

## 325. Interaction of Vibrating and Translational Motions

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**Abstract.** The mathematical model describing the interaction of a vibrating system and a system performing translational motion is proposed. Such motions take place in vibromotors. The main characteristics of the dynamic motion are defined. The model is investigated numerically and a number of graphical relationships are obtained. They are used in the process of design of systems with vibromotors like a walking inpipe robots.

**Keywords:** vibromotor, dynamical system, characteristics of motion, numerical integration, robot.

### Introduction

The mathematical model describing the interaction of a vibrating system and a system performing translational motion is proposed in the form of the matrix differential equation describing the motion of a nonlinear vibrating system with two degrees of freedom. Such motions take place in vibromotors [1, 2].

The main characteristics of the dynamic motion are defined. The model is investigated by using numerical methods [3]. The time histories of motion and the graphical relationships representing the characteristics of motion are obtained.

The obtained graphical relationships are used in the process of design of systems with vibromotors.

### Model of the system

The dynamics of interaction of a vibrating system and a system performing translational motion is described by the matrix differential equation of the second order:

$$[M] \begin{Bmatrix} \frac{d^2 u_1}{dt^2} \\ \frac{d^2 u_2}{dt^2} \end{Bmatrix} + [C] \begin{Bmatrix} \frac{du_1}{dt} \\ \frac{du_2}{dt} \end{Bmatrix} + [K] \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \{F\}, \quad (1)$$

where  $u_1$  is the displacement of the first mass and  $u_2$  is the displacement of the second mass,  $t$  is the time variable, the mass matrix has the form:

$$[M] = \begin{bmatrix} 1 & 0 \\ 0 & \mu \end{bmatrix}, \quad (2)$$

where  $\mu$  is the mass of the second element of the analyzed system, the damping matrix has the form:

$$[C] = \begin{bmatrix} h_1 & 0 \\ 0 & h_2 \end{bmatrix}, \quad (3)$$

where  $h_1$  is the coefficient of viscous friction of the first element of the analyzed system and  $h_2$  is the coefficient of viscous friction of the second element of the analyzed system, the stiffness matrix has the form:

$$[K] = \begin{bmatrix} n_1^2 & 0 \\ 0 & 0 \end{bmatrix}, \quad (4)$$

where  $n_1$  is the stiffness parameter of the first element of the analyzed system, the loading vector has the form:

$$\{F\} = \begin{Bmatrix} a \sin \omega t \\ f_2 \end{Bmatrix}, \quad (5)$$

where  $a$  is the amplitude of vibration excitation,  $\omega$  is the frequency of vibration excitation and  $f_2$  is the constant force.

When:

$$\frac{du_1}{dt} > \frac{du_2}{dt}, \quad (6)$$

the following matrix is added to the damping matrix:

$$[\hat{C}] = \begin{bmatrix} \lambda & -\lambda \\ -\lambda & \lambda \end{bmatrix}, \quad (7)$$

where  $\lambda$  is the large coefficient of viscous friction. Thus the constraint of motion of the two masses together is taken into account approximately by the penalty method. The numerical integration of the equations is performed by the method of constant average acceleration of Newmark. The acceptable large value of  $\lambda$  depends on the time step of numerical integration.

### Characteristics of motion of the system

The most important characteristics of dynamical motion of the analyzed system are:

1) the average velocity of the second mass:

$$\overline{\frac{du_2}{dt}} = \frac{1}{T} \int_T \frac{du_2}{dt} dt, \quad (8)$$

where the overbar denotes time averaging, the integral is over the period of steady state motion and the period is expressed as:

$$T = \frac{2\pi}{\omega}; \quad (9)$$

2) the useful work:

$$A_n = f_2 \int_T \frac{du_2}{dt} dt; \quad (10)$$

3) the work of the driving forces:

$$A_v = a \int_T \frac{du_1}{dt} \sin \omega t dt; \quad (11)$$

4) the coefficient of useful operation:

$$\eta = \frac{A_n}{A_v}. \quad (12)$$

### Results of analysis

The following parameters of the analyzed system are assumed:

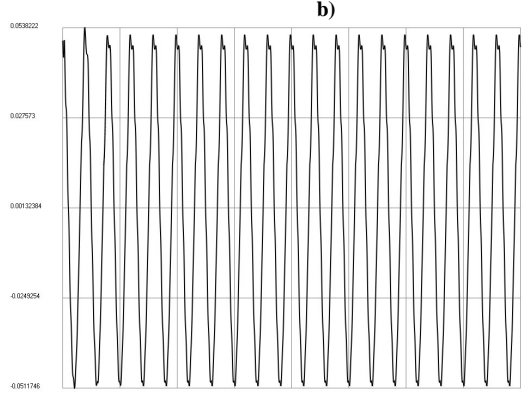
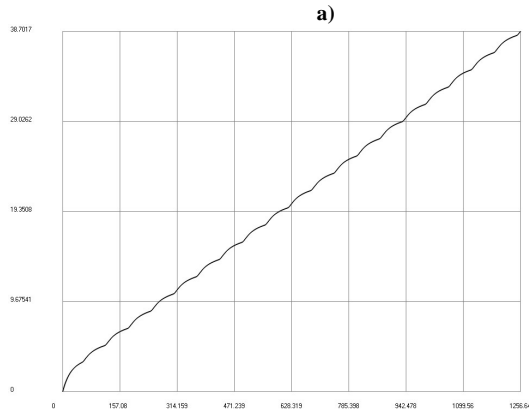
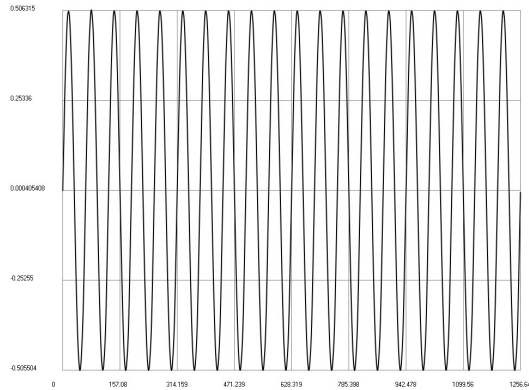
$$h_1 = 0.1, h_2 = 0.1, \mu = 2, n_1 = 1, a = 0.5, f_2 = .001, \lambda = 100. \quad (13)$$

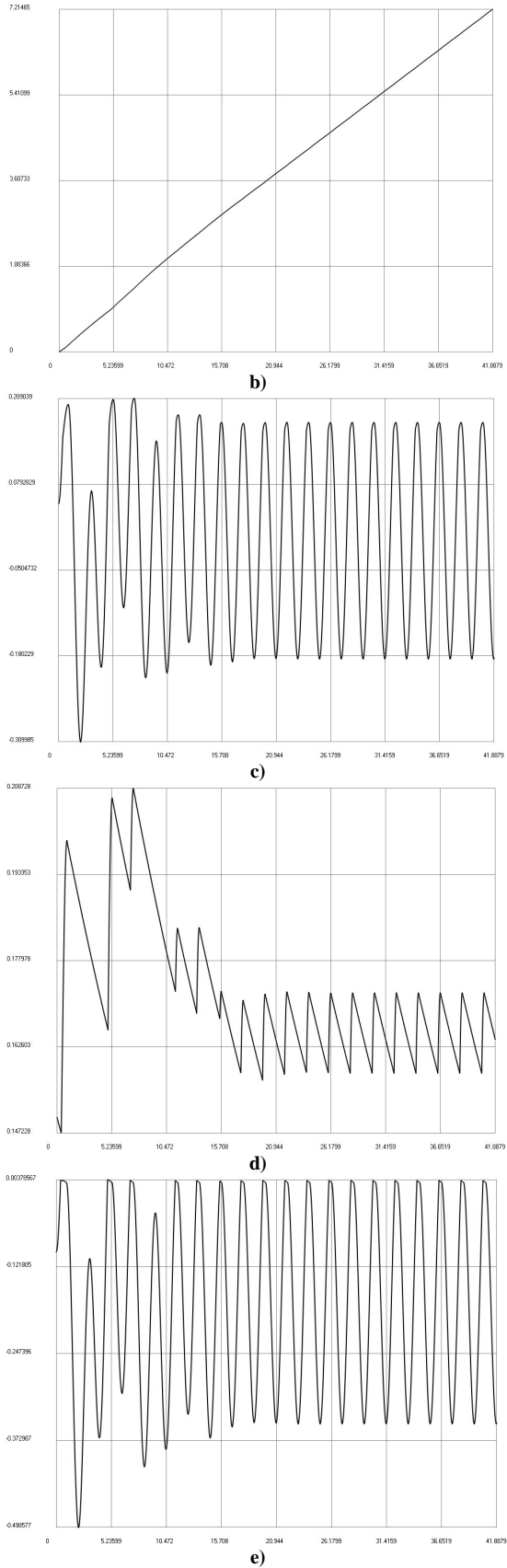
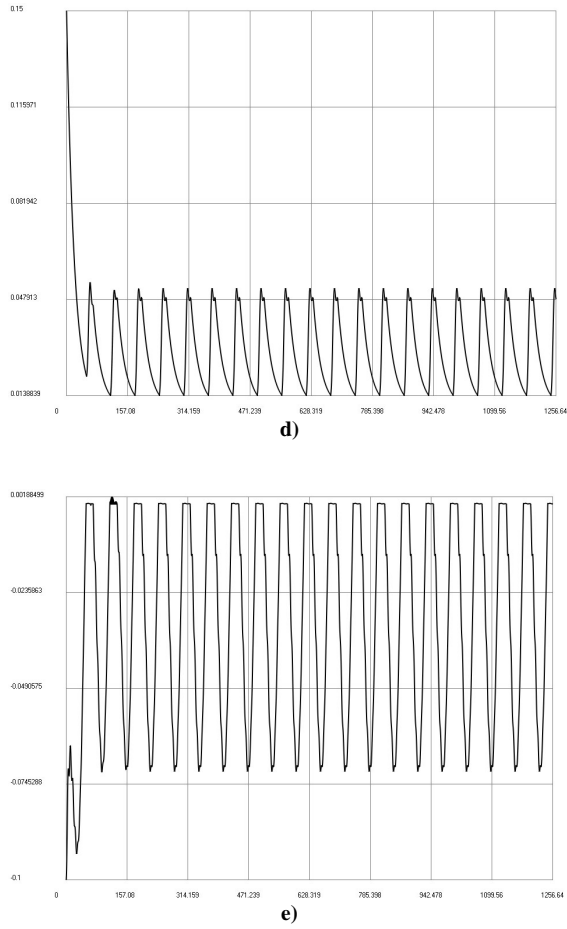
The initial conditions are taken as:

$$u_1(0) = 0, u_2(0) = 0, \frac{du_1}{dt}(0) = 0.05, \frac{du_2}{dt}(0) = 0.15. \quad (14)$$

When  $\omega = 0.1$  the time history of motion of the first element of the system is presented in Fig. 1a and of the second element in Fig. 1b. The corresponding velocities are presented in Fig. 1c and Fig. 1d. The quantity

$\frac{du_1}{dt} - \frac{du_2}{dt}$  is presented in Fig. 1e.





**Fig. 1.** When  $\omega=0.1$ : a)  $u_1$ , b)  $u_2$ , c)  $du_1/dt$ , d)  $du_2/dt$ , e)  $du_1/dt-du_2/dt$

When  $\omega=3$  the time history of motion of the first element of the system is presented in Fig. 2a and of the second element in Fig. 2b. The corresponding velocities are presented in Fig. 2c and Fig. 2d. The quantity  $\frac{du_1}{dt} - \frac{du_2}{dt}$  is presented in Fig. 2e.

**Fig. 2.** When  $\omega=3$ : a)  $u_1$ , b)  $u_2$ , c)  $du_1/dt$ , d)  $du_2/dt$ , e)  $du_1/dt-du_2/dt$

Further the main characteristics of motion as functions of the frequency of excitation are presented. The average velocity of the second mass is presented in Fig. 3. The useful work is presented in Fig. 4. The work of the driving forces is presented in Fig. 5. The coefficient of useful operation is presented in Fig. 6.

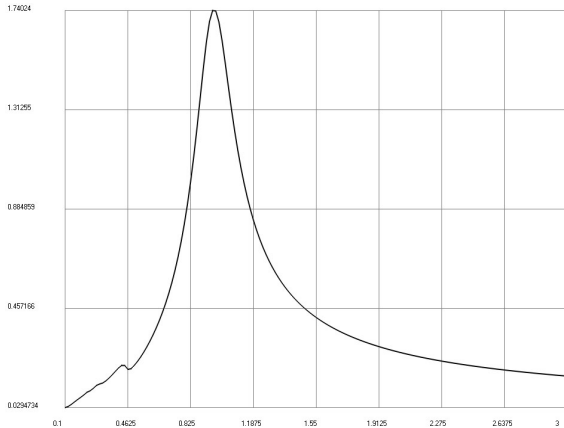


Fig. 3. Averaged  $du_2/dt$  as a function of  $\omega$

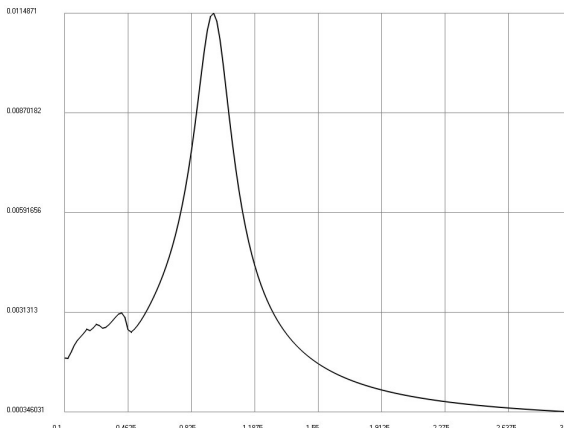


Fig. 4.  $A_n$  as a function of  $\omega$

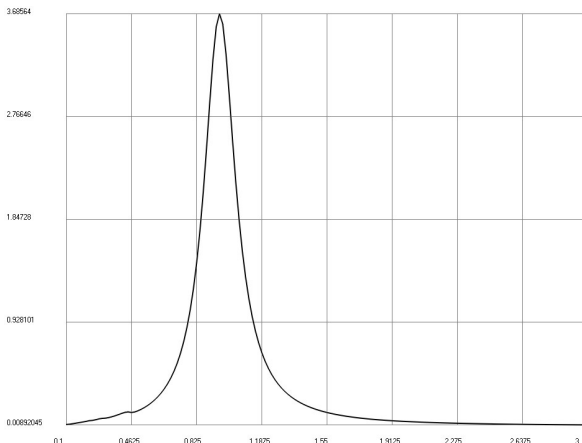


Fig. 5.  $A_v$  as a function of  $\omega$

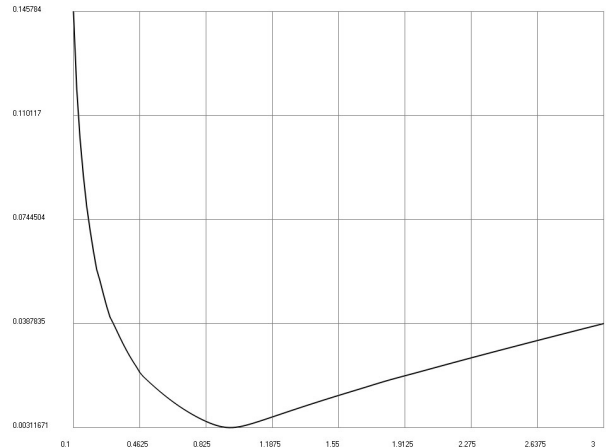


Fig. 6.  $\eta$  as a function of  $\omega$

### Construction Working Principle and characteristics of motion of the Robot vibromotor drive

A drive of the wall press walking inpipe robot with pneumatic vibromotor drive [4,5] is illustrated in Fig. 7 and Fig. 8. Robots drive consists of two locking blocks 1, which can turn by hinges 2 small angles one in point of other and interconnected, and propelled by two-way working pneumatic cylinder 3.

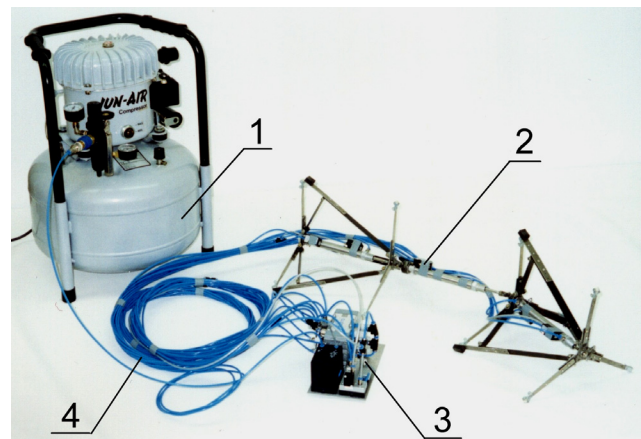
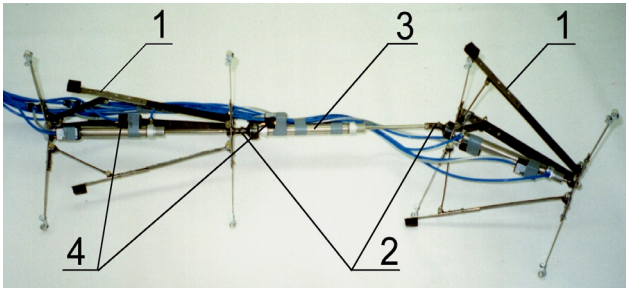


Fig. 7. The scheme of the walking inpipe robot with vibromotors: 1 – compressor, 2 – robots drive, 3 – operating block, 4 – pipes

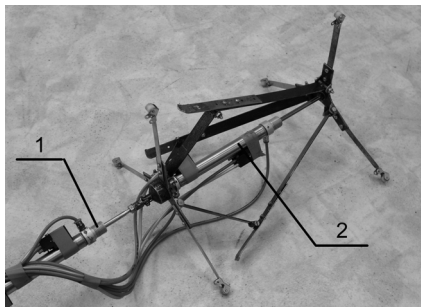
As on moving pneumatic cylinder as on locking blocks pneumatics cylinders are fixed pneumatic-magnetic operating sensors 4. They sending pneumatic signals to operating block and processed in operating block signals feed to cylinders of pneumatic drive.



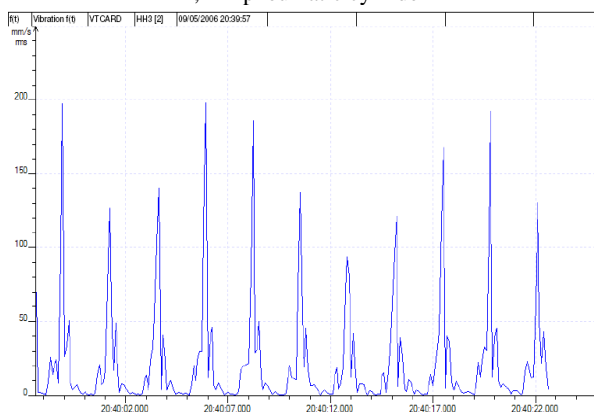
**Fig. 8.** A drive scheme of the wall press walking inpipe robot with vibromotor: 1 – locking block, 2 – hinge, 3 – pneumatic cylinder, 4 – operating sensors

Principle of robot working based on walking movement, periodically pressing the wall of pipeline with supporting elements of locking block. Pressured air gets into two-way working cylinder of locking block and its drawing stock in contact between supporting elements and pipeline wall fix a locking block. Than one of two locking blocks is fixed, pneumatic cylinder of other locking block working in reverse way. Supporting elements of second locking block is releasing by going out stock of cylinder. After stock of shift pneumatic cylinder going out/or in depend on which locking block contact with pipeline inside wall. In that way the wall press walking inpipe robot make a move forward.

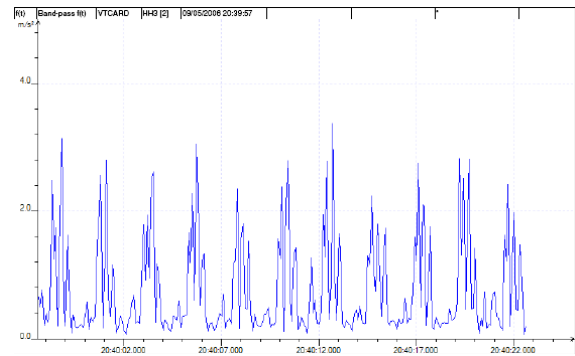
Having a physical experiment of this inpipe robot vibromotor drive pneumatic cylinder 1 presented in Fig. 9 was obtained the dynamical parameters. Average velocity and acceleration is presented in Fig. 10 and Fig. 11 .



**Fig. 9.** A scheme of the pneumatic vibromotor drive:  
1, 2 – pneumatic cylinder



**Fig. 10.** The velocity of the pneumatic vibromotor drive over the time



**Fig. 11.** The acceleration of the pneumatic vibromotor drive over the time

## Conclusions

The mathematical model describing the interaction of a vibrating system and a system performing translational motion is proposed in the form of the matrix differential equation describing the motion of a nonlinear vibrating system with two degrees of freedom. This model is convenient for performing the numerical integration of the equations of motion.

The main characteristics of the dynamic motion are defined. The time histories of motion and the graphical relationships representing the characteristics of motion are obtained.

The presented results are used in the process of design of systems with vibromotors.

The scheme of the construction of the walking inpipe robot with pneumatic vibromotor drive is given. The presented results of this drive physical experiment of dynamical parameters.

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