**Proceedings of 26th Annual Fellowship Symposium
of the Utah NASA Space Grant Consortium
UNSGC 2020**

ENSURING EXTRAMOBILE AND INTRAMOBILE MOTION ON CYLINDRICAL DEVELOPABLE MECHANISMS*

Jared Butler
Compliant Mechanisms Research
Dept. of Mechanical Engineering
Brigham Young University
Provo, Utah 84602
jaredbutler@byu.net

Spencer Magleby
Compliant Mechanisms Research
Dept. of Mechanical Engineering
Brigham Young University
Provo, Utah 84602
magleby@byu.edu

ABSTRACT

Developable mechanisms offer the ability to deploy to perform tasks then return to a hidden position along or interior to a predetermined developable surface. It is often advantageous for these mechanisms to not penetrate the surface along which they conform. This paper presents the limits of extramobile and intramobile motion (motion exterior to and interior to a developable surface). Three conditions are identified that determine a limit of extramobile and intramobile behavior. It is shown that the more difficult of these conditions to predict is never reached prior to the more simple cases. This is demonstrated for all possible Grashof and non-Grashof mechanisms, excluding change-point mechanisms.

1 INTRODUCTION

Developable mechanisms (Figure 1) are devices that are capable of conforming to a predetermined developable surface and then deploying from that surface to achieve a specified task [2]. Because of the prevalence of developable surfaces in almost all applications, developable mechanisms show potential for integration into many fields of use. They have been shown as feasible on both cylindrical [3] and conical [4] surfaces, and their usefulness has also been shown in minimally-invasive surgical applications [5].

It is often advantageous for developable mechanisms to not penetrate the surface along which they conform. For example, a device may be made to exist on the interior of a pressurized pipe and requires all parts to remain interior to the pipe during actuation. Another possibility would be mechanisms that lie on the outside of a rocket body where penetrating the pressure vessel would lead to catastrophic failure. While past work has established the ability to predict if a device is capable of such motion, the limits of such motion have yet to be explored.

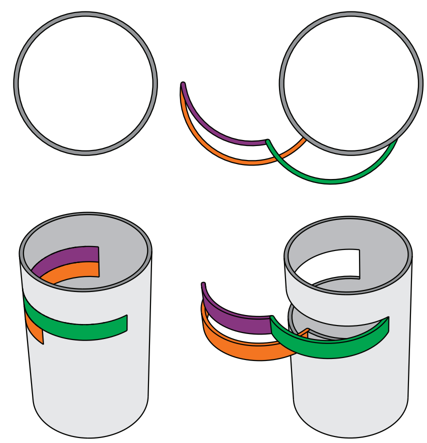


FIGURE 1: This four-bar linkage embedded within a cylinder illustrates a developable mechanism that conforms to and emerges from a cylinder.

This work presents three conditions that determine the limits of extramobile and intramobile motion (motion exterior or inte-

*This paper is a review of work published in the conference proceedings of IDETC 2020 [1] rior to a developable surface). It is shown that of the three conditions, one condition is more difficult to predict than the other two. However, under certain assumptions, the more difficult condition is never achieved prior to the simpler conditions.

2 DEVELOPABLE MECHANISM BACKGROUND

To be considered a developable mechanism (DM), a mechanism must conform to some predetermined developable surface (referred to as the reference surface). In the conformed position, all joint axis of a linkage must align with the ruling lines of the reference surface [2]. Because of the requirement to conform to the reference surface, motions that occur relative to this surface become of particular interest.

3 EXTRA/INTRAMOBILE MOTION

Previous work has defined terminology that describes the ability of a mechanism to entirely enter the exterior or interior of a reference surface, referred to as extramobility and intramobility [3]. However, this terminology refers only to the ability to move in those directions and says nothing about the large-scale motion of these devices.

We define *intramobile motion* to be motion of a developable mechanism where all moving parts of the mechanism remain interior to the reference surface, and we define *extramobile motion* to be motion of a developable mechanism where all moving parts of the mechanism remain exterior to the reference surface. This work investigates the limits of these motions and provide conditions to use during the design of these mechanisms to determine when a mechanism will no longer maintain intramobile or extramobile motion.

Cylindrical DMs are planar mechanisms. It is therefore advantageous to model these mechanisms from the view along the cylinder's centerline, as shown in Figure 2. While DMs with zero-thickness are created using curved links, it is also advantageous to model them with straight linkages as the kinematic behavior of the two are identical. For example, the two mechanisms shown in Figure 2 are kinematically equivalent. This paper will use both straight and curved linkages to demonstrate concepts.

3.1 Conditions for Extra/Intramobile Motion Limits

The requirement that a mechanism not have any moving part cross the reference surface to maintain extramobile or intramobile motion can be decomposed into the following three conditions:

1. No grounded link may rotate from the conformed position far enough to again intersect the reference surface.
2. No grounded link may rotate interior to (exterior to) the reference surface for extramobile (intramobile) motion.
3. No portion of the coupler may cross the reference surface.

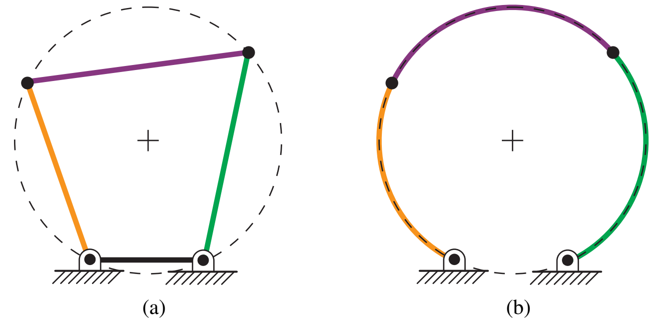


FIGURE 2: When modeling the kinematics of a developable mechanism, only the minimum distance between pivots (a) needs to be considered. However, when identifying potential contact between the links and the reference surface, the actual curved links (b) must be used.

While prediction the motion limits of grounded links (Conditions 1 and 2) is straightforward, predicting the motion limits of the coupler (Condition 3) can be much more complex. If any one of the three conditions is violated first, it becomes the limiting condition, meaning the other two conditions are no longer applicable since extramobile or intramobile motion has already reached a limit. If the first two conditions were to always occur prior to Condition 3, the design of cylindrical DMs that require intramobile and extramobile motion would be eased.

Any cylindrical DM that exhibits intramobile or extramobile motion will always violate Conditions 1 or 2 prior to Condition 3 given the following assumptions:

- A All links have an arclength $\leq \pi R$.
- B All links have the same curvature as the reference surface.
- C All links are modeled with zero thickness.
- D All grounded links only extend in one direction past their grounded pivot.
- E The coupler does not extend beyond either of the moving pivots.

Assumptions A and D ensure the mechanisms exhibit intramobility or extramobility [3]. Assumption B is necessary for all cylindrical developable mechanisms. Assumptions C and E build on previous work in this area [2,3] and provide a foundation for mechanisms with thickness and more complex geometries.

3.2 Conditions for Extramobile Motion

This section discusses Conditions 1, 2, and 3 for both Grashof and non-Grashof mechanisms. Though similar derivations can be shown for change-point mechanisms, they are not discussed in this work.

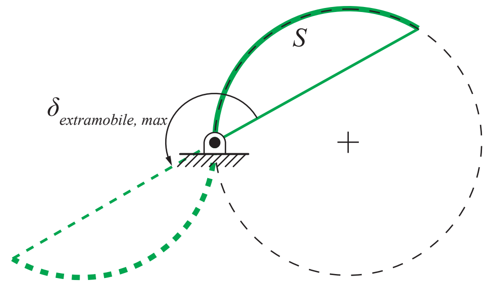


FIGURE 3: The maximum amount of rotation outside the reference surface for a grounded link.

3.2.1 Conditions 1 and 2

Links that are pinned to ground (links 2 and 4 in a traditional 4-bar mechanism) have a maximum exterior rotation (following Condition 1) that is shown in Figure 3 and defined mathematically by

$$\delta_{extramobile,max} = \pi \text{ for } (0 < S \leq \pi R) \quad (1)$$

where S is the arc length of the link.

To violate Condition 2, a link pinned to ground needs to move away from the conformed position, then return to the same position and continue motion across the reference surface. The combination of Conditions 1 and 2 are thus the limits of motion for all links pinned to ground, given by the conformed position and Equation 1.

3.2.2 Condition 3

Violation of Condition 3 would require the convex side of the coupler to cross the reference surface prior to either of its endpoints, as shown in Figure 4a. There are two scenarios that must simultaneously occur for Condition 3 to be met.

In the first scenario, the coupler must rotate to place its convex side adjacent to the reference surface while in the same circuit. Mechanisms that can achieve this position include double rockers and non-Grashof mechanisms.

The second scenario is that the coupler must intersect the reference surface prior to Conditions 1 or 2 being violated. This can be evaluated by looking at the rotation of the coupler, γ, when Condition 2 has been violated. If $\gamma < \pi$, the coupler has not yet crossed the reference surface when Condition 2 is met, as shown in Figure 4b. If $\gamma > \pi$, the coupler has already crossed the reference surace prior to Condition 2, as shown in Figure 4c.

Intramobile and extramobile mechanisms can be separated into three different classes of 4-bar mechanisms. There are a discrete number of mechanisms that can exist within each class [3]. These classes are showing in Figure 5. To show that Conditions 1 and 2 are always violated prior to Condition 3 for any intramobile or extramobile mechanism, we will look at each class separately and show geometrically the constraints that make this assertion true, so long as the previously stated assumptions hold true. Without loss of generality, we will assign θ_1, the angle of the ground link, to equal 0. There are also subcases within each class, but these are not discussed due to symmetry.

Class 1 Using Barker's classification for 4-bar linkages [6], it is possible to obtain GCCC (double crank), GCRR/GRRC (crank rocker), GRCR (double rocker), and RRR2/RRR3 (triple rocker) mechanisms (excluding change-point mechanisms) [3] that are extramobile in Class 1. Of these, only GRCR and RRR2/RRR3 mechanisms are capable of reaching both open and crossed positions within the same circuit (capable of placing the convex side of the coupler adjacent to the reference surface). The other mechanisms (GCCC and GCRR/GRRC) cannot reach both the open and crossed configurations in the same circuit and are therefore unable to violate Condition 3 prior to Conditions 1 and 2.

Figure 6 shows an example Class 1 mechanism in the open, conformed configuration (solid lines) and the correlated crossed position (dashed lines). Class 1 mechanisms must place all joints of the linkage within the same half of the reference surface while in the conformed position. This constrains the longest link l to be the link that lies closest to the center of the circle. The angles adjacent to l, α and ρ, must be less than $\pi/2$. When link 2 reconforms to the surface (reaches the crossed configuration for the same value of θ_2 at the conformed position) a symmetric polygon is formed by links 3 and 4 in the open and crossed positions.

For a Class 1 mechanism to exist (an open cyclic quadrilateral restricted to one side of a circle), ρ must always be less than $\pi/2$. Because the polygon formed between links 3 and 4 in the open and crossed configuration is symmetric, the angle opposite of ρ must equal ρ, making it also less than $\pi/2$. The angle γ must therefore always be less than π, preventing the coupler from moving past its tangent position and into the reference surface. It is therefore concluded that Class 1 mechanisms are incapable of violating Condition 3 prior to Conditions 1 and 2. The limits of extramobile motion are therefore established by Conditions 1 and 2, given the stated assumptions.

Class 2 It is possible to obtain GCCC and GRCR mechanisms under Class 2. Of these, only the GRCR can reach both the open and crossed configurations. Figure 7 illustrates this mechanism in its open, conformed (shown in solid) and correlated crossed (shown in dashed) configurations. In Class 2, link 4 may not cross over the center of the circle in order to maintain intramobility and extramobility. Therefore, α (the angle between links 1 and 4) must be less than $\pi/2$. The angle α subtends the same arc as the angle between links 1 and 3 (β). Therefore, $\alpha = \beta$ due to the inscribed angle theorem. Hence, $\beta > \pi/2$.

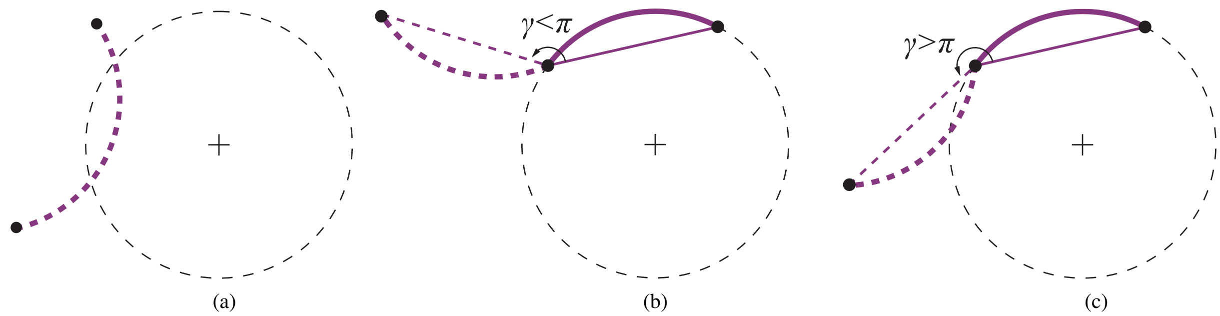
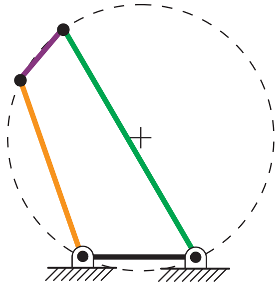
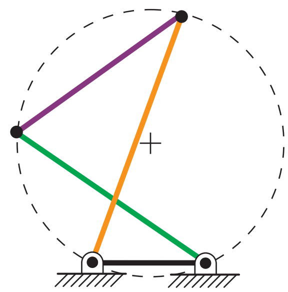


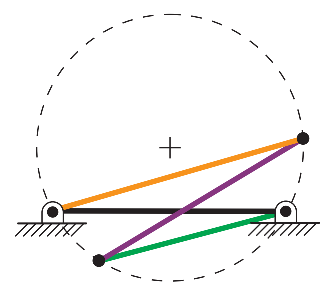
FIGURE 4: (a) The convex side of a coupler, shown in purple, may intersect the reference surface before the endpoints intersect the surface. The rotation of the coupler, γ, can help determine if Condition 3 is violated before Condition 1 or 2, as shown in (b) and (c).



(a) Class 1 mechanism. Possible configurations include GCCC, GRCC/GCCR, GRCR, and RRR2/RRR4 (excluding change-point mechanisms).



(b) Class 2 mechanism. are conformed when in their crossed configuration. Possible configurations include GCCC and GRCR (excluding change-point mechanisms).



(c) Class 3 mechanism. Possible configurations include GRCC/GCCR (excluding change-point mechanisms).

FIGURE 5: The three classes of extra/intramobile mechanisms. The black line represents the ground link and is constrained to have $\theta_1 = 0$. Class 1 mechanisms are conformed in their open configuration while Class 2 and 3 mechanisms are conformed in their crossed configuration. Note that there are 2 subclasses for each class but only 1 is shown here and discussed due to symmetry.

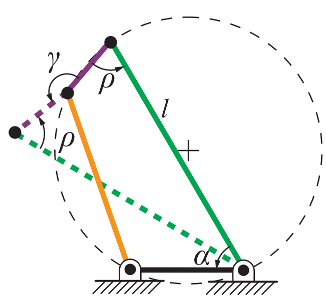


FIGURE 6: Class 1 mechanism in its open (solid, conformed) and crossed (dashed) configuration.

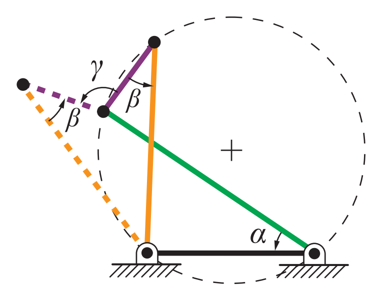


FIGURE 7: Class 2 mechanism in its crossed (solid, conformed) and open (dashed) configuration.

The symmetric polygon formed by links 2 and 3 in the open and crossed configurations provides the same logic as was used in Class 1. Since $\gamma < \pi$, it is concluded that Conditions 1 and 2 are the limiting conditions for extramobile motion for Class 2

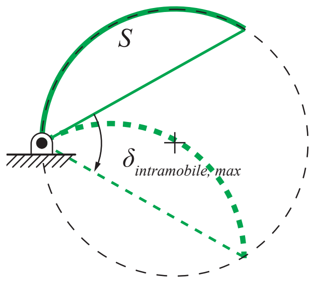


FIGURE 8: The maximum amount of rotation inside the reference surface for a grounded link.

mechanisms.

Class 3 Under Class 3, the only type of mechanism that can achieve extramobility and intramobility is a GCRR mechanism. Since this mechanism cannot reach both the open and crossed configurations within the same circuit, the coupler can never invert to place its convex side adjacent to the reference surface. Therefore, Conditions 1 and 2 are the limiting conditions for extramobile motion in Class 3 mechanisms given the stated assumptions.

3.3 Conditions for Intramobile Motion

For grounded links, (links 2 and 4 in traditional four-bar mechanisms), the maximum interior rotation for the link (the maximum rotation before violating Condition 1) can be calculated as described in the equation below and shown in Figure 8, where S is the arc length of the link.

$$\delta_{intramobile,max} = \pi - \frac{S}{R} \text{ for } (0 < S \leq \pi R) \quad (2)$$

Condition 2 can be violated if any grounded link moves away from its initial position on the reference surface then moves back to its initial position. At this point, a continuation of motion will move that link exterior to the reference surface.

In order to violate Condition 3, the convex side of the coupler would need to cross the reference surface. Because each link is shaped to the reference surface (see Assumption B above), the only way for any point on the coupler link (link 3 in traditional four-bar mechanisms) can cross the reference surface is if one or more of the endpoints has already crossed, as shown in Figure 9. Therefore, the intramobile motion for a regular cylindrical DM is bounded by the motion of links 2 and 4 (Conditions 1 and 2).

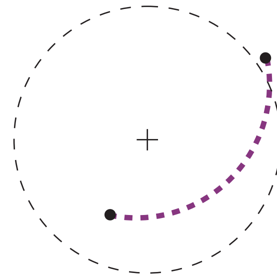


FIGURE 9: The coupler link has the same curvature as the reference surface. The only way for the coupler link to cross from the inside to the outside of the reference surface is if one of the coupler endpoints crosses first.

4 Conclusion

The observation that Condition 3 is never violated prior to Conditions 1 and 2 provides great flexibility to designers when creating developable mechanisms that do not penetrate the reference surface. This will always hold true so long as the assumptions asserted in this work are held. Future work will investigate relaxing some of these constraints to generalize the results to any form of developable mechanism. Additionally, since physical linkages must have some appreciable thickness, methods that account for link thickness when moving relative to the reference surface will be advantageous in the design of developable mechanisms.

ACKNOWLEDGMENT

This work was supported by the U.S. National Science Foundation through NSF grant no. 1663345 and the Utah NASA Space Grant Consortium.

REFERENCES

[1] Butler, J., Greenwood, J., Howell, L., and Magleby, S., 2020. "Limits of extramobile and intramobile motion of cylindrical developable mechanisms". In ASME 2020 International Design Engineering Technical Conferences and Computers and Information in Engineering Conference, American Society of Mechanical Engineers.

[2] Nelson, T. G., Zimmerman, T. K., Magleby, S. P., Lang, R. J., and Howell, L. L., 2019. "Developable mechanisms on developable surfaces". *Science Robotics,* ***4***(27), Feb, p. eaau5171.

[3] Greenwood, J. R., Magleby, S. P., and Howell, L. L., 2019. "Developable mechanisms on regular cylindrical surfaces". *Mechanism and Machine Theory,* ***142***, Dec, p. 103584.

[4] Hyatt, L. P., Magleby, S. P., and Howell, L. L., 2020. "Devel-

opable mechanisms on right conical surfaces". *Mechanism and Machine Theory,* ***149***, Jul, p. 103813.

[5] Seymour, K., Sheffield, J., Magleby, S. P., and Howell, L. L., 2019. "Cylindrical developable mechanisms for minimally invasive surgical instruments". In ASME 2019 International Design Engineering Technical Conferences and Computers and Information in Engineering Conference, American Society of Mechanical Engineers Digital Collection.

[6] Barker, C. R., 1985. "A complete classification of planar four-bar linkages". *Mechanism and Machine Theory,* ***20***(6), pp. 535–554.