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THE COMPLETENESS: FROM HENKIN'S PROPOSITION TO QUANTUM COMPUTER

Introduction. Both completeness and incompleteness are well distinguishable as to finiteness: Completeness supposes that any operations defined over any finite sets do not transcend them while incompleteness displays that they can do it sometimes. This legible boundary turns out to be unclear and even inconsistent jumping into infinity. One may say that there are two strategies or "philosophies" after that leap has been just made and any orientation in the unknown infinity is necessary for the thought to survive: one should keep either to completeness or to incompleteness for the infinity seems both complete and incomplete being as universal as open. Henkin and Gödel are recognized correspondingly as the symbolic personages embodying those two opposite types.

Prehistory and background. The paper retraces the strategy of completeness originating from Henkin's proposition [1] via Löb's proof [2] to the alleged incompleteness of quantum me-chanics [3], the "no hidden variables" theorem [12], [13], entanglement and quantum information and to the endpoint of a quantum computer [4], [5], which can solve any problem only if its user is situated inside it [6], [7].
An introductory reformulating roof sketch in the most concise relevant lan-guage is necessary for the above tentative itinerary to be able to be seen as a progressing whole rather than a heterogenic and eclectic mix. The man and scien-tist Leon Henkin can be a reliable guide implicitly:
Many efforts address the constructiveness of the proof in the so-called Gödel first incompleteness theorem [8]. However it is not only redundant but putting obstacles for the above objectivity. One should use the axiom of choice instead of this and abstract from that the theorem is formulated about statements, which will be replaced by the arbitrary elements of any infinite countable set. Furthermore all four theorems [2], [8], [9], [10] about the so-called metamathematical fixed points induced by the Gödel incompleteness theorems will be considered jointly and in a generalized way.

Strategy. An arbitrary infinite countable set "A" and another set "B" so that their intersection is empty are given. One constitutes their union "$C=A\cup B$", which will be an infinite set whatever "B" is. Utilizing the axiom of choice, a one-to-one mapping "$f = A \leftrightarrow C$" exists. One designates the image of "B" into "A" through "f " by "B_f " so that "$B_f \subset A$". If the axiom of choice holds, there always an internal and equivalent image like "B_f " for any external set like "B". Thus, if one accepts that "$B_f \equiv B$", whether an element "b" of "B" belongs or not to "A" is an undecidable problem as "$b_f \equiv b$".
However if the axiom of choice is not valid, one cannot guarantee that "f " exists and should display how a constructive analog of "f " can be built. If one shows how "f " to be constructed at least in one case, this will be a constructive proof of undecidability as what Gödel's is.
Another option is to prove that no constructive analog exists for any "f " guaranteed by the axiom of choice. In a sense, this would be an analog of the theorems about the absence of hidden variables in quantum mechanics [12], [13].
Coring the reformulation, the problem of (in)completeness can be generalized
as a property of all infinite sets. An infinite set unlike any finite one can be both
complete (universal) and incomplete (open) in a sense reminiscent to the "clopen" (both closed and open) sets in topology such as all discrete sets are. There is a hidden, but intimately link between discreteness and infinity. Quantum mechanics, forced to introduce quanta and thus discreteness, has therefore introduced infin-ity in an experimental and exact science such as physics. What is that bridge is what leads from Henkin's proposition to quantum computer, both being from the "internal side" of completeness.

Conclusion. The link between quantum mechanics, logic, and mathematics is much closer than it is commonly accepted. There is a special area of continuous transition between them, a "bridge" rather than a "gap". The solutions in each of those sciences may influence the formulations of problems as well as their solutions in the rest ones.
Particularly, the problem of completeness or incompleteness as well as the approaches for its solution in logic, mathematics, and quantum mechanics may be unified.

The paper addresses Leon Henkin's proposition as a "lighthouse", which can elucidate a vast territory of knowledge uniformly: logic, set theory, information theory, and quantum mechanics: Two strategies to infinity are equally relevant for it is as universal and thus complete as open and thus in-complete. Henkin's, Gödel's, Robert Jeroslow's, and Hartley Roger's proposition are reformulated so that both completeness and incompleteness to be unified and thus reduced as a joint property of infinity and of all infinite sets. However, only Henkin's proposition equivalent to an internal position to infinity is consistent. This can be retraced back to set theory and its axioms, where that of choice is a key. Quantum mechanics is forced to introduce infinity implicitly by Hilbert space, on which is founded its formalism. One can demonstrate that some essential properties of quantum information, entanglement, and quantum computer originate directly from infinity once it is involved in quantum mechanics. Thus, these phenomena can be elucidated as both complete and incomplete, after which choice is the border between them. A special kind of invariance to the axiom of choice shared by quantum mechanics is discussed to be involved that border between the completeness and incompleteness of infinity in a consistent way. The so-called paradox of Albert Einstein, Boris Podolsky, and Nathan Rosen is interpreted entirely in the same terms only of set theory. Quantum computer can demonstrate especially clearly the privilege of the internal position, or "observer", or "user" to infinity implied by Henkin's proposition as the only consistent ones as to infinity. An essential area of contemporary knowledge may be synthesized from a single viewpoint.

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