276. Slow Translation and Fast Rotation Motions of an Unbalanced Particle Subjected to Propagating Wave

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Abstract. The analysed system has three degrees of freedom in a plane: translation motion in the direction of wave propagation, oscillatory motion in the direction orthogonal to the direction of wave propagation, and the rotation motion. The dynamical parameters of the system are approximated for steady state motions. It is shown that the particle can be translated by the velocity much lower that the velocity of wave propagation. At the same time it can experience high angular frequency rotation motion. The condition of existence of such type of regime of motion is determined. Such modes of motion can exist in gaseous of liquid environment whenever an unbalanced particle is subjected to propagating longitudinal waves.

Keywords: unbalanced particle, propagating wave, translation and rotation motion

1. Introduction

Rotation motion occurs in different unbalanced systems subjected to waves and vibrations [1 – 4]. Transportation and manipulation of objects by propagating waves is analysed in a number of studies [5, 6]. Conveyance of particles and bodies by propagating waves is an important scientific and engineering problem with numerous applications. Particle segregation in suspensions subject to ac electric fields [7], transport of sand particles and oil spills in coastal waters [8], powder transport by piezoelectrically excited ultrasonic waves [9], transportation of thin films in biomedical applications [10] are just few examples of problems involving interaction between propagating waves and the transported objects. Conveyance of unbalanced rotors by propagating waves is an important problem. The object of this paper is to analyse steady state motions of such systems when its translation velocity in the direction of wave propagation is slow and rotation velocity is fast.

2. The model of the system

The schematic diagram of the system is presented in Fig. 1. Coordinates of points A and B are:

$$A(u + \eta, y)$$
$$B(u + \eta + r\sin\varphi, y - r\cos\varphi) \tag{1}$$

where $r = |AB|$; A is the axis of rotation of the rotor; B is the location of the concentrated unbalanced mass m; φ is the angle of deflection of the unbalanced mass from y-axis; $\eta = \eta(u,t)$ describes the longitudinal propagating wave.

The kinetic energy and dissipative system's function are:

$$2T = \left(I_0 + mr^2\right)\dot\varphi^2 + (m_0 + m)\left((\dot u + \dot\eta)^2 + \dot y^2\right) +$$
$$+2mr\left((\dot u + \dot\eta)\cos\varphi + \dot y\sin\varphi\right) \tag{2}$$

$$2D = H_x\left(1 + \eta'\right)\dot u^2 + H_y\dot y^2 + H_\varphi\dot\varphi^2$$

where m_0 is the mass of the particle without the unbalance; H_x, H_y and H_φ are viscous damping coefficients according to respective coordinates; top dot denotes full derivative by time;

$$\eta' = \partial\eta/\partial u\,;\ \eta'_t = \partial\eta/\partial t\,. \tag{3}$$

The governing differential equations of motion take the following form:

$$\left(I_0 + mr^2\right)\ddot\varphi + mr\left((\ddot u + \ddot\eta)\cos\varphi + \ddot y\sin\varphi\right) + H_\varphi\dot\varphi = M_\varphi$$
$$(m_0 + m)(\ddot u + \ddot\eta) + mr\left(\ddot\varphi\cos\varphi - \dot\varphi^2\sin\varphi\right) + H_x\left(1 + \eta'\right)\dot u = F_x \tag{4}$$
$$(m_0 + m)\ddot y + mr\left(\ddot\varphi\sin\varphi + \dot\varphi^2\cos\varphi\right) + H_y\dot y = 0$$

where M_φ is external force moment according to variable φ; F_x is external force according to coordinate x;

$$\dot\eta = \eta'\dot u + \eta'_t$$
$$\ddot\eta = \eta'\ddot u + \eta''\dot u^2 + 2\eta''_{ut}\dot u + \eta''_{tt} \tag{5}$$

The following notations

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$$\psi = \frac{u}{r} \; ; \; \alpha = \frac{\eta}{r} \; ; \; \beta = \frac{y}{r} \; ; \; h_x = \frac{H_x}{m_0 + m} \; ;$$

$$h_y = \frac{H_y}{m_0 + m} \; ; \; h_\varphi = \frac{H_\varphi}{I_0 + mr^2} \; ;$$

$$m_\varphi = \frac{M_\varphi}{I_0 + mr^2} \; ; \; f_x = \frac{F_x}{(m_0 + m)r} \; ;$$

$$\mu = \frac{m}{m_0 + m} \; ; \; \nu = \frac{mr^2}{I_0 + mr^2} \quad (6)$$

reduce eq. (4) to:

$$\ddot{\varphi} + h_\varphi \dot{\varphi} = \Phi$$
$$\ddot{\psi} + h_x \dot{\psi} = \Psi \quad (7)$$
$$\ddot{\beta} + h_y \dot{\beta} = B$$

where

$$\Phi = -\nu\left((\ddot{\psi} + \ddot{\alpha})\cos\varphi + \ddot{\beta}\sin\varphi\right) + m_\varphi$$
$$\Psi = -\ddot{\alpha} - \mu\left(\ddot{\varphi}\cos\varphi - \dot{\varphi}^2\sin\varphi\right) - h_x\alpha\dot{\psi} + f_x \quad (8)$$
$$B = -\mu\left(\ddot{\varphi}\sin\varphi + \dot{\varphi}^2\cos\varphi\right)$$

In case of harmonic propagating wave:

$$\eta = A\cos(\omega t - ku)$$
$$\alpha = a\cos(\omega t - kr\psi) \quad (9)$$
$$a = \frac{A}{r}$$

Further we will analyse steady-state motion with slow average velocity:

$$\overline{\dot{\psi}} = v << \frac{\omega}{kr}. \quad (10)$$

Equations (7 – 9) are transformed to the following form:

$$\Phi = \varepsilon\Phi$$
$$f_x - h_x v - h_x\alpha'\dot{\psi} \approx \varepsilon(f_x - h_x v) \quad (11)$$
$$\omega t - kr\psi = \omega t - \varepsilon kr\psi$$

where ε is a small parameter. The change of variables:

$$\psi = vt + \tilde{\psi} \quad (12)$$

where $\tilde{\psi}$ is motion which average in time equals to zero helps to construct the solution in the form of power series:

$$\varphi = \delta t + \overline{\varphi} + \varepsilon\varphi_1 + \dots$$
$$\tilde{\psi} = \tilde{\psi}_0 + \varepsilon\psi_1 + \dots \quad (13)$$
$$\beta = \beta_0 + \varepsilon\beta_1 + \dots$$

where

$$\delta = \omega - krv \; ; \quad (14)$$

$\overline{\varphi}$ is constant quantity in time; φ_i ($i = 1, 2, \dots$); $\tilde{\psi}_j$, β_j are functions which time averages are equal to zero.

The first approximations of differential equations from (7, 8, 10-13) take the following form:

$$\ddot{\varphi}_1 + h_\varphi\dot{\varphi}_1 = -\nu\left((\ddot{\psi}_0 + \ddot{\alpha}_0)\cos\varphi_0 + \ddot{\beta}_0\sin\varphi_0\right) + m_\varphi - h_\varphi\delta$$
$$\ddot{\tilde{\psi}}_0 + h_x\dot{\tilde{\psi}}_0 = -\ddot{\alpha}_0 + \mu\delta^2\sin(\delta t + \overline{\varphi})$$
$$\ddot{\tilde{\psi}}_1 + h_x\dot{\tilde{\psi}}_1 = kr\tilde{\psi}_0\left(\frac{\partial\ddot{\alpha}}{\partial\delta t}\right)_{\psi_0 = 0} + f_x - h_x v + \tilde{\psi}_1 \quad (15)$$
$$\ddot{\beta}_0 + h_y\dot{\beta}_0 = -\mu\delta^2\sin(\delta t + \overline{\varphi})$$

where $\ddot{\alpha}_0 = -a\delta^2\cos\delta t$; and zero order approximations are:

$$\tilde{\psi}_0 = \frac{\lambda_x}{\delta}\left(a(-\delta\cos\delta t + h_x\sin\delta t) - \mu(h_x\cos(\delta t + \overline{\varphi}) + \delta\sin(\delta t + \overline{\varphi}))\right)$$
$$\beta_0 = \frac{\lambda_y}{\delta}\mu(h\cos(\delta t + \overline{\varphi}) + \delta\sin(\delta t + \overline{\varphi})) \quad (16)$$

top line denotes time average; time average of function $\tilde{\psi}_1$ is equal to zero;

$$\lambda_x = \frac{\delta^2}{\delta^2 + h_x^2} \; ; \; \lambda_y = \frac{\delta^2}{\delta^2 + h_y^2} . \quad (17)$$

The two unknowns δ and $\overline{\varphi}$ are determined exploiting the condition of periodicity of φ_1 and $\tilde{\varphi}_1$:

$$\overline{\Phi} - h_\varphi\delta = 0$$
$$\overline{\Psi}_0 - h_x v = 0 \quad (18)$$

or:

$$0.5\nu a\delta(-\lambda_x h_x\sin\overline{\varphi} + (1 - \lambda_x)\delta\cos\overline{\varphi}) = h_\varphi\delta - m_\varphi + 0.5\nu\mu\delta(\lambda_x h_x - \lambda_y\delta) \quad (19)$$
$$0.5kra\mu\lambda_x\delta(h_x\sin\overline{\varphi} - s\cos\overline{\varphi}) = h_x v - f_x - 0.5kra^2\lambda_x h_x\delta$$

Eq. (19) produces the condition of existence of the analysed regime of motion:

$$\left| \frac{kr(h_\varphi\delta - m_\varphi) + \nu(h_x v - f_x) + 0.5\nu krs\left((\mu^2 - a^2)\lambda_x h_x - \mu^2\lambda_y\delta\right)}{0.5\nu kra\mu(1 - 2\lambda_x)\delta^2} \right| < 1 \quad (20)$$

where v is determined from Eq. (14).

Only one unknown variable δ is present in inequality (20) and the regions of the existence of the analysed regimes of motion can be easily identified from this inequality.

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3. Conclusions

It is shown that unbalanced particle can be slowly conveyed in the direction of wave propagation in the field of propagating waves and at the same time it can have a large spin. The condition of existence of such regime of motion is determined exploiting approximate analytical techniques.

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