following steps: (1) Initial shell geometry generation; (2) Application of the special technique to initialize and control the shape of the shell during the further optimization; (3) Performing the optimization analysis. Each step will be the detailed in the following chapters.

Modeling geometry of the mechanism

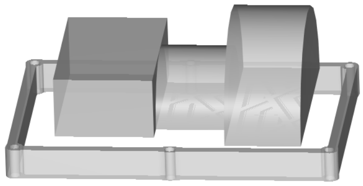
A part or multiple parts of a mechanism that are not fully enclosed in the base body need to be discretized. It means that the complicated form of the mechanism must be simplified and described in terms of primitive volumes. The choice of mechanism discretizing volume is a focal point in this research. It is a very important phase of the whole shell optimization process because selected discretized geometry of the mechanism will strongly influence the final optimal shape of the shell.

The discretization of the mechanism is to be performed by using three main types of primitive volumes: boxes, spheres and cylinders, or parts of them. The guidelines for this procedure are described below.

Firstly, selected volumes must geometrically approach the mechanism from outside as closely as possible without challenging technological limitations and requirements. Moving parts of the mechanism must fully fit inside the selected volumes. The selected primitives must have a minimal volume.



(a)



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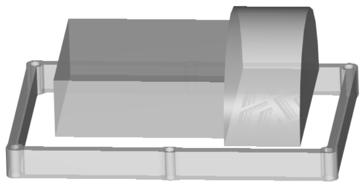


(c)

Fig. 1. Level of discretization. More complicated configuration illustrated in (a) should be replaced by simpler ones, as shown in (b) or (c)

The total number of discretizing volumes must be minimized. The coarser discretization is preferred over the detailed one as illustrated in Fig. 1. Detailed models would complicate the mathematical part of the optimization procedure and would significantly increase computational time. Therefore discretization should be performed using simpler configurations wherever possible (Fig. 1(b,c)).

The discretizing volumes must not intersect base body at fixation (shell ground) level and imaginary walls above it – such shell cannot be mounted. These requirements are purely geometrical.

All the primitives are then logically united into one body, called *discretizing body*. One of the main requirements for such a body is that any ray traced from the origin point may and must intersect the body surface in one and only point. This means that the discretizing body should not have hollow nor heavily concave regions. After the unification of the primitives, the discretizing body should contain minimum number of resulting surfaces. At the same time care should be taken in order to generate neighboring intersection lines of the comparable dimensions. Last but not least, the advantages of the symmetry must be exploited where possible.

The discretization procedure is based on experience and cannot be strictly expressed in terms of numbers or equations. The result also depends on human factor and the intuition.

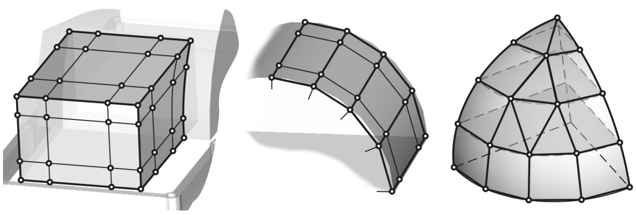
Initial shell model

The second important phase of the proposed shell optimization strategy is the selection of the initial shell geometry and its discretization. At this point, we have the discretizing body that is described as the union of some primitive volumes. Only a part of the body, which is above the baseline of the shell fixation, will be taken into consideration. The initial geometry of the shell is selected in a way that it replicates exactly the discretizing body that was generated in the first phase. The resulting surface is composed of three types of primitive surfaces or their parts: plane, cylindrical and spherical. This result follows from our previous choice of discretizing primitive volumes.

Since the final optimal geometry of the shell is unknown beforehand, we must provide a general description of the shell geometry to the finite element software in such a way that it could easily generate less or more possible shapes during the optimization process. At the same time, it must be able to generate smooth, transitional shapes. To accomplish that, the strategy of *master nodes* is proposed, relying on the generation of additional intermediate control points on each primitive surface of the initial shell. Each of three types of surfaces has different strategy of choosing the master nodes.

For plane surfaces the main control master nodes are located in the corners of plane areas. Additional master nodes are put at the intersections of auxiliary lines, and also at the intersections of auxiliary lines and the area boundary (Fig. 2 (a)).

For cylindrical surfaces, the master nodes on plane parts of cylinders are taken following the same procedure as for plane areas. The proposed locations of master nodes for cylindrical surfaces are shown in Fig. 2(b). The number of divisions on the arc of a cylinder depends on required accuracy.



(a) (b) (c)

Fig. 2. Location of master nodes for various types of surfaces: (a) plane, (b) cylindrical, (c) spherical

The choice of master nodes on spherical surfaces is more liberal. An example for a part of a sphere is shown in Fig. 2(c). The sphere is divided into several slices and several master nodes on each obtained arc are taken. It is in our interest to minimize their quantity since each additional master node increases computational time while optimizing.

Additional design master nodes must be taken on the perimeter of the base (body) line, which serves for shell fixation. These new master nodes are chosen at the minimal distance from the nearest point of the nearest surface. At the same time, the master nodes of the shell at the baseline level are dismissed. Having all these points, we may generate a thin shell model, which is composed of numerous triangular or rectangular areas.

Optimization

Naturally, each master node may move in three directions in space. Multiplying the number of optimization design variables by three is not an attractive perspective. Moreover, it would be very difficult to control mutual positions of the master nodes during the optimization. And finally, geometric limitations for location of the master nodes are quite difficult to describe and impose, especially for more complicated discretizing shapes. Therefore a new strategy is proposed.

The master node is considered as an end point of the vector $\overrightarrow{R_i}$, whose origin coincides with the global origin O, which lies at the gravity point of the baseline figure. The master node can only be displaced in one single direction, called *optimization direction*. This direction is collinear with the described vector (Fig. 3). Each master node is provided by a constant and individual optimization direction in space, described by spherical coordinates φ_i and θ_i. As the method requires choosing the master nodes on the mechanism discretizing volume surface, this is the limit position for the master node and the minimum length for the vector $\overrightarrow{R_i}$. One single common origin should be taken for all master nodes (though several origins may be present in the special cases).

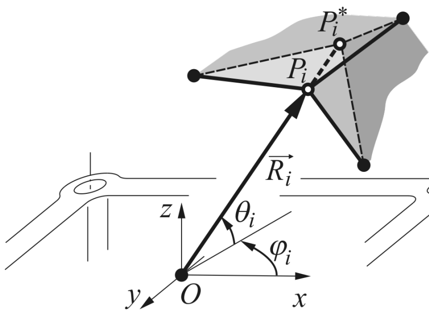


Fig. 3. The master node P_i is described by a vector $\overrightarrow{R_i}$ with constant spherical coordinates φ_i and θ_i. P_i^* shows one of the possible intermediate master node's P_i locations during optimization

The designed shell will serve as an acoustic shield for the mechanism beneath. Each mechanism has its nominal frequency or several frequencies, or a frequency range that is the most probable while functioning. It is our interest to design a shell that would not resonate at these working frequencies. That means that we must choose state variables outside normal operating frequencies of the mechanism. Therefore, state variables of the optimization procedure are the first m natural frequencies in working range.

As it was discussed before, the optimization objective function is to minimize the mass of the shell. We consider only the shells with uniform and constant thickness of the walls. In this case, the shell mass will be directly proportional to its surface area, and the objective function will be:

$$\Phi = min\, A(k_1 ... k_n), \tag{8}$$

subject to constrains:

$$1 \le k_i \le k_{i\,max}\,,\; i = \overline{1, n}\,, \tag{9}$$

and state variables:

$$\omega_{j\,min} \le \omega_j \le \omega_{j\,max}\,,\; j = \overline{1, m}\,, \tag{10}$$

yielded by vibration equation of the structure:

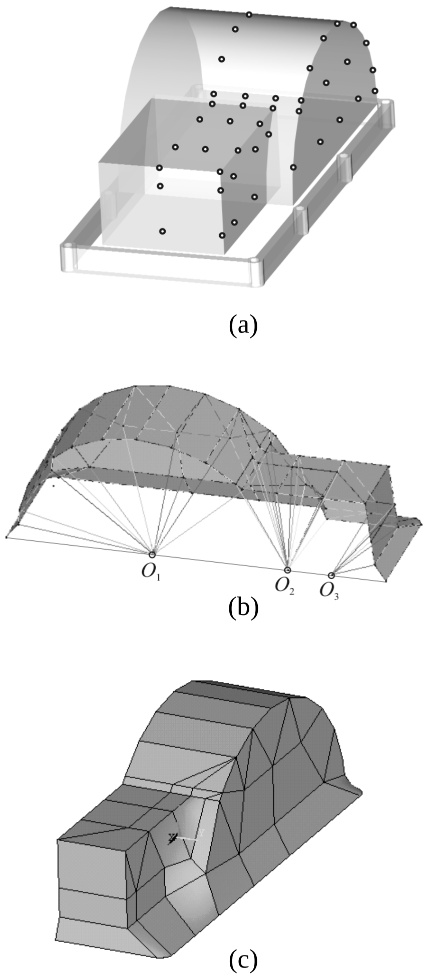


Fig. 4. Example of initial shell generation process: (a) the choice of discretizing volumes and the locations of master nodes; (b) three origins and the optimization directions; (c) discretization of the initial shell into triangular and rectangular areas

$$[K(k)]\varphi = \omega^2 [M(k)]\varphi\,. \tag{11}$$

where n is the number of master nodes, $k_i = OP_i^* / OP_i$ is the scaling factor for the i-th master vector (Fig. 3). So the optimization variables are relative elongations of the master vectors.

We may freely select simulation software and the optimization method for performing the optimization analysis. In most cases, finite element modeling software packages offer one or several algorithms for analysis. A multi-parameter optimization tool is necessary. Commands for the construction of the geometric model, for meshing it and then applying loads, are incorporated in a separate command file using ANSYS Parametric Design Language (APDL) included into this finite element package. Variations in the model are produced by altering the numerical parameters in APDL file. The values for the modal analysis are the required results extracted from the result file of ANSYS, since most software records very detailed information in a result file.

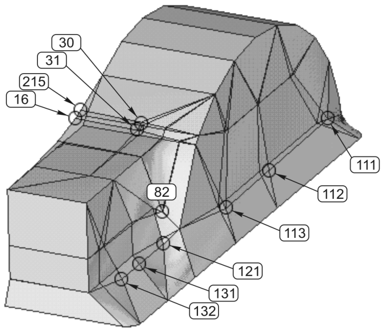
Example and results

Let us optimize a shell for a mechanism that may be similar to the differential slip mechanism. To begin with, the underlying mechanism is discretized by volumes as demonstrated in Fig. 4(a). The initial shell is generated, and the master nodes are selected on its walls. The master nodes on the baseline level are replaced by constant points on fixation perimeter of the shell. We choose three different origins for master vectors for groups of nearby located points to better describe their optimization directions (Fig. 4(b)). Shell symmetry condition is applied while modeling. The final step is to produce the initial shell that is composed of plane triangular and rectangular areas as indicated in Fig. 4(c).

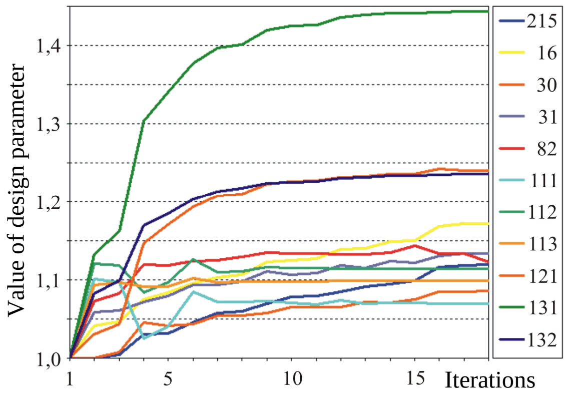
For illustration of this method ANSYS software is used. The method used is a first order approximation, where linear search step for each design variable gradient calculation is performed [6]. The task in this case is to minimize shell mass, while maintaining ω_1 higher than the first natural frequency of the non-optimized shell.

We have two discretizing volumes, for mechanism with base length approximately 23 cm, width 12 cm and height 9 cm. Triangular-type "SHELL63" element is used, which provides six degrees of freedom at each node, *ux*, *uy*, *uz*, rot*x*, rot*y* and rotz. Nylon is supposed as the material for the shell, with Young's modulus of 2100 MPa, Poisson's ratio of 0.4 and density equal to 1140 kg/m^3. Nylon is one of the most frequently used materials for such type of shells. Additional control condition is used: first natural frequency of the shell must not be lower than ω_1 = 100 Hz. It is the state variable during the optimization process.

After the optimization process, a shell with an optimal shape was obtained (Fig. 5(a)). Surface area (and mass) of the optimal shell was reduced by 2,9% in comparison to the initial shape, while the first natural frequency is by 58% larger than the required condition. Circles in Fig. 5(a)



(a)



(b)

Fig. 5. Comparison of initial and optimal shape of model A: (a) optimal shape (semitransparent) overlaid over initial shell. Circles indicate master nodes displaced the most during optimization; (b) Variation of design parameters that changed the most during optimization

indicate master nodes that changed position the most during optimization. It may be noticed that the most pronounced changes of shell shape occurred at intersections of discretization volumes as well as in places where the shell is joined with the base. Optimal shell is characterized by smooth transitions between planes. Furthermore, the edges and surfaces at the very top remain unchanged (this reminds vacuum package). Distribution density of master nodes is relatively small in places of the largest changes of design variables. In order to obtain smoother surface and more accurate result it is sensible to increase number of master nodes in these places (Fig. 6). In addition it is preferable to remove master nodes that are close to each other on different initial surfaces. Thus, in total, 12 master nodes are added in places of the largest changes of shell shape and 4 master nodes are eliminated from the initial shell. Some of the master nodes are repositioned so as to obtain similar spacing between them. Finally, location of centre of master vector O_2 is modified by displacing it towards O_1 (Fig. 4(b)). Thereby we obtain new model B. After performing optimization on this new model, an optimal shell shape is obtained (Fig. 7(a)):

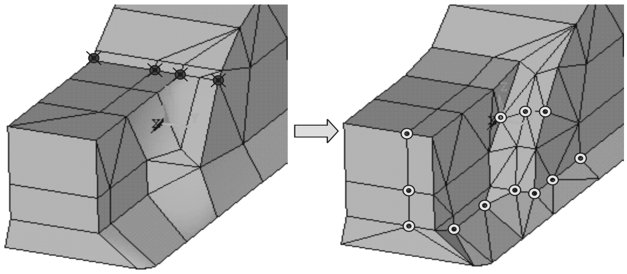
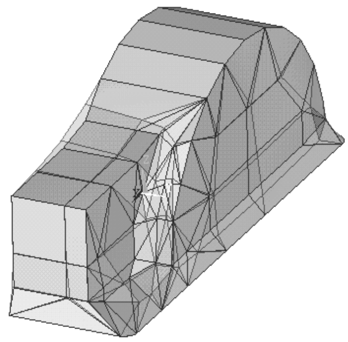
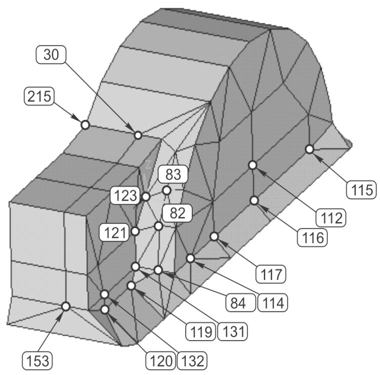


Fig. 6. Selection of additional master nodes (white rings on the right picture) and elimination of non-effective master nodes (black crossed circles on the left).

surface area as well as mass of the optimal shell was reduced by 5,2% in comparison to the initial shape, while the first natural frequency is by 55% larger than the required condition for state variable ω_1. Circles in Fig. 7(b) indicate master nodes that undergone the largest change of position during the optimization process.



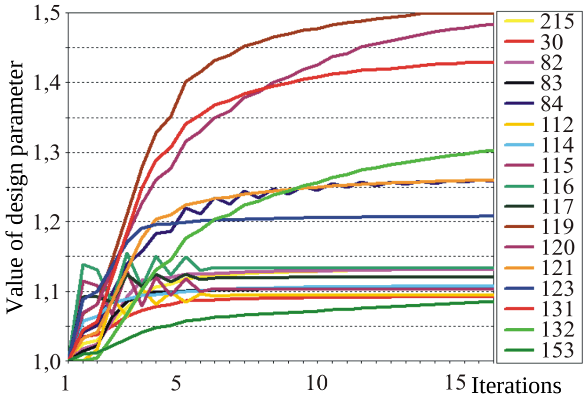
(a)



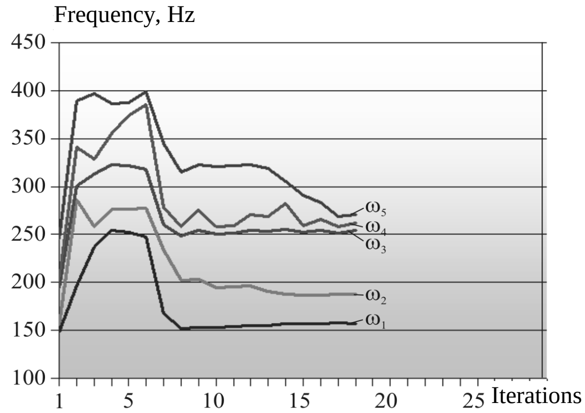
(b)

Fig. 7. Comparison of initial and optimal shape of model B: (a) optimal shape (semitransparent) overlaid over initial shell; (b) Circles indicate master nodes that were displaced the most during optimization.

Increase of density of distribution of master nodes resulted in a smoother description of geometrical shape of transitional zones. Similarly to the case with model A, the most pronounced changes of shell shape of model B are observed at intersections of discretization volumes as well as in places where the shell is joined with foundation (Fig. 7(b)). Circles in the figure indicate those master nodes that were displaced the most. Fig. 8(a) presents variations of design parameters that describe spatial position of master nodes in the course of optimization process.



(a)



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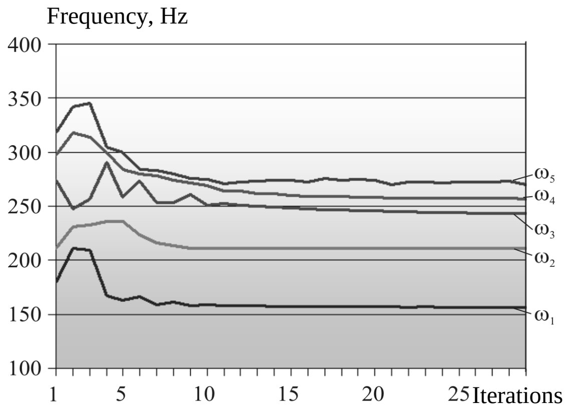

(c)

Fig. 8. (a) Variation of design parameters that changed the most during optimization. Variation of the first 5 natural frequencies of the shell during optimization in case of model A (b) and model B (c)

One of the defined objectives of the shell optimization was to retain required dynamical characteristics, i.e. the structure cannot be excited at pre-determined vibration frequency. Fig. 8(b,c) demonstrates variation of the first 5 natural frequencies of the shell during optimization. It may be observed that the character of variation is similar in the case of model A and model B. After 8-10 iterations the shell shape approaches an optimal one and the frequency curves attain constant value. The corresponding natural frequencies of the optimal shell are correlated in the case of different distributions of master nodes.

Conclusions

In this paper, a finite-element-based shape optimization program has been developed for the three-dimensional shell structures. To achieve shape optimization, different principles such as modal analysis, automatic mesh generation have been integrated. For the analysis of the models, finite element software ANSYS was used. The obtained optimal shapes have been presented in the paper.

General-purpose shape optimization program has been developed, which optimizes structures by controlling natural frequencies. The program is applicable for a variety of problems with little modification. Taking into account obtained results several main conclusions may be formulated. First, and most important, the presented method may be successfully employed for the optimization of the shells of pre-conceived mechanisms. The method for shell shape optimization presented here can decrease noticeably the final mass of the shell, while maintaining its natural frequencies in the initial or given range. The proposed method allows reducing the number of optimization variables by a factor of three and therefore results in reduction of required computational time, depending on the optimization routine used. Overall, the stability of such modeling approach is very high when compared with traditional 3D point displacement description. Shape generation stability and quality is far superior to the traditional methods. One of the main advantages of this method is the software-independent approach. The convergence relies on the optimization software and its internal numeric routines so finite element modeling software must have a reliable multi-parameter optimization tool. The method relies on changing locations in the space of multiple master nodes that is why some recommendations presented above must be followed.

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