256. Calculation of Vibrations of an Incompressible Elastic Structure

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(Received 20 January 2007; accepted 12 March 2007)

Abstract. Rubber and photo-elastic materials at elevated temperatures exhibit incompressible behavior. The vibrations of a structure by taking the incompressibility constraint into account by the penalty method are analyzed. Reduced order numerical integration of the penalty term is performed. Graphical representation of volumetric strains at the points of reduced integration is proposed for validation of the calculated eigenmodes.

Keywords: rubber, incompressibility, penalty method, reduced integration, finite elements.

Introduction

Rubber and photo-elastic materials at elevated temperatures exhibit incompressible behavior [1]. Vibrations of a structure in the state of plane strain are analyzed. The incompressibility constraint [1, 2, 3, 4] is taken into account by the penalty method [1, 3]. Reduced order numerical integration of the penalty term is performed [1, 3].

Graphical representation of volumetric strains at the points of reduced order numerical integration is proposed for validation of the calculated eigenmodes.

Finite element model of the structure

The nodal variables for the analyzed two dimensional problem of vibrations of the incompressible elastic body in the state of plane strain are the displacements u and v in the directions x and y of the orthogonal Cartesian system of coordinates.

The mass matrix has the form:

$$[M] = \int [N]^T \rho [N] dxdy, \qquad (1)$$

where ρ is the density of the material of the structure and:

$$[N] = \begin{bmatrix} N_1 & 0 & \dots \\ 0 & N_1 & \dots \end{bmatrix}, \qquad (2)$$

where N_i are the shape functions of the finite element.

The stiffness matrix has the form:

$$[K] = \int [B]^T [D][B] dxdy + \\ + \int [\overline{B}]^T \lambda [\overline{B}] dxdy, \qquad (3)$$

where:

$$[B] = \begin{bmatrix} \frac{\partial N_1}{\partial x} & 0 & \dots \\ 0 & \frac{\partial N_1}{\partial y} & \dots \\ \frac{\partial N_1}{\partial y} & \frac{\partial N_1}{\partial x} & \dots \end{bmatrix}, \qquad (4)$$

$$\left[\overline{B}\right]=\begin{bmatrix}\frac{\partial N_1}{\partial x} & \frac{\partial N_1}{\partial y} & \dots\end{bmatrix}, \tag{5}$$

$$\left[D\right]=\begin{bmatrix}\frac{4}{3}G & -\frac{2}{3}G & 0\\ -\frac{2}{3}G & \frac{4}{3}G & 0\\ 0 & 0 & G\end{bmatrix}, \tag{6}$$

where the shear modulus:

$$G=\frac{E}{3}, \tag{7}$$

where E is the Young's modulus and λ is the penalty parameter for the incompressibility constraint. The calculation of the second integral is performed using numerical integration rule of reduced order.

The volumetric strains for the eigenmode are calculated at the points of reduced integration order of the second integral of the stiffness matrix:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=\left[\overline{B}\right]\{\delta\}, \tag{8}$$

where $\{\delta\}$ is the eigenmode.

Results of analysis of vibrations of the incompressible structure

The rectangular elastic incompressible structure is analyzed. The length of the analyzed structure is equal to twice of its width. All the displacements on the left boundary are assumed equal to zero. The volumetric strains for the eigenmode are calculated at the points of reduced integration order and represented in a circle around this point as a black angle from the positive direction of the *x* axis. This angle is equal to the volumetric strain multiplied by a large constant. This constant is chosen for each eigenmode in order to achieve visible representation of the results.

The first eigenmode is presented in Fig. 1, the second eigenmode in Fig. 2, …, the tenth eigenmode in Fig. 10.

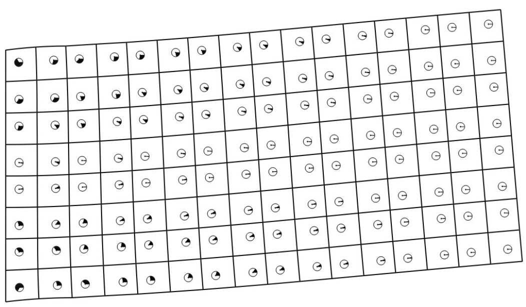

Fig. 1. The first eigenmode

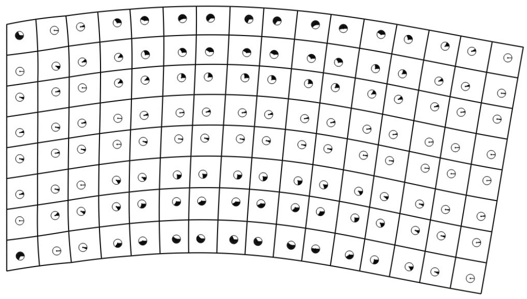

Fig. 2. The second eigenmode

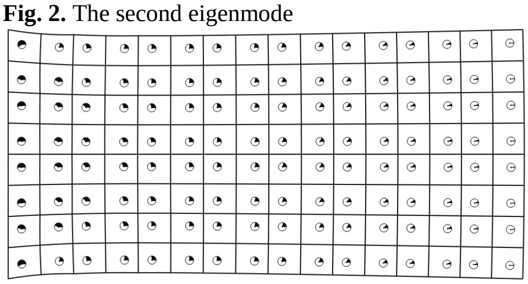

Fig. 3. The third eigenmode

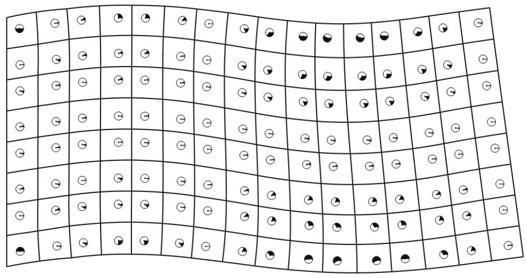

Fig. 4. The fourth eigenmode

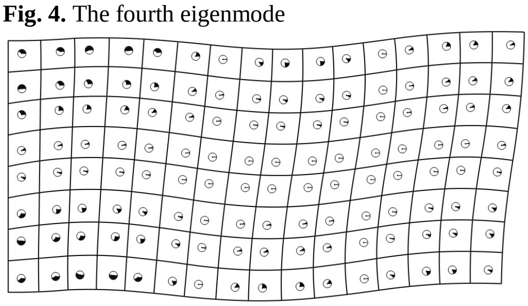

Fig. 5. The fifth eigenmode

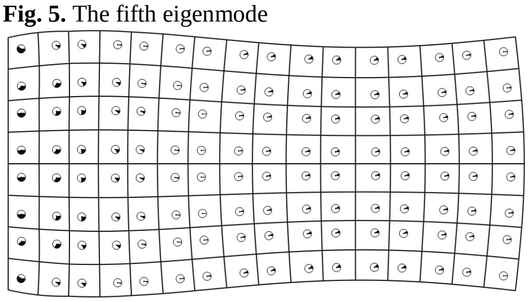

Fig. 6. The sixth eigenmode

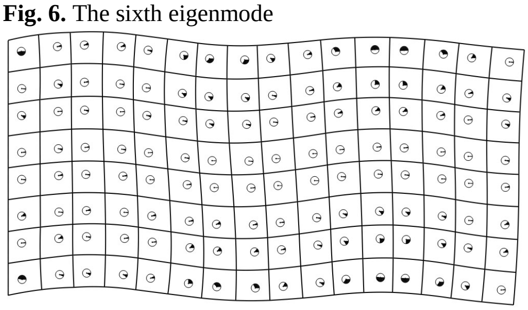

Fig. 7. The seventh eigenmode

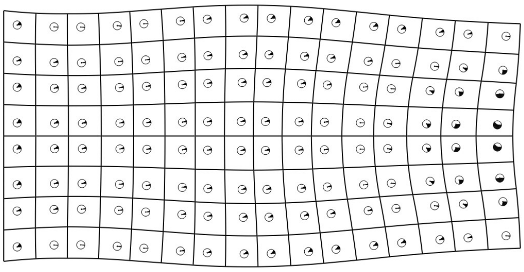

Fig. 8. The eighth eigenmode

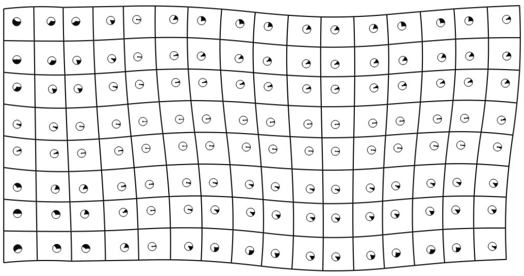

Fig. 9. The ninth eigenmode

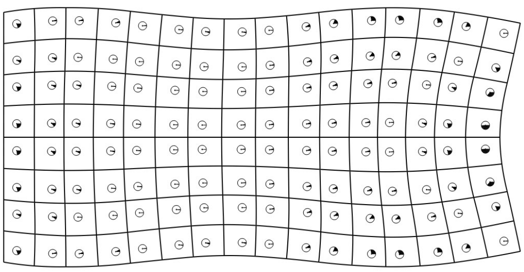

Fig. 10. The tenth eigenmode

This graphical representation shows the quality of satisfaction of the constraint on volumetric strains for the eigenmode over the whole structure and thus serves for the validation of the results of calculations.

Conclusions

Vibrations of a structure in the state of plane strain by taking the incompressibility constraint into account by the penalty method using reduced order numerical integration of the penalty term are analyzed.

Graphical representation of volumetric strains at the points of reduced order numerical integration is proposed for validation of the calculated eigenmodes. This graphical representation shows the quality of satisfaction of the constraint on volumetric strains over the whole structure.

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