

1 **Modelling Crack Initiation in Bituminous Binders under a Rotational Shear**  
2 **Fatigue Load**

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24 **Abstract:**

25 This study aims to model fatigue crack initiation in bituminous binders. An energy-  
26 based crack initiation criterion is developed for bitumen under a rotational shear fatigue  
27 load. Based on a damage mechanics analysis of fatigue cracking process, the crack  
28 initiation is defined and local energy redistribution around crack tips due to ‘factory-  
29 roof’ cracking is quantified. A quantitative energy criterion is proposed for the fatigue  
30 crack initiation in the bitumen using viscoelastic Griffith’s theory. The crack initiation  
31 criterion is validated through comparing the predicted and measured surface energy of  
32 the bitumen. The results show that bitumen fatigue cracking under the rotational shear  
33 fatigue load can be divided into two stages: the edge flow damage and the ‘factory-roof’  
34 cracking. The crack initiation is dependent of the shear modulus and surface energy of  
35 bituminous binders, critical crack size, and loading amplitude. The energy-based crack  
36 initiation criterion along with the DSR fatigue tests can be potentially used to determine  
37 the material surface energy.

38

39 **Key words:**

40 Dynamic shear rheometer (DSR); time sweep; energy-based crack initiation criterion;  
41 viscoelastic Griffith’s theory; surface energy

42 **1. Introduction**

43 Fatigue cracking in bituminous binders is one of the most major distresses resulting in  
44 the degradation of asphalt pavements. To optimise the material selection and  
45 consequently improve the pavement performance, research efforts have been made to  
46 characterise and quantify the fatigue performance of the bituminous binders. Based on  
47 Dynamic Shear Rheometer (DSR) tests, a number of fatigue parameters were proposed  
48 to evaluate the fatigue resistance of the binders, including SHRP fatigue factor  
49  $|G^*| \cdot \sin \delta$ , dissipated energy ratio (DER) [1-3], ratio of dissipated energy change  
50 (RDEC) [4-7] and fatigue law based on the viscoelastic continuum damage (VECD)  
51 mechanics [8-12]. Although these fatigue parameters have been employed to  
52 characterise the binders' fatigue behaviour, the fatigue cracking mechanism of the  
53 bituminous binders are not yet well understood, particularly on how the crack was  
54 initiated in the binders under a rotational shear fatigue load. Further studies from the  
55 fundamental mechanical perspective are very needed to understand the initiation of the  
56 fatigue damage for the bituminous binders under the shear fatigue load.

57

58 Three key questions have been raised by the authors as follows: (a) what the damage  
59 looks like in the bituminous binder under the shear fatigue load; (b) when the damage  
60 is initiated; and (c) how the damage evolves. The question (a) on what the damage is  
61 has been tentatively addressed by one of the authors' previous studies [13]. It was found  
62 that the fatigue damage in the bituminous binders under the rotational shear fatigue  
63 loads physically exhibited as an edge cracking, which was a circumferential crack that  
64 initiated at the edge of the sample and propagated toward the centre. The cracked  
65 surfaces in the bitumen samples after the rotational shear fatigue load are demonstrated  
66 as roughed 'factory-roof' like surfaces, which were also observed and characterised by  
67 the existing studies [13, 14]. Using damage mechanics, a DSR-cracking (DSR-C)  
68 model was developed to predict the crack length in the bitumen sample under the  
69 rotational shear fatigue load [13]. In the DSR-C model, the crack length can be  
70 calculated using the shear moduli and phase angles of the bitumen at the undamaged  
71 and damaged conditions, respectively. The proposed DSR-C model is proven to be  
72 capable of accurately predicting the crack length for the unaged and aged virgin or  
73 modified bituminous binders in the DSR fatigue testing (e.g., time sweep test) at

74 different temperatures, loading frequencies and cycles. Based on the analysis of fatigue  
75 cracking process with the DSR-C model, the question (c) on how the damage crack  
76 evolves has also been investigated using pseudo J-integral Paris' law in a separate work  
77 [15]. It was found the model coefficients ( $A$  and  $n$ ) from the Paris' law are material  
78 fundamental properties, which are function of temperature but independent of loading  
79 frequency or loading amplitude. A similar finding was reported for asphalt mixtures [16,  
80 17].

81

82 Following the understanding of the first and third questions, this study is focused on  
83 addressing the second question that is when the damage is initiated. Crack initiation  
84 refers to the onset of edge cracking, which is critical to estimate the crack propagation  
85 and fatigue life in a material under a cyclic load. Based on the microscopic observations,  
86 a number of studies have been carried out for metals and polymers to elucidate the  
87 mechanisms of crack initiation. It was found that the crack initiation of metallic  
88 materials is mainly caused by dislocations [18]. According to the dislocation movement  
89 and plastic deformation, dislocation-based models were proposed to characterise the  
90 crack initiation in the metallic materials [19-21]. Polymeric materials are composed of  
91 molecular chains, which present a viscoelastic fracture. Its crack initiation results from  
92 the chain scission or chain disentanglement. The plastic zone-based model [22, 23] or  
93 craze zone-based models [24-26] were developed to investigate the crack initiation of  
94 polymeric materials. Furthermore, the local crack growth models were proposed for  
95 viscoelastic materials, which provide the analytical solutions to crack initiation  
96 problems. Knauss [27] developed a crack speed criterion for viscoelastic materials by  
97 defining a critical crack speed transition using continuum mechanics. Schapery [28]  
98 defined a crack tip model with a failure zone behind the tip and developed a local energy  
99 criterion for viscoelastic materials to predict the time of crack initiation. While these  
100 models can be used to analyse the process of crack initiation, their application in  
101 engineering practice, e.g., asphalt mixtures, is limited due to complexity in the  
102 determination of the model inputs and the validations of the model predictions.

103

104 Griffith's energy criterion presented a more general manner to predict the crack  
105 initiation, which defined the energy condition at which the crack is initiated in brittle  
106 materials [29]. Recently, Griffith's crack initiation theory has been extended to develop

107 a fracture criterion for the viscoelastic materials [30-32]. The viscoelastic Griffith's  
108 criterion states that the potential energy of a viscoelastic body remains at equilibrium  
109 when cracks initiate. Dubois and Petit [33] used the viscoelastic Griffith's criterion to  
110 investigate the effects of viscoelastic characteristics on the creep crack initiation by  
111 developing a path-independent integral  $G\theta_v$ . Luo, Luo and Lytton [34] and Zhang, Luo,  
112 Luo and Lytton [35] applied viscoelastic Griffith's criterion to develop the crack  
113 initiation criteria for the asphalt mixtures in tension and compression, respectively. The  
114 tensile crack initiation criterion results in a simple way of measuring the endurance  
115 limit and the compressive one leads to a simple way of predicting compressive strength  
116 [36]. These researches have demonstrated that the viscoelastic Griffith's criterion is  
117 capable of characterising the crack initiation of the viscoelastic materials. In this study,  
118 the viscoelastic Griffith's criterion will be employed to investigate the crack initiation  
119 for the bituminous binders under the rotational shear fatigue loads.

120

121 The objective of this study is to develop an energy-based crack initiation criterion for  
122 the bituminous binders subjected to a rotational shear fatigue load. Laboratory tests  
123 including contact angle measurements and DSR tests were firstly conducted, which are  
124 followed by the viscoelastic characterisation of bituminous binders using the  
125 pseudostrain concept. Based on the analysis of the fatigue cracking process, crack  
126 initiation was defined and energy redistribution due to 'factory-roof' cracking was  
127 illustrated. Subsequently, an energy criterion for crack initiation was proposed and  
128 validated for bituminous binders under the rotational shear fatigue load using the  
129 viscoelastic Griffith's fracture mechanics. Finally, a potential application of the crack  
130 initiation criterion was presented, which is to determine the surface energy of the  
131 bituminous binders.

132

## 133 **2. Materials and Laboratory Tests**

### 134 *2.1 Materials*

135 Four types of bituminous binders were applied in this study, including 40/60 bitumen,  
136 X-70 bitumen, base X-70 bitumen and 100/150 bitumen. The 40/60 bitumen, base X-  
137 70 bitumen and 100/150 bitumen are unmodified binders with different penetration  
138 grades and the X-70 bitumen is a polymer-modified binder with 45/80 penetration grade.

139 These binders were all aged using standard methods including Rolling Thin Film Oven  
140 (RTFO) for short-term ageing [37] and Pressure Ageing Vessel (PAV) for long-term  
141 ageing [38]. Laboratory tests including contact angle measurements and Dynamic  
142 Shear Rheometer (DSR) tests were conducted on eight samples, i.e., unaged 40/60  
143 bitumen, RTFO aged 40/60 bitumen, PAV aged 40/60 bitumen, unaged X-70 bitumen,  
144 RTFO aged X-70 bitumen, PAV aged X-70 bitumen, PAV aged base X-70 bitumen and  
145 PAV aged 100/150 bitumen.

146

## 147 2.2 Contact angle measurements

148 In order to determine surface energy of the bituminous binders, contact angle  
149 measurements were performed using Sessile Drop method. This method provides a  
150 standard test for surface properties of solids and liquids, which has been widely applied  
151 for the study of the surface energy of bituminous binders [39-41]. In the Sessile Drop  
152 method, contact angles between probe liquids and bitumen sample are measured  
153 directly by capturing an image of a liquid drop on the sample surface.

154

155 The selection of probe liquids is critical for the contact angle measurements using the  
156 Sessile Drop method. The used liquids first need to be pure, homogenous and  
157 immiscible with the bituminous binders. Their surface energy components must be  
158 known to determine the surface energy of bituminous binders. In this study, distilled  
159 water, ethylene glycol and formamide were selected from the literature [41] as probe  
160 liquids for the contact angle experiments. Surface energy characteristics of the three  
161 probe liquids are summarized in **Table 1**. The magnitudes of their surface energy  
162 components are very distinctive, which is advantageous to accurately calculate the  
163 surface energy of bituminous binders later. The bitumen samples used for the contact  
164 angle measurements were prepared using glass slides with 25 mm × 75 mm × 1 mm in  
165 size, as shown in **Fig. 1(a)**. A small quantity of the heated bitumen was poured on a  
166 clean glass slide to form a bitumen substrate with a flat and smooth surface. The  
167 bitumen sample was cooled to room temperature before the contact angle measurements.  
168 Note that three coated slides for each bitumen are needed for three probe liquids.

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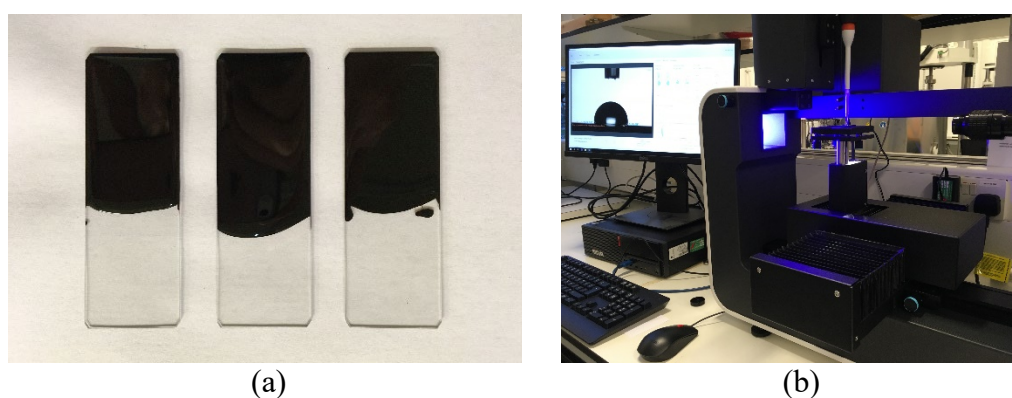
171 **Table 1** Surface energy components of probe liquids (mJ/m<sup>2</sup>).

Liquids	$\Gamma$	$\Gamma^{LW}$	$\Gamma^+$	$\Gamma^-$
Distilled Water (W)	72.80	21.80	25.50	25.50
Ethylene Glycol (EG)	48.00	29.00	1.92	47.00
Formamide (F)	58.00	39.00	2.28	39.60

172

173 Once the probe liquids were selected and the bitumen samples were prepared, the  
 174 contact angle measurements were conducted using an optical tensiometer from Biolin  
 175 Scientific shown in **Fig. 1(b)**. The glass slide coated with the bitumen was first placed  
 176 between the light source and the camera of the test device. A small drop of the probe  
 177 liquid was dispensed on the surface of the coated slide. An image of the drop was  
 178 captured by the camera and then the captured image was analysed automatically by  
 179 image processing software of the test device to obtain the angle between the baseline  
 180 and the edge of the drop. Five measurements were performed for each probe liquid.  
 181 Their average value was determined as the contact angle between the probe liquid and  
 182 the bitumen. **Table 2** shows the results of the contact angle measurements for all tested  
 183 bituminous binders. Based on the measured contact angles, the surface energy of the  
 184 bituminous binders can be calculated using the Good-van Oss-Chaudhury theory [42],  
 185 which will be discussed in detail later.

186



187

188

189 **Fig. 1.** Bitumen samples and test instrument used for contact angle measurements. (a) Glass  
 190 slide samples. (b) Tensiometer from Biolin Scientific.

191

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194

195 **Table 2** Contact angles between tested bitumen and liquid probes (°).

Bitumens	Distilled Water (W)	Ethylene glycol (EG)	Formamide (F)
Unaged 40/60	105.68±0.62	82.43±0.29	90.17±0.73
RTFO 40/60	103.80±0.18	81.98±0.80	89.44±0.64
PAV 40/60	107.09±0.37	81.77±0.93	91.45±0.65
Unaged X-70	104.10±0.39	79.59±0.45	89.04±0.58
RTFO X-70	101.57±1.42	80.87±0.49	88.85±1.66
PAV X-70	103.37±0.68	81.86±0.30	92.05±0.82
PAV base X-70	103.80±0.14	80.74±0.67	88.16±0.82
PAV 100/150	103.21±0.35	80.82±0.63	88.90±0.45

196

197 *2.3 Dynamic Shear Rheometer (DSR) tests*

198 To determine the linear viscoelastic properties and fatigue cracking behaviours of the  
 199 bituminous binders, viscoelastic frequency sweep tests and time sweep fatigue tests  
 200 were performed on all binders in this study. Bitumen samples were prepared for the  
 201 tests using silicone moulds and all the tests were conducted using a Kinexus DSR from  
 202 Malvern Panalytical with an 8 mm parallel plates with 2 mm gap as the sample height.  
 203 Before the bitumen samples were subjected to the testing load, they were heated to the  
 204 target temperature and then remained 5 min to allow temperature equilibrium and  
 205 material stability. Two replicates were tested for each testing condition and one more  
 206 replicate was required when their deviation was more than 10%.

207

208 To eliminate the effects of the initial stiffness of the bituminous binders, a fixed iso-  
 209 stiffness level at which  $|G^*| \cdot \sin \delta$  equals to 6.5 MPa at 10 Hz were used to determine  
 210 testing temperatures. Frequency sweep tests at multiple temperatures from 10 °C to 40 °C  
 211 were conducted to obtain the corresponding temperature for each tested sample. The  
 212 temperatures resulted from the iso-stiffness condition range from approximately 15 °C  
 213 to 35 °C, as shown in **Table 3**. It should be noted that the iso-stiffness temperature is  
 214 different for each tested binder in the following time sweep fatigue test. The shear  
 215 modulus ( $|G_0^*|$ ) and the phase angle ( $\delta_0$ ) of the undamaged binder can be obtained from  
 216 the frequency sweep test conducted at the iso-stiffness temperature.

217

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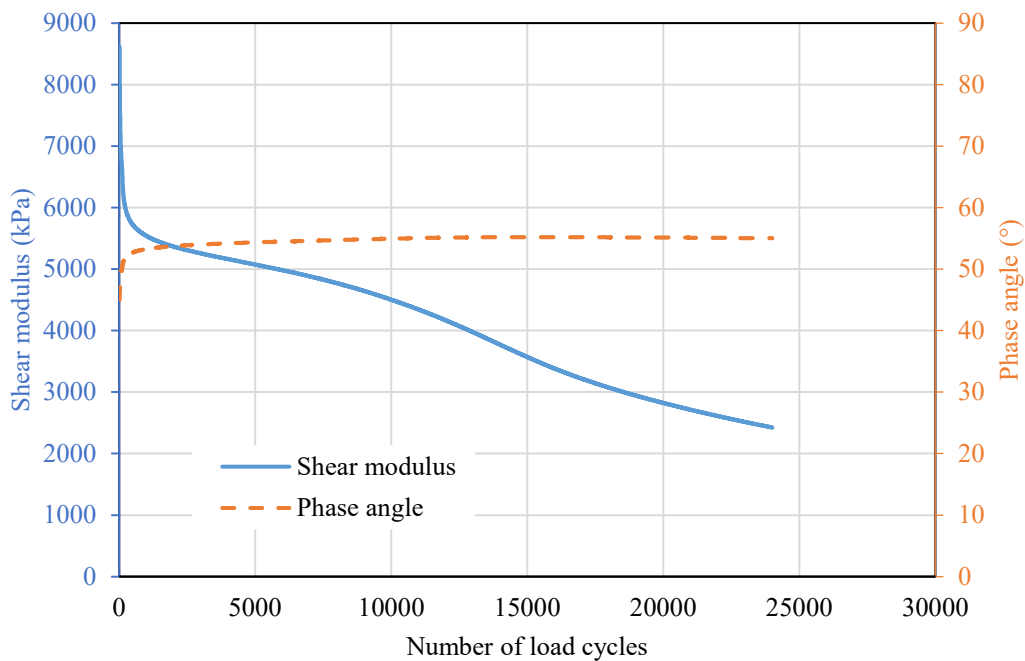


219 **Table 3** Iso-stiffness temperature for all the tested bitumens.

Bitumens	Iso-stiffness temperature (°C)
Unaged 40/60	24
RTFO 40/60	27
PAV 40/60	34
Unaged X-70	21
RTFO X-70	24
PAV X-70	28
PAV base X-70	28
PAV 100/150	26

220

221 To study the fatigue cracking behaviours of the binders, time sweep fatigue tests were  
 222 performed at the iso-stiffness temperature and the frequency of 10 Hz under a 5% shear  
 223 strain level corresponding to the rotation amplitudes of 0.025 rad. The strain level (5%)  
 224 have been employed for time sweep fatigue tests in the literature [13, 43] to investigate  
 225 the fatigue damage response of bituminous binders. **Fig. 2** illustrates a typical result of  
 226 the time sweep test for a PAV aged X-70 bitumen sample.



227

228 **Fig. 2.** Time sweep test results for a PAV aged X-70 bitumen sample at 28 °C.229 **3. Viscoelastic characterisation of bituminous binders with pseudostrain**

230 When a viscoelastic material is subjected to a rotational shear fatigue load, its stress-

231 strain curve forms a hysteresis loop in each load cycle. The area inside the loop is the  
 232 dissipated strain energy (DSE) density and the area between the stress-strain curve and  
 233 the horizontal (strain) axis is defined as the recoverable strain energy (RSE) density.  
 234 The DSE for the materials in the linear viscoelastic undamaged state is the energy  
 235 dissipated due to the viscoelastic relaxation of the material. In order to remove the  
 236 viscous effect, pseudostrain was proposed by Schapery [44] below

$$237 \quad \gamma^R(t) = \frac{1}{G_R} \int_{0^-}^t G(t-s) \frac{d\gamma(s)}{ds} ds \quad (1)$$

238 where  $\gamma^R$  is the pseudostrain;  $G_R$  is the shear reference modulus;  $G(t)$  is the shear  
 239 relaxation modulus;  $s$  is the time before the current time  $t$ ; and  $\gamma(s)$  is the measured  
 240 total strain. Pseudostrain has been used in the damage characterization of asphalt  
 241 mixtures [16, 35, 45-47]. After introducing pseudostrain, the stress-pseudostrain curve  
 242 at the linear viscoelastic state becomes a straight line, which indicates that the viscous  
 243 effect is eliminated and the stress-pseudostrain shows like an elastic behaviour. The  
 244 area below the stress-pseudostrain curve is the recoverable pseudostrain energy (RPSE)  
 245 density. RPSE is the density of the energy stored in the material during loading and can  
 246 be released during unloading, which is used for crack initiation analysis of the  
 247 bituminous binders under rotational a shear fatigue load in this study.

248

249 In a strain-controlled rotational shear fatigue test (e.g., time sweep test) of a cylindrical  
 250 bitumen sample, the applied shear strain is

$$251 \quad \gamma(t) = \gamma_0 \sin(\omega t) \quad (2)$$

252 where  $\gamma_0$  is the strain amplitude and  $\omega$  is the loading frequency. The strain amplitude  
 253 at a given radial position of the sample is expressed as

$$254 \quad \gamma_0(r) = \frac{\theta_0}{h} \cdot r \quad \text{with } 0 \leq r \leq r_0 \quad (3)$$

255 where  $\theta_0$  is the controlled amplitude of rotational angle,  $r_0$  and  $h$  are the original  
 256 radius and the height of the cylindrical sample, respectively. Thus, in the strain-  
 257 controlled cyclic test, the pseudostrain  $\gamma^R(t)$  in **Eq. (1)** at the linear viscoelastic  
 258 condition can be determined as

$$259 \quad \gamma^R(t) = \frac{\gamma_0}{G_R} |G_0^*| \sin(\omega t + \delta_0) \quad (4)$$

260 where  $\delta_0$  is the phase angle of the material in the undamaged condition. The  
 261 corresponding pseudostrain amplitude is written as

$$262 \quad \gamma_0^R = \frac{\gamma_0}{G_R} |G_0^*| \quad (5)$$

263

264 Based on the definition of RPSE density that is the area between the stress-pseudostrain  
 265 curve and the horizontal (pseudostrain) axis, the value of RPSE density at a given radial  
 266 position of the sample at the linear viscoelastic condition can be calculated as

$$267 \quad RPSE(r) = \frac{1}{2} \tau_0 \cdot \gamma_0^R \quad (6)$$

268 where  $\tau_0$  is the amplitude of the measured shear stress corresponding to the controlled  
 269 shear strain  $\gamma(t)$  shown in **Eq. (2)**. According to the linear viscoelastic stress-strain law,  
 270 the shear stress amplitude of the sample can be obtained by

$$271 \quad \tau_0 = |G_0^*| \gamma_0 = G_R \gamma_0^R \quad (7)$$

272

273 Substituting **Eqs. (5)** and **(7)** into **Eq. (6)** and considering **Eq. (3)**, RPSE density at a  
 274 given radial position of the cylindrical bitumen sample can be given as

$$275 \quad RPSE(r) = \frac{1}{2} \frac{(|G_0^*| \theta_0)^2}{G_R h^2} \cdot r^2 \quad (8)$$

276 From **Eq. (8)**, one can find that the RPSE density is non-uniform along the radial  
 277 direction of the cylindrical bitumen sample but a quadratic function of the radius  $r$  for  
 278 the sample. The maximum RPSE occurs at the location where the radius is the  
 279 maximum which is the edge of the sample. This again proves that the crack tends to be  
 280 initiated from the edge of the sample.

281

## 282 **4. Crack initiation modelling of bituminous binders**

### 283 *4.1 Crack initiation of bituminous binders under a rotational shear fatigue load*

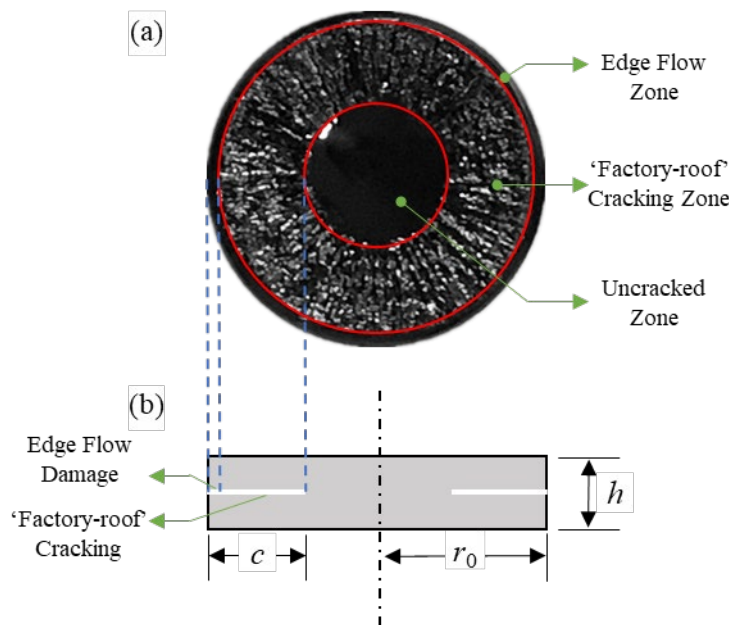
284 Under a rotational shear fatigue load, the crack in a cylindrical bitumen sample grows  
 285 as a circumferential crack that initiates at the edge of the sample and propagates toward  
 286 the centre [13, 14]. A typical crack surface in the bitumen samples is shown in **Fig. 3(a)**.  
 287 It can be seen that the crack surface exhibits three different zones based on the

288 morphology of the cracking surfaces:

- 289 1) Edge flow zone. The damage at the edge flow zone is known as ‘edge flow’ that  
 290 occurs at the edge of the sample and at the initial stage of fatigue cracking. The  
 291 edge flow is caused by edge instability with complex mechanisms.
- 292 2) ‘Factory-roof’ cracking zone. At the ‘factory-roof’ cracking zone, the crack is a  
 293 single ring crack that steadily propagates towards the centre of the sample,  
 294 resulting in a rough and ‘factory-roof’ like cracking surface under the shear-  
 295 mode loading.
- 296 3) Uncracked zone. The uncracked zone is the zone in the sample centre which is  
 297 not damaged during the rotational shear fatigue load. However, it is generated  
 298 and captured by pulling the top and bottom sample apart after the fatigue test at  
 299 a relatively low temperature (i.e., 3 °C). The low-temperature is used to  
 300 conserve the configuration of the crack surfaces including flow zone crack and  
 301 the ‘factory-roof’ crack generated during the fatigue test.

302 Thus, bituminous binders’ fatigue cracking under the rotational shear fatigue load can  
 303 be divided into two stages: edge flow damage and ‘factory-roof’ cracking. **Fig. 3(b)**  
 304 shows the schematic side view of a cylindrical sample with a circumferential crack.

305



306

307 **Fig. 3.** Fatigue cracking in bituminous binders under a rotational shear fatigue load. (a)  
 308 Cracking morphology. (b) Cracking model.  $c$  is the crack length,  $r_0$  is the original radius of the  
 309 sample, and  $h$  is the height of the sample.

310

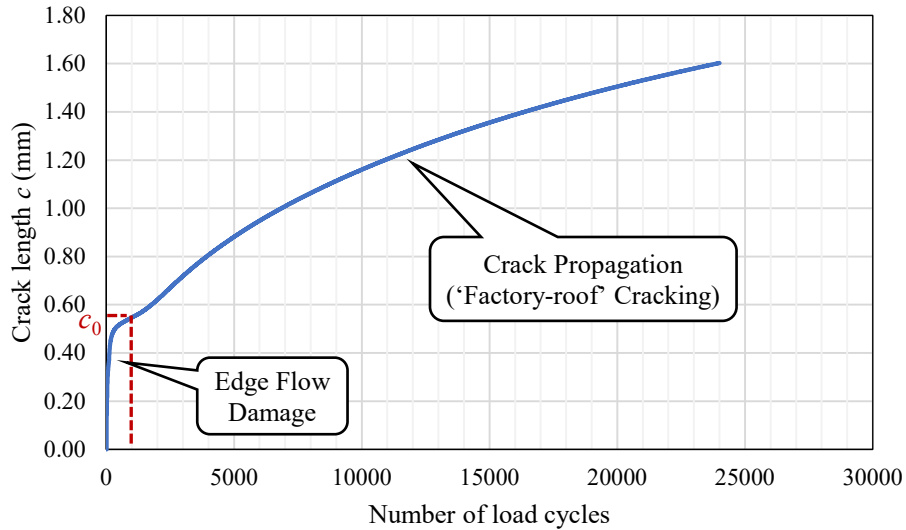
311 Based on the DSR-C model shown in **Eq. (9)** [13], the crack growth in bituminous  
 312 binders under a rotational shear fatigue load can be predicted using the moduli and  
 313 phase angles of the binders in the undamaged and damaged conditions.

$$314 \quad c = \left[ 1 - \left( \frac{\left( |G_N^*| / \sin(\delta_N) \right)^{\frac{1}{4}}}{\left( |G_0^*| / \sin(\delta_0) \right)} \right) \right] r_0 \quad (9)$$

315 where  $c$  is the crack length in the cylindrical bitumen sample;  $|G_0^*|$  and  $\delta_0$  are the  
 316 shear modulus and the phase angle in the undamaged condition, respectively;  $|G_N^*|$  and  
 317  $\delta_N$  are the shear modulus and the phase angle at the Nth load cycle in the damaged  
 318 condition, respectively; and  $r_0$  is the original radius of the bitumen sample.

319

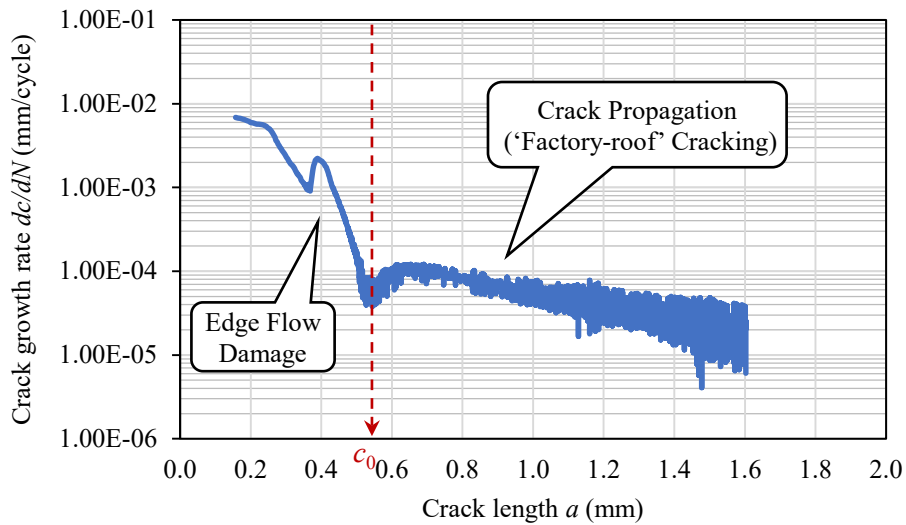
320 **Fig. 4** shows a typical crack growth curve with the number of load cycles, determined  
 321 using **Eq. (9)**. The corresponding crack growth rate with the crack length is plotted in  
 322 **Fig. 5**. It is very clear from **Figs. 4** and **5** that, before the 1000<sup>th</sup> load cycle, the crack  
 323 growth rate is decreasing but still much higher than that after the 1000<sup>th</sup> load cycle.  
 324 When the load cycle goes beyond the 1000<sup>th</sup> load cycle or the crack length is greater  
 325 than 0.55 mm, the crack presented a regular growth in a relatively lower growth rate  
 326 kept decreasing. This crack growth stage is referred to as the crack propagation  
 327 ('factory-roof' cracking), which has been studied in the previous work [15]. Before the  
 328 crack propagation stage, the initial portion of fatigue cracking is believed as the edge  
 329 flow damage.



330

331 **Fig. 4.** Crack growth with the number of load cycles for a cylindrical bitumen sample subjected  
 332 to a rotational shear fatigue load.

333



334

335 **Fig. 5.** Crack growth rate with the crack length for a cylindrical bitumen sample subjected to a  
 336 rotational shear fatigue load.

337

338 Edge flow damage is an unsteady cracking stage with complex cracking mechanisms.  
 339 It can be seen from **Figs. 4** and **5** that the crack length has a sharp increase and the crack  
 340 growth rate shows a significant fluctuation. A sequence of processes occurring in this  
 341 stage are described as follows:

342 (1) Molecular rearrangements. When the bituminous binders are subjected to the

343 external rotational shear fatigue load, their physical properties such as shear  
344 modulus and phase angle changes rapidly (see **Fig. 2**). This phenomenon is  
345 caused by the rearrangement of molecular chains in binders [48, 49], which is  
346 referred to as the thixotropic effect [50].

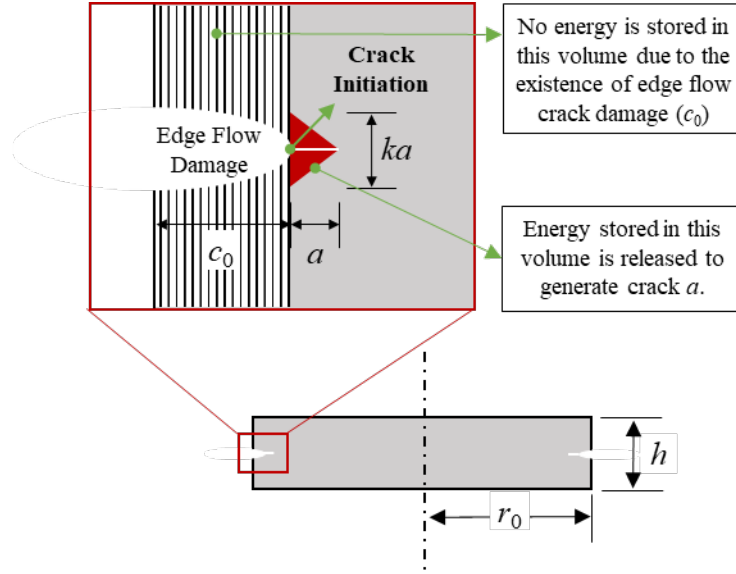
347 (2) Microcrack nucleation. After the molecular rearrangements, the microcracks are  
348 initiated at the edge of cylindrical bitumen sample either by the breakage of  
349 atomic bonds or from the pre-existing vacancies and flaws in the materials.  
350 These microcracks then occur nucleation and form small macrocracks [51].

351 (3) Circumferential crack formation. With the increasing load cycles, small  
352 macrocracks are eventually interconnected to generate a single circumferential  
353 crack at the edge of sample. The initial circumferential crack length  $c_0$  is defined  
354 as the edge flow damage size which separates the crack growth rate into  
355 completely two different stages. As shown in **Fig. 5**, the crack growth rate is  
356 much higher but drops much quicker in the edge flow damage stage than that in  
357 ‘factory-roof’ crack propagation stage.

358

359 Due to the edge instability and geometry dependence of samples, it becomes very  
360 challenging to develop a rigorous model or theory to characterise the molecular  
361 rearrangements or the microcrack nucleation during the edge flow damage. However,  
362 after the circumferential crack formation, the ‘factory-roof’ crack propagates at a  
363 relatively stable and more consistent manner as shown in **Figs. 4** and **5**. Thus in this  
364 study the “crack initiation” is defined as the moment at which the circumferential crack  
365 formation is complete, which is also the end of the edge flow damage and the start of  
366 the ‘factory-roof’ crack propagation. **Fig. 6** is proposed to model the crack initiation, at  
367 which the edge flow damage stage has been finished, thus the crack size has reached a  
368 length of  $c_0$ .

369



370

371 **Fig. 6.** Edge flow damage and crack initiation of ‘factory-roof’ cracking zone.  $c_0$  is the edge  
 372 flow damage size.  $a$  is the ‘factory-roof’ crack length for crack initiation.

373

374 *4.2 Energy redistribution due to ‘factory-roof’ cracking*

375 Cracking is a process of energy redistribution in material according to Griffith theory.  
 376 For bituminous binder, a viscoelastic material, pseudostrain is applied to replace the  
 377 physical strain so that a viscoelastic problem could be simplified as an elastic one.

378

379 According to the cracking pattern of bitumen binders under the rotational shear fatigue  
 380 load shown in **Figs. 3** and **6**, the total RPSE in bitumen sample with the edge flow  
 381 damage is redistributed due to the appearance of the circumferential ‘factory-roof’ crack  
 382 as follows: (1) RPSE is released from the intact material surrounding the ‘factory-roof’  
 383 crack and (2) surface energy is stored on the surfaces of the ‘factory-roof’ crack. Thus,  
 384 the energy redistribution resulting from the ‘factory-roof’ cracking is formulated as

385

386 
$$\varepsilon = \varepsilon_1 - \varepsilon_2 + \varepsilon_3 \tag{10}$$

387

388 where  $\varepsilon$  is the total pseudostrain energy after the energy redistribution in the  
 389 cylindrical bitumen sample;  $\varepsilon_1$  is the total RPSE in the sample with the edge flow  
 390 damage before the circumferential ‘factory-roof’ crack initiates;  $\varepsilon_2$  is the released  
 391 RPSE surrounding the circumferential ‘factory-roof’ crack; and  $\varepsilon_3$  is the total surface



392 energy stored on the ‘factory-roof’ crack surfaces.

393

394 To analyse the energy redistribution in bitumen sample, the three energy elements ( $\varepsilon_1$ ,  
395  $\varepsilon_2$  and  $\varepsilon_3$ ) were further derived based on the cracking model of bitumen sample shown  
396 in **Fig. 6**. The total RPSE in the sample with the edge flow damage before the ‘factory-  
397 roof’ crack initiates (i.e.,  $\varepsilon_1$ ) is defined as

398

$$399 \quad \varepsilon_1 = \iiint_{V_{total}} RPSE(r) dV = \int_0^{r_0-c_0} RPSE(r) \cdot (2\pi rh) dr \quad (11)$$

400

401 where  $V_{total}$  is the volume of the sample except the edge flow damage zone (the  
402 vertically shaded part shown in **Fig. 6**). Note that the edge flow damage zone was  
403 removed from the calculation of the  $V_{total}$  as no RPSE was stored in the zone due to the  
404 edge flow damage in the sample.

405

406 The released RPSE surrounding the ‘factory-roof’ crack (i.e.,  $\varepsilon_2$ ) is calculated as

407

$$408 \quad \varepsilon_2 = \iiint_{V_{released}} RPSE(r) dV \approx RPSE(r) \cdot V_{released} \quad (12)$$

409

410 where  $V_{released}$  is the volume of the sample that releases RPSE to generate the ‘factory-  
411 roof’ crack. As shown in **Fig. 6**, when a very small ‘factory-roof’ crack  $a$  ( $a \ll c_0$ ) is  
412 initiated, only the energy stored in the adjacent volume of the crack is released. It was  
413 assumed that this volume is a triangular ring region and highlighted as the red area as  
414 shown in **Fig. 6**. A similar assumption was used in the crack initiation modelling in  
415 rubber [52] and asphalt mixtures [34]. The horizontal height of the triangular region is  
416 the crack length  $a$  and the vertical height of the triangular region is  $ka$  according to the  
417 previous study [52]. Thus, the volume for releasing RPSE can be expressed as

418

$$419 \quad V_{released} = \frac{1}{2} \cdot ka \cdot a \cdot 2\pi(r_0 - c_0) \quad (13)$$

420

421 The total surface energy stored on the ‘factory-roof’ crack surfaces  $\varepsilon_3$  is determined as

422

$$423 \quad \varepsilon_3 = \Gamma \cdot 2A = \Gamma \cdot 2 \left[ \pi (r_0 - c_0)^2 - \pi (r_0 - c_0 - a)^2 \right] \quad (14)$$

424

425 where  $\Gamma$  is the total surface energy of the bituminous binders that is the energy (or  
426 work) done to create a unit area of the new surface;  $A$  is the surface area of a  
427 circumferential crack; and the coefficient 2 refers to that every crack has two surfaces.

428

#### 429 *4.3 Crack initiation criterion*

430 The Griffith crack initiation criterion states that the potential energy of an elastic body  
431 remains unchanged when cracks initiate or the existing cracks grow. Thus, the crack  
432 initiation follows an energy balance condition, where the rate of total energy to crack  
433 size equals to zero. Based on this criterion, the critical condition at the moment when  
434 the crack is initiated can be expressed as

435

$$436 \quad \frac{\partial \varepsilon}{\partial c} = 0 \quad (15)$$

437

438 where  $\varepsilon$  is the total pseudostrain energy after the energy redistribution in the sample  
439 shown in **Eq. (10)**. Then, substituting **Eq. (10)** with **Eqs. (11), (12), (13)**, and **(14)** into  
440 **Eq. (15)** and performing differentiation give

441

$$442 \quad RPSE(r) \cdot k \cdot a \cdot (r_0 - c_0) - 2\Gamma \cdot (r_0 - c_0 - a) = 0 \quad (16)$$

443

444 **Eq. (16)** is the energy-based crack initiation criterion for bituminous binders under the  
445 rotational shear fatigue load. It is noted that, in **Eq. (16)**, the recoverable pseudostrain  
446 energy density  $RPSE(r)$  is related to the shear modulus of materials and the loading  
447 amplitude (see **Eq. (8)**). Thus, the energy-based crack initiation criterion (**Eq. (16)**)  
448 shows that the crack initiation is dependent of the shear modulus and surface energy of  
449 bituminous binders, critical crack size, and loading amplitude.

450

451 **5. Results and discussions**452 *5.1 Surface energy of bituminous binders*

453 Surface energy of a material is defined as the energy required to create a unit area of  
 454 the new surface in the material. According to the acid-base theory [53], the total surface  
 455 energy of a material is mainly comprised of a nonpolar Lifshitz-van der Waals (LW)  
 456 component and an acid-base (AB) component, as expressed in **Eq. (17)**.

457

$$458 \quad \Gamma = \Gamma^{\text{LW}} + \Gamma^{\text{AB}} \quad (17)$$

459

460 where  $\Gamma$  is the total surface energy,  $\Gamma^{\text{LW}}$  is the Lifshitz-van der Waals (LW) component,  
 461 and  $\Gamma^{\text{AB}}$  is the acid-base component.

462

463 To predict the acid-base interaction, the acid-base term in **Eq. (17)** was divided into a  
 464 Lewis acid component and a Lewis base component, as described in **Eq. (18)**.

465

$$466 \quad \Gamma^{\text{AB}} = 2\sqrt{\Gamma^+ \Gamma^-} \quad (18)$$

467

468 where  $\Gamma^+$  is the Lewis acid surface parameter and  $\Gamma^-$  is the Lewis base surface  
 469 parameter.

470

471 The surface energy components of the bituminous binders were determined indirectly  
 472 using contact angle measurements in this study. **Table 2** shows the contact angles  
 473 between bitumen and three liquid probes. Based on the contact angle values, the surface  
 474 energy components ( $\Gamma_S^{\text{LW}}$ ,  $\Gamma_S^+$  and  $\Gamma_S^-$ ) of bitumen can be obtained by the Young-Dupre  
 475 equation shown in **Eq. (19)** [54].

476

$$477 \quad (1 + \cos \theta) \Gamma_L = 2 \left( \sqrt{\Gamma_S^{\text{LW}} \Gamma_L^{\text{LW}}} + \sqrt{\Gamma_S^+ \Gamma_L^-} + \sqrt{\Gamma_S^- \Gamma_L^+} \right) \quad (19)$$

478

479 where  $\theta$  is the contact angle between bitumen (S) and probe liquid (L).

480

481 Once the surface energy components ( $\Gamma_S^{\text{LW}}$ ,  $\Gamma_S^+$  and  $\Gamma_S^-$ ) of bitumen were obtained, the

482 total surface energy of bitumen was determined using **Eqs. (17) and (18)**. **Table 4** shows  
 483 the total surface energy of the tested binders with their surface energy components.

484 **Table 4** Surface energy components of tested bitumens (mJ/m<sup>2</sup>).

Bitumens	$\Gamma$	$\Gamma^{\text{LW}}$	$\Gamma^{\text{AB}}$	$\sqrt{\Gamma^+}$	$\sqrt{\Gamma^-}$
Unaged 40/60	11.51	8.15	3.37	1.49	1.13
RTFO 40/60	12.56	8.79	3.77	1.39	1.36
PAV 40/60	8.89	4.80	4.09	2.08	0.98
Unaged X-70	10.80	5.98	4.82	1.96	1.23
RTFO X-70	12.91	7.93	4.98	1.51	1.65
PAV X-70	10.59	3.48	7.11	2.21	1.61
PAV base X-70	13.11	9.73	3.39	1.36	1.24
PAV 100/150	12.32	7.99	4.33	1.56	1.39

485

## 486 5.2 Determination of model parameters

487 To implement the energy-based crack initiation criterion for the bitumen in **Eq.**  
 488 **(16)**, the model parameters are discussed and determined in this section. The  
 489 recoverable pseudostrain energy density  $RPSE(r)$  is calculated using **Eq. (8)** with the  
 490 crack initiation position  $r = r_0 - c_0$ . More specifically, the shear modulus  $|G_0^*|$  in the  
 491 undamaged condition is obtained from the frequency sweep tests. The controlled  
 492 amplitude of rotational angle  $\theta_0$  is 0.025 rad. The shear reference modulus  $G_R$  is  
 493 determined as the shear modulus  $|G_0^*|$ . The authors' previous study [45] proved that,  
 494 when assigning the material modulus to the reference modulus, the physical meaning  
 495 of the pseudostrain becomes the difference between the viscous strain and the total  
 496 strain. It is noted that some studies [55, 56] used a unit (1 Pa) as the reference modulus,  
 497 however, this will result in an unclear physical meaning of the pseudostrain as the unit  
 498 reference modulus has normalised the pseudostrain when the stress-pseudostrain  
 499 relations are modelled.

500

501 The surface energy  $\Gamma$  was calculated based on the contact angle measurements that have  
 502 been presented in **Section 5.1**. The edge flow damage size  $c_0$  was determined as the  
 503 corresponding crack length at which the minimum crack growth rate was obtained

504 before the “factory-roof” crack propagation is started in the TS tests, as shown in **Fig.**  
 505 **5**. The radius  $r_0$  and the height  $h$  of the cylindrical bitumen sample were 4 mm and 2  
 506 mm, respectively.

507

508 The coefficient  $k$  of the vertical height  $ka$  in the triangular region shown in **Fig. 6** is  
 509 determined as  $2\pi$  [34]. Note that the ‘factory-roof’ crack length  $a \ll c_0$  at the critical  
 510 moment of the crack initiation, thus it is assumed that the crack length  $a = \alpha \cdot c_0$  where  
 511  $\alpha$  is within a range from 0.0004 to 0.0012 for the tested bitumen samples. It is noted  
 512 that the triangular region that releases the RPSE is strongly related to the material  
 513 viscoelastic property, leading to the small difference in the values of the parameter  $\alpha$   
 514 for the different bituminous binders. It is very challenging to directly measure the  
 515 parameter  $\alpha$  in the lab tests due to the critical moment of the fatigue crack initiation.

516

### 517 5.3 Validation of crack initiation criterion

518 To validate the crack initiation criterion, **Eq. (16)** is used to predict the surface energy  
 519 of the bitumen. Based on **Eq. (8)**,  $RPSE(r)$  at the crack initiation position ( $r_0 - c_0$ ) is  
 520 written as

521

$$522 \quad RPSE = \frac{1}{2} \frac{\left(|G_0^*| \theta_0\right)^2}{G_R h^2} \cdot (r_0 - c_0)^2 \quad (20)$$

523

524 Then, substituting **Eq. (20)** into **Eq. (16)** and using  $r_0 - c_0 - a \approx r_0 - c_0$  due to  $a \ll c_0$   
 525 for the crack initiation, the surface energy of the bitumen can be calculated by

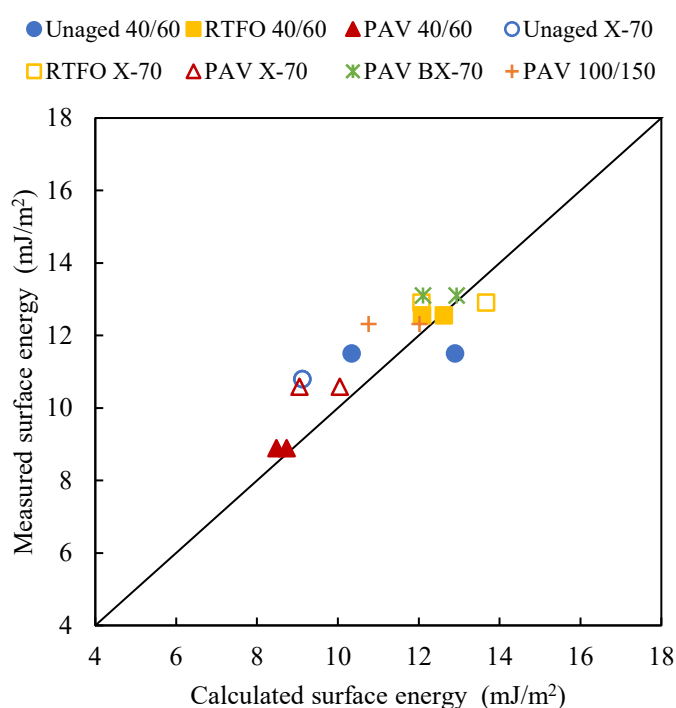
526

$$527 \quad \Gamma = \frac{\left(|G_0^*| \theta_0\right)^2 \cdot k \cdot a \cdot (r_0 - c_0)^2}{4G_R h^2} \quad (21)$$

528

529 **Fig. 7** presents the comparison between the calculated results from **Eq. (21)** and the  
 530 tested results shown in **Table 3**. The quality line ( $y = x$ ) is added in Fig.7 to clearly show

531 the difference between the predicted and measured results. Closer the data points are to  
 532 the quality line, smaller the difference between the predicted and measured values is.  
 533 The difference ratio  $R$  is also calculated to show a measure of how close the data are to  
 534 the quality line. It is defined as the absolute difference between the predicted and  
 535 measured values divided by the measured value. The average  $R$  for all tested samples  
 536 is 7.51%, which means that the predicted results agree well with the tested data. Thus,  
 537 the energy-based crack initiation criterion is validated for all the tested bituminous  
 538 binders. Furthermore, it is concluded that the crack initiation criterion can be used to  
 539 determine the surface energy of the bituminous binders by the DSR fatigue tests.



540  
 541 **Fig. 7.** Comparison between the calculated results and the tested data from the contact angle  
 542 measurements.

543

#### 544 5.4 Crack initiation analysis of bituminous binders

545 After validation, the fatigue crack initiation of bituminous binders can be analysed  
 546 using the developed energy-based criterion. In this study, eight types of bituminous  
 547 binders are tested, including the unmodified and polymer-modified samples under the  
 548 unaged and aged conditions. The experimental results of DSR fatigue tests are  
 549 presented, including the shear modulus and phase angle (**Fig. 2**), cracking morphology

550 (Fig. 3), crack length (Fig. 4), and crack growth rate (Fig. 5). According to these  
551 experimental results, the fatigue cracking process in the bituminous binders is analysed  
552 and discussed to reveal the cracking mechanism and thus to model the fatigue crack  
553 initiation illustrated in Fig. 6. Based on the cracking mechanism and the developed  
554 criterion (Eq. (16)), the edge flow damage size  $c_0$ , the ‘factory-roof’ crack length  $a$  and  
555 the number of load cycles to initiation are calculated and presented in Table 5 for all  
556 tested bituminous binders.

557

558 It can be seen from Table 5 that the polymer-modified X-70 bitumen exhibits a smaller  
559 edge flow damage size than the unmodified 40/60 bitumen in the unaged or aged  
560 condition, indicating a better resistance to the edge flow damage. The edge damage size  
561 for both the 40/60 and X-70 binders increases with the increasing ageing level, which  
562 means that the ageing can accelerate the edge flow damage. At the moment of  
563 circumferential crack initiation, the ‘factory-roof’ crack length is found within one  
564 micrometre for all tested bituminous binders. It is suggested that the microscopic  
565 damage in the bituminous binders subjected to the rotational shear fatigue loads needs  
566 to be examined and explored to understand the role of microstructure in the fatigue  
567 crack initiation.

568

569 Table 5 also shows the number of load cycles until crack initiation for the tested binders.  
570 For both the 40/60 and X-70 binders, the number of load cycles grows as the ageing  
571 level increases. This is because that the aged binders have a greater shear modulus.  
572 However, a higher value of load cycles to crack initiation does not mean a longer  
573 fatigue life that consists of not only the crack initiation life but also the crack  
574 propagation life. It has been reported by the authors’ previous study [15] that the ageing  
575 accelerates the crack propagation and reduce the fatigue crack propagation life. Among  
576 the four kinds of PAV aged binders (i.e., PAV 40/60, PAV X-70, PAV base X-70, PAV  
577 100/150), the number of load cycles for the polymer-modified X-70 bitumen is the  
578 largest (4314), which demonstrates that the polymer modification can improve the  
579 binder resistance to the crack initiation and extend the fatigue crack initiation life.

580

581 **Table 5** Predicted results of fatigue crack initiation for all tested bituminous binders using  
 582 developed energy-based crack initiation criterion.

Bitumens	Edge flow damage $c_0$ (mm)	'Factory-roof' crack $a$ ( $\mu\text{m}$ )	Cycles to initiation
Unaged 40/60	0.47	0.46	839
RTFO 40/60	0.68	0.48	1509
PAV 40/60	0.83	0.34	1062
Unaged X-70	0.32	0.45	1750
RTFO X-70	0.55	0.55	2003
PAV X-70	0.68	0.45	4314
PAV base X-70	0.59	0.49	1713
PAV 100/150	0.59	0.51	1791

583

584 *5.5 Potential application of crack initiation criterion*

585 This energy-based criterion can be potentially applied to determine the critical shear  
 586 stress for the crack initiation of bituminous binders subjected to the rotational shear  
 587 fatigue loads. Based on **Eqs. (6)** and **(7)**,  $RPSE(r)$  at the crack initiation position  
 588  $(r_0 - c_0)$  is written as

589

$$590 \quad RPSE = \frac{1}{2} \frac{\tau_c^2}{G_R} \quad (22)$$

591

592 where  $\tau_c$  is the critical shear stress.

593

594 Substituting **Eq. (22)** into **Eq. (16)** and then considering  $r_0 - c_0 - a \approx r_0 - c_0$  ( $a \ll c_0$ ),  
 595 the critical shear stress can be obtained as

596

$$597 \quad \tau_c = 2 \sqrt{\frac{\Gamma G_R}{\pi a}} \quad (23)$$

598

599 **Eq. (23)** shows a direct relation of the critical shear stress  $\tau_c$  with the surface energy  $\Gamma$ ,  
 600 the shear reference modulus  $G_R$  and the critical crack size  $a$ . It is noted that the form of



601 **Eq. (23)** is analogous to the well-known solution for the critical tensile stress of an  
602 elastic material shown in **Eq. (24)**.

603

$$604 \quad \sigma_c = \sqrt{\frac{G_c E}{\pi a}} \quad (24)$$

605

606 where  $\sigma_c$  is the critical tensile stress,  $a$  is the critical crack size,  $E$  is elastic modulus,  
607 and  $G_c$  is the energy release ratio (surface energy of the crack for brittle materials).

608 Similarly, the critical shear strain can be expressed as

609

$$610 \quad \gamma_c = \frac{2}{|G_0^*|} \sqrt{\frac{\Gamma G_R}{\pi a}} \quad (25)$$

611

612 The critical shear stress (**Eq. (23)**) or strain (**Eq. (25)**) identifies the critical condition  
613 when the crack initiation occurs in the bituminous binders subjected to the rotational  
614 shear fatigue loads. The critical condition for crack initiation is called the endurance  
615 limit, which is the maximum amplitude of a cyclic load applied to a material that does  
616 not lead to fatigue failure of the material. The critical condition, or endurance limit, for  
617 bituminous binders can be measured by a linear amplitude sweep (LAS) test, which is  
618 an accelerated bituminous binder fatigue test consisting of a series of cyclic loads with  
619 systematically linearly increasing loading amplitudes at a constant frequency.  
620 Following the theoretical modelling of crack initiation in this study, the future research  
621 will be focused on further validation and engineering application of the fatigue crack  
622 initiation criterion in predicting the endurance limit of bituminous binders using the  
623 LAS test.

624

## 625 **6. Conclusions**

626 In this study, an energy-based crack initiation criterion is developed for the bituminous  
627 binders subjected to a rotational shear fatigue load based on mechanics principles and  
628 physical facts during the crack initiation process. Laboratory tests including contact  
629 angle measurements and DSR tests are conducted to determine the model parameters.

630 The crack initiation criterion is validated through comparing the predicted and  
631 measured surface energy of the bitumen. The major contributions of this study are  
632 summarised as follows:

- 633 (1) Bitumen fatigue cracking under a rotational shear fatigue load can be divided  
634 into two stages: the edge flow damage and the ‘factory-roof’ cracking.
- 635 (2) The edge flow damage can be regarded as a process including the molecular  
636 rearrangements, microcrack nucleation and circumferential crack formation.
- 637 (3) The local energy redistribution due to the circumferential ‘factory-roof’ crack is  
638 quantified by the pseudostrain energy. A crack initiation criterion is developed  
639 for bitumen under a rotational shear fatigue load based on viscoelastic Griffith’s  
640 fracture mechanics.
- 641 (4) The crack initiation is dependent of the shear modulus and surface energy of  
642 bituminous binders, critical crack size, and loading amplitude.
- 643 (5) The energy-based crack initiation criterion along with the DSR fatigue tests can  
644 be used to determine the surface energy of bituminous binders.
- 645 (6) The polymer modification can improve the binder resistance to the crack  
646 initiation and extend the fatigue crack initiation life.

647

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652

## 653 **References**

- 654 [1] K.A. Ghuzlan, S.H. Carpenter, Energy-derived, damage-based failure criterion for fatigue  
655 testing, Transportation research record, 1723 (2000) 141-149.
- 656 [2] D.A. Anderson, Y.M. Le Hir, M.O. Marasteanu, J.-P. Planche, D. Martin, G. Gauthier,  
657 Evaluation of fatigue criteria for asphalt binders, Transportation Research Record, 1766  
658 (2001) 48-56.
- 659 [3] K.S. Bonnetti, K. Nam, H.U. Bahia, Measuring and defining fatigue behavior of asphalt  
660 binders, Transportation Research Record, 1810 (2002) 33-43.
- 661 [4] S. Shen, S.H. Carpenter, Application of the dissipated energy concept in fatigue endurance

- 662 limit testing, *Transportation Research Record*, 1929 (2005) 165-173.
- 663 [5] S. Shen, G.D. Airey, S.H. Carpenter, H. Huang, A dissipated energy approach to fatigue  
664 evaluation, *Road Materials and Pavement Design*, 7 (2006) 47-69.
- 665 [6] S. Shen, H.-M. Chiu, H. Huang, Characterization of fatigue and healing in asphalt binders,  
666 *Journal of Materials in Civil Engineering*, 22 (2010) 846-852.
- 667 [7] A. Akbari, A. Modarres, Evaluating the effect of nano-clay and nano-alumina on the fatigue  
668 response of bitumen using strain and time sweep tests, *International Journal of Fatigue*, 114  
669 (2018) 311-322.
- 670 [8] C. Hintz, R. Velasquez, C. Johnson, H. Bahia, Modification and validation of linear  
671 amplitude sweep test for binder fatigue specification, *Transportation Research Record*,  
672 2207 (2011) 99-106.
- 673 [9] C. Hintz, H. Bahia, Simplification of linear amplitude sweep test and specification  
674 parameter, *Transportation Research Record*, 2370 (2013) 10-16.
- 675 [10] J.Y.M. Nuñez, E.D. Leonel, A.L. Faxina, Fatigue characteristics of modified asphalt  
676 binders using fracture mechanics, *Engineering Fracture Mechanics*, 154 (2016) 1-11.
- 677 [11] W. Cao, C. Wang, Fatigue performance characterization and prediction of asphalt binders  
678 using the linear amplitude sweep based viscoelastic continuum damage approach,  
679 *International Journal of Fatigue*, 119 (2019) 112-125.
- 680 [12] B.S. Underwood, A continuum damage model for asphalt cement and asphalt mastic  
681 fatigue, *International Journal of Fatigue*, 82 (2016) 387-401.
- 682 [13] Y. Zhang, Y. Gao, Predicting crack growth in viscoelastic bitumen under a rotational shear  
683 fatigue load, *Road Materials and Pavement Design*, (2019) 1-20.
- 684 [14] C. Hintz, H. Bahia, Understanding mechanisms leading to asphalt binder fatigue in the  
685 dynamic shear rheometer, *Road Materials and Pavement Design*, 14 (2013) 231-251.
- 686 [15] Y. Gao, L. Li, Y. Zhang, Modeling Crack Propagation in Bituminous Binders under a  
687 Rotational Shear Fatigue Load using Pseudo J-Integral Paris' Law, *Transportation Research*  
688 *Record*, (2020) 0361198119899151.
- 689 [16] Y. Zhang, R. Luo, R.L. Lytton, Anisotropic characterization of crack growth in the tertiary  
690 flow of asphalt mixtures in compression, *Journal of Engineering Mechanics*, 140 (2013)  
691 04014032.
- 692 [17] Y. Zhang, F. Gu, B. Birgisson, R.L. Lytton, Modelling cracking damage of asphalt mixtures  
693 under compressive monotonic and repeated loads using pseudo J-integral Paris' law, *Road*  
694 *Materials and Pavement Design*, 19 (2018) 525-535.
- 695 [18] L. Brown, Dislocation substructures and the initiation of cracks by fatigue, *Metal Science*,  
696 11 (1977) 315-320.
- 697 [19] K.S. Chan, A microstructure-based fatigue-crack-initiation model, *Metallurgical and*

- 698 Materials Transactions A, 34 (2003) 43-58.
- 699 [20] K. Tanaka, T. Mura, A dislocation model for fatigue crack initiation, Journal of Applied  
700 Mechanics, 48 (1981) 97-103.
- 701 [21] G. Venkataraman, Y.-W. Chung, T. Mura, Application of minimum energy formalism in a  
702 multiple slip band model for fatigue—I. Calculation of slip band spacings, Acta  
703 Metallurgica et Materialia, 39 (1991) 2621-2629.
- 704 [22] W. Bascom, R. Cottingham, R. Jones, P. Peyser, The fracture of epoxy-and elastomer-  
705 modified epoxy polymers in bulk and as adhesives, Journal of Applied Polymer Science,  
706 19 (1975) 2545-2562.
- 707 [23] Y.-W. Mai, B. Cotterell, On the essential work of ductile fracture in polymers, International  
708 Journal of Fracture, 32 (1986) 105-125.
- 709 [24] H. Brown, I. Ward, Craze shape and fracture in poly (methyl methacrylate), Polymer, 14  
710 (1973) 469-475.
- 711 [25] C. Hui, A. Ruina, C. Creton, E. Kramer, Micromechanics of crack growth into a craze in a  
712 polymer glass, Macromolecules, 25 (1992) 3948-3955.
- 713 [26] R.P. Kambour, Mechanism of fracture in glassy polymers. III. Direct observation of the  
714 craze ahead of the propagating crack in poly (methyl methacrylate) and polystyrene, Journal  
715 of Polymer Science Part A-2: Polymer Physics, 4 (1966) 349-358.
- 716 [27] W. Knauss, Stable and unstable crack growth in viscoelastic media, Transactions of the  
717 Society of Rheology, 13 (1969) 291-313.
- 718 [28] R.A. Schapery, A theory of crack initiation and growth in viscoelastic media, International  
719 Journal of Fracture, 11 (1975) 141-159.
- 720 [29] A.A. Griffith, VI. The phenomena of rupture and flow in solids, Philosophical transactions  
721 of the royal society of london. Series A, containing papers of a mathematical or physical  
722 character, 221 (1921) 163-198.
- 723 [30] R. Christensen, A rate-dependent criterion for crack growth, International Journal of  
724 Fracture, 15 (1979) 3-21.
- 725 [31] R.Y. Liang, J. Zhou, Energy based approach for crack initiation and propagation in  
726 viscoelastic solid, Engineering Fracture Mechanics, 58 (1997) 71-85.
- 727 [32] L. Nikitin, Application of the Griffith's approach to analysis of rupture in viscoelastic  
728 bodies, International Journal of Fracture, 24 (1984) 149-157.
- 729 [33] F. Dubois, C. Petit, Modelling of the crack growth initiation in viscoelastic media by the  
730  $G\theta v$ -integral, Engineering Fracture Mechanics, 72 (2005) 2821-2836.
- 731 [34] X. Luo, R. Luo, R.L. Lytton, Energy-based crack initiation criterion for viscoelastoplastic  
732 materials with distributed cracks, Journal of Engineering Mechanics, 141 (2014) 04014114.
- 733 [35] Y. Zhang, X. Luo, R. Luo, R.L. Lytton, Crack initiation in asphalt mixtures under external

- 734 compressive loads, *Construction and Building Materials*, 72 (2014) 94-103.
- 735 [36] R.L. Lytton, Y. Zhang, F. Gu, X. Luo, Characteristics of damaged asphalt mixtures in  
736 tension and compression, *International Journal of Pavement Engineering*, 19 (2018) 292-  
737 306.
- 738 [37] ASTM, Standard test method for effect of heat and air on a moving film of asphalt (rolling  
739 thin-film oven test), in, ASTM International, United States 2012.
- 740 [38] ASTM, Standard practice for accelerated aging of asphalt binder using a pressurized aging  
741 vessel (PAV), in, ASTM International, United States 2013.
- 742 [39] Y. Tan, M. Guo, Using surface free energy method to study the cohesion and adhesion of  
743 asphalt mastic, *Construction and Building Materials*, 47 (2013) 254-260.
- 744 [40] M. Koc, R. Bulut, Assessment of a sessile drop device and a new testing approach  
745 measuring contact angles on aggregates and asphalt binders, *Journal of Materials in Civil  
746 Engineering*, 26 (2013) 391-398.
- 747 [41] R. Moraes, R. Velasquez, H. Bahia, Using bond strength and surface energy to estimate  
748 moisture resistance of asphalt-aggregate systems, *Construction and Building Materials*, 130  
749 (2017) 156-170.
- 750 [42] C.J. van Oss, M. Chaudhury, R.J. Good, Monopolar surfaces, *Advances in colloid and  
751 interface science*, 28 (1987) 35-64.
- 752 [43] C. Wang, H. Zhang, C. Castorena, J. Zhang, Y.R. Kim, Identifying fatigue failure in asphalt  
753 binder time sweep tests, *Construction and Building Materials*, 121 (2016) 535-546.
- 754 [44] R.A. Schapery, Correspondence principles and a generalized J integral for large  
755 deformation and fracture analysis of viscoelastic media, *International Journal of Fracture*,  
756 25 (1984) 195-223.
- 757 [45] Y. Zhang, R. Luo, R.L. Lytton, Characterizing permanent deformation and fracture of  
758 asphalt mixtures by using compressive dynamic modulus tests, *Journal of Materials in Civil  
759 Engineering*, 24 (2011) 898-906.
- 760 [46] Y. Zhang, R. Luo, R.L. Lytton, Mechanistic modeling of fracture in asphalt mixtures under  
761 compressive loading, *Journal of Materials in Civil Engineering*, 25 (2012) 1189-1197.
- 762 [47] Y. Zhang, X. Luo, Y. Deng, S. Hou, X. Shi, R.L. Lytton, Evaluation of rutting potential of  
763 flexible pavement structures using energy-based pseudo variables, *Construction and  
764 Building Materials*, 247 (2020) 118391.
- 765 [48] A. Schmets, N. Kringos, A. Scarpas, C. Duif, G. Schitter, T. Pauli, First-principles  
766 investigation of the multiple phases in bituminous materials: the case of asphaltene stacking,  
767 *Advanced Testing and Characterisation of Bituminous Materials*, 1 (2009) 143-150.
- 768 [49] R. Jahangir, D. Little, A. Bhasin, Evolution of asphalt binder microstructure due to tensile  
769 loading determined using AFM and image analysis techniques, *International Journal of*

- 770 Pavement Engineering, 16 (2015) 337-349.
- 771 [50] L. Shan, Y. Tan, S. Underwood, Y.R. Kim, Application of thixotropy to analyze fatigue and  
772 healing characteristics of asphalt binder, Transportation Research Record, 2179 (2010) 85-  
773 92.
- 774 [51] R. Doremus, Cracks and energy—Criteria for brittle fracture, Journal of Applied Physics,  
775 47 (1976) 1833-1836.
- 776 [52] A. Gent, O. Yeoh, Crack growth in twisted rubber disks. Part 3. Effects of crack depth and  
777 location, Rubber Chemistry and Technology, 76 (2003) 1276-1289.
- 778 [53] C.J. Van Oss, M.K. Chaudhury, R.J. Good, Interfacial Lifshitz-van der Waals and polar  
779 interactions in macroscopic systems, Chemical Reviews, 88 (1988) 927-941.
- 780 [54] C.J. van Oss, Use of the combined Lifshitz–van der Waals and Lewis acid–base approaches  
781 in determining the apolar and polar contributions to surface and interfacial tensions and free  
782 energies, Journal of Adhesion Science and Technology, 16 (2002) 669-677.
- 783 [55] H.-J. Lee, Y.R. Kim, Viscoelastic constitutive model for asphalt concrete under cyclic  
784 loading, Journal of Engineering Mechanics, 124 (1998) 32-40.
- 785 [56] N.H. Gibson, C.W. Schwartz, R.A. Schapery, M.W. Witzak, Viscoelastic, viscoplastic,  
786 and damage modeling of asphalt concrete in unconfined compression, Transportation  
787 Research Record, 1860 (2003) 3-15.
- 788