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Digital twins: Understanding the added value of integrated models for through-life engineering services

Rok Vrabič^{a,*}, John Ahmet Erkoyuncu^b, Peter Butala^a, Rajkumar Roy^b

^aUniversity of Ljubljana, Department of Manufacturing Systems and Control, Slovenia ^bCranfield University, Through-life Engineering Services Centre, United Kingdom

Abstract

Digital twins are digital representations of physical products or systems that consist of multiple models from various domains describing them on multiple scales. By means of communication, digital twins change and evolve together with their physical counterparts throughout their lifecycle. Domain-specific partial models that make up the digital twin, such as the CAD model or the degradation model, are usually well known and provide accurate descriptions of certain parts of the physical asset. However, in complex systems, the value of integrating the partial models increases because it facilitates the study of their complex behaviours which only emerge from the interactions between various parts of the system. The paper proposes that the partial models of the digital twin share a common model space that integrates them through a definition of their interrelations and acts as a bridge between the digital twin and the physical asset. The approach is illustrated in a case of a mechatronic product – a differential drive mobile robot developed as a testbed for digital twin research. It is demonstrated how the integrated models add value to different stages of the lifecycle, allowing for evaluation of performance in the design stage and real-time reflection with the physical asset during its operation.

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Keywords: digital twin ; modelling ; multi-domain model

1. Introduction

First coined in a product lifecycle management context in 2003 [1], digital twin is a concept that was envisioned well before the technology necessary for its implementation caught up. Initially, the digital twin was conceptualised as a *virtual product* - a rich representation of a product that is virtually indistinguishable from its physical counterpart. The initial concept included a data connection tying the virtual and the real product together throughout the lifecycle. Since the concept was conceived, tremendous advances have been achieved in the field of digital representation as

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^{*} Corresponding author. Tel.: +386-1-4771-753.

E-mail address: rok.vrabic@fs.uni-lj.si

well as in data collection and management technologies, and today the concept is receiving a lot of attention from the industry [2].

A recent pan-industry survey [3] found that much of the digital-twin-related technology is already available on the market, but that its integration still presents a challenge. In addition, the in-service phase has been highlighted by the industry as an area of significant potential benefit for the digital twin. A new definition that emphasises the through-life engineering aspects was proposed: "A digital twin is a digital representation of a physical item or assembly using integrated simulations and service data. The digital representation holds information from multiple sources across the product lifecycle. This information is continuously updated and is visualised in a variety of ways to predict current and future conditions, in both design and operational environments, to enhance decision making" [3].

Digital twins are nowadays considered as more than just collections of digital artefacts, but rather, as collections of *linked* digital artefacts. This distinction is a key one, and integrating and linking the models and the data still presents a research challenge. While partial models are able to address specific parts of the product, new behaviours can emerge when the models are intertwined. It is argued that the value of the digital twin primarily lies in the elimination of information inefficiencies which result in a waste of resources [4]. The assumption here is that, even though the information is never free to acquire or use, the cost of the information will be less than the cost of the wasted resources. But additional value can be derived from the integration of the models because this allows for a consideration of scenarios which exceed the descriptive capabilities of any single model.

The paper presents a concept of model integration within the digital twin, using a mobile robot as a test bed, and discusses the implications of the integration for added value in the context of through-life engineering services.

2. Integration of models within the digital twin

An often cited definition of the digital twin is that a digital twin is an ultra-high fidelity simulation, that when coupled with the specific vehicle health monitoring data, maintenance history and historical fleet data, can help to improve safety and reliability [5]. However other authors argue that the digital twin does not need to be an ultra-high fidelity simulation, but rather a reasonable estimation of the product operation [6]. They report that a digital twin of a complete airframe of an aircraft has 10^{12} degrees of freedom and that if microstructure is considered, those models have 10^7 degrees of freedom at each location. This makes it impossible to simulate the behaviour of the system in real time even with the current strongest supercomputers. Instead of focusing on the computational aspect, i.e. going into depth, not much is found in the literature about model complexity and combining the simulation and other models in a meaningful way, i.e. going into breadth [4].

This can be achieved by creating a common model space that allows the models to exchange the data, defines the relations between the models and acts as a bridge between the digital twin and the physical asset. Two kinds of relations between the models are defined. An *input/output* relation exists when a model needs the output of another model as its input. For example, a CAM model takes the output of a part's CAD model as its input and is then able to produce the NC code for part machining. Another kind of relation is an *influences* relation which specifies that a change in one of the model influences another model. For example, a tool wear model does not influence the CAD model but can be used to improve the CAM one. The common model space, therefore, defines how the models are connected as well as how and when they are updated.

The integrated model defined through the common model space also provides the capability to explore what-if scenarios. This is realised by a *learning model*. In addition to providing the simulation capability, the integrated model can use the learning model to support real-time reflection, interaction along the lifecycle, and self-evolution [7]. The learning model should, therefore, define *how* to run the simulation, i.e. with what parameters. Consider, for example, how to ensure that the digital twin continues to represent the ever-changing physical asset. If a change is detected in the physical asset or its environment, the learning model should define simulation scenarios to explore which and how the model parameters have to be changed to mirror the observed behaviour. In other words, the learning model allows for real-time reflection through a simulation-based search through the parameter space.

The rest of the paper illustrates this idea and shows how it can be employed in various stages of a product lifecycle. A differential drive mobile robot with line following capability is considered. While it is a complex mechatronic product, its parts are simple enough to be analytically modelled. This, along with its ability to communicate in real time, makes the mobile robot a suitable test bed for digital twin research.

3. Digital twin of a mobile robot

The robot is based on an 8-bit microcontroller and is actuated by two geared DC motors. The robot's sensors include an array of eight reflectance sensors facing downwards for line following, six reflectance sensors facing sideways for obstacle detection, a 9-axis inertial measurement unit (IMU), and encoders for detecting motor velocities. A Bluetooth module is used for real-time communication.

In this investigation, a line following task is considered. Although the principle of line following is simple, optimisation of the robot's speed presents a challenge because of the many factors in play ranging from the robot's geometry and dynamics, its hardware design, the control algorithms used, to responding to changes in the environment conditions while the task is performed.

3.1. Digital twin architecture

The architecture of the digital twin is based around loosely-coupled models that describe various aspects of the robot and its environment. Each of the models can be independently tested and calibrated to reflect the corresponding physical asset as closely as possible. To build the digital twin of the robot, Robot Operating System (ROS) is chosen as the basis for the common model space [8]. Fig. 1 shows the models along with the tools and formats that are used for their implementation. The rest of the section then details the models and shows how they can be integrated in the common model space.



Fig. 1: Partial models of the digital twin.

The models include the CAD model, exported from the electronics design automation (EDA) software, in which the robot was designed. The environment model includes assets such as the track that are needed for line follower simulation. The motion model is implemented in Gazebo simulator and uses ODE physics engine. DC motors are simulated using a custom-made Gazebo plugin. A software model is developed that allows that the same code to be uploaded to the digital twin and the physical robot and makes sure that the code execution speeds are comparable. Line sensors are simulated using Gazebo's camera plugin, while encoders are simulated by measuring the joint positions and velocities from the physics engine. A control model is developed for line following, along with a learning model which is able to optimise the control parameters and synchronise the digital twin with the physical asset. In order to understand the relations between the models shown in Fig. 1 and to be able to integrate them, a deeper understanding of their inputs, outputs, and parameters is necessary.

3.2. CAD model

The CAD model is created in EDA software. All component models are imported or created in .STEP format. This allows for the final model to be exported to any other CAD software for further processing and analysis. Fig. 2 shows the physical robot along with its CAD models in EDA software and in ROS, converted to Universal Robot Description Format (URDF) and visualised in ROS's visualisation component RViZ. The CAD model defines the robot's dimensions (L, R, S), and can be used to estimate masses and inertias of the robot platform and the wheels (m_c, m_w, I_c, I_w), and the location of the centre of gravity (d).

3.3. Model of motion

The model of motion describes robot's kinematics and dynamics and is a core part of the robot's digital twin. It is important for this model to be as accurate as possible in order to get the realistic behaviour of the robot in a



Fig. 2: a) Real robot. b) CAD model in EDA software. c) CAD model in RViZ, showing robot's frames.

simulation. The kinematic model describes the relationship between the velocities of the wheels and the translational and rotational velocities of the robot as well as the relation between the robot frame and the inertial frame. The coordinate systems are shown in Fig. 3.



Fig. 3: Differential drive mobile robot.

The position of the robot's centre of rotation is denoted as (x_a, y_a) , and it's centre of gravity (COG) as (x_c, y_c) . The robot frame (X_r, Y_r) moves with the robot and is rotated by ϕ with respect to the inertial frame (X_I, Y_I) . The linear velocity of a differential drive mobile robot in the robot frame can be expressed as the average of the linear velocities of its wheels (Eq. 1). The angular velocity of the robot depends on the difference of the linear speeds and the distance between the wheels L (Eq. 2). The robot kinematics in the inertial frame can be expressed with the translational v and angular ω velocities as shown in Eq. 3. The dynamic model is derived by using Lagrangian mechanics [9]. It is shown in [10] that the equations of motion can be expressed in terms of v and ω as shown in Eqs. 4 and 5.

$$v = \frac{v_R + v_L}{2} = \frac{R(\dot{\theta}_R + \dot{\theta}_L)}{2}$$
(1)

$$\omega = \frac{v_R - v_L}{2L} = \frac{R(\dot{\theta_R} - \dot{\theta_L})}{2L} \tag{2}$$

$$\begin{bmatrix} \dot{x}_a \\ \dot{y}_a \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} \cos(\phi) & 0 \\ \sin(\phi) & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix}$$
(3)

$$(m + \frac{2I_w}{R^2})\dot{v} - m_c d\omega^2 = \frac{1}{R}(\tau_R + \tau_L)$$
(4)

$$(I + \frac{2 * L^2}{R^2} I_w) \dot{\omega} + m_c d\omega v = \frac{L}{R} (\tau_r - \tau_L)$$
(5)

Where $m = m_c + 2m_w$ is the total mass of the robot and $I = I_c + m_c d^2 + 2m_w L^2 + 2I_m$ is the total equivalent inertia. Thus, the model of motion can be thought of as having the two motor torques (τ_r, τ_l) as the inputs, the translational and rotational speeds (v, ω) as the outputs, subject to the parameters describing the mass, inertia, and the geometry of the robot $(m_c, m_w, I_c, I_w, d, L, R)$. Besides, motion models are often executed by physics engines that enable a relaxation

of the kinematic constraints, e.g. by providing a simulation of friction, characterised by the friction coefficient *mu*.

3.4. DC motor model

The torques for the model of motion are produced by DC motors with permanent magnets. For an ideal DC motor, the motor torque is proportional to the armature current *i* by a constant factor of *K*. The back emf U_{EMF} , i.e. the voltage that the motor is generating while its rotor is spinning in the magnetic field of the stator, is proportional to the angular velocity $\dot{\theta}$ by the same factor (in SI units) *K*. The electric and the mechanical equations of an ideal DC motor is shown in Eqs. 6 and 7 respectively [11].

$$U = L_M \frac{\mathrm{d}i}{\mathrm{d}t} + R_M i + U_{EMF} = L_M \frac{\mathrm{d}i}{\mathrm{d}t} + R_M i + K\dot{\theta} \tag{6}$$

$$I_w \ddot{\theta} + b_M \dot{\theta} = \tau = Ki \tag{7}$$

Where U is the input voltage, L_M and R_M are inductance and resistance of the rotor windings, and b_M is the viscous friction. The DC motor model therefore has the voltage U as the input, the rotational velocity $\dot{\theta}$ as the output, and the following electrical and mechanical parameters (R_M , L_M , I_W , b_M , K).

3.5. Sensor model

The line sensor is able to measure the position of the line below the robot by using an array of reflectance sensors made up from an infrared light source (LED) and a phototransistor. Since each sensor is only able to measure the changes in the amount of reflected light, the sensors need to be calibrated in the beginning. This is achieved by rotating the robot around its centre of rotation so that each sensor crosses the line multiple times. Minimum $s_{i,min}$ and maximum $s_{i,max}$ sensor values are stored for each sensor *i*. The raw sensor value $s_{i,raw}$ is then scaled between 0 and 1000 to obtain the calibrated sensor value s_i as shown in Eq. 8. To estimate the position of the line, a weighted average is calculated as shown in Eq. 9. The angle of the line with respect to the robot's centre of rotation $line_{angle}$ is considered to be the error signal *e* for the control model.

$$s_i = \frac{(s_{i,raw} - s_{i,min}) \cdot 1000}{s_{i,max} - s_{i,min}} \tag{8}$$

$$line_{angle} = e = \operatorname{atan2}\left(\left(\frac{\sum_{i=0}^{N} s_i \cdot i \cdot 1000}{\sum_{i=0}^{N} s_i} - 3500\right)/100000, S\right)$$
(9)

3.6. Control model

For the line following task, translational and rotational velocities are controlled. The chosen control policies are shown in Eqs. 10 and 11, respectively. The translational velocity controller calculates the desired velocity as the absolute error *e* multiplied by a control parameter K_B and subtracted from a maximum velocity constant v_{max} . The rotational velocity is controlled by a standard PID controller. The control loop is executed every Δt . The control model takes the error *e* as the input signal, and generates translational *v* and rotational ω velocity signals to be executed by the robot's microcontroller. The control is subject to five parameters: $(v_{max}, K_B, K_P, K_I, K_D)$.

$$v = v_{max} - K_B \cdot |e| \tag{10}$$

$$\omega = K_P \cdot e + K_I \cdot \int_0^t e \, \mathrm{d}t + K_D \cdot \frac{\mathrm{d}e}{\mathrm{d}t} \tag{11}$$

3.7. Learning model

The role of the learning model is to enable simulation of what-if scenarios, which can then be used for different tasks such as optimisation of control parameters or synchronisation of the digital twin models with the physical asset. For the line following task, a gradient descent algorithm (Eq. 12) with parameters as shown in Eq. 13 is chosen for the learning model.

$$\boldsymbol{x}_{n+1} = \boldsymbol{x}_n - \gamma_n \nabla F(\boldsymbol{x}_n) \tag{12}$$

$$\boldsymbol{x} = (v_{max}, K_B, K_P, K_I, K_D) \tag{13}$$

The reason for choosing gradient descent as opposed to other machine learning techniques, such as genetic algorithms or neural networks, is that the problem space is mostly monotonic. For example, increasing the translational speed will almost always improve the performance of the line following task (measured by e.g. the lap time), up until the point that the task itself becomes unachievable, i.e. up to the point when the robot goes off track. Similarly, the lap time will almost certainly decrease if the tyre friction coefficient is reduced. Optimised values of parameters can, therefore, be found quickly with gradient descent, i.e. in a lower amount of completed laps and consequently in a lower amount of time. The learning model implementation is as follows. The robot completes the first lap with initial parameters. The parameters are varied one by one in the following laps to estimate the gradient. The robot then completes a lap with the varied parameters and estimates parameters for a new round of gradient measurements (Eq. 12). The procedure repeats for a fixed number of laps or until the parameters converge to their final values.

3.8. Model integration

The integration of the above models within the common model space is essential if the digital twin is to accurately represent the changing behaviour of its physical counterpart. Fig. 4 shows how the models are related in the case of a line following task.



Fig. 4: Model interactions.

The learning model takes care of synchronisation of the digital twin with the real robot through real-time communication. The two inputs to the learning model are the sensor data from the sensor model and the sensor data from the robot. These are compared, and if there is a discrepancy, gradient descent is used to re-synchronise them through an exploration of what-if scenarios. Control parameters as well as physics parameters of the motion model, such as the coefficient of friction μ , can be explored. The motion model converts the motor torques τ , defined by the control model and generated by the DC motor model, into translational ν and rotational ω velocities, which can be integrated by the physics engine to obtain the position of the robot in time, i.e. its trajectory. The sensor model then uses the current position of the robot and the track shape to generate simulated sensor data, closing the control loop.

The partial models are thus linked together into an integrated model that can be used to explore scenarios that no partial model can. The added value of this is that changes in any of the models or changes in the physical asset can be considered holistically, be it from the viewpoint of design or the viewpoint of real-time reflection. The cases that follow illustrate these examples.

4. Case studies

To illustrate the potential uses of the approach, two case studies are presented. The first one explores how the integrated models can be used to assess performance in the design stage using the learning model for control parameter optimisation. The second case study shows how changing conditions can be captured during operation and then reflected in the digital twin using the learning model.

4.1. Exploring the effect of design parameters on performance

It is difficult to assess the performance of complex products and systems in the design stage because of the interconnectedness of their parts and models. Typically, designs are modified using existing product knowledge. For example, an aircraft engine manufacturer almost never designs the product from scratch, but rather modifies the design parameters of an existing one incrementally, based on specific tests designed to improve part performance. Digital twins can assist in this process, but only if they are able to accurately represent the physical asset. Simulating the behaviour of the digital twin can then be used to explore what-if design scenarios. This case explores how the geometrical parameters of the design of the mobile robot influence its performance. As the geometry changes, so must the parameters of the control model, because, for example, a wider robot might need a stronger response to the error. The performance of modified designs can only be measured through concurrent optimisation of the control parameters. To do this, the learning model (Eq. 12) is employed. The robot performing the line following task is simulated. Each lap, the control parameters are optimised using gradient descent until they converge to their final values. Four scenarios, each with a different set of design parameters, are simulated. The results are shown in Fig. 5.



Fig. 5: a) The digital twin in the simulated environment. b) A typical gradient descent run. c) A distribution chart showing the effect of design parameters on performance.

In Fig. 5 a) the digital twin is shown in a simulated environment. A typical run of the simulation is shown in 5 b). The control parameters are changed using gradient descent and, in time, converge to their local optimums. Lap times from lap 100 forward, i.e. when the parameters converge and the lap times stabilize, are taken as the performance metric. Fig. 5 c) shows how the final lap time depends on the modified geometrical design parameters. The results show that the initial design would benefit from increasing the distance between the wheels L as well as from increasing the distance between the line sensors and the centre of rotation S. This is because the first change increases the overall torque for rotation which is the limiting factor (not the velocity of the rotation), and the second change increases the time the robot has to react to changes in track curvature, as placing the sensors further upfront can be thought of as looking further forward into the future from the perspective of the control model. In this case, the integrated model is used to link the design parameters, through simulation and learning, to the performance of the robot. An accurate simulation is not enough; software, control, and optimisation models also need to be linked and considered together in order to accurately asses the effect of design parameters on performance.

4.2. Digital twin alignment during operation

An important question is how to align the digital twin with the physical asset during operation. This is especially relevant in the context of through-life engineering services, as the service offering needs to consider the immediate state of the physical asset as well as predictions about future behaviour. If the digital twin simulation is running alongside the operation of the physical asset, it is possible to measure and communicate discrepancies in the behaviour of the two in real time. Aligning the two then becomes a question of finding the corresponding model parameter values and modifying them accordingly. The case demonstrates how this can be achieved through the learning model. Suppose that the robot reports that it is achieving slower times than those predicted by the simulation. The most probable cause in case of line following is that the robot's tyres have accumulated some dirt and are therefore slipping due to a decreased friction coefficient. The simulation model has to be modified to take that into account. Fig. 6 shows how the friction coefficient μ is modified until the simulated lap time is aligned with the measured one $t_{measured} = 15700$ ms.



Fig. 6: a) The robot in the real environment. b) Lap times during gradient descent. c) Friction coefficient μ during gradient descent.

The use of the integrated model, in this case, allows for the exploration of the parameter space in order to align the simulation parameters with the observed behaviour of the physical asset. This can subsequently be used for optimisation of the control model parameters as shown in the first case study.

5. Discussion and conclusion

As demonstrated by the cases, integrating the partial models allows for consideration of new scenarios that cannot be evaluated without the digital twin, i.e. by using any single model. The approach proposes a common model space that includes a learning model which guides the simulation when exploring what-if scenarios. The value of the integration is especially evident if the whole lifecycle of the product is considered. For example, modifying the shape of turbine blades or an aircraft engine might result in better performance of the engine, but cannot be executed in reality because it would pose additional dimensional, stress, and control constraints on other parts of the system throughout its lifetime. Integrating the models provides an additional insight with which undesired behaviours emerging from the interconnectedness of the models are avoided. The integration of models impacts the whole product lifecycle:

- Simulation experiments in the design stage can be used to simulate the behaviour of the product during operation and can help to reduce the time to market and improve the cost [12].
- The digital twin extends the simulation-based design, augmenting it with real data, improving the fidelity of the simulation.
- Maintenance aspects such as safety, reliability, and availability can be considered during the design stage [13].
- The integration of the models can also impact the product during operation. Real-time reflection can be achieved through learning, allowing for further optimisation during operation.

Future work will focus on reducing the effort needed for model integration, as currently, an expert is needed to parametrise the models and define their interconnections. We envision that each model should only be connected to the common model space, while a learning mechanism would be able to automatically discern how the model is connected to others through observation of the behaviour of the physical counterpart.

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