Combining Non-orthogonal Transmission with Network-Coded Cooperation: Performance Analysis under Nakagami-m Fading

Chao Wang, Xun Li, Ping Wang, and Geyong Min

Abstract

This paper investigates efficient transmission design in a class of multi-user cooperation networks, in which multiple information sources intend to distribute their messages to a sufficient proportion of ambient destinations with the assistance of multiple relays, under Nakagami-m fading. We apply a relaying scheme that combines non-orthogonal transmission with network coding techniques to efficiently utilize available channel resources. Specifically, the sources and relays are divided into clusters, terminals within each of which are allowed to non-orthogonally access the same channel. A class of finite-field network codes are adopted in the relays. We provide the methods to derive the system decoding error probability and diversity-multiplexing tradeoff (DMT). Through error probability and finite-SNR DMT analysis for certain clustering strategies, and infinite-SNR DMT analysis for general situations, we show that the considered relaying scheme can notably improve system performance over the conventional approach that demands only orthogonal transmission in network-coded cooperation networks.

Index Terms

Multi-user relay networks, Network-coded cooperation, Non-orthogonal transmission

I. Introduction

Due to the rapid development of wireless communication technologies, a large variety of novel mobile Internet and Internet-of-things (IoT) applications and services have emerged in recent years. A very important future mobile communication application scenario is the content distribution network where information sources (content generators) broadcast their messages to ambient destinations (content subscribers). For instance, Fig. 1(a) illustrates a potential example in intelligent transportation systems (ITS). Allowing roadside unit (source S_1) and vehicles (sources S_2 and S_3) to share their sensing data regarding a complex traffic environment with other vehicles (destinations D_1, D_2, and D_3), under the framework of vehicular cloud networks [1], can enable vehicles to access others' sensors. The accuracy and reliability of individual environment perception can be greatly enhanced. Other promising applications include advertisement broadcasting in shopping malls and video sharing in stadiums supported by the 5G device-to-device (D2D) technology [2]. These wireless systems in general consist of several sources and potentially many destinations. The content-rich source messages (e.g., in the form of multimedia files) should be delivered in severe signal propagation environments (e.g., high-speed or indoor environments). Providing a high quality of message distribution can be challenging.

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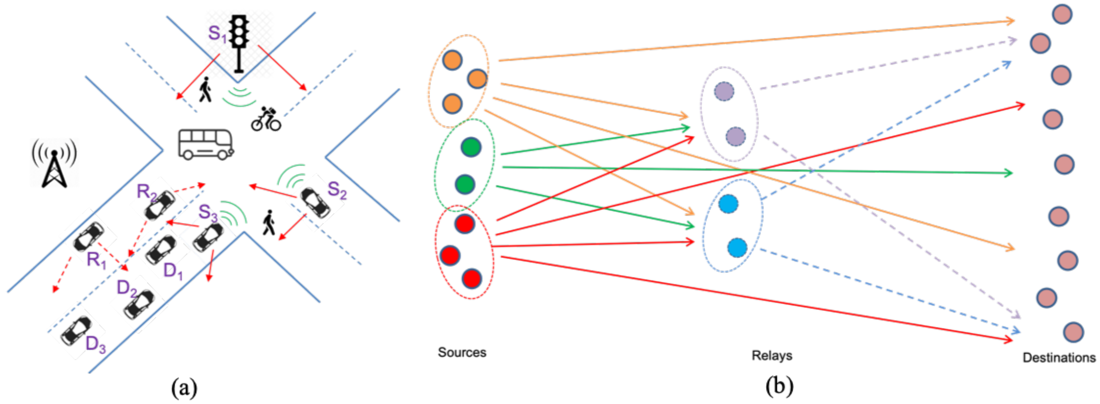


Fig. 1: (a) An ITS application scenario of (b) a multi-source multi-relay multi-destination network

Introducing cooperative relays to improve communication performance in wireless networks has attracted much recent interest [3]. When a number of relays (e.g., R_1 and R_2 in Fig. 1(a)) are utilized to assist in the data distribution from several sources to several destinations, the network structure can be abstracted as a multi-source multi-relay multi-destination network shown in Fig. 1(b). From an information-theoretic viewpoint, multi-hop networks are fundamentally different from single-hop networks, since relays bring complicated interference and signal processing issues. The capacity of even very small relay networks is unknown. Efficient transmission designs and their performances in many types of networks are far from being well-understood.

In addition to providing power gain, the advantages of cooperative relays have been exploited mainly from two aspects. First, recent achievements in network information theory have shown that relays can provide degrees of freedom (DoF) gain to multi-user wireless networks (see e.g., [4]–[7]). This implies that, the capacity region of multi-hop relay networks may be much larger than that of single-hop networks, at least for the high signal-to-noise ratio (SNR) regime. The basic idea behind high-DoF transmission designs is to properly coordinate multiple non-orthogonally activated terminals, with a sufficient knowledge of the network channel state information (CSI), such that the inter-user interference can be effectively eliminated or mitigated.

Another research trend takes more practical conditions, including imperfect channel knowledge and limited node coordination, into consideration and exploits relays' capability of providing diversity gain for improving transmission reliability [8], [9]. Extensive investigations have been conducted to design efficient relaying schemes for *link-level* information delivery between a single source-destination pair. These schemes can be readily employed in multiple-user networks when different users' transmissions are orthogonalized. However, each relay is normally forced to separately forward different sources' messages, using, for example, repetition coding or distributed space-time coding. Network resources are inefficiently utilized.

Applying the network coding (NC) technique [10] at relays is capable of realizing efficient *network-level* transmissions in wireless cooperative networks. For instance, the binary NC can provide excellent performance in multi-source single-relay [11] and single-source multi-relay [12] networks. Nevertheless, directly employing the binary XOR operation to combine source messages in multi-source multi-relay networks may not be able to attain full diversity. Summation in high-order finite field has been considered as a solution. The maximum distance separable network codes (MDS-NC) [13]–[15] and random linear network codes (RLNC) [16] developed following this principle are proven to be maximal-diversity-achievable in the uplink cooperative networks with multiple sources, multiple relays, and one destination. Further NC designs suitable for such a network structure have been proposed in [17]–[21]. The system model is also extended to reuse-mode D2D networks with arbitrary sources, relays, and destinations in [22]–[24].

To concentrate on novel coding strategies design and evaluation, most of existing related works demand all sources and relays to be orthogonally activated, so that inter-user interference can be avoided [13]–[20]. Message transmissions are commonly assumed to be conducted under Rayleigh fading [13]–[22], due to mathematical tractability. In a single-hop multi-source single-destination network, it is known that orthogonalization leads to an inferior achievable rate region compared with permitting all sources to non-orthogonally share the channel (i.e., a multiple-access (MAC) channel) [25]. Reference [26] studies the case that two sources are activated concurrently (but relays are still orthogonalized) and shows that it can outperform orthogonalizing all sources. Our earlier work [27] proposes a novel NC-based relaying scheme that permits arbitrary sources (and relays) to transmit their messages together. Through the infinite-SNR diversity-multiplexing tradeoff (DMT) analysis, it is shown that the performance of the repetition-coding-based and orthogonal-NC-based relaying schemes can be improved. These observations demonstrate the potential of combining non-orthogonal transmission with NC techniques in multi-user cooperation networks. But the applicability of such results is still relatively limited.

First, same as [15]–[21], the system model considers uplink transmission with only one destination. Many future content distribution applications may need to deliver services to a number of ambient content subscribers. Due to the fact that the decoding results of destinations are dependent (affected by the relays), the system performance cannot be straightforwardly attained from the single-destination case. Second, similar to [13]–[22], analytical results are obtained under only Rayleigh fading. In many application scenarios (e.g., vehicular and indoor transmissions), the wireless signal propagation characteristics can be better modelled by the Nakagami-m fading [28]. Transmission design and performance in these two environments can be very different, especially when the source-destination, source-relay, and relay-destination channels have diverse fading properties. Finally, performance gain is discovered using only a high-SNR indicator, the infinite-SNR DMT. Hence one may ask if the advantage of combing non-orthogonal transmissions with NC still exists, in more general conditions.

In this paper, we provide investigations on this question. Our main contributions are as follows.

1) We study the communication problem in a class of wireless multi-user network-coded cooperation networks where several sources intend to broadcast their messages to a sufficient proportion of multiple destinations, with the assistance of a number of relays, in a Nakagami-m fading environment. Such a general system model can contain those considered in many related works as special cases. We adopt the idea proposed in [27] and apply a novel *cluster NC (C-NC)* relaying scheme to conduct message delivery. The scheme divides sources and relays into clusters, and allows terminals within each cluster to transmit messages together. The relays employ a class of finite-field NC to simultaneously forward multiple source messages. By this means, the advantages of NC and non-orthogonal transmission techniques can both be exploited.

2) We provide a framework to calculate the achievable system decoding error probability, through which the finite-SNR/infinite-SNR DMT and certain other performance indicators can also be identified. The error probability is a function of the clustering strategy and various system parameters. For the finite-SNR regime, we focus on the case that the number of terminals in each cluster is bounded by three. By finding the individual decoding error probabilities in the two-user and three-user MAC channels, we can derive the closed-form expressions of our C-NC scheme's achievable error probability and finite-SNR DMT. It is shown that the system performance can be significantly better than requiring orthogonal transmission among all sources and relays.

3) For more general clustering strategies that allow arbitrary numbers of terminals in each cluster, we derive the achievable infinite-SNR DMT, and also provide discussions regarding the impact of node partition approaches in several special network structures. The results can be used to facilitate finding the best clustering solution for different system setups. A notable performance gain over orthogonal transmission can also be observed. Consequently, a more complete picture of the potentials of combining non-orthogonal transmission with NC is exhibited.

The remainder of the paper is organized as follows. Section II introduces our system model and the C-NC relaying scheme. In Section III, we first present the method to calculate the system error probability. The individual decoding probabilities in two-user and three-user MAC channels are derived and then applied to the performance analysis of the C-NC scheme. In Section IV, we present the achievable infinite-SNR DMT for general clustering strategies and provide discussions. Section V concludes the paper.

Notations: We use $|\mathcal{A}|$ and $\bar{\mathcal{A}}$ to denote the cardinality and complement set of set $\mathcal{A}$. For a Gamma distributed random variable $X \sim \text{Gamma}(\alpha, \beta)$ with integer shape parameter α and rate parameter β, its PDF is denoted by $f_X(x) = \frac{\beta^\alpha x^{\alpha-1} e^{-\beta x}}{\Gamma(\alpha)}$ and its CDF is denoted by $F(x; \alpha, \beta) = 1 - \sum_{i=0}^{\alpha-1} \frac{(\beta x)^i}{i!} e^{-\beta x}$, where $\Gamma(\alpha) = (\alpha - 1)!$ is the gamma function. $(a)^+$ is used to denote $\max\{0, a\}$. The exponential equality $X \doteq \rho^b$ means $b = \lim_{\rho \to \infty} \frac{\log_2 X}{\log_2 \rho}$.

II. SYSTEM MODEL AND TRANSMISSION SCHEME

We investigate data distribution in a class of wireless single-antenna networks in which a set of M sources $\mathcal{S} = \{\mathrm{S}_1, \cdots, \mathrm{S}_M\}$ intend to broadcast their messages $I_{\mathrm{S}_1}, \cdots, I_{\mathrm{S}_M}$ to a set of N destinations $\mathcal{D} = \{\mathrm{D}_1, \cdots, \mathrm{D}_N\}$. To enhance performance, a set of K half-duplex decode-and-forward relays $\mathcal{R} = \{\mathrm{R}_1, \cdots, \mathrm{R}_K\}$ are used to assist in the message delivery. We denote the set of source messages by $\mathcal{I} = \{I_{\mathrm{S}_1}, \cdots, I_{\mathrm{S}_M}\}$. It is desired to guarantee a sufficient proportion, $\sigma\%$ $(0 < \sigma \leq 100)$, of the destinations to receive the complete set $\mathcal{I}$. Otherwise, if less than $\lceil N \cdot \sigma\% \rceil$ destinations can do so, we say the transmission fails and an error event is declared. Such a *multi-source multi-relay multi-destination* cooperative network is illustrated in Fig. 1(b).

Assume that source messages are encoded using capacity-achieving Gaussian random codes with data rate R bit/codeword. The transmission of each codeword spans one time slot, in a narrow-band Nakagami-m slow-fading environment. The channel fading coefficient between any transmitter a and receiver b is denoted by $h_{b,a}$. The channel power gains between different sources and destinations are modeled as independent random variables following Gamma distribution with *integer* shape parameter m_{sd} and rate parameter $\frac{m_{\text{sd}}}{\Omega_{\mathrm{D}_n,\mathrm{S}_j}}$, i.e., $|h_{\mathrm{D}_n,\mathrm{S}_j}|^2 \sim \text{Gamma}\left(m_{\text{sd}}, \frac{m_{\text{sd}}}{\Omega_{\mathrm{D}_n,\mathrm{S}_j}}\right)$, for $j \in \{1, \cdots, M\}$ and $n \in \{1, \cdots, N\}$. The value m_{sd} measures the small-scale source-destination channel fading phenomenon. The special case $m_{\text{sd}} = 1$ represents Rayleigh fading. $\Omega_{\mathrm{D}_n,\mathrm{S}_j} = \mathbb{E}\{|h_{\mathrm{D}_n,\mathrm{S}_j}|^2\}$ quantifies the impact of large-scale fading between S_j and D_n. The similar consideration holds for the source-relay channels and relay-destination channels, i.e., $|h_{\mathrm{R}_k,\mathrm{S}_j}|^2 \sim \text{Gamma}\left(m_{\text{sr}}, \frac{m_{\text{sr}}}{\Omega_{\mathrm{R}_k,\mathrm{S}_j}}\right)$ and $|h_{\mathrm{D}_n,\mathrm{R}_k}|^2 \sim \text{Gamma}\left(m_{\text{rd}}, \frac{m_{\text{rd}}}{\Omega_{\mathrm{D}_n,\mathrm{R}_k}}\right)$ for $k \in \{1, \cdots, K\}$. For the whole transmission period, $h_{b,a}$ remains fixed and is known at only the receiver b. Due to the lack of transmitter-side channel knowledge, power adaptation is not considered. The sources transmit signals with the same power ρ_{s}, and the relays transmit with power ρ_{r}.

To efficiently conduct message distribution, we borrow the concept presented in [27] and propose combining non-orthogonal transmission with NC techniques in the considered multi-user cooperation network. Specifically, as illustrated in Fig. 1(a), we partition the M sources into μ $(1 \leq \mu \leq M)$ non-overlapping clusters $\mathcal{S}_1, \cdots, \mathcal{S}_\mu$, where $\mathcal{S}_j = \left\{\mathrm{S}_1^{[j]}, \cdots, \mathrm{S}_{|\mathcal{S}_j|}^{[j]}\right\}$ and $\mathrm{S}_i^{[j]}$ represents the ith source node in $\mathcal{S}_j$. The relays are also divided into κ $(1 \leq \kappa \leq K)$ clusters $\mathcal{R}_1, \cdots, \mathcal{R}_\kappa$, in which $\mathcal{R}_k = \left\{\mathrm{R}_1^{[k]}, \cdots, \mathrm{R}_{|\mathcal{R}_k|}^{[k]}\right\}$ and $\mathrm{R}_i^{[k]}$ is the ith relay terminal in $\mathcal{R}_k$. Clearly, $\sum_{j=1}^{\mu} |\mathcal{S}_j| = M$ and $\sum_{k=1}^{\kappa} |\mathcal{R}_k| = K$.

We allow the terminals within each cluster to be activated *non-orthogonally* in order to reduce overall channel consumption, while different clusters accessing different channels (i.e., time slots) to avoid unnecessarily large inter-user interference. Therefore, a total of $\mu + \kappa$ time slots are consumed to deliver $\mathcal{I}$ from the sources to the destinations. The first μ time slots are assigned to the μ source clusters. At the jth time slot $(j \in \{1, \cdots, \mu\})$, all nodes within $\mathcal{S}_j$ broadcast their messages together to the relays and destinations. Afterwards, each of the remaining κ time slots is reserved for an individual relay cluster. At each time slot, every receiving node applies successive interference cancellation (SIC) to its received signal for facilitating decoding.

At the relays we employ a class of finite-field network codes, the MDS-NC [13], [15]. (The RLNC can also be applied. In this case, the coding coefficients are randomly generated.) The network coding function applied at the ith relay R_i ($i \in \{1, \cdots, K\}$) aims to combine the sources' messages in a certain finite field to generate a new network codeword $I_{\mathrm{R}_i} = \sum_{j=1}^{M} \gamma_{i,j} I_{\mathrm{S}_j}$. The coding construction follows that for an MDS code and the coding coefficients $\gamma_{i,j}$ are designed to guarantee the global encoding kernels to be linearly independent. (Such network codes always exist if the coding field size is sufficiently large.) Therefore, the complete source message set $\mathcal{I}$ can be fully recovered using *any* M messages among $I_{\mathrm{S}_1}, I_{\mathrm{S}_2}, \cdots, I_{\mathrm{S}_M}, I_{\mathrm{R}_1}, I_{\mathrm{R}_2}, \cdots, I_{\mathrm{R}_K}$.

To avoid error propagation, a relay is activated only if it attains all messages in $\mathcal{I}$ from the transmissions of sources. Otherwise, it does not participate in the cooperative message distribution process. Use $\hat{\mathcal{R}}_k$ ($\hat{\mathcal{R}}_k \subseteq \mathcal{R}_k$) to denote the set of activated relays in $\mathcal{R}_k$. The received signal of node b ($b \in \mathcal{R} \cup \mathcal{D}$ when $t \leq \mu$, and $b \in \mathcal{D}$ when $t > \mu$) at time slot t is expressed as:

$$y_b[t] = \sum_{a \in \mathcal{T}_t} h_{b,a} x_a + \epsilon_b[t], \tag{1}$$

in which $\mathcal{T}_t = \mathcal{S}_t$ for $t \in \{1, \cdots, \mu\}$, $\mathcal{T}_t = \hat{\mathcal{R}}_{t-\mu}$ for $t \in \{\mu+1, \cdots, \mu+\kappa\}$, x_a is the signal sent by a, and $\epsilon_b[t]$ denotes the unit-power additive white Gaussian noise (AWGN).

We term our transmission scheme *cluster network-coded (C-NC) relaying*. Clearly, if we set $\mu = M$ and $\kappa = K$, each source and relay cluster contains a single node. All terminals in the network are thus activated orthogonally. This special case is termed *orthogonal NC (O-NC) relaying*. It is the scheme applied to multi-user network-coded cooperation networks in most existing related works. Inter-user interference is avoided at the cost of a large channel consumption of $M+K$ time slots. If $\mu = 1$ and $\kappa = 1$, all sources and relays are non-orthogonally activated in two time slots respectively. This is termed *non-orthogonal NC (NO-NC) relaying*. For a single-hop multi-transmitter single-receiver network, demanding all transmitters to non-orthogonally send their signals can achieve better performance (in terms of e.g., capacity region and infinite-SNR DMT) than activating a subset of transmitters at a time [25], [29]. But in our considered network, this may not be the case because the system performance is determined by the joint impacts of source-destination, source-relay, and relay-destination transmissions.

We aim to evaluate the performances of our C-NC scheme for different clustering strategies (including O-NC and NO-NC), to reveal the advantages of combining non-orthogonal transmission with NC techniques. Our main performance metric is the error probability. Let the number of destinations that are able to correctly recover all the messages in $\mathcal{I}$ be ζ ($0 \leq \zeta \leq N$). The message distribution is considered to be successful only if $\zeta \geq \lceil N \cdot \sigma\% \rceil$. The *system error probability* P_{err}, i.e., the probability of occurring an error event, is hence defined as

$$P_{\text{err}} = \Pr\left\{\zeta < \lceil N \cdot \sigma\% \rceil\right\}. \tag{2}$$

We will show that when the number of terminals in each cluster is no more than three, closed-form expressions of P_{err} can be attained. To evaluate performance simultaneously from the perspectives of channel usage efficiency and communication reliability, using P_{err} one can further derive the achievable DMT. To this end, we assume that the sources' and relays' transmit powers can be expressed as $\rho_{\text{s}} = a_{\text{s}}\rho$ and $\rho_{\text{r}} = a_{\text{r}}\rho$ for some constants a_s and a_r. If we allow the *average* transmission data rate of each source $\bar{R} = \frac{R}{\mu+\kappa}$ (in bit/source/time slot) to be chosen according to $r\log_2(1+\rho)$ with multiplexing gain r, the achievable diversity gain $d(r,\rho)$ measures the negative slope of the log-log plot of system error probability versus the common operating SNR ρ. The complete tradeoff between r and $d(r,\rho)$ is termed *finite-SNR DMT*, i.e., [30]:

$$r = \frac{\bar{R}}{\log_2(1+\rho)} \quad \text{and} \quad d(r,\rho) = -\frac{\rho}{P_{\text{err}}}\frac{\partial P_{\text{err}}}{\partial \rho}. \tag{3}$$

Further, if we allow $\rho \to \infty$, the performance of any clustering approach with arbitrary cluster sizes can be evaluated asymptotically through the *infinite-SNR DMT* [31]:

$$r^* = \lim_{\rho\to\infty} \frac{\bar{R}}{\log_2 \rho} \quad \text{and} \quad d^*(r^*) = -\lim_{\rho\to\infty} \frac{\log_2 P_{\text{err}}}{\log_2 \rho}. \tag{4}$$

$d^*(r^*)$ captures the slope of the curve $\log_2 P_{\text{err}}$ plotted against $\log_2 \rho$ in the high-SNR region.

III. Finite-SNR Performance Analysis

We start our performance analysis by presenting the general method of deriving P_{err}.

A. System Error Probability Derivation

The decoding behaviors of the destinations are dependent, and cannot be separately considered. To find P_{err}, we introduce a binary vector $\boldsymbol{\pi} = [\pi_1^{[1]}, \cdots, \pi_{|\mathcal{R}_1|}^{[1]}, \cdots, \pi_1^{[\kappa]}, \cdots, \pi_{|\mathcal{R}_\kappa|}^{[\kappa]}]$ indicating the decoding status of the K relays, i.e., whether they can be activated. If $\mathrm{R}_i^{[k]}$, the ith relay in $\mathcal{R}_k$, fully recovers $\mathcal{I}$, then $\pi_i^{[k]} = 1$. Otherwise, $\pi_i^{[k]} = 0$. The set of all realizations of $\boldsymbol{\pi}$ is denoted by

$$\mathcal{V}_\pi = \left\{ \boldsymbol{\pi} : \pi_i^{[k]} \in \{0, 1\}, 1 \le k \le \kappa, 1 \le i \le |\mathcal{R}_k| \right\}. \tag{5}$$

Let $\Pr\{\boldsymbol{\pi}\}$ denote the probability that a particular realization of $\boldsymbol{\pi} \in \mathcal{V}_\pi$ occurs, and $P_{\text{err}|\boldsymbol{\pi}}$ denote the conditional probability that the desired message distribution fails (i.e., less than $\lceil N \cdot \sigma\% \rceil$ destinations recover $\mathcal{I}$) given $\boldsymbol{\pi}$. The overall system error probability (2) thus is:

$$P_{\text{err}} = \sum_{\boldsymbol{\pi} \in \mathcal{V}_\pi} \Pr\{\boldsymbol{\pi}\} P_{\text{err}|\boldsymbol{\pi}}. \tag{6}$$

We can consider $\boldsymbol{\pi}$ as a sequence of K independent Bernoulli random variables. Therefore,

$$\Pr\{\boldsymbol{\pi}\} = \prod_{k=1}^{\kappa} \prod_{i=1}^{|\mathcal{R}_k|} \left(\Pr\left\{ \pi_i^{[k]} = 1 \right\} \right)^{\pi_i^{[k]}} \left(\Pr\left\{ \pi_i^{[k]} = 0 \right\} \right)^{1 - \pi_i^{[k]}}. \tag{7}$$

The event $\pi_i^{[k]} = 1$ occurs only if $\mathrm{R}_i^{[k]}$ correctly decodes all source signals, in each of the first μ time slots. From (1) we can see that at time slot t ($t \in \{1, \cdots, \mu\}$), the sources in $\mathcal{S}_t$ and $\mathrm{R}_i^{[k]}$ form an $|\mathcal{S}_t|$-user MAC channel. We use $Q_{\mathcal{A},b}^{[s]}$ to denote the probability that, in a MAC channel formed by a set of transmitters $\mathcal{A} = \{a_1, \cdots, a_{|\mathcal{A}|}\}$ and a common receiver b, the terminal b can successfully decode exactly s ($0 \le s \le |\mathcal{A}|$) messages through SIC. Now, the probability that $\mathrm{R}_i^{[k]}$ correctly recovers all messages sent from $\mathcal{S}_t$ can be expressed as $Q_{\mathcal{S}_t, \mathrm{R}_i^{[k]}}^{[|\mathcal{S}_t|]}$. This results in

$$\Pr\left\{ \pi_i^{[k]} = 1 \right\} = \prod_{t=1}^{\mu} Q_{\mathcal{S}_t, \mathrm{R}_i^{[k]}}^{[|\mathcal{S}_t|]}. \tag{8}$$

For each $\boldsymbol{\pi}$, substituting (8) and $\Pr\left\{ \pi_i^{[k]} = 0 \right\} = 1 - \Pr\left\{ \pi_i^{[k]} = 1 \right\}$ into (7) leads to $\Pr\{\boldsymbol{\pi}\}$.

Given $\boldsymbol{\pi}$, the destinations' decoding behaviors become independent. Define a binary vector $\boldsymbol{\tau}_{|\boldsymbol{\pi}} = [\tau_{1|\boldsymbol{\pi}}, \cdots, \tau_{N|\boldsymbol{\pi}}]$ to indicate their decoding status: $\tau_{n|\boldsymbol{\pi}} = 1$ if D_n obtains $\mathcal{I}$ and $\tau_{n|\boldsymbol{\pi}} = 0$ otherwise. The set of all realizations of $\boldsymbol{\tau}_{|\boldsymbol{\pi}}$ that result in failure of the desired transmission is:

$$\mathcal{V}_{\tau_{|\boldsymbol{\pi}}} = \left\{ \boldsymbol{\tau}_{|\boldsymbol{\pi}} : \sum_{n=1}^{N} \tau_{n|\boldsymbol{\pi}} < \lceil N \cdot \sigma\% \rceil, \tau_{n|\boldsymbol{\pi}} \in \{0, 1\} \right\}. \tag{9}$$

This leads to

$$P_{\text{err}|\boldsymbol{\pi}} = \sum_{\boldsymbol{\tau}_{|\boldsymbol{\pi}} \in \mathcal{V}_{\tau_{|\boldsymbol{\pi}}}} \prod_{n=1}^{N} \left(\Pr\{\tau_{n|\boldsymbol{\pi}} = 1\} \right)^{\tau_{n|\boldsymbol{\pi}}} \left(\Pr\{\tau_{n|\boldsymbol{\pi}} = 0\} \right)^{1 - \tau_{n|\boldsymbol{\pi}}}. \tag{10}$$

To find $\Pr\{\tau_{n|\boldsymbol{\pi}} = 1\}$, the conditional probability that destination D_n is capable of attaining all source messages given relay decoding status $\boldsymbol{\pi}$, we denote the cluster of nodes that transmit signals at time slot t ($t \in \{1, \cdots, \mu + \kappa\}$) by $\mathcal{T}_{t|\boldsymbol{\pi}}$. The probability that D_n can successfully decode the messages of s_t ($0 \le s_t \le |\mathcal{T}_{t|\boldsymbol{\pi}}|$) nodes in $\mathcal{T}_{t|\boldsymbol{\pi}}$ is the probability $Q_{\mathcal{T}_{t|\boldsymbol{\pi}}, \mathrm{D}_n}^{[s_t]}$. After $\mu + \kappa$ time slots, the probability that D_n

can obtain $\mathcal{I}$ is the probability that it correctly decodes at least M signals from all the $\sum_{t=1}^{\mu+\kappa}|\mathcal{T}_{t|\boldsymbol{\kappa}}|$ signals it received. Defining vector $\boldsymbol{s}=[s_1,\cdots,s_{\mu+\kappa}]$,

$$\mathcal{V}_{s|\boldsymbol{\pi}}=\left\{\boldsymbol{s}:\sum_{t=1}^{\mu+\kappa}s_t\geq M, 0\leq s_t\leq|\mathcal{T}_{t|\boldsymbol{\pi}}|\right\} \tag{11}$$

represents the set of all potential decoding status that allows a destination to recover $\mathcal{I}$. Then

$$\Pr\left\{\tau_{n|\boldsymbol{\pi}}=1\right\}=\sum_{\boldsymbol{s}\in\mathcal{V}_{s|\boldsymbol{\pi}}}\prod_{t=1}^{\mu+\kappa}Q^{[s_t]}_{\mathcal{T}_{t|\boldsymbol{\pi}},\mathrm{D}_n}. \tag{12}$$

Substituting (12) into (10), and then substituting (7) and (10) into (6) lead to P_{err}. From the above derivation process, we can see that if the probabilities $Q^{[s]}_{\mathcal{A},b}$ in MAC channels are known, the error performance of our C-NC scheme can be attained. Using P_{err}, we can further derive the finite-SNR DMT, and potentially other performance indicators such as spectral efficiency and energy efficiency [32]. When $|\mathcal{A}|=1$, $Q^{[0]}_{\mathcal{A},b}$ and $Q^{[1]}_{\mathcal{A},b}$ can be easily found by using the Gamma CDF. Hence identifying P_{err} of the O-NC scheme is straightforward. However, deriving the closed-form expressions of $Q^{[s]}_{\mathcal{A},b}$ can be very involved when the size of $\mathcal{A}$ is large. [33] studies the 2-user MAC channel under Rayleigh fading. In the following subsection, we present the results for $|\mathcal{A}|=2$ and $|\mathcal{A}|=3$ under Nakagami-m fading, and then use the results to find the performance of the C-NC scheme when the number of nodes in each cluster is upper-bounded by 3. The more general situations are analyzed through the infinite-SNR DMT in Section IV.

B. Individual Decoding Probabilities in MAC Channels

Consider a MAC channel formed by a set of simultaneously activated transmitters $\mathcal{A}=\{a_1,\cdots,a_{|\mathcal{A}|}\}$ and a common receiver b. Each transmitter $a\in\mathcal{A}$ uses independent Gaussian random codes to encode its message with rate R and transmitter-side SNR ρ_a. b applies SIC for decoding. Let $|h_{b,a}|^2\sim\text{Gamma}(m_a,\frac{m_a}{\Omega_{b,a}})$ denote the channel fading power gain between a and b. If for a given subset $\mathcal{G}\subseteq\mathcal{A}$, the following two events $E_{1,\mathcal{G}}$ and $E_{2,\bar{\mathcal{G}}}$ occur [33],

$$E_{1,\mathcal{G}}\text{: }\log_2\!\left(1+\frac{\sum_{a\in\mathcal{J}}\rho_a|h_{b,a}|^2}{1+\sum_{a'\in\bar{\mathcal{G}}}\rho_{a'}|h_{b,a'}|^2}\right)>|\mathcal{J}|R,\ \forall\mathcal{J}\subseteq\mathcal{G}, \tag{13}$$

$$E_{2,\bar{\mathcal{G}}}\text{: }\log_2\!\left(1+\frac{\sum_{a\in\mathcal{K}}\rho_a|h_{b,a}|^2}{1+\sum_{a'\in\bar{\mathcal{K}}\cap\bar{\mathcal{G}}}\rho_{a'}|h_{b,a'}|^2}\right)<|\mathcal{K}|R,\ \forall\mathcal{K}\subseteq\bar{\mathcal{G}}, \tag{14}$$

then b can decode signals from nodes in $\mathcal{G}$, but not those from the remaining nodes in $\bar{\mathcal{G}}$. The probability of occurring such events, denoted by $Q^{\mathcal{G}}_{\mathcal{A},b}=\Pr\{E_{1,\mathcal{G}},E_{2,\bar{\mathcal{G}}}|\mathcal{G},\mathcal{A}\}$, is termed *individual decoding probability* that b recovers the messages of the nodes in $\mathcal{G}$. Consequently, the probability that b recovers *exactly* s $(0\leq s\leq|\mathcal{A}|)$ messages, i.e., $Q^{[s]}_{\mathcal{A},b}$, is calculated by:

$$Q^{[s]}_{\mathcal{A},b}=\sum_{|\mathcal{G}|=s}Q^{\mathcal{G}}_{\mathcal{A},b}, \tag{15}$$

where the summation is taken over all possible $\mathcal{G}$ that satisfy $|\mathcal{G}|=s$.

If $\mathcal{A}=\{a_1\}$, i.e., a point-to-point channel, it is simple to have $Q^{[0]}_{\mathcal{A},b}=F(2^R-1;m_a,\frac{m_a}{\rho_a\Omega_{b,a}})$ and $Q^{[1]}_{\mathcal{A},b}=1-Q^{[0]}_{\mathcal{A},b}$, in which $F(x;\alpha,\beta)$ denotes the Gamma CDF. The following proposition provides the expressions of $Q^{\mathcal{G}}_{\mathcal{A},b}$ in a two-user MAC channel. Those in a symmetric three-user MAC channel, where $m_a=m$ and $\rho_a\Omega_{b,a}=\rho\Omega$ for all $a\in\mathcal{A}$, are presented in Appendix A.[1]

[1]Due to paper page limit, we consider only the symmetric network case. The individual decoding probabilities in general 3-user MAC channels can be derived by following the same procedure presented in Appendix A.

Proposition 1: Consider a 2-user MAC channel with $\mathcal{A} = \{a_1, a_2\}$. Let $m_1 = m_{a_1}$, $m_2 = m_{a_2}$, $\beta_1 = \frac{m_{a_1}}{\rho_{a_1}\Omega_{b,a_1}}$, $\beta_2 = \frac{m_{a_2}}{\rho_{a_2}\Omega_{b,a_2}}$, and $\eta_s = 2^{sR} - 1$ for any integer s. When $\beta_1 = \beta_2$, the probability that b successfully recovers both transmitters' messages, $Q_{\mathcal{A},b}^{\{a_1,a_2\}}$, is

$$Q_{\mathcal{A},b}^{\{a_1,a_2\}} = \sum_{i=0}^{m_1-1}\sum_{j=0}^{i} \frac{(-1)^j \beta_1^{m_2+i} \eta_1^{m_2+j} \eta_2^{i-j} \eta_{m_2+j} e^{-\beta_1\eta_2}}{(m_2-1)!j!(i-j)!(m_2+j)} + \sum_{i=0}^{m_2-1}\sum_{j=0}^{m_1-1} \frac{\beta_1^{i+j}(\eta_2-\eta_1)^i \eta_1^j e^{-\beta_1\eta_2}}{i!j!}. \tag{16}$$

When $\beta_1 \neq \beta_2$, the probability $Q_{\mathcal{A},b}^{\{a_1,a_2\}}$ is

$$\begin{aligned} Q_{\mathcal{A},b}^{\{a_1,a_2\}} &= \sum_{i=0}^{m_1-1}\sum_{j=0}^{i}\sum_{l=0}^{m_2+j-1} \frac{(-1)^j \beta_1^i \beta_2^{m_2} \eta_2^{i-j} e^{-\beta_1\eta_2}\left(\eta_1^l e^{-(\beta_2-\beta_1)\eta_1} - (\eta_2-\eta_1)^l e^{-(\beta_2-\beta_1)(\eta_2-\eta_1)}\right)}{(m_2-1)!j!(i-j)!l!(\beta_2-\beta_1)^{m_2+j-l}/(m_2+j-1)!} \\ &+ \sum_{i=0}^{m_2-1}\sum_{j=0}^{m_1-1} \frac{\beta_1^j \beta_2^i (\eta_2-\eta_1)^i \eta_1^j e^{-\beta_1\eta_1} e^{-\beta_2(\eta_2-\eta_1)}}{i!j!}. \end{aligned} \tag{17}$$

The individual decoding probabilities that b recovers the message of only one transmitter are

$$Q_{\mathcal{A},b}^{\{a_1\}} = \sum_{i=0}^{m_1-1}\sum_{j=0}^{i} \frac{\beta_1^i \beta_2^{m_2} \eta_1^i (m_2+j-1)! e^{-\beta_1\eta_1}}{j!(i-j)!(m_2-1)!\left(\beta_1\eta_1+\beta_2\right)^{m_2+j}} \left(1 - \sum_{l=0}^{m_2+j-1} \frac{(\beta_1\eta_1^2+\beta_2\eta_1)^l}{l!} e^{-(\beta_1\eta_1^2+\beta_2\eta_1)}\right), \tag{18}$$

$$Q_{\mathcal{A},b}^{\{a_2\}} = \sum_{i=0}^{m_2-1}\sum_{j=0}^{i} \frac{\beta_2^i \beta_1^{m_1} \eta_1^i (m_1+j-1)! e^{-\beta_2\eta_1}}{j!(i-j)!(m_1-1)!\left(\beta_2\eta_1+\beta_1\right)^{m_1+j}} \left(1 - \sum_{l=0}^{m_1+j-1} \frac{(\beta_2\eta_1^2+\beta_1\eta_1)^l}{l!} e^{-(\beta_2\eta_1^2+\beta_1\eta_1)}\right). \tag{19}$$

Finally, for an empty decoded message set ϕ, $Q_{\mathcal{A},b}^{\phi} = 1 - Q_{\mathcal{A},b}^{\{a_1\}} - Q_{\mathcal{A},b}^{\{a_2\}} - Q_{\mathcal{A},b}^{\{a_1,a_2\}}$.

Proof: Denote $H_i = \rho_{a_i}|h_{b,a_i}|^2 \sim \text{Gamma}(m_i, \beta_i)$ for $i \in \{1,2\}$. The binomial expansion $(x+y)^n = \sum_{i=0}^{n}\binom{n}{i}x^{n-i}y^i$ and the relationship between the PDF and CDF of random variable $X \sim \text{Gamma}(\alpha,\beta)$, i.e., $\int_0^x f_X(x')\mathrm{d}x' = F(x;\alpha,\beta) = 1 - \sum_{i=0}^{\alpha-1}\frac{(\beta x)^i}{i!}e^{-\beta x}$ will be used.

The basic idea is to divide the sample space of the bivariate Gamma distribution (H_1 and H_2) into sub-regions based on the decoding events defined in (13) and (14), and then take integrals regarding the PDF to reach the corresponding individual decoding probabilities. Specifically,

$$Q_{\mathcal{A},b}^{\{a_1,a_2\}} = \Pr\{H_1 > \eta_1, H_2 > \eta_1, H_1 + H_2 > \eta_2\} = \mathcal{P}\{A_1\} + \mathcal{P}\{A_2\}. \tag{20}$$

A_1 and A_2 denote two sub-regions, the union of which corresponds to the events $H_1 > \eta_1, H_2 > \eta_1, H_1 + H_2 > \eta_2$, and $\mathcal{P}\{A_i\}$ is the probability that a random sample locates in A_i [33], [34]:

$$\begin{aligned} \mathcal{P}\{A_1\} &= \int_{\eta_1}^{\eta_2-\eta_1}\left(\int_{\eta_2-x_2}^{\infty} f_{H_1}(x_1)\mathrm{d}x_1\right) f_{H_2}(x_2)\mathrm{d}x_2 = \int_{\eta_1}^{\eta_2-\eta_1}\big(1-F(\eta_2-x_2;m_1,\beta_1)\big) f_{H_2}(x_2)\mathrm{d}x_2 \\ &= \frac{\beta_2^{m_2}e^{-\beta_1\eta_2}}{\Gamma(m_2)}\sum_{i=0}^{m_1-1}\sum_{j=0}^{i}\binom{i}{j}\frac{\beta_1^i(-1)^j\eta_2^{i-j}}{i!}\cdot\int_{\eta_1}^{\eta_2-\eta_1} x_2^{m_2-1+j}e^{-(\beta_2-\beta_1)x_2}\mathrm{d}x_2. \end{aligned}$$

$$\mathcal{P}\{A_2\} = \int_{\eta_2-\eta_1}^{\infty}\left(\int_{\eta_1}^{\infty} f_{H_1}(x_1)\mathrm{d}x_1\right) f_{H_2}(x_2)\mathrm{d}x_2 = \big(1 - F(\eta_2-\eta_1; m_2, \beta_2)\big)\big(1 - F(\eta_1; m_1, \beta_1)\big).$$

If $\beta_1 = \beta_2$, we can complete the integral in $\mathcal{P}\{A_1\}$ and attain

$$\mathcal{P}\{A_1\} = \sum_{i=0}^{m_1-1}\sum_{j=0}^{i}\frac{(-1)^j\beta_1^i\beta_2^{m_2}\eta_2^{i-j}e^{-\beta_1\eta_2}}{(m_2-1)!j!(i-j)!}\cdot\frac{(\eta_2-\eta_1)^{m_2+j}-\eta_1^{m_2+j}}{m_2+j}.$$

When $\beta_1 \neq \beta_2$, we have

$$\mathcal{P}\{A_1\} = \sum_{i=0}^{m_1-1} \sum_{j=0}^{i} \frac{(-1)^j \beta_1^i \beta_2^{m_2} e^{-\beta_1 \eta_2} \eta_2^{i-j} \left(F(\eta_2-\eta_1; m_2+j, \beta_2-\beta_1) - F(\eta_1; m_2+j, \beta_2-\beta_1)\right)}{(m_2-1)! j! (i-j)! \left(\beta_2-\beta_1\right)^{m_2+j} / (m_2+j-1)!}.$$

Substituting the expressions of $\mathcal{P}\{A_1\}$ and $\mathcal{P}\{A_2\}$ into (20) leads to (16) and (17).

The probability that b correctly decodes only a_1 but not a_2, $Q_{\mathcal{A},b}^{\{a_1\}}$, can be calculated as

$$\begin{aligned} Q_{\mathcal{A},b}^{\{a_1\}} &= \Pr\left\{H_2 < \eta_1, H_1 > \eta_1(H_2+1)\right\} = \int_0^{\eta_1} \left(\int_{\eta_1(x_2+1)}^{\infty} f_{H_1}(x_1) \mathrm{d}x_1 \right) f_{H_2}(x_2) \mathrm{d}x_2 \\ &= \frac{\beta_2^{m_2} e^{-\beta_1 \eta_1}}{\Gamma(m_2)} \sum_{i=0}^{m_1-1} \sum_{j=0}^{i} \binom{i}{j} \frac{\beta_1^i \eta_1^i \Gamma(m_2+j)}{i! \left(\beta_1 \eta_1 + \beta_2\right)^{m_2+j}} \cdot F(\eta_1; m_2+j, \beta_1 \eta_1 + \beta_2). \end{aligned} \tag{21}$$

This is the expression in (18). Due to symmetry, the probability that b correctly decodes only a_2, i.e., the individual decoding probability $Q_{\mathcal{A},b}^{\{a_2\}}$, can be obtained by swapping m_1 and m_2, and swapping β_1 and β_2 in the above equation. This leads to (19) and completes the proof. ■

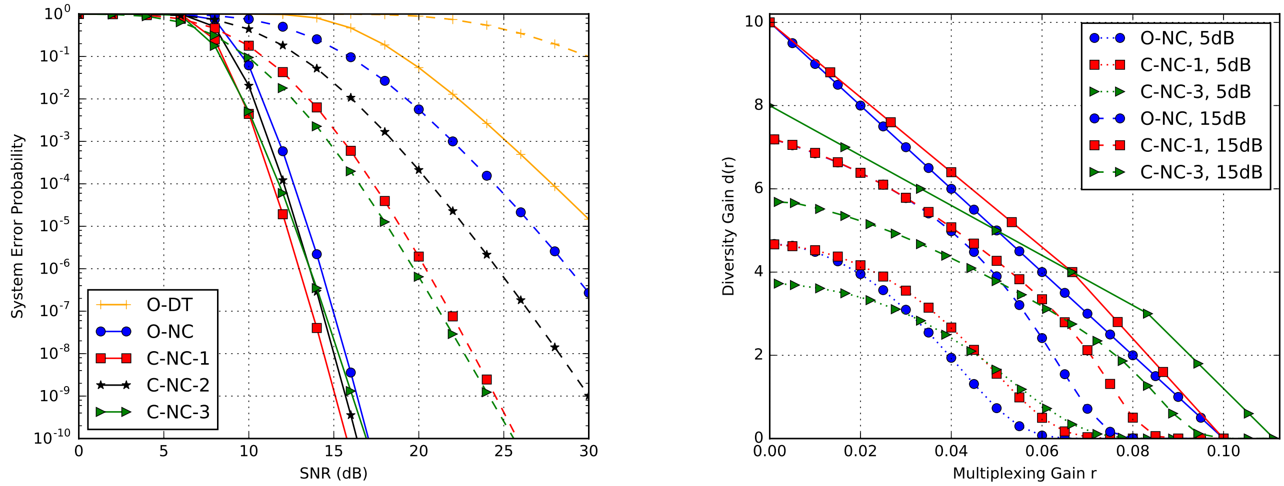
Using Proposition 1, we can attain the probabilities that the receiver b decodes two, one, and zero transmitters in $\mathcal{A}$ as $Q_{\mathcal{A},b}^{[2]} = Q_{\mathcal{A},b}^{\{a_1,a_2\}}$, $Q_{\mathcal{A},b}^{[1]} = Q_{\mathcal{A},b}^{\{a_1\}} + Q_{\mathcal{A},b}^{\{a_2\}}$, and $Q_{\mathcal{A},b}^{[0]} = Q_{\mathcal{A},b}^{\phi}$, respectively. These results can be used to derive $\Pr\{\boldsymbol{\pi}\}$ in (7) and $P_{\text{err}|\boldsymbol{\pi}}$ in (10), and thus to identify the system error probability P_{err} of our C-NC scheme in (6), if each cluster has no more than two terminals. Consider a 6-source, 4-relay, 25-destination example network. We can divide the 6 sources into three 2-node clusters and divide the 4 relays into two 2-node clusters. The C-NC scheme uses 5 time slots to complete the distribution of the 6 source messages. If the O-NC scheme is adopted, a total of 10 time slots would be required to deliver the same amount of information. In Appendix A, we also present the individual decoding probabilities in a special symmetric three-user MAC channel. Using these results, one can attain $Q_{\mathcal{A},b}^{[3]}$, $Q_{\mathcal{A},b}^{[2]}$, $Q_{\mathcal{A},b}^{[1]}$, and $Q_{\mathcal{A},b}^{[0]}$ for $\mathcal{A} = \{a_1, a_2, a_3\}$. Clustering strategies that have 3-node clusters can also be analyzed. In the above network, one can partition the 6 sources into two clusters each with 3 nodes, while keeping two 2-node relay clusters. 4 time slots suffice for completing the message distribution. Comparison between these clustering methods is presented in the following subsection.

Now, if we consider the special case that $m_{a_1} = m_{a_2} = 1$, the results in Proposition 1 become $Q_{\mathcal{A},b}^{\{a_1,a_2\}} = (\beta_1 \eta_1^2 + 1) e^{-\beta_1 \eta_2}$ when $\rho_{a_1} \Omega_{b,a_1} = \rho_{a_2} \Omega_{b,a_2}$, $Q_{\mathcal{A},b}^{\{a_1,a_2\}} = \frac{\beta_2 e^{-\beta_1 \eta_2} \left(e^{-(\beta_2-\beta_1)\eta_1} - e^{-(\beta_2-\beta_1)(\eta_2-\eta_1)}\right)}{\beta_2-\beta_1} + e^{-\beta_1 \eta_1} e^{-\beta_2(\eta_2-\eta_1)}$ when $\rho_{a_1} \Omega_{b,a_1} \neq \rho_{a_2} \Omega_{b,a_2}$, $Q_{\mathcal{A},b}^{\{a_1\}} = \frac{\beta_2 e^{-\beta_1 \eta_1}}{\beta_1 \eta_1 + \beta_2} \left(1 - e^{-(\beta_1 \eta_1^2 + \beta_2 \eta_1)}\right)$, and finally $Q_{\mathcal{A},b}^{\{a_2\}} = \frac{\beta_1 e^{-\beta_2 \eta_1}}{\beta_2 \eta_1 + \beta_1} \left(1 - e^{-(\beta_2 \eta_1^2 + \beta_1 \eta_1)}\right)$. They are consistent with [33]. Hence all results presented in our paper can be directly applied to Rayleigh fading environments.

C. Numerical Results

In this subsection, we use an example network with $M = 6$, $K = 4$, and $N = 25$ to demonstrate the performance of our C-NC scheme. For presentation simplicity, the sources and cooperative relays are considered to have the same power $\rho_{\text{s}} = \rho_{\text{r}} = \rho$, and the large-scale fading power gains are normalized to $\Omega_{b,a} = 1$ for all transmitter-receiver pairs. We first consider the case with Nakagami shape (small-scale fading) parameters $m_{\text{sd}} = 1$, $m_{\text{sr}} = 3$, and $m_{\text{rd}} = 2$. The destination decoding proportion parameter $\sigma = 85$, which means at least 85% or 22 of the 25 destinations are expected to successfully attain the 6 source messages. For fair comparison, we require different schemes to have the same average transmission rate $\bar{R}$ bit/source/time slot.

There are multiple ways to partition the sources and relays. For instance, we can set $\mu = 3$, $\kappa = 2$, and $\mathcal{S}_1 = \{\mathrm{S}_1, \mathrm{S}_2\}$, $\mathcal{S}_2 = \{\mathrm{S}_3, \mathrm{S}_4\}$, $\mathcal{S}_3 = \{\mathrm{S}_5, \mathrm{S}_6\}$, $\mathcal{R}_1 = \{\mathrm{R}_1, \mathrm{R}_2\}$, $\mathcal{R}_2 = \{\mathrm{R}_3, \mathrm{R}_4\}$. The sources and relays are divided into five 2-node clusters. We term the C-NC scheme with this clustering strategy "C-NC-1."



(a) $\sigma = 85$, $m_{\text{sd}} = 1$, $m_{\text{sr}} = 3$, $m_{\text{rd}} = 2$. Solid and dash curves denote the cases $\bar{R} = \frac{1}{6}$ and $r = \frac{1}{16}$ respectively.

(b) $\sigma = 100$, $m_{\text{sd}} = 2$, $m_{\text{sr}} = 3$, $m_{\text{rd}} = 2$. Solid curves denote the case with infinitely large SNR.

Fig. 2: (a) Error probability and (b) DMT in 6-source, 4-relay, 25-destination networks.

Fig. 2(a) illustrates the error performance comparison of the C-NC-1 scheme (using 5 time slots) with the orthogonal direct transmission (O-DT) without relaying (using 6 time slots) and the O-NC scheme (using 10 time slots), when $\bar{R}$ is fixed at $\bar{R} = 1/6$. Clearly, both C-NC-1 and O-NC achieve much higher diversity gain and smaller error probability than the O-DT scheme. Network-coded cooperation provides protection to the source message distribution. Our C-NC-1 scheme has lower error probability than the O-NC scheme due to its efficient usage of available channel through non-orthogonal transmission. The performance advantage can become more significant if we allow the transmission data rate to scale with SNR. For example, we also display the error performance when the multiplexing gain in (3) is $r = \frac{1}{16}$, i.e., $\bar{R} = \frac{1}{16}\log(1+\rho)$. The C-NC-1 scheme's error probability is much smaller than the O-NC scheme's now, especially when SNR increases. A larger achievable diversity gain is observed.

We can also divide the 6 sources into two 3-node clusters, while keeping the relays as two 2-node clusters. Four time slots are used to complete the transmission. This scheme is termed "C-NC-2" and its error probability can be derived using the results presented in Proposition 1 and Appendix A. From Fig. 2(a), it is seen that such a clustering method does not provide performance improvement over C-NC-1, for the considered rates $\bar{R} = \frac{1}{6}$ and $\bar{R} = \frac{1}{16}\log(1+\rho)$, because of large inter-user interference. In addition, we can even discard one relay. This reduces the maximally achievable diversity gain, but may demand a smaller channel consumption. For instance, in such a 6-source 3-relay network, we treat the sources and relays as three 3-node clusters, and 3 time slots are sufficient to finish the delivery of source messages. This scheme is termed "C-NC-3." We can see from Fig. 2(a) that it achieves lower error probability than the C-NC-1 scheme for the scenario $\bar{R} = \frac{1}{6}$ when SNR is small, and for $r = \frac{1}{16}$ when ρ is not too large. Therefore, discarding certain relays can even be advantageous.

Using the closed-form expressions of P_{err}, we can further apply (3) to derive the entire finite-SNR DMT, to obtain a more complete description of the performances of the C-NC scheme. In Fig. 2(b), we plot some results for the same 6-source 4-relay 25-destination network structure, but with a different decoding proportion coefficient, $\sigma = 100$, and different fading characteristics, $m_{\text{sd}} = 2$, $m_{\text{sr}} = 3$, and $m_{\text{rd}} = 2$. All destinations are expected to fully recover all the six source messages. It is seen that for the two chosen operating SNR $\rho = 5$ dB and $\rho = 15$ dB, the C-NC-1 scheme achieves strictly better finite-SNR DMT than the O-NC scheme, which implies both higher channel usage efficiency and higher communication reliability. The results shown in the figure provide a positive answer to the question raised in Section I: Combining non-orthogonal transmission with NC techniques is able to achieve performance improvements over orthogonal transmission in more general network-coded cooperation networks with multiple destinations, under Nakagami-m fading and finite SNR. In addition, comparing the two C-NC schemes with different clustering strategies, one of them can be better for different operating r.

Clearly, for different system parameters (network structure, transmission rate, operating SNR and multiplexing gain, channel fading statistics, and communication quality requirement, etc.), choosing different clustering strategies can lead to various performance. Properly partitioning the sources and relays is thus of importance, especially when the large-scale fading between different nodes (i.e., $\Omega_{b,a}$) has diverse properties. This can be directed by the error probability and DMT results presented in our paper. One can simply enumerate all possible clustering approaches (including discarding a subset of relays) and compare their performance to make the selection decision. Note that now the closed-form expressions of P_{err} and $d(r,\rho)$ can only be obtained for clustering solutions with limited cluster sizes. For more general cases, we can consider resorting to the asymptotic performance analysis by letting $\rho \to \infty$.

IV. High-SNR Asymptotic Performance Analysis

In this section, we present the achievable infinite-SNR DMT of our C-NC scheme. Clustering strategies in certain special network structures are also discussed.

A. Achievable Infinite-SNR DMT

To help presentation, we first define some notations. Use M_1, M_2, $\cdots$, M_μ to denote the ordered sizes of the source clusters such that $M_1 \geq M_2 \geq \cdots \geq M_\mu$ (i.e., M_1 is the size of the largest source cluster and M_μ is that of the smallest). Similarly, use K_1, K_2, $\cdots$, K_κ to denote the ordered sizes of the relay clusters with $K_1 \geq K_2 \geq \cdots \geq K_\kappa$. Further, let $M_0 = K_0 = 0$, and for each value $U \in \{0, 1, \cdots, K\}$, we can find the integer κ_U such that $\sum_{k=0}^{\kappa_U - 1} K_k < U \leq \sum_{k=0}^{\kappa_U} K_k$. Set $\tilde{K}_k^{[U]} = K_k$ for $k \in \{0, 1, \cdots, \kappa_U - 1\}$, and $\tilde{K}_{\kappa_U}^{[U]} = U - \sum_{k=0}^{\kappa_U - 1} K_k$. Finally, letting $\tilde{U}_{uv}^{[U]} = U + 1 - \sum_{j=0}^{u} M_j - \sum_{k=0}^{v} \tilde{K}_k^{[U]}$ be a function of U and non-negative integers u and v, define two sets regarding the integer two-tuple (u, v):

$$\mathcal{V}_1^{[U]} = \left\{(u,v) : 0 < \tilde{U}_{uv}^{[U]} \leq M_{u+1}\right\} \quad \text{and} \quad \mathcal{V}_2^{[U]} = \left\{(u,v) : 0 < \tilde{U}_{uv}^{[U]} \leq \tilde{K}_{v+1}^{[U]}\right\}. \tag{22}$$

For instance, consider a 6-source 4-relay example network with $\mu = 2$, $\kappa = 2$, and $M_1 = 4$, $M_2 = 2$, $K_1 = 3$, $K_2 = 1$. For $U = 1$, we have $\kappa_1 = 1$ since $\sum_{k=0}^{0} K_k < U \leq \sum_{k=0}^{1} K_k$. Using $\tilde{K}_0^{[1]} = K_0 = 0$, $\tilde{K}_1^{[1]} = U - K_0 = 1$, and expression $\tilde{U}_{uv}^{[1]} = 2 - \sum_{j=0}^{u} M_j - \sum_{k=0}^{v} \tilde{K}_k^{[1]}$, we obtain $\mathcal{V}_1^{[1]} = \{(0,0), (0,1)\}$ and $\mathcal{V}_2^{[1]} = \phi$. For $U = 4$, we have $\kappa_4 = 2$. Using $\tilde{K}_0^{[4]} = K_0 = 0$, $\tilde{K}_1^{[4]} = K_1 = 3$, $\tilde{K}_2^{[4]} = U - K_1 - K_0 = 1$, and $\tilde{U}_{uv}^{[4]} = 5 - \sum_{j=0}^{u} M_j - \sum_{k=0}^{v} \tilde{K}_k^{[4]}$, we have $\mathcal{V}_1^{[4]} = \{(1,0), (0,1), (0,2)\}$ and $\mathcal{V}_2^{[4]} = \{(1,0)\}$. Using these notations, the achievable infinite-SNR DMT, as defined in (4), of our C-NC scheme is summarized in the following proposition.

***Proposition** 2:* Applying the C-NC scheme in the considered M-source, K-relay, N-destination network with source clusters $\mathcal{S}_1$, $\cdots$, $\mathcal{S}_\mu$ and relay clusters $\mathcal{R}_1$, $\cdots$, $\mathcal{R}_\kappa$, the achievable DMT

$$d^*(r^*) = \min_{U \in \{0,1,\cdots,K\}} \left\{(K-U)\,\hat{d}(r') + (\lfloor (1-\sigma\%)N \rfloor + 1)\,\tilde{d}(r', U)\right\}, \tag{23}$$

where $r' = (\mu + \kappa) r^*$ is the effective multiplexing gain for source message (i.e., $R = r' \log_2 \rho$), and $\hat{d}(r')$ and $\tilde{d}(r', U)$ reflect respectively the diversity order at each relay and each destination:

$$\hat{d}(r') = m_{\text{sr}} \min\left\{(1 - r')^+, M_1 (1 - M_1 r')^+\right\}, \tag{24}$$

$$\tilde{d}(r', U) = \min\left\{\min_{(u,v)\in\mathcal{V}_1^{[U]}} \left\{\tilde{d}_{uv,1}^{[U]}(r')\right\}, \min_{(u,v)\in\mathcal{V}_2^{[U]}} \left\{\tilde{d}_{uv,2}^{[U]}(r')\right\}\right\}, \tag{25}$$

$$
\begin{aligned}
\tilde{d}^{[U]}_{uv,1}(r') \;=\; & m_{\rm sd} \min\left\{ \tilde{U}^{[U]}_{uv} \left(1-\tilde{U}^{[U]}_{uv} r'\right)^{+}, M_{u+1}\left(1-M_{u+1}r'\right)^{+} \right\} \\
& + \sum_{j=0}^{u} m_{\rm sd} M_j \left(1-M_j r'\right)^{+} + \sum_{k=0}^{v} m_{\rm rd} \tilde{K}^{[U]}_{k} \left(1-\tilde{K}^{[U]}_{k} r'\right)^{+}, \qquad (26)
\end{aligned}
$$

$$
\begin{aligned}
\tilde{d}^{[U]}_{uv,2}(r') \;=\; & m_{\rm rd} \min\left\{ \tilde{U}^{[U]}_{uv} \left(1-\tilde{U}^{[U]}_{uv} r'\right)^{+}, \tilde{K}^{[U]}_{v+1}\left(1-\tilde{K}^{[U]}_{v+1}r'\right)^{+} \right\} \\
& + \sum_{j=0}^{u} m_{\rm sd} M_j \left(1-M_j r'\right)^{+} + \sum_{k=0}^{v} m_{\rm rd} \tilde{K}^{[U]}_{k} \left(1-\tilde{K}^{[U]}_{k} r'\right)^{+}. \qquad (27)
\end{aligned}
$$

Proof: See Appendix B. ■

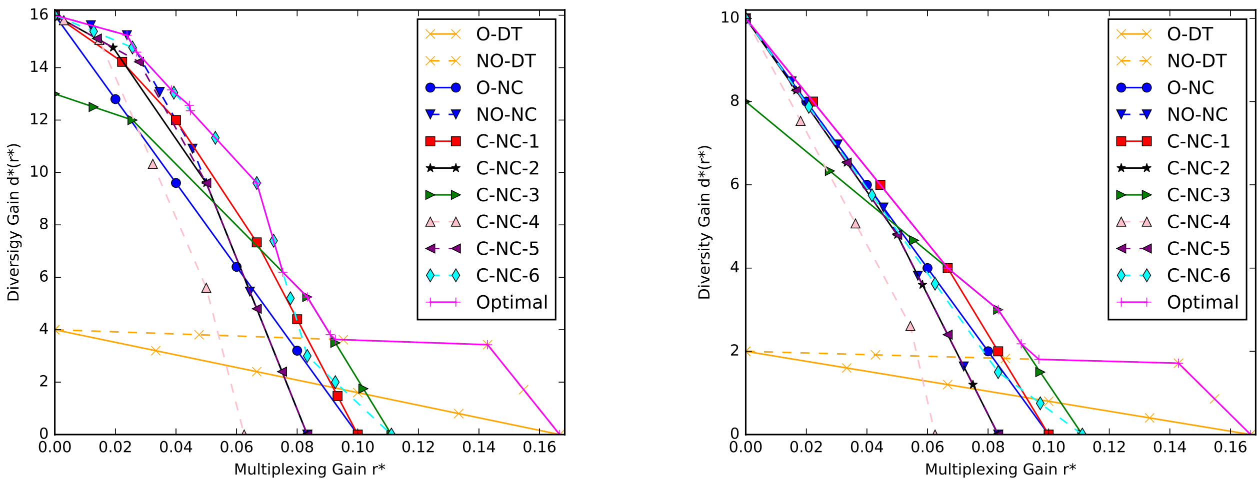
It can be clearly seen that, similar to $P_{\rm err}$ presented in Section III, the achievable DMT is also affected by system parameters including network structure (M, K, N), operating multiplexing gain (r^*), channel fading statistics ($m_{\rm sd}$, $m_{\rm sr}$, $m_{\rm rd}$), communication quality requirement (σ), and the clustering strategy (μ, κ, $|\mathcal{S}_1|, \cdots, |\mathcal{S}_\mu|, |\mathcal{R}_1|, \cdots, |\mathcal{R}_\kappa|$). Proposition 2 provides the method of deriving the DMT, for each clustering approach. Consider again the aforementioned example network with $M=6$, $K=4$, $N=25$, $\sigma=85$, $m_{\rm sd}=1$, $m_{\rm sr}=3$, and $m_{\rm rd}=2$. Let us set $\mu=\kappa=2$, and divide the sources into $\mathcal{S}_1=\{{\rm S}_1,{\rm S}_2,{\rm S}_3,{\rm S}_4\}$ and $\mathcal{S}_2=\{{\rm S}_5,{\rm S}_6\}$, and the relays into $\mathcal{R}_1=\{{\rm R}_1\}$ and $\mathcal{R}_2=\{{\rm R}_2,{\rm R}_3,{\rm R}_4\}$ (we term it "C-NC-4"). Using $M_1=4$, $M_2=2$, $K_1=3$, and $K_2=1$, we have $r'=4r^*$, $\lfloor(1-\sigma\%)N\rfloor+1=4$, and $\hat{d}(r')=3\min\left\{(1-r')^+,4(1-4r')^+\right\}$.

For $U=0$, we have $\kappa_0=0$, $\tilde{K}^{[0]}_0=0$, and $\tilde{U}^{[0]}_{uv}=1-\sum_{j=0}^{u}M_j-\sum_{k=0}^{v}\tilde{K}^{[0]}_k$. It is easy to see that the sets $\mathcal{V}^{[0]}_1=\{(0,0)\}$ and $\mathcal{V}^{[0]}_2=\phi$. In this case, $\tilde{d}(r',0)=m_{\rm sd}\min\left\{\tilde{U}^{[0]}_{00}(1-\tilde{U}^{[0]}_{00}r')^+,M_1(1-M_1r')^+\right\}=\min\left\{(1-r')^+,4(1-4r')^+\right\}$. For $U=1$, following earlier discussion, we have $\kappa_1=1$, $\tilde{K}^{[1]}_0=0$, $\tilde{K}^{[1]}_1=1$, $\tilde{U}^{[1]}_{uv}=2-\sum_{j=0}^{u}M_j-\sum_{k=0}^{v}\tilde{K}^{[1]}_k$, $\mathcal{V}^{[1]}_1=\{(0,0),(0,1)\}$, and $\mathcal{V}^{[1]}_2=\phi$. It is straightforward to show $\tilde{d}(r',1)=\min\left\{2(1-2r')^+,4(1-4r')^+\right\}$.

Similarly, we can also find $\tilde{d}(r',U)$ for $U\in\{2,3,4\}$. Applying (23) leads to $d^*(r^*)=\min\{16-64r^*,19-268r^*,28-448r^*\}$ for $0\le r^*\le\frac{1}{16}$. This result is displayed in Fig. 3(a). We can compare it with $d^*(r^*)=4(1-6r^*)^+$ and $d^*(r^*)=\min\{4(1-r^*)^+,24(1-6r^*)^+\}$, achieved respectively by O-DT and non-orthogonal DT (NO-DT). For small r^*, our C-NC-4 scheme significantly increases achievable diversity gain, due to the utilization of cooperative relays. Compared with $d^*(r^*)=16(1-10r^*)^+$, attained by the O-NC scheme, allowing non-orthogonal transmission among terminals obtains better diversity gain, for small values of r^*.

To achieve high diversity gain for a wider range of multiplexing gain, one can adopt other clustering strategies. For example, we can choose to evenly partition the sources, i.e., $|\mathcal{S}_1|=|\mathcal{S}_2|=3$, while keeping $|\mathcal{R}_1|=1$ and $|\mathcal{R}_2|=3$ (termed "C-NC-5"). It strictly outperforms the C-NC-4 scheme, in terms of DMT. Further, we can also set $\mu=\kappa=1$, which leads to the NO-NC scheme. Different from the case in single-hop networks, requiring all terminals to non-orthogonally send messages does not always result in better DMT than O-NC. In this example, the NO-NC scheme achieves $d^*(r^*)=\min\{16-32r^*,20-200r^*,24-288r^*\}$ for $0\le r^*\le\frac{1}{12}$, and outperforms the O-NC scheme only when $0<r^*<\frac{1}{16}$. In Fig. 3(b) we consider the same network structure but with different channel statistics, $m_{\rm sd}=m_{\rm rd}=2$ and $m_{\rm sr}=3$, and decoding requirement, $\sigma=100$ (corresponding to the case shown in Fig. 2(b)). Now the NO-NC scheme is always worse than the O-NC scheme. In both figures, we can see that if we divide the sources into three 2-node clusters and also divide the relays into two 2-relay clusters (i.e., the C-NC-1 scheme), a strictly better performance than O-NC can be attained.

It should still be noted that $d^*(r^*)$ is a high-SNR performance metric. If one clustering approach attains a higher value of $d^*(r^*)$ than another method, the error probability of the former can be guaranteed to be smaller when the SNR ρ is sufficiently large. But for the low- and median- SNR regimes, it may not be true. This can be seen from Fig. 2(a) and Fig. 3(a), by comparing the achievable $P_{\rm err}$ and $d^*(r^*)$



(a) $\sigma = 85$, $m_{\text{sd}} = 1$, $m_{\text{sr}} = 3$, $m_{\text{rd}} = 2$

(b) $\sigma = 100$, $m_{\text{sd}} = 2$, $m_{\text{sr}} = 3$, $m_{\text{rd}} = 2$

Fig. 3: Achievable infinite-SNR DMT in 6-source, 4-relay, 25-destination networks.

of the C-NC-1 and C-NC-3 schemes for $r = \frac{1}{16}$. In addition, $d^*(r^*)$ is related to only the number of terminals in each cluster. A clustering strategy with $\mathcal{R}_1 = \{\text{R}_1, \text{R}_2\}$, $\mathcal{R}_2 = \{\text{R}_3, \text{R}_4\}$ obtaines the same infinite-SNR DMT as another strategy with $\mathcal{R}_1 = \{\text{R}_1, \text{R}_4\}$, $\mathcal{R}_2 = \{\text{R}_2, \text{R}_3\}$. But the actual system error probabilities of these two methods can be different, even through they have the same high-SNR slopes. This is true especially if the channels between terminals exhibit diverse fading characteristics. Therefore, DMT provides a relatively coarse indicator for selecting clustering solutions. Properly deciding non-orthogonal transmission among sources/relays according to system parameters is also important.

In what follows, we provide further discussions regarding Proposition 2, to gain more insights into the performance of the C-NC scheme. To simplify notation we set $z = \lfloor (1 - \sigma\%)N \rfloor + 1$.

B. Maximally Achievable DMT

The *maximally achievable diversity gain* of the C-NC scheme can be readily calculated by setting $r^* = 0$. Following Proposition 2, it is seen that $\hat{d}(0) = m_{\text{sr}}$, $\tilde{d}^{[U]}_{uv,1}(0) = m_{\text{sd}}(U+1) + \sum_{k=0}^{v}(m_{\text{rd}} - m_{\text{sd}})\tilde{K}_k^{[U]}$, and $\tilde{d}^{[U]}_{uv,2}(0) = m_{\text{rd}}(U+1) + \sum_{j=0}^{u}(m_{\text{sd}} - m_{\text{rd}})M_j$. Clearly, if $m_{\text{rd}} \leq m_{\text{sd}}$:

$$d^*(0) = \min_{U\in\{0,\cdots,K\}} \Big\{(K-U)m_{\text{sr}} + z(Um_{\text{rd}} + m_{\text{sd}})\Big\} = zm_{\text{sd}} + \min\{Km_{\text{sr}}, Kzm_{\text{rd}}\}. \tag{28}$$

Otherwise, for the case $m_{\text{rd}} > m_{\text{sd}}$, if $K < M$,

$$d^*(0) = \min_{U\in\{0,\cdots,K\}} \Big\{(K-U)m_{\text{sr}} + z(U+1)m_{\text{sd}}\Big\} = zm_{\text{sd}} + \min\{Km_{\text{sr}}, Kzm_{\text{sd}}\}. \tag{29}$$

Finally, if $m_{\text{rd}} > m_{\text{sd}}$ but $K \geq M$, we have

$$\begin{aligned} d^*(0) &= \min\Big\{\min_{U\in\{0,\cdots,M-1\}}\Big\{(K-U)m_{\text{sr}}+z(U+1)m_{\text{sd}}\Big\}, \min_{U\in\{M,\cdots,K\}}\Big\{(K-U)m_{\text{sr}}+z(Mm_{\text{sd}}+(U+1-M)m_{\text{rd}})\Big\}\Big\} \\ &= \begin{cases} zm_{\text{sd}}+Km_{\text{sr}} & \text{if } zm_{\text{rd}} > zm_{\text{sd}} \geq m_{\text{sr}} \\ zMm_{\text{sd}}+(K-M+1)m_{\text{sr}} & \text{if } zm_{\text{rd}} \geq m_{\text{sr}} > zm_{\text{sd}}. \\ zMm_{\text{sd}}+(K-M+1)zm_{\text{rd}} & \text{if } m_{\text{sr}} > zm_{\text{rd}} > zm_{\text{sd}} \end{cases} \end{aligned} \tag{30}$$

It is seen that $d^*(0)$ is irrelevant to $M_1, \cdots, M_\mu$ and $K_1, \cdots, K_\kappa$. This means that, if the transmission data rate $\bar{R}$ is fixed (i.e., $r^* = 0$), any approach of partitioning the M sources and K relays would result in the same high-SNR error probability curve slope. This can be seen in Fig. 2(a) by comparing the P_{err} results of the O-NC, C-NC-1, and C-NC-2 schemes for the case $\bar{R} = \frac{1}{6}$. In addition, $d^*(r^*)$ becomes zero when $r^* \geq \frac{1}{(\mu+\kappa)M_1}$. The *maximally achievable multiplexing gain* is affected by the terminal partition, due to inter-user interference. The complete tradeoff between the two gains for different clustering strategies can be very different.

To attain the highest diversity gain for each operating multiplexing gain r^*, one can enumerate all clustering strategies to find the method that provides the largest $d^*(r^*)$. For example, in the network considered in Fig. 3(a), when $r^* = \frac{1}{15}$, applying our C-NC scheme by dividing the six sources into $\mu = 2$ identical clusters each with 3 nodes, and keeping all relays as a single 4-node cluster (termed "C-NC-6") outperforms all other approaches. But in the scenario considered in Fig. 3(b), C-NC-1 is the best scheme for $r^* = \frac{1}{15}$.

Moreover, we can even choose using only a subset of the available relays to assist the sources, to avoid having to reserve channel resources (i.e., time slots) for all half-duplex relays. For instance, the C-NC-3 scheme neglects one relay and sets $\mu = 2$, $\kappa = 1$, and $|\mathcal{S}_1| = |\mathcal{S}_2| = |\mathcal{R}_1| = 3$. It can lead to better performance than other methods in both scenarios in Fig. 3, for some median values of r^*. Certainly, if none of the available relays is chosen to help the sources, the best way of activating the sources is to set $\mu = 1$, i.e., the NO-DT scheme which achieves positive diversity gain when $r^* < \frac{1}{M}$. Therefore, for each possible multiplexing gain $0 \leq r^* \leq \frac{1}{M}$, we can try different numbers of relays and enumerate all clustering strategies to find the highest achievable diversity gain, as summarized in the following corollary. For the considered example network, the results are shown in Fig. 3 by the curves labelled "Optimal."

Corollary 1: Define sets $\mathcal{M} = \{(M_1, \cdots, M_\mu) : M_1 \geq \cdots \geq M_\mu > 0, \sum_{j=1}^{\mu} M_j = M, 1 \leq \mu \leq M\}$ and $\mathcal{K}_k = \{(K_1, \cdots, K_\kappa) : K_1 \geq \cdots \geq K_\kappa > 0, \sum_{j=1}^{\kappa} K_j = k, 1 \leq \kappa \leq k\}$ for $k \in \{1, \cdots, K\}$. Let $d^*_{\mathcal{K}_0}(r^*) = m_{\text{sd}} \min\{(1-r^*)^+, M(1-Mr^*)^+\}$ be the DMT of the NO-DT scheme. The maximally achievable infinite-SNR DMT in the considered network is

$$d^*_{\max}(r^*) = \max\left\{d^*_{\mathcal{K}_0}(r^*), \max_{\mathcal{M},\mathcal{K}_1}\{d^*(r^*)\}, \max_{\mathcal{M},\mathcal{K}_2}\{d^*(r^*)\}, \cdots, \max_{\mathcal{M},\mathcal{K}_K}\{d^*(r^*)\}\right\}, \tag{31}$$

for $0 \leq r^* \leq \frac{1}{M}$, and $\max_{\mathcal{M},\mathcal{K}_k}\{d^*(r^*)\}$ denotes the maximization of $d^*(r^*)$ calculated by (23) over all $M_1, \cdots, M_\mu, K_1, \cdots, K_\kappa$ in sets $\mathcal{M}$ and $\mathcal{K}_k$.

In certain network structures, some clustering solutions can strictly outperform others. This is discussed in the following subsection.

C. Clustering Strategies in Special Network Structures

Consider a special network structure in which $M = \mu\Theta$ holds for some integer Θ. One can evenly divide the sources into μ clusters with the same size Θ. Now $\hat{d}(r')$ in (24) becomes

$$\hat{d}(r') = m_{\text{sr}} \min\left\{(1-r')^+, \Theta(1-\Theta r')^+\right\}. \tag{32}$$

For each choice of $(u, v) \in \mathcal{V}_1^{[U]}$, the value $\tilde{d}_{uv,1}^{[U]}(r')$ in (26) becomes

$$\tilde{d}_{uv,1}^{[U]}(r') = m_{\text{sd}} \min\left\{\tilde{U}_{uv}^{[U]}\left(1-\tilde{U}_{uv}^{[U]}r'\right)^+, \Theta(1-\Theta r')^+\right\} + u m_{\text{sd}}\Theta(1-\Theta r')^+ + \sum_{k=0}^{v} m_{\text{rd}}\tilde{K}_k^{[U]}\left(1-\tilde{K}_k^{[U]}r'\right)^+,$$

and for each choice of $(u, v) \in \mathcal{V}_2^{[U]}$, the value $\tilde{d}_{uv,2}^{[U]}(r')$ in (27) becomes

$$\tilde{d}_{uv,2}^{[U]}(r') = m_{\text{rd}} \min\left\{\tilde{U}_{uv}^{[U]}\left(1-\tilde{U}_{uv}^{[U]}r'\right)^+, \tilde{K}_{v+1}^{[U]}\left(1-\tilde{K}_{v+1}^{[U]}r'\right)^+\right\} + u m_{\text{sd}}\Theta(1-\Theta r')^+ + \sum_{k=0}^{v} m_{\text{rd}}\tilde{K}_k^{[U]}\left(1-\tilde{K}_k^{[U]}r'\right)^+.$$

Substituting these equations into (23) leads to the achievable DMT.

With the same value of μ, instead of evenly dividing the sources, one can also choose other clustering solutions, which would cause $M_1 > \Theta$. Following the proof procedure in Appendix B, it can be shown that when $0 < r^* < \frac{1}{(\mu+\kappa)M_1}$, their achievable DMT would always be smaller than that attained by the above approach. For example, in the scenarios shown in Fig. 3, the C-NC-5 scheme (with $M_1 = 3$ and $M_2 = 3$) performs strictly better than the C-NC-4 scheme (with $M_1 = 4$ and $M_2 = 2$). This result is summarized in the following corollary.

***Corollary** 2:* In networks with $M = \mu\Theta$, dividing the sources into equal-size Θ-node clusters achieves better infinite-SNR DMT than other source partition methods with the same μ.

Now, consider the situation that the number of relays K can be written as $K = \kappa\Theta$ for some integer Θ. We can also evenly partition the relays into κ Θ-node clusters. However, this approach may not be able to outperform uneven clustering strategies. For instance, in a network with $M = 6$, $K = 10$, $N = 25$, $\sigma = 100$, $m_{\rm sd} = m_{\rm sr} = m_{\rm rd} = 2$, given $M_1 = M_2 = 3$ and $\kappa = 2$, one may set $K_1 = 9$ and $K_2 = 1$, or set $K_1 = K_2 = 5$. Following Proposition 2, when $\frac{9}{188} \leq r^* \leq \frac{1}{16}$, the former strategy achieves higher diversity.

Finally, for networks where both $M = \mu\Theta$ and $K = \kappa\Theta$ apply, we can set the sources to μ clusters and relays to κ clusters, each with Θ nodes. For example, in the aforementioned 6-source 4-relay network, one can choose $\mu = 3$, $\kappa = 2$, $\Theta = 2$ (i.e., C-NC-1), or $\mu = 6$, $\kappa = 4$, $\Theta = 1$ (i.e., O-NC). When only $K = 3$ relays are considered, we can choose $\mu = 2$, $\kappa = 1$, $\Theta = 3$ (i.e., C-NC-3). Although in general these approaches do not necessarily lead to the maximal DMT presented in Corollary 1 for all operating multiplexing gain, the expressions of their DMT performance are relatively simple. Specifically, for each $U \in \{0, 1, \cdots, K\}$, we have $\kappa_U = \lceil \frac{U}{\Theta} \rceil$. Let $\tilde{\Theta} = U - (\kappa_U - 1)\Theta$. When $m_{\rm rd} \leq m_{\rm sd}$, it is easy to show that

$$\tilde{d}(r', U) = (\kappa_U - 1) m_{\rm rd} \Theta (1 - \Theta r')^+ + m_{\rm rd} \tilde{\Theta} (1 - \tilde{\Theta} r')^+ + m_{\rm sd} \min\{(1 - r')^+, \Theta(1 - \Theta r')^+\}.$$

Otherwise, when $m_{\rm rd} > m_{\rm sd}$, define $\tau_U = \lceil \frac{U+1}{\Theta} \rceil$ and $\hat{\Theta} = U + 1 - (\tau_U - 1)\Theta$. Then

$$\tilde{d}(r', U) = (\tau_U - 1) m_{\rm sd} \Theta (1 - \Theta r')^+ + m_{\rm sd} \min\{\hat{\Theta}(1 - \hat{\Theta} r')^+, \Theta(1 - \Theta r')^+\},$$

if $\tau_U \leq \mu$ (i.e., $U + 1 \leq M$). If $\tau_U > \mu$, we have

$$\tilde{d}(r', U) = \mu m_{\rm sd} \Theta (1 - \Theta r')^+ + (\tau_U - \mu - 1) m_{\rm rd} \Theta (1 - \Theta r')^+ + m_{\rm rd} \min\{\hat{\Theta}(1 - \hat{\Theta} r')^+, \Theta(1 - \Theta r')^+\}.$$

Substituting these equations into (23) leads to the achievable DMT.

When both $\frac{M}{\mu_1} = \frac{K}{\kappa_1} = \Theta_1$ and $\frac{M}{\mu_2} = \frac{K}{\kappa_2} = \Theta_2$ can apply, for integers $\Theta_1 > \Theta_2$, following the proof procedure in Appendix B, it can be shown that dividing sources and relays into $(\mu_1 + \kappa_1)$ Θ_1-node clusters achieves better DMT performance than that with $(\mu_2 + \kappa_2)$ Θ_2-node clusters. This means, partitioning the terminals into larger-size clusters can produce better performance, in terms of infinite-SNR DMT. For example, in a network with $M = 4$ and $K = 4$, setting $\mu = \kappa = 1$ and $|\mathcal{S}_1| = |\mathcal{R}_1| = 4$ (i.e., the NO-NC scheme) attains higher DMT than setting $\mu = \kappa = 2$ and $|\mathcal{S}_1| = |\mathcal{S}_2| = |\mathcal{R}_1| = |\mathcal{R}_2| = 2$. They both strictly outperform the case with $\mu = \kappa = 4$ and $|\mathcal{S}_i| = |\mathcal{R}_i| = 1$ for all $i \in \{1, 2, 3, 4\}$, which is the O-NC scheme. This also explains the observation that the DMT curves of C-NC-1 are strictly higher than those of O-NC in Fig. 3. We summarize such a result as follows.

***Corollary** 3:* In networks with $M = \mu\Theta$ and $K = \kappa\Theta$ ($\Theta > 1$), the C-NC scheme that allows Θ nodes to transmit non-orthogonally achieves higher infinite-SNR DMT than the O-NC scheme.

All the above results exhibit that properly permitting non-orthogonal transmission in network-coded cooperation networks has clear advantages over demanding only orthogonal transmissions.

V. Conclusion

We investigate the advantages of combining non-orthogonal transmission with network coding techniques in wireless cooperative networks. A general form of multi-user relay networks is considered, in which several relays are used to assist multiple sources to distribute messages to a sufficient proportion of ambient destinations, under Nakagami-m fading. We apply an efficient C-NC relaying scheme that, in addition to employing a class of maximum-diversity-achieving finite-field network codes in the relays, divides sources and relays into clusters such that multiple terminals can share the same channel resource. We have presented the error probabilities and finite-SNR DMT of the C-NC scheme with cluster sizes bounded by three, and also the infinite-SNR DMT for general situations. Our results have shown that permitting proper non-orthogonal transmission can provide notable performance improvement over the conventional approach that orthogonalizes all terminals. Such a design may potentially be applied in many other types of cooperation networks with different network coding solutions and fading characteristics.

Appendix

A. *Individual Decoding Probabilities in a* 3*-user MAC Channel*

Denote $H_i = \rho_{a_i}|h_{b,a_i}|^2 \sim \text{Gamma}(m_i, \beta_i)$, for $i \in \{1,2,3\}$. In what follows, we focus on the case with $m_i = m$ and $\beta_i = \frac{m_i}{\Omega_{b,a_i}\rho_{a_i}} = \beta$, $\forall i$. The derivations of the individual decoding probabilities follow the similar procedure in the 2-user case. The sample space of three Gamma-distributed random variables H_1, H_2, and H_3 are partitioned into non-overlapping sub-regions according to the decoding events defined in (13) and (14). The probability that a random sample locates in each sub-region A can be attained by taking integrals of the following form:

$$\mathcal{P}(A) = \int_{\chi_{31}}^{\chi_{32}} \left(\int_{\chi_{21}}^{\chi_{22}} \left(\int_{\chi_{11}}^{\chi_{12}} f_{H_1}(x_1)\mathrm{d}x_1 \right) f_{H_2}(x_2)\mathrm{d}x_2 \right) f_{H_3}(x_3)\mathrm{d}x_3, \tag{33}$$

where χ_{i1} and χ_{i2} represent the integral boundaries of H_i for $i \in \{1,2,3\}$.

Consider first the case that b is capable of decoding all three transmitters. According to (13) and (14), this requires: $H_1 > \eta_1, H_2 > \eta_1, H_3 > \eta_1, H_1+H_2 > \eta_2, H_1+H_3 > \eta_2, H_2+H_3 > \eta_2, H_1+H_2+H_3 > \eta_3$, where $\eta_s = 2^{sR}-1$. These events can be proven to correspond to six non-overlapping sub-regions in the three-dimensional sample space. Hence we can write

$$Q_{\mathcal{A},b}^{\{a_1,a_2,a_3\}} = \mathcal{P}\{A_1\} + \mathcal{P}\{A_2\} + \mathcal{P}\{A_3\} + \mathcal{P}\{A_4\} + \mathcal{P}\{A_5\} + \mathcal{P}\{A_6\}. \tag{34}$$

The six-tuple $[(\chi_{11},\chi_{12}),(\chi_{21},\chi_{22}),(\chi_{31},\chi_{33})]$ for $\mathcal{P}\{A_1\}$-$\mathcal{P}\{A_6\}$ can be respectively expressed as $[(\eta_3 - x_2 - x_3, \infty), (\eta_2 - x_3, \eta_3 - \eta_2), (\eta_1, \eta_2 - \eta_1)]$, $[(\eta_2 - x_3, \infty), (\eta_3 - \eta_2, \infty), (\eta_1, \eta_2 - \eta_1)]$, $[(\eta_3 - x_2 - x_3, \infty), (\eta_1, \eta_3 - \eta_1 - x_3), (\eta_2 - \eta_1, \eta_3 - \eta_2)]$, $[(\eta_1, \infty), (\eta_3 - \eta_1 - x_3, \infty), (\eta_2 - \eta_1, \eta_3 - \eta_2)]$, $[(\eta_2 - x_3, \infty), (\eta_1, \eta_2 - \eta_1), (\eta_3 - \eta_2, \infty)]$, and $[(\eta_1, \infty), (\eta_2 - \eta_1, \infty), (\eta_3 - \eta_2, \infty)]$.

$\mathcal{P}\{A_1\}$-$\mathcal{P}\{A_6\}$ can be derived with closed-form expressions by completing the integrals according to (33). When $\beta_1 = \beta_2 = \beta_3 = \beta$, the decoding probability $Q_{\mathcal{A},b}^{\{a_1,a_2,a_3\}}$ is

$$\begin{aligned}
Q_{\mathcal{A},b}^{\{a_1,a_2,a_3\}} &= \sum_{i=0}^{m-1}\sum_{j=0}^{i}\sum_{l=0}^{j} \frac{(-1)^{i-j+l}\beta^{2m+i}e^{-\beta\eta_3}\eta_3^{j-l}}{l!(i-j)!(j-l)!(m+i-j)(m-1)!(m-1)!} \\
&\quad \cdot \left(\frac{\eta_1^{2m+i+l-j}\left(\eta_{2m+2i-2j} - \eta_{m+l}\right)\eta_{m+l}}{m+l} + \sum_{q=0}^{m+i-j} \frac{(-1)^q \eta_1^{m+l+q}\eta_2^{m+i-j-q}\eta_{m+l+q}\eta_{2m+i-j+l}}{q!(m+i-j-q)!(m+l+q)/(m+i-j)!} \right) \\
&\quad + \sum_{i=0}^{m-1}\sum_{j=0}^{i} \frac{2(-1)^j\beta^{m+i}\eta_1^{m+j}\eta_2^{i-j}\eta_{m+j}e^{-\beta\eta_2}}{j!(i-j)!(m-1)!(m+j)} \cdot \big(1 - F\left(\eta_3 - \eta_2; m, \beta\right)\big) \\
&\quad + \sum_{i=0}^{m-1}\sum_{j=0}^{i} \frac{(-1)^j\beta^{m+i}\eta_1^{m+j}(\eta_{2m+2j} - \eta_{m+j})(\eta_3 - \eta_1)^{i-j}e^{-\beta(\eta_3-\eta_1)}}{j!(i-j)!(m-1)!(m+j)} \big(1 - F\left(\eta_1; m, \beta\right)\big) \\
&\quad + \big(1 - F\left(\eta_1; m, \beta\right)\big)\big(1 - F\left(\eta_2 - \eta_1; m, \beta\right)\big)\big(1 - F\left(\eta_3 - \eta_2; m, \beta\right)\big).
\end{aligned}$$

The event that b can decode a_1 and a_2 but not a_3 is guaranteed as long as the following conditions are satisfied: $H_1 > \eta_1(1+H_3)$, $H_2 > \eta_1(1+H_3)$, $H_1 + H_2 > \eta_2(1+H_3)$, $H_3 < \eta_1$. Comparing these with the conditions for calculating $Q_{\mathcal{A},b}^{\{a_1,a_2\}}$ in the two-user MAC channel in (20), we can see that $Q_{\mathcal{A},b}^{\{a_1,a_2\}}$ can in fact be derived using

$$Q_{\mathcal{A},b}^{\{a_1,a_2\}} = \int_0^{\eta_1} \tilde{P}_{12|x_3} f_{H_3}(x_3)\mathrm{d}x_3, \tag{35}$$

where $\tilde{P}_{12|x_3}$ denotes the probability defined in (20) but replacing η_1 and η_2 with $\eta_1(1+x_3)$ and $\eta_2(1+x_3)$ respectively. When $\beta_1 = \beta_2 = \beta_3 = \beta$, solving the integral provides us with

$$\begin{aligned} Q_{\mathcal{A},b}^{\{a_1,a_2\}} =& \sum_{i=0}^{m-1}\sum_{j=0}^{i}\sum_{l=0}^{m+i} \frac{(m+i)!(m+l-1)!\beta^{m+i-l}\eta_1^{m+j}\eta_2^{i-j}\eta_{m+j}e^{-\beta\eta_2}F(\eta_1;m+l,\beta(1+\eta_2))}{(-1)^j j!(i-j)!l!(m+i-l)!(m-1)!(m-1)!(m+j)(\eta_{2m+2l}+1)} \\ &+ \sum_{i=0}^{m-1}\sum_{j=0}^{m-1}\sum_{l=0}^{i+j} \frac{(i+j)!(m+l-1)!\beta^{i+j-l}(\eta_2-\eta_1)^i\eta_1^j e^{-\beta\eta_2}F(\eta_1;m+l,\beta(1+\eta_2))}{i!j!l!(i+j-l)!(m-1)!(\eta_{2m+2l}+1)}. \end{aligned}$$

By symmetry, $Q_{\mathcal{A},b}^{\{a_1,a_3\}}$ and $Q_{\mathcal{A},b}^{\{a_2,a_3\}}$ have the same form as the above expression.

Furthermore, when $\frac{H_1}{1+H_2+H_3} > \eta_1$, $\frac{H_2}{1+H_3} < \eta_1$, $\frac{H_3}{1+H_2} < \eta_1$, and $H_2 + H_3 < \eta_2$, the receiver b can decode only a_1. The corresponding sample space can be shown to include two non-overlapping sub-regions, denoted by A_7 and A_8. Therefore, we have $Q_{\mathcal{A},b}^{\{a_1\}} = \mathcal{P}\{A_7\} + \mathcal{P}\{A_8\}$, in which $\mathcal{P}\{A_7\}$ and $\mathcal{P}\{A_8\}$ can be derived by calculating the integrals in (33) with $[(\chi_{11},\chi_{12}),(\chi_{21},\chi_{22}),(\chi_{31},\chi_{33})]$ being $[(\eta_1(1+x_2+x_3),\infty),(0,\eta_1(1+x_3)),(0,\eta_1)]$ and $[(\eta_1(1+x_2+x_3),\infty),(\frac{x_3}{\eta_1}-1,\eta_2-x_3),(\eta_1,\eta_2-\eta_1)]$. When $m_i = m$ and $\beta_i = \beta$, $\forall i \in \{1,2,3\}$, we have

$$\begin{aligned} Q_{\mathcal{A},b}^{\{a_1\}} =& \sum_{i=0}^{m-1}\sum_{j=0}^{i}\sum_{l=0}^{j} \frac{\beta^l\eta_1^i e^{-\beta\eta_1}(m+i-j-1)!(m+j-l-1)!F(\eta_1;m+j-l,\beta(\eta_1+1))}{l!(j-l)!(i-j)!(m-1)!(m-1)!(\eta_1+1)^{2m+i-l}} \\ &- \sum_{i=0}^{m-1}\sum_{j=0}^{i}\sum_{q=0}^{m+i-j-1}\sum_{l=0}^{q+j} \frac{(q+j)!(m+i-j-1)!(m+q+j-l-1)!\beta^l\eta_1^{q+i}e^{-\beta\eta_2}}{j!l!(q+j-l)!(i-j)!q!(m-1)!(m-1)!(\eta_1+1)^{3m+i+q+j-2l}} \\ &\cdot F(\eta_1;m+q+j-l,\beta(\eta_2+1)) + \sum_{i=0}^{m-1}\sum_{j=0}^{i}\sum_{q=0}^{m+i-j-1} \frac{(m+i-j-1)!\beta^{m+j+q}\eta_1^i}{(i-j)!(m-1)!(m-1)!(1+\eta_{m+i-j-q})} \\ &\cdot\Bigg(\sum_{l=0}^{j}\sum_{s=0}^{q} \frac{(m+l+s-1)!\big(F(\eta_2-\eta_1;m+l+s,\frac{\beta(\eta_2+1)}{\eta_1}) - F(\eta_1;m+l+s,\frac{\beta(\eta_2+1)}{\eta_1})\big)}{(-1)^{q-s}l!(j-l)!s!(q-s)!e^{-\beta}\eta_1^s(\beta(2+\eta_1+\frac{1}{\eta_1}))^{m+l+s}} \\ &\quad - \sum_{l=0}^{j}\sum_{s=0}^{q} \frac{(-1)^s e^{-\beta\eta_3}\eta_1^{m+l+s}\eta_2^{q-s}\eta_{m+l+s}}{l!(j-l)!s!(q-s)!(m+l+s)}\Bigg). \end{aligned}$$

Again, due to symmetry, $Q_{\mathcal{A},b}^{\{a_2\}}$ and $Q_{\mathcal{A},b}^{\{a_3\}}$ have the same form as the above equation.

For the more general cases where $\beta_1 = \beta_2 = \beta_3$ does not hold, the derivation of these decoding probabilities can follow the same procedure. We can respectively consider $\beta_1 = \beta_2 \neq \beta_3$ and $\beta_1 \neq \beta_2 \neq \beta_3$ to calculate the above integrals. We omit the results due to page length limit.

B. Proof of Proposition 2

We start our proof by finding the achievable DMT in a MAC channel formed by a cluster of transmitters $\mathcal{A} = \{a_1,\cdots,a_{|\mathcal{A}|}\}$ and a receiver b. Let the channel power gain between $a \in \mathcal{A}$ and b follow Gamma distribution with integer shape parameter m and rate parameter $\frac{m}{\Omega_{b,a}}$. Consider that all transmitters send messages with the same transmitter-side SNR ρ. Following Section III-B, the probability that b recovers only the first s ($s \in \{0,1,\cdots,|\mathcal{A}|-1\}$) transmit messages is $\Pr\big\{E_{1,\mathcal{G}},E_{2,\bar{\mathcal{G}}}|\mathcal{G}=\{a_1,\cdots,a_s\}\big\}$. It is upper-bounded by $\Pr\big\{E_{2,\bar{\mathcal{G}}}|\mathcal{G}\big\}$, which is further upper-bounded by $\Pr\{\log_2(1+\sum_{a\in\bar{\mathcal{G}}}\rho|h_{b,a}|^2) < (|\mathcal{A}|-s)R\} =$

$\Pr\{H_{a_{s+1}} + H_{a_{s+2}} + \cdots + H_{a_{|\mathcal{A}|}} < 2^{(|\mathcal{A}|-s)R} - 1\}$, with $H_{a_i} = \rho|h_{b,a_i}|^2 \sim \text{Gamma}(m, \frac{m}{\rho\Omega_{b,a}})$. Since the error probability upper bound $\Pr\{\sum_{i=s+1}^{|\mathcal{A}|} H_{a_i} < 2^{(|\mathcal{A}|-s)R} - 1\}$ is bounded between the associated probabilities when all H_{a_i} have the same rate parameters $\frac{m}{\rho \max_a\{\Omega_{b,a}\}}$ and $\frac{m}{\rho \min_a\{\Omega_{b,a}\}}$, its diversity order equals that when all channels have a same value of $\Omega_{b,a}$. Now let $\Omega_{b,a} = \Omega$, $\forall a$. The sum of n i.i.d. Gamma distributed random variables with shape parameter α and rate parameter β is a Gamma distributed random variable with shape parameter $n\alpha$ and rate parameter β. Hence the upper bound can be written as $F(2^{(|\mathcal{A}|-s)R} - 1; (|\mathcal{A}|-s)m, \frac{m}{\rho\Omega})$. Let $R = r' \log_2 \rho$ for $\rho \to \infty$. Using the expression of $F(x;\alpha,\beta)$, $Q^{[s]}_{\mathcal{A},b}$ is upper-bounded by $\binom{|\mathcal{A}|}{s} \sum_{i=(|\mathcal{A}|-s)m}^{\infty} \frac{(\frac{m}{\Omega}\rho^{(|\mathcal{A}|-s)r'-1})^i}{i!} e^{-\frac{m}{\Omega}\rho^{(|\mathcal{A}|-s)r'-1}}$. When $(|\mathcal{A}|-s)r' - 1 < 0$, it is easy to see $e^{-\frac{m}{\Omega}\rho^{(|\mathcal{A}|-s)r'-1}} \to 1$ and the upper-bound is dominated by $(\frac{m}{\Omega}\rho^{(|\mathcal{A}|-s)r'-1})^{(|\mathcal{A}|-s)m}$ for $\rho \to \infty$. Therefore, asymptotically we can express $Q^{[s]}_{\mathcal{A},b}$ as

$$Q^{[s]}_{\mathcal{A},b} \doteq \rho^{-m(|\mathcal{A}|-s)(1-(|\mathcal{A}|-s)r')^+}.$$

Since $\min_{i\in\{\bar{s},\bar{s}+1,\cdots,|\mathcal{A}|\}}\{i(1-ir')^+\} = \min\{\bar{s}(1-\bar{s}r')^+, |\mathcal{A}|(1-|\mathcal{A}|r')^+\}$ [29], the probability that b cannot decode *at least* $\bar{s} \in \{1, 2, \cdots, |\mathcal{A}|\}$ messages is $\sum_{i=\bar{s}}^{|\mathcal{A}|} Q^{[|\mathcal{A}|-i]}_{\mathcal{A},b} \doteq \rho^{-d(r')}$ with

$$d(r') = \min\{m\bar{s}(1-\bar{s}r')^+, m|\mathcal{A}|(1-|\mathcal{A}|r')^+\}. \tag{36}$$

The probability that b recovers all the $|\mathcal{A}|$ transmitted messages can be found by $1 - \sum_{i=1}^{|\mathcal{A}|} Q^{[|\mathcal{A}|-i]}_{\mathcal{A},b}$.

We extend the system to two clusters of transmitters $\mathcal{A}_1$ and $\mathcal{A}_2$, intending to send their messages to a common receiver b. All transmitters within the same cluster are activated simultaneously, but the two clusters are orthogonalized. Without loss of generality, assume $|\mathcal{A}_1| \geq |\mathcal{A}_2|$. We aim to find the error probability that b cannot decode at least $\bar{s}$ messages from the $|\mathcal{A}_1|+|\mathcal{A}_2|$ transmitters. It is the probability that b cannot decode at least $\bar{s}_1 \in \{\max\{0, \bar{s}-|\mathcal{A}_2|\}, \cdots, \min\{\bar{s}, |\mathcal{A}_1|\}\}$ messages from $\mathcal{A}_1$, and at least $\bar{s}_2 = \bar{s}-\bar{s}_1$ messages from $\mathcal{A}_2$. This probability can be written as $\sum_{\bar{s}_1=\max\{0,\bar{s}-|\mathcal{A}_2|\}}^{\min\{\bar{s},|\mathcal{A}_1|\}} \left(\sum_{i=\bar{s}_1}^{|\mathcal{A}_1|} Q^{[|\mathcal{A}_1|-i]}_{\mathcal{A}_1,b}\right)\left(\sum_{j=\bar{s}_2}^{|\mathcal{A}_2|} Q^{[|\mathcal{A}_2|-j]}_{\mathcal{A}_2,b}\right)$. When $\rho \to \infty$, it is dominated by the probability of the case $\bar{s}_1 = \min\{\bar{s}, |\mathcal{A}_1|\}$. Therefore, it can be expressed as $\rho^{-d(r')}$, where $d(r') = \min\left\{m\bar{s}(1-\bar{s}r')^+, m|\mathcal{A}_1|(1-|\mathcal{A}_1|r')^+\right\}$ if $\bar{s} \leq |\mathcal{A}_1|$, and $d(r') = m|\mathcal{A}_1|(1-|\mathcal{A}_1|r')^+ + \min\left\{m\hat{s}(1-\hat{s}r')^+, m|\mathcal{A}_2|(1-|\mathcal{A}_2|r')^+\right\}$ otherwise with $\hat{s} = \bar{s}-|\mathcal{A}_1|$. The probability that errors come first from the larger cluster dominates.

In addition, we consider the case $|\mathcal{A}_1| + |\mathcal{A}_2| = A$ for a fixed integer A, and the size of cluster $\mathcal{A}_1$ can be chosen between $\lceil \frac{A}{2} \rceil$ and A. For each value of $|\mathcal{A}_1|$, when $\rho \to \infty$ the probability that b cannot decode at least $\bar{s}$ messages can be written as $\rho^{-d(|\mathcal{A}_1|,r')}$ where the diversity order $d(|\mathcal{A}_1|, r')$ is a function of both $|\mathcal{A}_1|$ and r'. It can be shown that when $0 \leq r' \leq \frac{1}{|\mathcal{A}_1|}$, we have $d(A, r') \leq d(|\mathcal{A}_1|, r') \leq d(\lceil \frac{A}{2} \rceil, r')$. This means, unevenly clustering the available transmitters leads to smaller achievable diversity order than evenly dividing them for $0 \leq r' \leq \frac{1}{|\mathcal{A}_1|}$. The worst case is that all transmitters are set to the same cluster and interfere each other.

The above results can be extended further to systems with more clusters. Specifically, consider that there are C orthogonalized transmitter clusters $\mathcal{A}_1$, $\cdots$, $\mathcal{A}_C$ with $|\mathcal{A}_1| \geq \cdots \geq |\mathcal{A}_C|$. It can be shown that when the SNR is sufficiently high, the probability that the common receiver b cannot decode at least $\bar{s}$ messages is dominated by the probability that all errors are from the largest clusters. To derive the achievable DMT, we let $|\mathcal{A}_0| = 0$ and find the integer τ such that $\sum_{i=0}^{\tau-1} |\mathcal{A}_i| < \bar{s} \leq \sum_{i=0}^{\tau} |\mathcal{A}_i|$. The asymptotic behavior of the error probability can be expressed as $\rho^{-d(r')}$, where $d(r') = \sum_{i=0}^{\tau-1} m|\mathcal{A}_i|(1 - |\mathcal{A}_i|r')^+ + \min\{m\hat{s}(1-\hat{s}r')^+, m|\mathcal{A}_\tau|(1-|\mathcal{A}_\tau|r')^+\}$ and $\hat{s} = \bar{s} - \sum_{i=0}^{\tau-1} |\mathcal{A}_i|$. In addition, if the total number of transmitters is fixed, evenly partitioning the transmitters results in higher achievable diversity order than any uneven clustering strategy.

Now, we consider another situation with two sets of transmitter clusters $\mathcal{C}_1 = \{\mathcal{A}_1, \cdots, \mathcal{A}_{C_1}\}$, and $\mathcal{C}_2 = \{\mathcal{B}_1, \cdots, \mathcal{B}_{C_2}\}$, intending to send information to a common destination b. All clusters are orthogonalized, but the nodes in each cluster are activated non-orthogonally. Use m_i and Ω_i to denote Nakagami fading parameters of the channels between clusters in $\mathcal{C}_i$ and b. Again, we are interested in the probability that b

cannot decode at least a certain number of $\bar{s} \in \{1, 2, \cdots, \sum_{\mathcal{A}\in\mathcal{C}_1} |\mathcal{A}| + \sum_{\mathcal{B}\in\mathcal{C}_2} |\mathcal{B}|\}$ transmitted messages. Without loss of generality, assume $|\mathcal{A}_1| \geq |\mathcal{A}_2| \cdots| \geq |\mathcal{A}_{C_1}|$ and $|\mathcal{B}_1| \geq |\mathcal{B}_2| \cdots \geq |\mathcal{B}_{C_2}|$. Following the above discussion, we know that within each set of transmitter clusters, the probability that b cannot decode at least some number of messages is dominated by the probability that all errors come from the largest clusters. Therefore, consider an integer $v \in \{0, 1, \cdots, C_2\}$. If all the messages from the first v clusters in set $\mathcal{C}_2$ are decoded with errors, then b should be unable to decode at least $\bar{s} - \sum_{k=0}^{v} |\mathcal{B}_k|$ messages from clusters $\mathcal{A}_1, \cdots, \mathcal{A}_{C_1}$. Find the integer u such that $0 < \bar{s} - \sum_{j=0}^{u} |\mathcal{A}_j| - \sum_{k=0}^{v} |\mathcal{B}_k| \leq |\mathcal{A}_{u+1}|$. Define $\tilde{U}_{uv} = \bar{s} - \sum_{j=0}^{u} |\mathcal{A}_j| - \sum_{k=0}^{v} |\mathcal{B}_k|$. When $\rho \to \infty$, the probability of seeing this event can be expressed as $\rho^{-d_{uv,1}^{[\bar{s}]}(r')}$, where

$$d_{uv,1}^{[\bar{s}]}(r') = \sum_{k=0}^{v} m_2 |\mathcal{B}_k| (1 - |\mathcal{B}_k| r')^+ + \sum_{i=0}^{u} m_1 |\mathcal{A}_i| (1 - |\mathcal{A}_i| r')^+ \\ + \min\{m_1 \tilde{U}_{uv}^{[\bar{s}]} (1 - \tilde{U}_{uv}^{[\bar{s}]} r')^+, m_1 |\mathcal{A}_{u+1}| (1 - |\mathcal{A}_{u+1}| r')^+\}. \tag{37}$$

Similarly, consider integer $u \in \{0, 1, \cdots, C_1\}$. If all the messages from the first u clusters in set $\mathcal{C}_1$ are decoded with errors, then b should be unable to decode at least $\bar{s} - \sum_{k=0}^{u} |\mathcal{A}_k|$ messages from clusters $\mathcal{B}_1, \cdots, \mathcal{B}_{C_2}$. Therefore, find the integer v such that $0 < \bar{s} - \sum_{j=0}^{u} |\mathcal{A}_j| - \sum_{k=0}^{v} |\mathcal{B}_k| \leq |\mathcal{B}_{v+1}|$. The probability of seeing this event, when $\rho \to \infty$, is $\rho^{-d_{uv,2}^{[\bar{s}]}(r')}$, where

$$d_{uv,2}^{[\bar{s}]}(r') = \sum_{i=0}^{u} m_1 |\mathcal{A}_i| (1 - |\mathcal{A}_i| r')^+ + \sum_{k=0}^{v} m_2 |\mathcal{B}_k| (1 - |\mathcal{B}_k| r')^+ \\ + \min\{m_2 \tilde{U}_{uv}^{[\bar{s}]} (1 - \tilde{U}_{uv}^{[\bar{s}]} r')^+, m_2 |\mathcal{B}_{v+1}| (1 - |\mathcal{B}_{v+1}| r')^+\}. \tag{38}$$

In the high-SNR region, the overall error probability is dominated by that with the smallest diversity order. This means, we can define two sets

$$\mathcal{V}_1 = \{(u, v) : 0 < \tilde{U}_{uv} \leq |\mathcal{A}_{u+1}|\} \text{ and } \mathcal{V}_2 = \{(u, v) : 0 < \tilde{U}_{uv} \leq |\mathcal{B}_{v+1}|\}. \tag{39}$$

The probability that b cannot decode at least $\bar{s}$ messages is $\rho^{-\tilde{d}(r')}$ with diversity order

$$\tilde{d}(r') = \min\Big\{ \min_{(u,v)\in\mathcal{V}_1} \{d_{uv,1}(r')\}, \min_{(u,v)\in\mathcal{V}_2} \{d_{uv,2}(r')\} \Big\}. \tag{40}$$

Armed with the above results, we can start proving Proposition 2. We aim to find the dominant factor in the system error probability P_{err} in (6). First, focus on $\Pr(\boldsymbol{\pi})$ for a particular relay decoding status $\boldsymbol{\pi}$. Let $||\boldsymbol{\pi}||$ denote the norm of vector $\boldsymbol{\pi}$. The event $\boldsymbol{\pi}$ means that after the sources complete broadcasting their messages, a total of $||\boldsymbol{\pi}||$ relays can obtain the source message set $\mathcal{I}$ and thus can participate in the following message forwarding process. In other words, each of the remaining $K - ||\boldsymbol{\pi}||$ relays has at least one source message that is not successfully recovered. For each relay, the probability that it cannot decode at least one message from $\mathcal{S}_1, \cdots, \mathcal{S}_\mu$, i.e., $\Pr\left\{\pi_i^{[k]} = 0\right\}$, can be expressed as $\Pr\left\{\pi_i^{[k]} = 0\right\} \doteq \rho^{-\min\{m_{\text{sr}}(1-r')^+, m_{\text{sr}} M_1 (1-M_1 r')^+\}}$, with $r' = (\mu + \kappa) r^*$. Substituting $\Pr\left\{\pi_i^{[k]} = 0\right\}$ and $\Pr\left\{\pi_i^{[k]} = 1\right\} = 1 - \Pr\left\{\pi_i^{[k]} = 0\right\}$ into (7) gives

$$\Pr\{\boldsymbol{\pi}\} \doteq \rho^{-(K - ||\boldsymbol{\pi}||)\hat{d}(r)}, \tag{41}$$

in which $\hat{d}(r) = \min\{m_{\text{sr}}(1 - r')^+, m_{\text{sr}} M_1 (1 - M_1 r')^+\}$. Therefore, different realizations of $\boldsymbol{\pi}$ with the same value of $||\boldsymbol{\pi}||$ (asymptotically) have the same occurring probability. Let $U = ||\boldsymbol{\pi}||$. Then for each value of $U \in \{0, 1, \cdots, K\}$, we aim to find the realization of $\boldsymbol{\pi}$ that causes the largest value of $P_{\text{err}|\boldsymbol{\pi}}$. Such a $P_{\text{err}|\boldsymbol{\pi}}$ would dominate the decoding error probability given U.

Assume that a particular relay decoding status $\boldsymbol{\pi}$ occurs and $||\boldsymbol{\pi}|| = U$. For each destination D_n, the probability $\Pr\{\tau_{n|\boldsymbol{\pi}} = 0\}$ is the probability that D_n cannot decode at least $U + 1$ messages sent from the M sources and U activated relays. Following the above discussion, the worst case occurs when all the activated relays come from the largest relay clusters, i.e., all the relays in the first $\tau_U - 1$ relay clusters and $(U - \sum_{k=0}^{\tau_U-1} K_k)$ relays in the τ_Uth largest relay cluster are activated to assist in the source message transmissions. This means, D_n sees two sets of transmitter clusters. The first set is the source clusters, with sizes M_1, $\cdots$, M_μ respectively. The second set is the activated relay set, with sizes $\tilde{K}_1$, $\cdots$, $\tilde{K}_{\tau_U}$, respectively. The probability $\Pr\{\tau_{n|\boldsymbol{\pi}} = 0\}$ can thus be calculated by the above process with $\mathcal{C}_1 = \{\mathcal{A}_1, \cdots, \mathcal{A}_\mu\}$ with $|\mathcal{A}_i| = M_i$ for $i \in \{1, \cdots, \mu\}$, $\mathcal{C}_2 = \{\mathcal{B}_1, \cdots, \mathcal{B}_{\tau_U}\}$ with $|\mathcal{B}_k| = \tilde{K}_k$ for $k \in \{1, \cdots, \tau_U\}$, $r' = (\mu+\kappa)r^*$, and $\bar{s} = U+1$. The system error probability is defined as the probability that less than $\lceil N \cdot \sigma\% \rceil$ destinations can recover $\mathcal{I}$. Substituting $\Pr\{\tau_{n|\boldsymbol{\pi}} = 0\}$ and $\Pr\{\tau_{n|\boldsymbol{\pi}} = 1\} = 1 - \Pr\{\tau_{n|\boldsymbol{\pi}} = 0\}$ into equation (10) leads to the worst case conditional error probability, and the term $(\lfloor (1-\sigma\%)N \rfloor + 1)\tilde{d}(r', U)$ shown in Proposition 2. Therefore, for each value of $U \in \{0, \cdots, K\}$, the worst case error probability has diversity order $(K-U)\hat{d}(r') + (\lfloor (1-\sigma\%)N \rfloor + 1)\tilde{d}(r', U)$. Finally, for all possible values of U, the largest error probability value (with the smallest diversity order) dominates the overall error probability and leads to the result in Proposition 2. The proof is complete.

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REFERENCES

[1] M. Whaiduzzaman, M. Sookhak, A. Gani, and R. Buyya, "A survey on vehicular cloud computing," *Journal of Network and Computer Applications*, vol. 40, pp. 325–344, Apr. 2014.

[2] E. Datsika, A. Antonopoulos, N. Zorba, and C. Verikoukis, "Green cooperative device-to-device communication: A social-aware perspective," *IEEE ACCESS*, vol. 4, pp. 3697–3707, Aug. 2016.

[3] E. Ahmed and H. Gharavi, "Cooperative vehicular networking: A survey," *IEEE Trans. Intell. Transport. Syst.*, vol. 19, no. 3, pp. 996–1014, Mar. 2018.

[4] S.-W. Jeon, S.-Y. Chung, and S. Jafar, "Degrees of freedom region of a class of multi-source Gaussian relay networks," *IEEE Trans. Inform. Theory*, vol. 57, no. 5, pp. 3032–3044, May 2011.

[5] S. Mohajer, S. Diggavi, C. Fragouli, and D. Tse, "Approximate capacity of a class of Gaussian interference-relay networks," *IEEE Trans. Inform. Theory*, vol. 57, no. 5, pp. 2837–2864, May 2011.

[6] I. Shomorony and A. Avestimehr, "Degrees of freedom of two-hop wireless networks: 'everyone gets the entire cake'," *IEEE Trans. Inform. Theory*, vol. 60, no. 5, pp. 2417–2431, May 2014.

[7] C. Wang, P. Wang, F. Liu, and G. Min, "New achievable sum degrees of freedom in half-duplex single-antenna multi-user multi-hop networks," *IEEE Trans. Commun.*, vol. 66, no. 7, pp. 2840–2854, Jul. 2018.

[8] J. Laneman, D. Tse, and G. Wornell, "Cooperative diversity in wireless networks: Efficient protocols and outage behavior," *IEEE Trans. Inform. Theory*, vol. 50, no. 12, pp. 3062–3080, Dec. 2004.

[9] A. Bletsas, A. Khisti, D. Reed, and A. Lippman, "A simple cooperative diversity method based on network path selection," *IEEE J. Select. Areas Commun.*, vol. 24, no. 3, pp. 659–672, Mar. 2006.

[10] R. Yeung, S.-Y. Li, N. Cai, and Z. Zhang, *Network Coding Theory*. MA, USA: NOW publishers, Inc, Hanover, 2006.

[11] S. Yang and R. Koetter, "Network coding over a noisy relay: A belief propagation approach," in *IEEE International Symposium on Information Theory (ISIT)*, Toronto, Canada, Jul. 2008.

[12] A. Douik, S. Sorour, T. Al-Naffouri, H.-C. Yang, and M.-S. Alouini, "Delay reduction in multi-hop device-to-device communication using network coding," *IEEE Trans. Wireless Commun.*, vol. 17, no. 10, pp. 7040–7053, Oct. 2018.

[13] M. Xiao and M. Skoglund, "Multiple-user cooperative communications based on linear network coding," *IEEE Trans. Commun.*, vol. 58, no. 12, pp. 3345–3351, Dec. 2010.

[14] H. Topakkaya and Z. Wang, "Wireless network code design and performance analysis using diversity-multiplexing tradeoff," *IEEE Trans. Commun.*, vol. 16, no. 8, pp. 488–496, Feb. 2011.

[15] M. Xiao, J. Kliewer, and M. Skoglund, "Design of network codes for multiple-user multiple-relay wireless networks," *IEEE Trans. Commun.*, vol. 60, no. 12, pp. 3755–3766, Dec. 2012.

[16] S. Basaran, S. Gokceli, G. Kurt, E. Ozdemir, and E. Yaraneri, "Error performance analysis of random network coded cooperation systems," *IEEE Trans. Wireless Commun.*, vol. 16, no. 8, pp. 5325–5337, Aug. 2017.

[17] J. Li, J. Yuan, R. Malaney, M. Xiao, and W. Chen, "Full-diversity binary frame-wise network coding for multiple-source multiple-relay networks over slow-fading channels," *IEEE Trans. Veh. Technol.*, vol. 61, no. 3, pp. 1346–1360, Mar. 2012.

[18] M. Renzo, M. Iezzi, and F. Graziosi, "Error performance and diversity analysis of multi-source multi-relay wireless networks with binary network coding and cooperative MRC," *IEEE Trans. Wireless Commun.*, vol. 12, no. 6, pp. 2883–2903, Jun. 2013.

[19] J.-T. Seong and H.-N. Lee, "Predicting the performance of cooperative wireless networking schemes with random network coding," *IEEE Trans. Commun.*, vol. 62, no. 8, pp. 2951–2964, Aug. 2014.

[20] A. Heidarpour, M. Ardakani, and C. Tellambura, "Multiuser diversity in network-coded cooperation: Outage and diversity analysis," *IEEE Commun. Lett.*, vol. 23, no. 3, pp. 550–553, Mar. 2019.

[21] L. Wei and W. Chen, "Compute-and-forward network coding design over multi-source multi-relay channels," *IEEE Trans. Wireless Commun.*, vol. 11, no. 9, pp. 3348–3357, Sep. 2012.

[22] W. Xing, F. Liu, C. Wang, M. Xiao, and P. Wang, "Multi-source network-coded D2D cooperative content distribution systems," *J. Commun. Netw-s Kor.*, vol. 20, no. 1, pp. 69–84, Feb. 2018.

[23] Y. Zhao, Y. Li, X. Chen, and N. Ge, "Joint optimization of resource allocation and relay selection for network coding aided device-to-device communications," *IEEE Commun. Lett.*, vol. 19, no. 5, pp. 807–810, May 2015.

[24] J. Huang, H. Gharavi, H. Yan, and C.-C. Xing, "Network coding in relay-based device-to-device communications," *IEEE Netw.*, vol. 31, no. 4, pp. 102–107, Jul. 2017.

[25] D. Tse, P. Viswanath, and L. Zheng, "Diversity-multiplexing tradeoff in multiple-access channels," *IEEE Trans. Inform. Theory*, vol. 50, no. 9, pp. 1859–1874, Sep. 2004.

[26] A. Khan and I. Chatzigeorgiou, "Non-orthogonal multiple access combined with random linear network coded cooperation," *IEEE Signal Process. Lett.*, vol. 24, no. 9, pp. 1298–1302, Sep. 2017.

[27] C. Wang, M. Xiao, and M. Skoglund, "Diversity-multiplexing tradeoff analysis of coded multi-user relay networks," *IEEE Trans. Commun.*, vol. 59, no. 7, pp. 1995–2005, Jul. 2011.

[28] L. Cheng, B. Henty, D. Stancil, F. Bai, and P. Mudalige, "Mobile vehicle-to-vehicle narrow-band channel measurement and characterization of the 5.9 GHz dedicated short range communication (DSRC) frequency band," *IEEE J. Select. Areas Commun.*, vol. 25, no. 8, pp. 1501–1516, Oct. 2007.

[29] D. Tse and P. Viswanath, *Fundamentals of Wireless Communication*. Cambridge, UK: Cambridge University Press, 2005.

[30] E. Stauffer, O. Oyman, R. Narasimhan, and A. Paulraj, "Finite-SNR diversity-multiplexing tradeoffs in fading relay channels," *IEEE J. Select. Areas Commun.*, vol. 25, no. 2, pp. 245–257, Feb. 2007.

[31] L. Zheng and D. Tse, "Diversity and multiplexing: A fundamental tradeoff in multiple-antenna channels," *IEEE Trans. Inform. Theory*, vol. 49, no. 5, pp. 1073–1096, May 2003.

[32] R. Mahapatra, Y. Nijsure, G. Kaddoum, N. Hassan, and C. Yuen, "Energy efficiency tradeoff mechanism towards wireless green communication: A survey," *IEEE Comm. Surveys Tut.*, vol. 18, no. 1, pp. 686–705, First Quarter 2016.

[33] R. Narasimhan, "Individual outage rate regions for fading multiple access channels," in *IEEE International Symposium on Information Theory (ISIT)*, Nice, France, Jun. 2007.

[34] X. Li, C. Wang, P. Wang, and F. Liu, "Efficient transmission in multi-user relay networks with node clustering and network coding," in *IEEE Wireless Communications & Networking Conference (WCNC)*, Marrakech, Morocco, Apr. 2019.