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A New Achievable Sum DoF Result in Multi-user Half-duplex Relay Networks

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Abstract

This paper investigates the achievable sum degrees of freedom (DoF) in a wireless single-antenna multi-user relay network, which consists of multiple sources, multiple destinations, and multiple layers of half-duplex relays in between. A cluster successive relaying (CSR) transmission scheme is applied to efficiently deliver information in this network. Different from existing works on the CSR scheme which normally divide each layer of relays into two alternatively activated equal-size clusters, we allow the clusters to contain different numbers of terminals in order to properly involve all available relays into the transmission process. Using the channel-extension based interference alignment technique, it is shown that the asymptotically achievable sum DoF can be larger than the previously known results and hence can serve as a new lower bound to the optimally achievable sum DoF in the considered relay network.

I. Introduction

In the past few years the communication theory research community's knowledge towards the performance limits in large wireless communication networks has been greatly advanced. A number of discoveries have been made to show that the achievable sum degrees of freedom (DoF) in multi-user networks can be related to the number of users sharing the transmission medium (see, e.g., [1]). This indicates that the conventional orthogonal transmission design's data rate can actually be significantly improved.

Recently the research attention has been extended from single-hop networks to multi-hop networks. Consider a single-antenna multi-user relay network with M dedicated source-destination pairs. If there is no direct source-destination signal propagation link and when the number of intermediate relays is sufficiently large, the achievable sum DoF can be as high as M, as if joint signal processing among the terminals within each layer is possible [2]–[6]. Since the optimally achievable sum DoF (i.e., the network sum capacity's scaling factor regarding changing SNR) in a single-antenna M-user interference channel is $\frac{M}{2}$, relays bring not only information delivery paths but also DoF gain over single-hop networks.

But the above results are attained based on an *ideal full-duplex* relaying assumption that *implicitly eliminates several interference issues*. Specifically, when relays conduct their receiving and forwarding operations simultaneously in the same frequency band, the transmitted signals would not affect their own receptions. In addition, signals forwarded by a relay may not interfere the other relays in the same layer and the prior layer. Thus these results do not reflect the system performance when all these interference signals cannot be simply avoided. Half-duplex relaying serves as a natural solution to tackling such interference issues. Nevertheless, half-duplex operation itself demands extra channel usage and hence also has the potential to reduce DoF performance. For example, in two-hop networks, utilizing the schemes proposed by [2]–[6] with half-duplex relays would halve the achievable sum DoF. If the source messages have to be delivered through more layers of relays, the results would be further decreased.

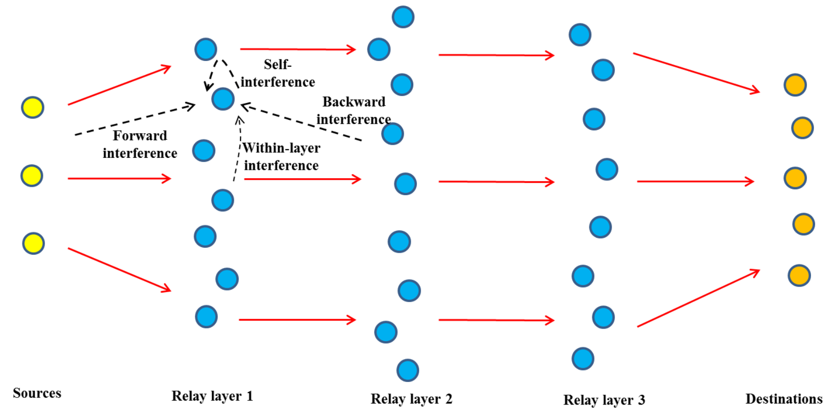

Fig. 1. System model: A $\{3,7,9,8,5\}^{(4)}$ example network.

To handle this issue, our previous work [7] proposed a cluster successive relaying (CSR) strategy to carry out information delivery in single-antenna multi-user networks with multiple layers of half-duplex relays. (The idea is also applicable in two-hop networks [8], [9].) Different from most conventional relaying schemes that demand all relays in the same layer to operate in the same mode together, the CSR scheme divides each relay layer into two identical clusters and alternatively activate them. It is shown that the asymptotically achievable sum DoF can be larger than the results attained by directly replacing ideal full-duplex relays with half-duplex relays in the aforementioned conventional relaying schemes. If the number of relays in each layer approaches infinity, the achievable sum DoF approaches the result in full-duplex systems. This indicates that taking the aforementioned interference issues into consideration may not necessarily diminish relay systems's DoF gain. Properly designed half-duplex relaying schemes can also serve as effective approaches to identify wireless multi-hop networks' performance limits.

However, the work in [7] requires the sizes of the two clusters in the same layer to be identical. If a relay layer contains an odd number of terminals, one of them would be discarded from the system. This may cause a waste of the system hardware resources, especially when the number of available relays is limited. In this paper, we relax such a demand and permit the two clusters to have different sizes. All available relays can be properly clustered to participate in the transmission process. Based on the interference alignment technique, we find a new achievable sum DoF which can be better than previously known results. Hence the result presented in this paper can serve as a new lower bound for the optimally achievable sum DoF in multi-user relay networks.

Notations: $|\mathcal{A}|$ denotes the cardinality of set $\mathcal{A}$. $\lfloor\cdot\rfloor$ and $\lceil\cdot\rceil$ represent the floor and ceiling functions respectively. span$(\mathbf{A})$ denotes the space spanned by the column vectors of matrix $\mathbf{A}$.

II. System Model and Main Results

A. System model and previous results

We intend to study the achievable sum DoF in a wireless single-antenna multi-user relay network. Such a network contains $M_s \geq 2$ information sources and $M_d \geq 2$ information destinations. Every source attempts to send one independent message to every destination. There is no direct signal propagation path between any source and any destination. $N \geq 2$ layers of decode-and-forward (DF) relays are deployed to carry out information delivery. The network exhibits a *layered* topology with $N+1$ hops, and is denoted by the form $\{M_s, K_1, \cdots, K_N, M_d\}^{(N+1)}$, in which $K_i \geq 4$ ($i \in \{1,2,\cdots,N\}$) is the number of relays in the ith relay layer. A simple four-hop $\{3,7,9,8,5\}^{(4)}$ example network is illustrated in Fig. 1.

A terminal operating in the listening mode can always overhear the transmissions of terminals located in the same layer and the adjacent layers. Due to hardware limitations, no relay can effectively shield its reception from its own transmission, if its listening and forwarding modes operate simultaneously in the same frequency band. Consequently, each relay in the ith layer may experience four types of interference, generated by terminals located in the $(i-1)$th, ith, and $(i+1)$th layers, and by the relay itself. We respectively term them *forward interference*, *within-layer interference*, *backward interference*, and *self-interference*, as shown in Fig. 1.

The message transmission process is conducted via a slotted fashion, in a narrow-band time-varying block-fading environment. The channel fading coefficients are modelled by independent random variables generated from a continuous distribution, with absolute values bounded away from zero and infinity. They would change independently across different (unit-length) time slots. The channel knowledge regarding the whole network is causally available at all terminals.

Use $\rho \to \infty$ to denote SNR and d_Σ to denote the achievable sum DoF in the $\{M_s, K_1, \cdots, K_N, M_d\}^{(N+1)}$ network such that a sum transmission rate $R_\Sigma = d_\Sigma \log_2 \rho + o(\log_2 \rho)$ can be attained. It is straightforward to see, via cut-set bound analysis, that $d_\Sigma \le \min\{M_s, K_1, \cdots, K_N, M_d\}$. This upper bound is known to be attainable in certain special cases. For instance, when the within-cluster interference, backward interference and self-interference do not exist, by activating $\min\{M_s, K_1, \cdots, K_N, M_d\}$ terminals in each layer, the transmission strategy proposed in [5] allows the (ideal full-duplex) relays to manipulate their forwarding directions such that the source messages are mapped from the source transmit space to the destination receive space using a diagonal linear transformation. The message delivery can hence be free of interference. The DoF $\min\{M_s, K_1, \cdots, K_N, M_d\}$ is achieved.

But if all four types of interference are present, the above result is no longer attainable. One can use *half-duplex relays* to avoid the self-interference and within-layer interference, and demand terminals operating in the transmitting mode to be separated by three hops (i.e., the ith and $(i+3)$th layers can transmit together) to eliminate backward-interference. This means that the transmissions of three consecutive hops should be orthogonalized. It would cause three times of channel consumption compared with the ideal case considered in [5] and thus lead to the achievable sum DoF $\frac{\min\{M_s, K_1, \cdots, K_N, M_d\}}{3}$. Although this result can still be much better than that attained by conventional transmission strategies which tend to orthogonalize all sources' transmissions and achieve d_Σ at most 1, it is clearly far away from $\min\{M_s, K_1, \cdots, K_N, M_d\}$.

This observation does not necessarily mean that the optimally achievable sum DoF in multi-user multi-hop networks subject to the interference issues shown in Fig. 1 would be limited to a small level. In fact, the above result can be improved in many network topologies by applying more *spectrally-efficient half-duplex relaying* transmission protocols. Specifically, in our previous work [7], we proposed a CSR scheme, which, in stead of demanding all relays in the same layer to operate in the same mode simultaneously, divides each relay layer into two equal-size clusters and take turns activating them. (The detailed transmission process will be presented in Section III-A.) Its asymptotically achievable sum DoF is as follows

$$d_\Sigma = \min\left\{\frac{M_s\left\lfloor\frac{K_1}{2}\right\rfloor}{M_s + \left\lfloor\frac{K_1}{2}\right\rfloor}, \frac{\left\lfloor\frac{\min\{K_1,\cdots,K_N\}}{2}\right\rfloor}{2}, \frac{\left\lfloor\frac{K_N}{2}\right\rfloor M_d}{\left\lfloor\frac{K_N}{2}\right\rfloor + M_d - 1}\right\}. \tag{1}$$

For given values of M_s and M_d, if $K_1, \cdots, K_N \to \infty$, we have $d_\Sigma \to \min\{M_s, K_1, \cdots, K_N, M_d\} = \min\{M_s, M_d\}$. Again, the sum DoF upper bound is achieved. This means that the optimally achievable sum DoF is identified and the network's sum capacity can be characterized as $C_\Sigma = \min\{M_s, M_d\}\log_2 \rho + o(\log_2 \rho)$, no matter how many layers of relays have to be used to conduct transmissions. For more general situations with a finite number of relays, the optimally achievable sum DoF is still unknown. As long as the numbers of relays in each layer is sufficiently large, the result shown in (1) can be higher than $\frac{\min\{M_s, K_1, \cdots, K_N, M_d\}}{3}$ and thus provide a lower bound to the optimally achievable sum DoF.

B. New achievable sum DoF results

For the CSR scheme studied in [7], the two relay clusters in each layer are required to have the same size. It means that if a relay layer has an odd number of terminals, one relay has to be discarded from the transmission process. Such a demand simplifies the transmission design, but does not fully exploit the hardware resources of the considered network. In this paper, we will show that the achievable sum DoF in (1) can be further improved, when some relay layers contain odd numbers of terminals. This is done by allowing each relay layer to be divided into unequal clusters, so that all relays can be properly used to assist in the data transmission.

The new achievable sum DoF can be summarized in the following theorem and the proof is presented in Section III.

***Theorem** 1:* With time-varying fading and global CSI at all terminals, an achievable sum DoF of applying the CSR scheme in a single-antenna $(N+1)$-hop $\{M_s, K_1, \cdots, K_N, M_d\}^{(N+1)}$ network is expressed in (2).

It is easy to show that when $K_1, \cdots, K_N$ are all even values, the expressions in (1) and (2) are identical. However, if some relay layers contain odd numbers of terminals, the sum DoF achieved in (2) can be larger. Consider the situation that the number of relays in each layer is not too small so that (2) is higher than $\frac{\min\{M_s, K_1, \cdots, K_N, M_d\}}{3}$. Then to the best of our knowledge, the result shown in Theorem 1 is by far the highest achievable sum DoF in such single-antenna multi-user $\{M_s, K_1, \cdots, K_N, M_d\}^{(N+1)}$ networks when all the four types of interference are taken into account. Hence it can serve as a new lower bound for the optimally achievable sum DoF.

We will use a few simple examples to demonstrate such a result. Firstly, consider a 3-hop $\{5, 8, 8, 3\}^{(3)}$ example network. It is seen that $\frac{\min\{M_s, K_1, \cdots, K_N, M_d\}}{3} = 1$, much smaller than the sum DoF upper bound of 3. The result provided in [7], i.e. equation (1), shows that, by dividing each relay layer into two 4-relay clusters, the achievable sum DoF of the CSR scheme can be 2. It is clearly larger than 1. Now consider that each relay layer contains 9 relays. Using equation (1) the result remains to be 2, because although an extra relay exists in each layer it will be discarded. Theorem 1, however, shows that a larger sum DoF $d_\Sigma = \frac{60}{29}$ can actually be achieved.

The above observations hold in $\{5, 9, \cdots, 9, 3\}^{(N+1)}$ networks for any value of $N \geq 2$. To see the impact of the number of available relays on the achievable sum DoF, we consider $\{5, 2K+1, \cdots, 2K+1, 3\}^{(N+1)}$ networks (i.e., all relay layers contain the same odd number of relays), and display d_Σ versus K in Fig. 2. The performance improvement of (2) over (1) can be clearly seen. When the number of relays in each layer increases, d_Σ also increases to approach the sum DoF upper bound 3. This can be straightforwardly shown by setting all $K_i \to \infty$ in (2). The result is in line with [7].

The sizes of different relay layers need not be identical. Theorem 1 is applicable to the general $\{M_s, K_1, \cdots, K_N, M_d\}^{(N+1)}$ networks. For instance, for a 5-hop $\{3, 13, 15, 9, 12, 5\}^{(5)}$ network, the new achievable sum DoF is $d_\Sigma = \frac{84}{41}$, which is larger than 2, obtained using equation (1).

In the next section, we will prove Theorem 1. We will first describe the transmission process of the CSR scheme, for a fixed relay separation strategy. The original multi-hop network can be transformed into an equivalent single-hop network. Based on the interference alignment technique, we will identify the number of independent Gaussian codeword streams that can be successfully transmitted in the equivalent network. Using this result we will obtain the achievable sum DoF of the original network. Finally, we will discuss how to properly divide each layer of relays to maximize the system sum DoF. This will be the result shown in Theorem 1.

$$d_\Sigma = \min\Bigg\{ \frac{2}{\frac{2}{M_s} + \frac{1}{\lfloor \frac{K_1}{2} \rfloor} + \frac{1}{\lceil \frac{K_1}{2} \rceil}}, \frac{2}{\frac{1}{M_s} + \frac{1}{\lfloor \frac{K_1}{2} \rfloor} + \frac{1}{\lfloor \frac{\min\{K_1, \cdots, K_N\}}{2} \rfloor} + \frac{1}{\lceil \frac{\min\{K_1, \cdots, K_N\}}{2} \rceil}}, \frac{2}{\frac{1}{M_s} + \frac{1}{\lfloor \frac{K_1}{2} \rfloor} + \frac{\lfloor \frac{K_N}{2} \rfloor + M_d - 1}{\lfloor \frac{K_N}{2} \rfloor M_d}},$$
$$\frac{1}{\frac{1}{\lfloor \frac{\min\{K_1, \cdots, K_N\}}{2} \rfloor} + \frac{1}{\lceil \frac{\min\{K_1, \cdots, K_N\}}{2} \rceil}}, \frac{2}{\frac{\lfloor \frac{K_N}{2} \rfloor + M_d - 1}{\lfloor \frac{K_N}{2} \rfloor M_d} + \frac{1}{\lfloor \frac{\min\{K_1, \cdots, K_N\}}{2} \rfloor} + \frac{1}{\lceil \frac{\min\{K_1, \cdots, K_N\}}{2} \rceil}}, \frac{2}{\frac{\lceil \frac{K_N}{2} \rceil + M_d - 1}{\lceil \frac{K_N}{2} \rceil M_d} + \frac{\lfloor \frac{K_N}{2} \rfloor + M_d - 1}{\lfloor \frac{K_N}{2} \rfloor M_d}} \Bigg\} \quad (2)$$

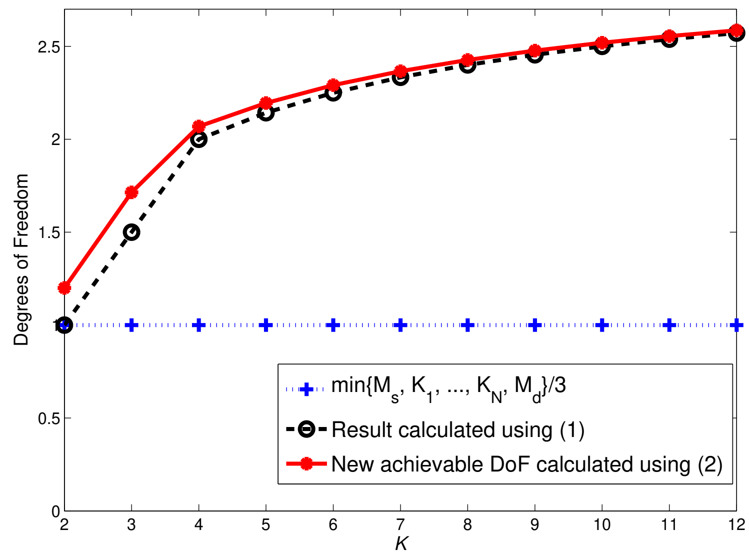

Fig. 2. The achievable sum DoF in $\{5, 2K+1, \cdots, 2K+1, 3\}^{(N+1)}$ networks.

III. The CSR Scheme and Achievable DoF Analysis

A. *Transmission process and equivalent network*

In the CSR scheme, we divide each layer of relays into two clusters and *take turns* activating them. Let us for now fix the clustering strategy. Specifically, the ith ($i \in \{1, \cdots, N\}$) relay layer is divided into an R_i-relay ($2 \leq R_i \leq K_i - 2$) cluster, denoted by $\mathcal{R}_{i,1}$, and a $(K_i - R_i)$-relay cluster, denoted by $\mathcal{R}_{i,2}$. (Each cluster should have at least two terminals.)

Since every source intends to send one independent message to every destination, we divide each source message into L ($L \geq 1$) *sub-message sets*. Every set contains the same number of multiple independent sub-messages, each of which can be encoded into a Gaussian codeword stream with rate $\log_2 \rho + o(\log_2 \rho)$. Hence every source has LM_d sub-message sets to transmit and every destination expects LM_s sub-message sets.

The overall message transmission process consumes a total of $L+N$ *time intervals*. Every time interval contains multiple (unit-length) time slots. During each of the first L intervals, every source broadcasts M_d sub-message sets, each of which is intended for one destination, to the first layer of relays. At any relay layer, the two clusters successively operate in the listening and forwarding modes: At an odd time interval, terminals within $\mathcal{R}_{i,1}$ listen to the transmissions from the prior layer, and terminals within $\mathcal{R}_{i,2}$ forward the signals they received in the previous interval to the next layer; At the next (i.e., an even) time interval, the two clusters exchange their functioning so that $\mathcal{R}_{i,2}$ listens and $\mathcal{R}_{i,1}$ forwards. After $L+N$ time intervals all the sub-message sets would reach the destinations. This transmission process operated in a $\{3, 7, 9, 8, 5\}^{(4)}$ example network is illustrated in Fig. 3(a).

It can be seen that, at any time interval, the transmission in the considered $(N+1)$-hop network is actually similar to that in a *single-hop equivalent network*. The equivalent network contains $N+1$ pairs of transmitter-receiver *clusters*, respectively denoted as $\mathcal{S}_i$ and $\mathcal{D}_i$ ($i = 1, 2, \cdots, N+1$). Clearly, $|\mathcal{S}_1| = M_s$ and $|\mathcal{D}_{N+1}| = M_d$. At odd time intervals, $|\mathcal{S}_i| = |\mathcal{R}_{i-1,2}| = K_{i-1} - R_{i-1}$ and $|\mathcal{D}_{i-1}| = |\mathcal{R}_{i-1,1}| = R_{i-1}$ ($i \in \{2, \cdots, N+1\}$). At even time intervals, $|\mathcal{S}_i| = |\mathcal{R}_{i-1,1}| = R_{i-1}$ and $|\mathcal{D}_{i-1}| = |\mathcal{R}_{i-1,2}| = K_{i-1} - R_{i-1}$.

For $i \in \{1, 2, \cdots, N-1\}$, in addition to the desired signals and forward interference coming from $\mathcal{S}_i$, the receptions in $\mathcal{D}_i$ experience two types of *inter-cluster interference*: the within-layer interference generated by $\mathcal{S}_{i+1}$ and the backward interference generated by $\mathcal{S}_{i+2}$. The receptions in $\mathcal{D}_N$ are interfered by the transmissions in one unintended cluster $\mathcal{S}_{N+1}$, i.e., the within-layer interference. Finally, receivers

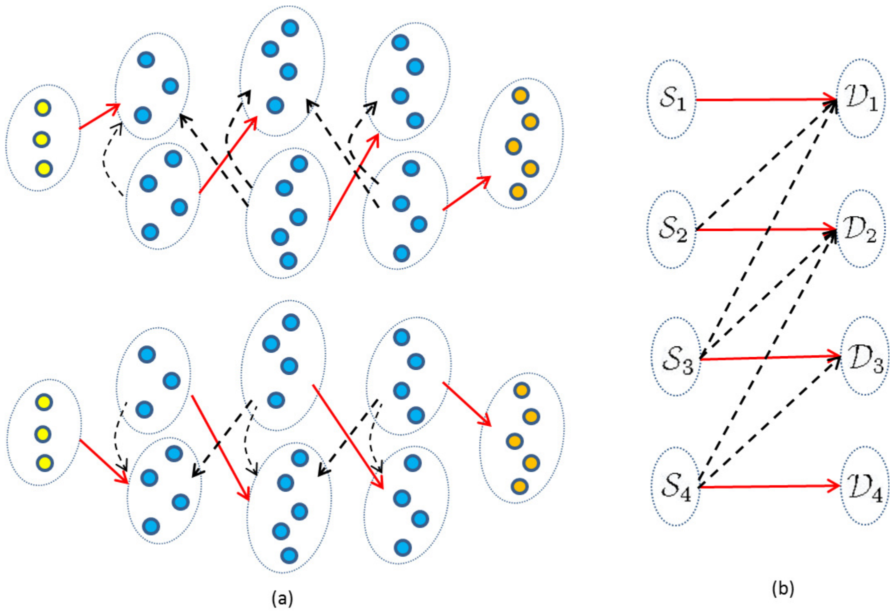

Fig. 3. (a) The CSR scheme and (b) the equivalent network. Solid lines represent intended transmission directions. Dashed lines represent within-layer interference and backward interference.

in $\mathcal{D}_{N+1}$ do not experience inter-cluster interference. Fig. 3(b) illustrates the equivalent network for the $\{3, 7, 9, 8, 5\}^{(4)}$ example network.

In what follows, we will focus on the equivalent network and use the interference alignment technique to design transmission strategy. It allows identifying the number of Gaussian codeword streams that can be successfully delivered within each hop at each time interval in the original network. Using this result we will be able to derive the asymptotically achievable sum DoF of the considered multi-hop network.

Note that in the equivalent network the transmitters in $\mathcal{S}_2$, $\mathcal{S}_3$, $\cdots$, $\mathcal{S}_{N+1}$ are actually relay terminals. The messages that they transmit can only be those they received in the past time interval. Since the sources broadcast the same number of sub-messages at different time intervals, in the equivalent network, the number of codeword streams to be delivered between each transmitter-receiver cluster pair $\mathcal{S}_i$ and $\mathcal{D}_i$ must be identical for all $i \in \{1, 2, \cdots, N+1\}$. Our interference alignment design would be constructed subject to this constraint.

B. Interference alignment design in the equivalent network

Consider transmissions in the equivalent network. Assuming that a total of T time slots are use, we use the $T \times T$ diagonal matrix $\mathbf{H}_{q,p}^{[i_2,i_1]}$ ($i_1, i_2 \in \{1, \cdots, N+1\}$, $p \in \{1, \cdots, |\mathcal{S}_{i_1}|\}$, $q \in \{1, \cdots, |\mathcal{D}_{i_2}|\}$) to denote the channel matrix between the pth transmitter in $\mathcal{S}_{i_1}$ and the qth receiver in $\mathcal{D}_{i_2}$.

In the original $\{M_s, K_1, \cdots, K_N, M_d\}^{(N+1)}$ network, let each source divide its message dedicated to each destination into L sub-message sets, and each set contain $\prod_{j=1}^{N} R_j(K_j - R_j)c^{\Gamma}$ independent sub-messages, in which c and Γ are integer constants. In the equivalent network, this means that the number of independent codeword streams to be delivered between each transmitter-receiver cluster pair is $M_s M_d \prod_{j=1}^{N} R_j(K_j - R_j)c^{\Gamma} = \prod_{j=1}^{N+1} |\mathcal{S}_j||\mathcal{D}_j|c^{\Gamma}$. Define $B_i = \frac{\prod_{j=1}^{N+1} |\mathcal{S}_j||\mathcal{D}_j|}{|\mathcal{S}_i||\mathcal{D}_i|}$ for $i \in \{1, 2, \cdots, N+1\}$. Treat each cluster pair as a wireless X channel so that the pth transmitter in $\mathcal{S}_i$ intends to send $B_i c^{\Gamma}$ independent codeword streams to the qth receiver in $\mathcal{D}_i$. These codeword streams are denoted by B_i different $c^{\Gamma} \times 1$ vectors $\mathbf{x}_{p,[(q-1)B_i+1]}^{[i]}$, $\mathbf{x}_{p,[(q-1)B_i+2]}^{[i]}$, $\cdots$, $\mathbf{x}_{p,[qB_i]}^{[i]}$, each element of which represents a stream with rate $\log_2 \rho + o(\log_2 \rho)$. Their transmitter-side beamforming matrices are denoted by $T \times c^{\Gamma}$ matrices $\mathbf{V}_{p,[(q-1)B_i+1]}^{[i]}$, $\mathbf{V}_{p,[(q-1)B_i+2]}^{[i]}$, $\cdots$, $\mathbf{V}_{p,[qB_i]}^{[i]}$.

$$\mathbf{y}_q^{[i]} = \sum_{p=1}^{|\mathcal{S}_i|} \mathbf{H}_{q,p}^{[i,i]} \left(\sum_{\kappa=(q-1)B_i+1}^{qB_i} \mathbf{V}_{p,[\kappa]}^{[i]} \mathbf{x}_{p,[\kappa]}^{[i]} \right) + \sum_{p=1}^{|\mathcal{S}_i|} \mathbf{H}_{q,p}^{[i,i]} \left(\sum_{\kappa=1}^{(q-1)B_i} \mathbf{V}_{p,[\kappa]}^{[i]} \mathbf{x}_{p,[\kappa]}^{[i]} + \sum_{\kappa=qB_i+1}^{B_i|\mathcal{D}_i|} \mathbf{V}_{p,[\kappa]}^{[i]} \mathbf{x}_{p,[\kappa]}^{[i]} \right)$$
$$+ \sum_{p=1}^{|\mathcal{S}_{i+1}|} \mathbf{H}_{q,p}^{[i,i+1]} \left(\sum_{\kappa=1}^{B_{i+1}|\mathcal{D}_{i+1}|} \mathbf{V}_{p,[\kappa]}^{[i+1]} \mathbf{x}_{p,[\kappa]}^{[i+1]} \right) + \sum_{p=1}^{|\mathcal{S}_{i+2}|} \mathbf{H}_{q,p}^{[i,i+2]} \left(\sum_{\kappa=1}^{B_{i+2}|\mathcal{D}_{i+2}|} \mathbf{V}_{p,[\kappa]}^{[i+2]} \mathbf{x}_{p,[\kappa]}^{[i+2]} \right)$$
$$+ \mathbf{H}_q^{[i,0]} \left(\sum_{\kappa=1}^{\max_{j=0,1,2}\{B_{i+j}|\mathcal{D}_{i+j}|\}} \mathbf{V}_{0,[\kappa]}^{[0]} \mathbf{x}_{0,[\kappa]}^{[0]} \right) + \mathbf{z}_q^{[i]}. \tag{3}$$

To facilitate presentation, following [7], we create a virtual transmitter S_0 in the equivalent network. S_0 is assumed to broadcast $\max\{B_1|\mathcal{D}_1|, \cdots, B_{N+1}|\mathcal{D}_{N+1}|\}(c+1)^\Gamma$ dummy codeword streams (known at all terminals), denoted by $(c+1)^\Gamma \times 1$ vectors $\mathbf{x}_{0,[j]}^{[0]}$ for $j \in \{1, 2, \cdots, \max\{B_1|\mathcal{D}_1|, \cdots, B_{N+1}|\mathcal{D}_{N+1}|\}\}$. Their beamforming matrices are denoted by $T \times (c+1)^\Gamma$ matrices $\mathbf{V}_{0,[j]}^{[0]}$. The channel matrix between S_0 and the qth receiver in $\mathcal{D}_i$ is denoted by $T \times T$ diagonal matrix $\mathbf{H}_q^{[i,0]}$.

Now the received signal at the qth receiver in $\mathcal{D}_i$ can be expressed in (3), shown on the top of the next page. The first term on the right hand side is the $|\mathcal{S}_i|B_i c^\Gamma$ desired codeword streams, the second term denotes the undesired interference signals generated by $\mathcal{S}_i$, the third and fourth terms represent the inter-cluster interference from $\mathcal{S}_{i+1}$ and $\mathcal{S}_{i+2}$ respectively, the fifth term is the dummy interference signals sent from $\mathcal{S}_0$, and $\mathbf{z}_q^{[i]}$ is additive white Gaussian noise. Our interference alignment construction targets aligning the interference at each receiver to the subspace decided by the dummy interference.

Specifically, for every $\kappa \in \{1, 2, \cdots, (q-1)B_i, qB_i+1, qB_i+2, \cdots, |\mathcal{D}_i|B_i\}$, we intend to align the undesired signals $\mathbf{x}_{p,[\kappa]}^{[i]}$ ($p = 1, 2, \cdots, |\mathcal{S}_i|$), appeared in the second term of (3), in the $(c+1)^\Gamma$-dimensional subspace decided by $\mathbf{H}_q^{[i,0]}\mathbf{V}_{0,[\kappa]}^{[0]}$. This means, for any $i \in \{1, 2, \cdots, N+1\}$, let

$$\text{span}\left(\mathbf{H}_{q,p}^{[i,i]}\mathbf{V}_{p,[\kappa]}^{[i]}\right) \subset \text{span}\left(\mathbf{H}_q^{[i,0]}\mathbf{V}_{0,[\kappa]}^{[0]}\right), \ \forall 1 \le p \le |\mathcal{S}_i|. \tag{4}$$

For every value of $\kappa \in \{1, 2, \cdots, |\mathcal{D}_{i+1}|B_{i+1}\}$, we align the inter-cluster interference signals $\mathbf{x}_{p,[\kappa]}^{[i+1]}$ ($p = 1, 2, \cdots, |\mathcal{S}_{i+1}|$), appeared in the third term of (3), to the $(c+1)^\Gamma$-dimensional subspace decided by $\mathbf{H}_q^{[i,0]}\mathbf{V}_{0,[\kappa]}^{[0]}$. That is, for $i \in \{1, \cdots, N\}$:

$$\text{span}\left(\mathbf{H}_{q,p}^{[i,i+1]}\mathbf{V}_{p,[\kappa]}^{[i+1]}\right) \subset \text{span}\left(\mathbf{H}_q^{[i,0]}\mathbf{V}_{0,[\kappa]}^{[0]}\right), \ \forall 1 \le p \le |\mathcal{S}_{i+1}|. \tag{5}$$

Finally, for every $\kappa \in \{1, \cdots, |\mathcal{D}_{i+2}|B_{i+2}\}$, align the inter-cluster interference signals $\mathbf{x}_{p,[\kappa]}^{[i+2]}$ ($p = 1, 2, \cdots, |\mathcal{S}_{i+2}|$), appeared in the fourth term of (3), to the $(c+1)^\Gamma$-dimensional subspace decided by $\mathbf{H}_q^{[i,0]}\mathbf{V}_{0,[\kappa]}^{[0]}$, i.e., for $i \in \{1, \cdots, N-1\}$:

$$\text{span}\left(\mathbf{H}_{q,p}^{[i,i+2]}\mathbf{V}_{p,[\kappa]}^{[i+2]}\right) \subset \text{span}\left(\mathbf{H}_q^{[i,0]}\mathbf{V}_{0,[\kappa]}^{[0]}\right), \forall 1 \le p \le |\mathcal{S}_{i+2}|. \tag{6}$$

Clearly, in order to eliminate all interference and recover the $|\mathcal{S}_i|B_i c^\Gamma$ desired codeword streams, the receive space's dimension should be no less than $|\mathcal{S}_i|B_i c^\Gamma + \max\{|\mathcal{D}_i|B_i, |\mathcal{D}_{i+1}|B_{i+1}, |\mathcal{D}_{i+2}|B_{i+2}\}(c+1)^\Gamma$ for any receiver in $\mathcal{D}_i$ ($i \in \{1, \cdots, N-1\}$) . For each receiver in $\mathcal{D}_N$, the receive space should have dimensions no less than $|\mathcal{S}_N|B_N c^\Gamma + \max\{|\mathcal{D}_N|B_N, |\mathcal{D}_{N+1}|B_{N+1}\}(c+1)^\Gamma$. And for each receiver in

$\mathcal{D}_{N+1}$, the receive space's dimensions should be at least $|\mathcal{S}_{N+1}|B_{N+1}c^{\Gamma} + (|\mathcal{D}_{N+1}|-1)B_{N+1}(c+1)^{\Gamma}$. These indicate that the minimum value of T must be chosen as

$$\begin{aligned} T &= \max\Bigg\{ \max_{i=1,\cdots,N-1} \left\{ |\mathcal{S}_i|B_i c^{\Gamma} + \max_{j=0,1,2}\left\{|\mathcal{D}_{i+j}|B_{i+j}\right\}(c+1)^{\Gamma} \right\}, \\ & \qquad |\mathcal{S}_N|B_N c^{\Gamma} + \max_{j=0,1}\{|\mathcal{D}_{N+j}|B_{N+j}\}(c+1)^{\Gamma}, \\ & \qquad |\mathcal{S}_{N+1}|B_{N+1}c^{\Gamma} + (|\mathcal{D}_{N+1}|-1)B_{N+1}(c+1)^{\Gamma} \Bigg\}. \end{aligned} \tag{7}$$

Set the integer constant $\Gamma = \sum_{j=1}^{N+1} |\mathcal{S}_j|\left(|\mathcal{D}_j|-1\right) + \sum_{j=1}^{N} |\mathcal{S}_{j+1}||\mathcal{D}_j| + \sum_{j=1}^{N-1} |\mathcal{S}_{j+2}||\mathcal{D}_j|$, and for each value of $i \in \{1,2,\cdots,N+1\}$, set $\mathbf{V}_{[\kappa]} = \mathbf{V}^{[i]}_{1,[\kappa]} = \mathbf{V}^{[i]}_{2,[\kappa]} = \cdots = \mathbf{V}^{[i]}_{|\mathcal{S}_i|,[\kappa]}$, $\forall \kappa \in \{1,2,\cdots,|\mathcal{D}_i|B_i\}$. One can use the conditions (4)-(6) to construct the beamforming matrices $\mathbf{V}_{[\kappa]}$ and $\mathbf{V}^{[0]}_{0,[\kappa]}$, which guarantee, with probability one, that at the qth receiver in $\mathcal{D}_i$ the $|\mathcal{S}_i|$ different $B_i c^{\Gamma}$-dimensional subspaces for the desired $B_i|\mathcal{S}_i|c^{\Gamma}$ codeword streams are independent to that for the aligned interference signals and also independent to each other.[1] Since the dummy interference signals are known to the terminal and hence can be directly cancelled, a linear zero-forcing filter suffices to eliminate unknown interference and recover all the desired codeword streams.

Therefore, for the single-hop equivalent network, across T unit time slots, each transmitter cluster $\mathcal{S}_i$ can deliver a total of $\prod_{j=1}^{N+1} |\mathcal{S}_j||\mathcal{D}_j|c^{\Gamma}$ independent codeword streams to its receiver cluster $\mathcal{D}_i$. This means that in the original multi-hop network, during each time interval a total of $\prod_{j=1}^{N+1} |\mathcal{S}_j||\mathcal{D}_j|c^{\Gamma} = M_s M_d \prod_{j=1}^{n} R_j\left(K_j - R_j\right)c^{\Gamma}$ independent Gaussian codeword streams can be successfully delivered within every hop. Armed with this result, we are ready to analyze the achievable sum DoF of applying the CSR scheme in the considered $\{M_s, K_1, \cdots, K_N, M_d\}^{(N+1)}$ network.

C. Achievable sum DoF in the original network

Following the above discussions, the required unit time slots at an odd time interval is denoted by T_o and can be found by substituting $|\mathcal{S}_1| = M_s$, $|\mathcal{D}_{N+1}| = M_d$, $|\mathcal{D}_i| = R_i$, and $|\mathcal{S}_{i+1}| = K_i - R_i$ into (7). Similarly, the number of required time slots at an even time interval, denoted by T_e, can be calculated by substituting $|\mathcal{S}_1| = M_s$, $|\mathcal{D}_{N+1}| = M_d$, $|\mathcal{D}_i| = K_i - R_i$ and $|\mathcal{S}_{i+1}| = R_i$ into (7).

Hence the total number of time slots consumed by the CSR scheme to complete the transmission of $LM_sM_d\prod_{j=1}^{N} R_j\left(K_j-R_j\right)c^{\Gamma}$ codeword streams is $\lceil\frac{L+N}{2}\rceil T_o + \lfloor\frac{L+N}{2}\rfloor T_e$. The asymptotically achievable sum DoF can be found as follows by letting $L \to \infty$ and $c \to \infty$:

$$\begin{aligned} d_{\mathrm{CSR}(R_1,\cdots,R_N)} &= \frac{LM_sM_d\prod_{j=1}^{N} R_j\left(K_j-R_j\right)c^{\Gamma}}{\lceil\frac{L+N}{2}\rceil T_o + \lfloor\frac{L+N}{2}\rfloor T_e} \\ &\overset{(a)}{\approx} \frac{2M_sM_d\prod_{j=1}^{N} R_j\left(K_j-R_j\right)c^{\Gamma}}{T_o+T_e} \\ &\overset{(b)}{\approx} \frac{2}{t_{(R_1,\cdots,R_N)} + t_{(K_1-R_1,\cdots,K_N-R_N)}}, \end{aligned} \tag{8}$$

[1] The channel-extension based interference alignment construction approach is adapted from those presented in [7], [10]. Due to the paper length limit the details are not provided here, but will be available in our full version paper.

where (a) follows from $\frac{\lceil \frac{L+N}{2} \rceil}{L} \approx \frac{\lfloor \frac{L+N}{2} \rfloor}{L} \approx \frac{1}{2}$ when $L \to \infty$, (b) follows from $\frac{c^\Gamma}{(c+1)^\Gamma} \approx 1$ when $c \to \infty$, and the function

$$\begin{aligned} t_{(r_1,\cdots,r_N)} &= \max\left\{\frac{1}{r_1} + \max\left\{\frac{1}{M_s}, \frac{1}{K_1 - r_1}, \frac{1}{K_2 - r_2}\right\},\right. \\ &\qquad \max_{i=2,\cdots,N-1}\left\{\frac{1}{r_i} + \max_{j=-1,0,1}\left\{\frac{1}{K_{i+j} - r_{i+j}}\right\}\right\}, \\ &\qquad \left.\frac{1}{r_N} + \max\left\{\frac{1}{K_{N-1} - r_{N-1}}, \frac{1}{K_N - r_N}\right\}, \frac{K_N - r_N + M_d - 1}{(K_N - r_N)M_d}\right\}. \end{aligned}$$

Equation (8) identifies the achievable sum DoF for a particular separation of the relay layers. To make the best use of all the relays, one should properly choose the values of R_1, $\cdots$, R_N to maximize $d_{\text{CSR}(R_1,\cdots,R_N)}$, i.e.,

$$\begin{aligned} &\text{maximize} \quad \frac{2}{t_{(R_1,\cdots,R_N)} + t_{(K_1-R_1,\cdots,K_N-R_N)}} \\ &\text{s.t.} \quad 2 \le R_i \le K_i - 2,\ \forall i \in \{1,2,\cdots,N\}. \end{aligned} \tag{9}$$

Use $\min_{\mathbf{R}}\{g(R_1,\cdots,R_N)\}$ to denote the minimization of a function $g(R_1,\cdots,R_N)$ by choosing among $2 \le R_1 \le K_1 - 2$, $\cdots$, $2 \le R_N \le K_N - 2$. Hence solving the problem (9) is equivalent to finding the solution of $\min_{\mathbf{R}}\{t_{(R_1,\cdots,R_N)} + t_{(K_1-R_1,\cdots,K_N-R_N)}\}$. Define

$$\tilde{T} = \max\left\{\max_{2\le i\le N}\left\{\frac{1}{R_i} + \frac{1}{K_{i-1} - R_{i-1}}\right\}, \max_{1\le i\le N}\left\{\frac{1}{R_i} + \frac{1}{K_i - R_i}\right\}, \max_{1\le i\le N-1}\left\{\frac{1}{R_i} + \frac{1}{K_{i+1} - R_{i+1}}\right\}\right\}.$$

We can show that

$$\begin{aligned} \min_{\mathbf{R}}\left\{\tilde{T}\right\} &\ge \max\left\{\frac{1}{\lfloor\frac{K_1}{2}\rfloor} + \frac{1}{\lceil\frac{K_1}{2}\rceil}, \cdots, \frac{1}{\lfloor\frac{K_N}{2}\rfloor} + \frac{1}{\lceil\frac{K_N}{2}\rceil}\right\} \\ &= \frac{1}{\left\lfloor\frac{\min\{K_1,\cdots,K_N\}}{2}\right\rfloor} + \frac{1}{\left\lceil\frac{\min\{K_1,\cdots,K_N\}}{2}\right\rceil}. \end{aligned}$$

The lower bound $\frac{1}{\left\lfloor\frac{\min\{K_1,\cdots,K_N\}}{2}\right\rfloor} + \frac{1}{\left\lceil\frac{\min\{K_1,\cdots,K_N\}}{2}\right\rceil}$ is achievable when we choose $R_i = \lfloor\frac{K_i}{2}\rfloor$ for all $i \in \{1,\cdots,N\}$.

After some mathematical manipulations, the function $t_{(R_1,\cdots,R_N)}$ can be expressed as $t_{(R_1,\cdots,R_N)} = \max\left\{\frac{1}{R_1} + \frac{1}{M_s}, \tilde{T}, \frac{K_N-R_N+M_d-1}{(K_N-R_N)M_d}\right\}$. Therefore,

$$\begin{aligned} &\min_{\mathbf{R}}\{t_{(R_1,\cdots,R_N)} + t_{(K_1-R_1,\cdots,K_N-R_N)}\} \\ = &\min_{\mathbf{R}}\left\{\max\left\{\frac{1}{K_1 - R_1} + \frac{1}{R_1} + \frac{2}{M_s}, \frac{1}{R_1} + \frac{1}{M_s} + \tilde{T}, \frac{1}{R_1} + \frac{1}{M_s} + \frac{R_N + M_d - 1}{R_N M_d}, \frac{1}{K_1 - R_1} + \frac{1}{M_s} + \tilde{T},\right.\right. \\ &\qquad 2\tilde{T}, \frac{K_N - R_N + M_d - 1}{(K_N - R_N)M_d} + \frac{1}{K_1 - R_1} + \frac{1}{M_s}, \frac{R_N + M_d - 1}{R_N M_d} + \tilde{T}, \frac{K_N - R_N + M_d - 1}{(K_N - R_N)M_d} + \tilde{T}, \\ &\qquad \left.\left.\frac{K_N - R_N + M_d - 1}{(K_N - R_N)M_d} + \frac{R_N + M_d - 1}{R_N M_d}\right\}\right\} \end{aligned}$$

When we always try to evenly divide each layer of relays and set $|\mathcal{R}_{i,1}| = \lfloor\frac{K_i}{2}\rfloor$ and $|\mathcal{R}_{i,2}| = \lceil\frac{K_i}{2}\rceil$ for all $i \in \{1,2,\cdots,N\}$, the terms $\frac{1}{K_1-R_1} + \frac{1}{M_s} + \tilde{T}$, $\frac{K_N-R_N+M_d-1}{(K_N-R_N)M_d} + \frac{1}{K_1-R_1} + \frac{1}{M_s}$, and $\frac{K_N-R_N+M_d-1}{(K_N-R_N)M_d} + \tilde{T}$ can be removed from the above expression. It can be shown that an achievable lower bound of the maximized value of $d_{\text{CSR}(R_1,\cdots,R_N)}$ shown in (8) is expressed as (2). If all the relay layers contain even numbers of nodes, then this achievable sum DoF is identical to that shown in (1). Otherwise, possibly the new achievable sum DoF can be higher. Theorem 1 is proved.

IV. Conclusion

We have investigated the achievable sum DoF in a wireless single-antenna multi-user relay network, with multiple sources, multiple destinations, and multiple layers of relays. We consider a general situation that the relay self-interference and inter-relay interference cannot be ignored, so that half-duplex relaying must be adopted. We have studied combining a half-duplex CSR transmission scheme with the interference alignment technique to carry out information delivery. Allowing the two clusters in each relay layer to contain different numbers of nodes, all available relays can be involved and properly clustered. It has been shown that the asymptotically achievable sum DoF can be larger than previously known results and hence can serve as a new lower bound for the optimally achievable sum DoF in the considered network.

Acknowledgment

This work was funded by National Natural Science Foundation of China (61401314), the EU H2020 Marie Sklodowska-Curie IF MACROSS project (H2020-MSCA-IF-2016-752979), and the EU FP7 QUICK project (PIRSES-GA-2013-612652).

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