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Key Points:

- A method to assess the effect of the variability of input variables on sediment transport estimates is presented
- The method indicates to which input variable a given transport relation is most sensitive
- Results may be used to prioritize field measurement efforts in areas where available data are scarce or nonexistent

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Input-variable sensitivity assessment for sediment transport relations

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Abstract A methodology to assess input-variable sensitivity for sediment transport relations is presented. The Mean Value First Order Second Moment Method (MVFOSM) is applied to two bed load transport equations showing that it may be used to rank all input variables in terms of how their specific variance affects the overall variance of the sediment transport estimation. In sites where data are scarce or nonexistent, the results obtained may be used to (i) determine what variables would have the largest impact when estimating sediment loads in the absence of field observations and (ii) design field campaigns to specifically measure those variables for which a given transport equation is most sensitive; in sites where data are readily available, the results would allow quantifying the effect that the variance associated with each input variable has on the variance of the sediment transport estimates. An application of the method to two transport relations using data from a tropical mountain river in Costa Rica is implemented to exemplify the potential of the method in places where input data are limited. Results are compared against Monte Carlo simulations to assess the reliability of the method and validate its results. For both of the sediment transport relations used in the sensitivity analysis, accurate knowledge of sediment size was found to have more impact on sediment transport predictions than precise knowledge of other input variables such as channel slope and flow discharge.

Plain Language Summary Estimation of accurate sediment loads in rivers is challenging; particularly in areas where data needed to use sediment transport relations are scarce. Variability of the input parameters implies difficulty in the ability to constrain the range of possible sediment transport estimates obtained. Are all input parameters equally responsible for the variability or is there one that contributes more to it? This paper presents a simple and useful methodology to rank the importance of each variable in a given sediment transport relation. The results of the method, applicable to any sediment transport equation, specifically indicate how sensitive the relation is to each input parameter. This knowledge provides useful guidance when planning or conducting field measurements within a research or engineering project, in order to prioritize available resources so as to better constrain the variability of the input parameters and, therefore, that of the sediment transport estimates.

1. Introduction

Assessing variable sensitivity and variability in sediment transport estimates has proven challenging to engineers and scientists for many years [*Garcia*, 2008; *Wilcock et al.*, 2009]. No sediment transport equation has universal applicability [*Gomez and Church*, 1989] and estimates obtained with them may span orders of magnitude [*Recking et al.*, 2012] due mainly to their nonlinear nature [*Recking*, 2013a] and difficulties in determining the critical shear stress for a given combination of sediment type and hydraulic conditions [*Wilcock*, 2010]. In many research projects and engineering applications, particularly in developing countries, data are very limited or nonexistent. Even where data are available, variability plays an important role particularly in coarse-bed channels [*Niño and Garcia*, 1994, 1998; *Cienciala and Hassan*, 2016; *D'Agostino and Lenzi*, 1999]. These factors highlight the need for quick, easy to use, techniques to assess input-variable sensitivity of sediment transport equations.

Previous studies in sediment transport sensitivity have mainly focused on the variability of the transport estimates. Monte Carlo simulations have been commonly used as a tool to assess the uncertainty of sediment transport estimates by allowing input variables to change within a specified probability distribution [*Osidele et al*., 2003; *Wilcock et al*., 2009]. Such analyses have the advantage of determining the variability in the transport estimates for many combinations of the input variables but this comes with a computational cost that grows with the number of input variables and required number of simulations. Other authors [e.g., *Chang et al*., 1993; *Ruark et al*., 2011; *Judi Sani et al*., 2015] have approached the problem employing traditional sensitivity analysis which focuses on the variability of the deterministic transport estimate by changing specific variables, one at a time, within a range of observed or assumed values. This technique has the advantage of being straightforward to use but is slow to implement and output variability might be assessed without taking into account the probability distributions of the input variables and the variability due to the interactions between them.

Recently, Bayesian statistics have been used to propose a model for sediment transport estimates [*Schmelter et al*., 2011] and to analyze the uncertainty associated with model parameters as well as the sediment transport computations themselves [*Schmelter et al*., 2012; *Sabatine et al*., 2015]. The Bayesian approach has the advantage of incorporating the variability of input variables by formulating the sediment transport relations using Bayes' theorem, which allows taking into account real values (measurements) with simulated data to constrain the possible values of input parameters and the likelihood of the outputs. Notwithstanding this ability, the posterior distributions (results) are dependent on the prior distributions used, knowledge about the possible values that input variables can take and knowledge about the likelihood of outputs. Such knowledge, expressed in terms of credibility intervals, benefits from the availability of real data but, where no data are available, educated guesses are needed thus introducing subjectivity into the analyses. The latter limitation is not exclusive to the Bayesian approach though, it is in fact a global problem that affects all methodologies discussed. However, a methodology devised specifically to rank the contribution of each parameter's variance is still needed.

The Mean Value First Order Second Moment (MVFOSM) method is presented herein as an alternative to assess input-variable sensitivity and to fill that gap. The method was implemented by the authors in the context of a recent study conducted in four tropical mountain rivers, in Costa Rica where a clear need to estimate bed load transport rates and evaluate input-variable sensitivity and uncertainty of bed load transport relations with no data available for validation existed. Selection of the MVFOSM method was based on the following aspects: (i) it is quick and easy to implement for any transport relation, especially with the aid of symbolic math packages such as the Symbolic Math toolbox and MuPad [*The Mathworks, Inc*., 2016a, 2016b], or open source libraries as SymPy [*Meurer et al*., 2017] which are widely available; (ii) allows assessing sensitivity for a single input only or multiple input variables at the same time; (iii) requires only two values for each input variable, namely a mean and a variance which can be estimated from basic knowledge of the river in question and a one day visit to the site; (iv) as a local sensitivity analysis, it allows evaluating the effect of variations (perturbations) about a base state, i.e., the river's current conditions (slope and grain size distribution), on the variability of the transport estimates; and (v) in places where there is sufficient information for one or more variables, but there is still need to better constrain others, the method is able to point out which of the remaining variables will introduce the largest variability in the transport estimates thus helping with the design of field campaigns in such a way as to better invest the available resources. The methodology, developed to assess input-variable sensitivity and to better inform field measurement campaigns to complement the transport estimates and associated uncertainty is presented next.

2. Methodology

2.1. Mean Value First Order Second Moment (MVFOSM) Method

The MVFOSM method has been used in structural engineering to determine reliability indices [e.g., *Elishakoff et al*., 1987] as well as in hydraulics to assess variable sensitivity and error propagation of Manning's equation [e.g., *Yen et al*., 1986; *Melching*, 1992], and laboratory suspended sediment concentration measurements with acoustic concentration profilers [*Admiraal and Garcia*, 2000]. Application of this methodology to sediment transport relations is proposed as a tool to assess the relative importance of specific input variables in a sediment transport equation.

Given a function f of independent variables $x_i (i=1, 2, \ldots, n)$, i.e., $f=f(x_1, x_2, \ldots, x_n)$, the MVFOSM method can be expressed as shown in equation (1). The method allows to determine the variance of the function based on the mean values of the input variables and their standard deviations. It must be noted that the focus here is not on the variance of the function estimate but on the contribution of each variable to the overall variance, i.e., the sensitivity of the equation to a specific input variable. This sensitivity is determined by dividing each addend in equation (1) by the sum, as shown in equation (2). For more details regarding the method, the reader might refer to *Tung et al*. [2006] for a thorough description of the theory behind it. The relevant relations are as follows:

$$\sigma_f^2 = \left(\frac{\partial f}{\partial x_1}\right)^2 \sigma_{x_1}^2 + \cdots + \left(\frac{\partial f}{\partial x_n}\right)^2 \sigma_{x_n}^2 \tag{1}$$

$$C_{x_i} = \frac{\left(\frac{\partial f}{\partial x_i}\right)^2 \sigma_{x_i}^2}{\sigma_f^2}; \quad i=1,\ldots,n \tag{2}$$

where $\sigma_{x_i}^2$ is the variance of the *i*th variable; σ_f^2 is the total variance of function *f*; and C_{x_i} is the contribution of the *i*th variable to the total variance.

2.2. Application of the MVFOSM Method to Sediment Transport Relations

The MVFOSM method is applied here to two sediment transport relations, the historical one originally proposed by *Schoklitsch* [1962] and the more modern one proposed by *Recking* [2013b]. Both have been found to produce appropriate results in tropical, mountain rivers in Costa Rica. Although the presentation is limited to these two transport equations for pedagogical purposes, the proposed analysis can be easily extended to other transport relations.

2.2.1. MVFOSM Method for the Schoklitsch [1962] Transport Relation

The sediment transport relation of *Schoklitsch* [1962] was developed from experimental and field data for channels with slopes of $\sim$1% or less [*Bathurst*, 2007]. It works well in supply-limited mountain rivers and in rivers where some particle size classes are supply limited [*Bathurst et al*, 1987; *Lopes et al*, 2001]. It is given by equations (3) and (4) where q_s $[L^2T^{-1}]$ is the sediment transport rate, S is the channel slope, q_w $[L^2T^{-1}]$ is the specific flow discharge, q_c $[L^2T^{-1}]$ is the specific critical flow discharge (below which no transport occurs), D_{40} $[L]$ is the characteristic particle size for which 40% of the grains in the distribution are smaller, and R is the submerged specific gravity of the sediment (usually $R=1.65$). The *Schoklitsch* [1962] equation reads

$$q_s = \frac{5}{2}\frac{S^{3/2}(q_w - q_c)}{(R+1)} \tag{3}$$

where

$$q_c = \frac{1}{12}\frac{g^{1/2}R^{5/3}D_{40}^{3/2}}{S^{7/6}} \tag{4}$$

The relation is expressed in dimensionless terms (5) with the use of the Einstein number $q^* = q_s/\sqrt{gRD_{40}^3}$.

$$q^* = \frac{5}{2}\frac{S^{3/2}(q_w - q_c)}{(R+1)\sqrt{gRD_{40}^3}} \tag{5}$$

To use the MVFOSM method with the *Schoklitsch* [1962] relation, (4) must be substituted into (5). The expanded result is shown in equation (6).

$$q^* = \frac{5}{2}\frac{q_w S^{3/2}}{(R+1)\sqrt{gRD_{40}^3}} - \frac{5}{24}\frac{R^{7/6}S^{1/3}}{(R+1)} \tag{6}$$

The derivatives with respect to the independent variables slope (S), characteristic grain size (D_{40}), and specific discharge (q_w) are shown in equations (7)–(9).

$$\frac{\partial q^*}{\partial S} = \frac{15}{4}\frac{q_w S^{1/2}}{(R+1)\sqrt{gRD_{40}^3}} - \frac{5}{72}\frac{R^{7/6}}{S^{2/3}(R+1)} \tag{7}$$

$$\frac{\partial q^*}{\partial D_{40}} = -\frac{15}{4}\frac{q_w g R D_{40}^2 S^{3/2}}{(R+1)\left(gRD_{40}^3\right)^{3/2}} \tag{8}$$

$$\frac{\partial q^*}{\partial q_w} = \frac{5}{2}\frac{S^{3/2}}{(R+1)\sqrt{gRD_{40}^3}} \tag{9}$$

The MVFOSM method relation of the *Schoklitsch* [1962] bed load transport relation is presented in equation (10):

$$\sigma_{q^*}^2 = \left(\frac{\partial q^*}{\partial S}\right)^2 \sigma_S^2 + \left(\frac{\partial q^*}{\partial D_{40}}\right)^2 \sigma_{D_{40}}^2 + \left(\frac{\partial q^*}{\partial q_w}\right)^2 \sigma_{q_w}^2 \tag{10}$$

The contribution of each independent variable to the total variance is determined with equations (11)–(13) shown below.

$$C_S = \frac{\left(\frac{\partial q^*}{\partial S}\right)^2 \sigma_S^2}{\sigma_{q^*}^2} \tag{11}$$

$$C_{D_{40}} = \frac{\left(\frac{\partial q^*}{\partial D_{40}}\right)^2 \sigma_{D_{40}}^2}{\sigma_{q^*}^2} \tag{12}$$

$$C_{q_w} = \frac{\left(\frac{\partial q^*}{\partial q_w}\right)^2 \sigma_{q_w}^2}{\sigma_{q^*}^2} \tag{13}$$

2.2.2. MVFOSM Method for the *Recking* [2013b] Sediment Transport Relation

The transport relation proposed by *Recking* [2013b] was developed with field data for rivers with slopes ranging between 0.07% and 5.5.% and median particle sizes between 0.5 and 100 mm. It is shown in equations (14)–(17) where τ_{84}^* is the dimensionless Shields stress based on the characteristic particle size D_{84} $[L]$ for which 84% of the grains in the distribution are smaller, τ_m^* is the dimensionless mobility Shields stress, D_{50} $[L]$ is the characteristic particle size for which 50% of the grains in the distribution are smaller, W $[L]$ is the channel width, R_h $[L]$ is the hydraulic radius, and p is an empirical constant. The Shields stresses in this relation allow for four different combinations. τ_m^* is given for both sand-bed and gravel bed rivers and τ_{84}^* may be computed with the hydraulic radius R_h or the specific discharge q_w. The latter option is an empirical fit proposed by *Recking* [2013b] who indicates that in equation (15), the relation that uses the specific discharge, $\tau_{84}^* = \tau_{84}^*(q_w)$, provides better results than the one that uses the hydraulic radius R_h due to either difficulties in measuring R_h adequately or due to the fact that in mountain rivers bed load transport may not always occur over the entire length of the bed within a given cross section.

$$q^* = \frac{q_s}{\sqrt{gRD_{84}^3}} = \frac{14\tau_{84}^{*2.5}}{\left[1+\left(\frac{\tau_m^*}{\tau_{84}^*}\right)^4\right]} \tag{14}$$

$$\tau_{84}^* = \frac{R_h S}{RD_{84}} = \frac{1}{\left[\frac{2}{W} + 74p^{2.6}(gS)^p q_w^{-2p} D_{84}^{3p-1}\right]}\frac{S}{RD_{84}} \tag{15}$$

$$\tau_m^* = \begin{cases} (5S+0.06)\left(\dfrac{D_{84}}{D_{50}}\right)^{4.4\sqrt{S}-1.5} & \text{for gravel} \\ 0.045 & \text{for sand} \end{cases} \tag{16}$$

$$p = \begin{cases} 0.23 & \text{for } \dfrac{q_w}{\sqrt{gSD_{84}^3}} < 100 \\ 0.30 & \text{otherwise} \end{cases} \tag{17}$$

The MVFOSM method may be applied to any of the four combinations but here the focus will be in the relation that uses the specific discharge in gravel bed streams. This relation allows for a more thorough presentation and analysis of the potential of the MVFOSM method as compared to the relation for sand-bed rivers for which the mobility number is a constant (equation (16)).

Application of the MVFOSM method to the *Recking* [2013b] equation with specific discharge for gravel bed streams requires substitution of (15) and (16) into (14). The result shown in (18) is then differentiated with respect to each independent variable (Appendix A) to obtain all relations required to compute the total variance (19) and the contributions of each input variable (20)–(24). Note that equations (19) and (23) assume that both the channel width and the specific discharge are independent variables although, commonly, the specific discharge is computed as $q_w = Q/W$ where $Q\,[L^3\,T^{-1}]$ is the flow discharge. This is true for straight channels and is considered a good assumption for natural river reaches with riffles but not for pools and river bends where the specific discharge per unit width, and thus the shear stresses, vary within the cross section as has been recently observed by *Clayton* [2012] in a bend of the Colorado River. Here the variables are assumed to be independent of each other to assess their first-order specific contributions.

$$q^* = \frac{14\left(\frac{1}{\left[\frac{2}{W}+74p^{2.6}(gS)^p q_w^{-2p} D_{84}^{3p-1}\right]}\frac{S}{RD_{84}}\right)^{2.5}}{\left[1+\left(\frac{RD_{84}}{S}\left[\frac{2}{W}+74p^{2.6}(gS)^p q_w^{-2p} D_{84}^{3p-1}\right]\right)^4\left(\frac{D_{84}}{D_{50}}\right)^{17.6\sqrt{S}-6.0}(5S+0.06)^4\right]} \tag{18}$$

$$\sigma_{q^*}^2 = \left(\frac{\partial q^*}{\partial D_{84}}\right)^2 \sigma_{D_{84}}^2 + \left(\frac{\partial q^*}{\partial S}\right)^2 \sigma_S^2 + \left(\frac{\partial q^*}{\partial q_w}\right)^2 + \left(\frac{\partial q^*}{\partial W}\right)^2 \sigma_W^2 + \left(\frac{\partial q^*}{\partial D_{50}}\right)^2 \sigma_{D_{50}}^2 \tag{19}$$

$$C_{D_{84}} = \frac{\left(\frac{\partial q^*}{\partial D_{84}}\right)^2 \sigma_{D84}^2}{\sigma_{q^*}^2} \tag{20}$$

$$C_S = \frac{\left(\frac{\partial q^*}{\partial S}\right)^2 \sigma_S^2}{\sigma_{q^*}^2} \tag{21}$$

$$C_{q_w} = \frac{\left(\frac{\partial q^*}{\partial q_w}\right)^2 \sigma_{q_w}^2}{\sigma_{q^*}^2} \tag{22}$$

$$C_W = \frac{\left(\frac{\partial q^*}{\partial W}\right)^2 \sigma_W^2}{\sigma_{q^*}^2} \tag{23}$$

$$C_{D_{50}} = \frac{\left(\frac{\partial q^*}{\partial D_{50}}\right)^2 \sigma_{D50}^2}{\sigma_{q^*}^2} \tag{24}$$

3. Input-Variable Sensitivity Test Case

To assess the input-variable sensitivity of the transport relations, data from the Reventazón River in Costa Rica [*Castro*, 2004] are used. The coordinates of the site under consideration are 09°49′37″N and 83°41′23″W (Figure 1). Table 1 shows the hydraulic variables considered for the example; Figure 2 shows the average grain size distribution; Table 2 shows the range of characteristic grain sizes; Figures 3a and 3b show two pictures of the site and Figures 3c and 3d show images taken 200 m upstream of the site of interest where the material available for transport may be observed.

3.1. Input-Variable Sensitivity for the *Schoklitsch* [1962] Sediment Transport Relation

The *Schoklitsch* [1962] relation is assessed first, assuming that the standard deviations of all input variables (σ_{q_w}, σ_S, $\sigma_{D_{40}}$) are an equal percentage of their mean value. If no information regarding the variability in input variables is available, this assumption allows for determining which variable contributes more to the total variance of the result. Before looking at a scenario where all variables are allowed to deviate from the mean, three cases are presented for which one variable's standard deviation, and therefore its variance, is assumed to be zero and the other two are allowed to vary between zero and the mean value.

Figure 1. Site location, Reventazón River, Costa Rica.

1. Case 1: $\sigma_{q_w}=0$ and $0<(\sigma_{D_{40}}/\mu_{D_{40}},\ \sigma_S/\mu_S)\leq 1$
   Figure 4a shows a contour plot where the colors indicate the contribution of the slope variance C_S (11) with grain size and slope standard deviations ranging between zero and their mean value. The dashed white line indicates the condition for which both slope and grain size standard deviations are an equal percentage of the mean value. In such a case, the contribution of the slope to the overall variance is slightly below 50%. In general, this indicates that both slope and grain size contribute to the overall variance a similar amount when both variables deviate the same percentage away from the mean.
2. Case 2: $\sigma_S=0$ and $0<(\sigma_{q_w}/\mu_{q_w},\ \sigma_{D_{40}}/\mu_{D_{40}})\leq 1$
   The contour levels in Figure 4b indicate the contribution of the grain size variance $C_{D_{40}}$ (12) with specific discharge and grain size standard deviations ranging between zero and their mean value. The dashed white line indicates the condition for which both specific discharge and grain size standard deviations are an equal percentage of the mean value. In such a case, the contribution of the grain size to the overall variance is slightly below 70%. The remaining 30% corresponds to the specific discharge alone. Therefore, the grain size is responsible for a larger percentage of the overall variance.
3. Case 3: $\sigma_{D_{40}}=0$ and $0<(\sigma_{q_w}/\mu_{q_w},\ \sigma_S/\mu_S)\leq 1$
   Figure 4c shows the contribution of the slope under the assumption that the grain size has no variance. The white dashed line indicates that if both specific discharge and slope deviate equal percentages from their mean value, the slope contributes slightly less than 70% to the total variance. Under such conditions, the specific discharge contributes the remaining 30% to the total variance.

In summary, the slope and the grain size contribute similar amounts to the total variance when the standard deviations of the variables are assumed to be an equal percentage of the mean value, whereas the specific discharge contributes less. Table 3 shows the contributions of each variable for a scenario in which all three variables deviate an equal percentage from the mean. The result confirms the results inferred from the analysis of the three cases above. The grain size and the slope contribute the most to the total variance. Being able to characterize this variance or narrow it by spending some time in the field conducting measurements will reduce the overall variance in the result.

Table 1. Hydraulic Variables, Reventazón River, Costa Rica

Variable	Q (m^3/s)	W (m)	q_w (m^2/s)	S (%)
Value	180	27	6.7	1.3

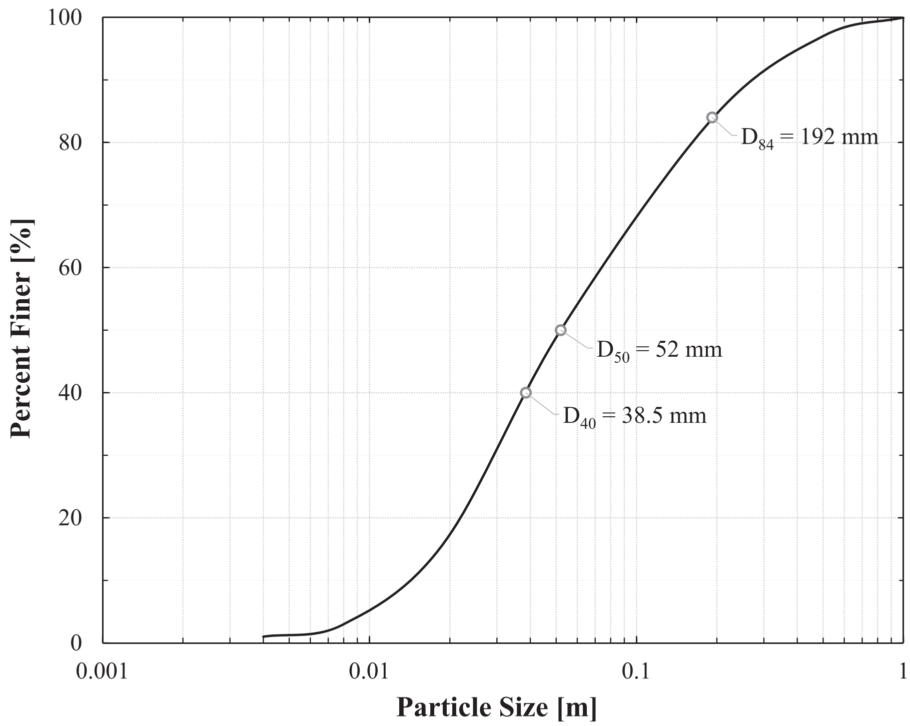

Figure 2. Grain size distribution, Reventazón River, Costa Rica.

In the case of the *Schoklitsch* [1962] relation, simple inspection of (6) suggests these results given that in the first term, both the grain size and the slope have exponents of 1.5 whereas the specific discharge has an exponent of one, and the second term is comparatively small. The contributions of each term might not be as clear though when the variances of all three variables are different. In the case of the Reventazón River in Costa Rica, at the site of interest the reach is relatively straight. Therefore, the specific discharge can be assumed to have little to no variance within the cross section. Information regarding the slopes is not readily available but for this analysis, it was assumed that its standard deviation is 10% of the mean for the flow conditions presented in Table 1. This kind of variability has been observed in the Cordon River, in Italy, during floods [*D'Agostino and Lenzi*, 1999]. Since the Reventazón and Cordon are relatively similar mountain streams, the 10% deviation is taken as a good first-order estimate in the absence of more data. The grain sizes shown in Table 2 suggest that the standard deviation for D_{40} can be assumed to be 30% of the mean value since under a normal distribution assumption, 99.7% of the possible values lie within three standard distributions. Using these values, contributions are calculated and the results are shown in Table 4. The variation in the grain size contributes more than 90% to the total variance of the transport estimate and being able to quantify its true variability in the field is very important in order to assess the variability of the transport estimates.

3.2. Input-Variable Sensitivity for the *Recking* [2013b] Transport Relation Using the Specific Discharge and Gravel Bed River Combination

Input-variable sensitivity assessment of the *Recking* [2013b] transport relation is first conducted by assuming that all variables deviate from the mean an equal percentage. Table 5 shows the contributions of each variable to the total variance for this scenario. The results indicate that the three most important variables in this equation are q_w, D_{84}, and S with the grain size being the one with the largest contribution. The channel width and the grain size D_{50} contribute very little to the total variance.

Table 2. Grain Size Ranges Measured by *Castro* [2004] for the Characteristic Diameters Required in the Bed Load Transport Equations

	Range of Values		
Diameter (mm)	Min.	Mean	Max.
D_{84}	52	192	276
D_{50}	27	52	119
D_{40}	22	39	92

Following the approach used to evaluate the sensitivity of each variable in the *Schoklitsch* [1962] relation, the three variables that contribute the most to the overall variance (q_w, D_{84}, S)

Figure 3. Reventazón River in Costa Rica. Images looking downstream-upstream (a, b) at the site of interest and (c, d) 200 m upstream, respectively.

will be paired up assuming that the variable that was left out has zero variance and the other two are allowed to vary between zero and their mean value.

1. Case 1: $\sigma_{q_w}=0$ and $0<(\sigma_{D_{84}}/\mu_{D_{84}},\ \sigma_S/\mu_S)\leq 1$ Figure 4d shows the contribution of the slope to the total variance under the assumption that the specific discharge has no variance. The white dashed line indicates the condition for which both slope and D_{84} deviate an equal percentage from their mean value. It indicates that under such conditions, the slope contributes less than 40% to the total variance and the remaining 60% corresponds to the contribution of D_{84}.
2. Case 2: $\sigma_S=0$ and $0<(\sigma_{q_w}/\mu_{q_w},\ \sigma_{D_{84}}/\mu_{D_{84}})\leq 1$ The contribution of D_{84} (20) to the total variance under the assumption that the slope variance is zero is shown in Figure 4e. In such conditions, $C_{D_{84}}=66\%$ when both specific discharge and grain size deviate an equal percentage from their mean value. The remaining 34% corresponds to the contribution of the specific discharge.
3. Case 3: $\sigma_{D_{84}}=0$ and $0<(\sigma_{q_w}/\mu_{q_w},\ \sigma_S/\mu_S)\leq 1$ This case is shown in Figure 4f where D_{84} is assumed to have zero variance and the variances of the specific discharge and slope range between zero and their mean value. The white dashed line indicates that when both q_w and S deviate an equal percentage from their mean value the slope contributes 55% to the total variance and the specific discharge 45%.

In summary, D_{84} is the input variable which contributes the most to the overall variance of the *Recking* [2013b] transport relation in gravel bed rivers when the specific discharge is used instead of the hydraulic radius to estimate the dimensionless shear stress τ^*_{84}. When assessed against both slope and specific discharge it contributed more than 60% to the total variance.

In the case of the Reventazón River, most of the variance comes from the grain sizes as shown in Table 2. Table 6 shows the contributions of each variable to the total variance for the site. The variance of the channel width is not considered because it is relatively constant for the hydraulic conditions presented by *Castro* [2004] (Table 1). For the other variables, the variances are equal to those used to assess input-variable sensitivity of the *Schoklitsch* [1962] relation. As in that relation, a grain size is the variable that contributes more to the total variance. Here D_{84} is responsible for 90% of the variance and D_{50} contributes only 4% to the

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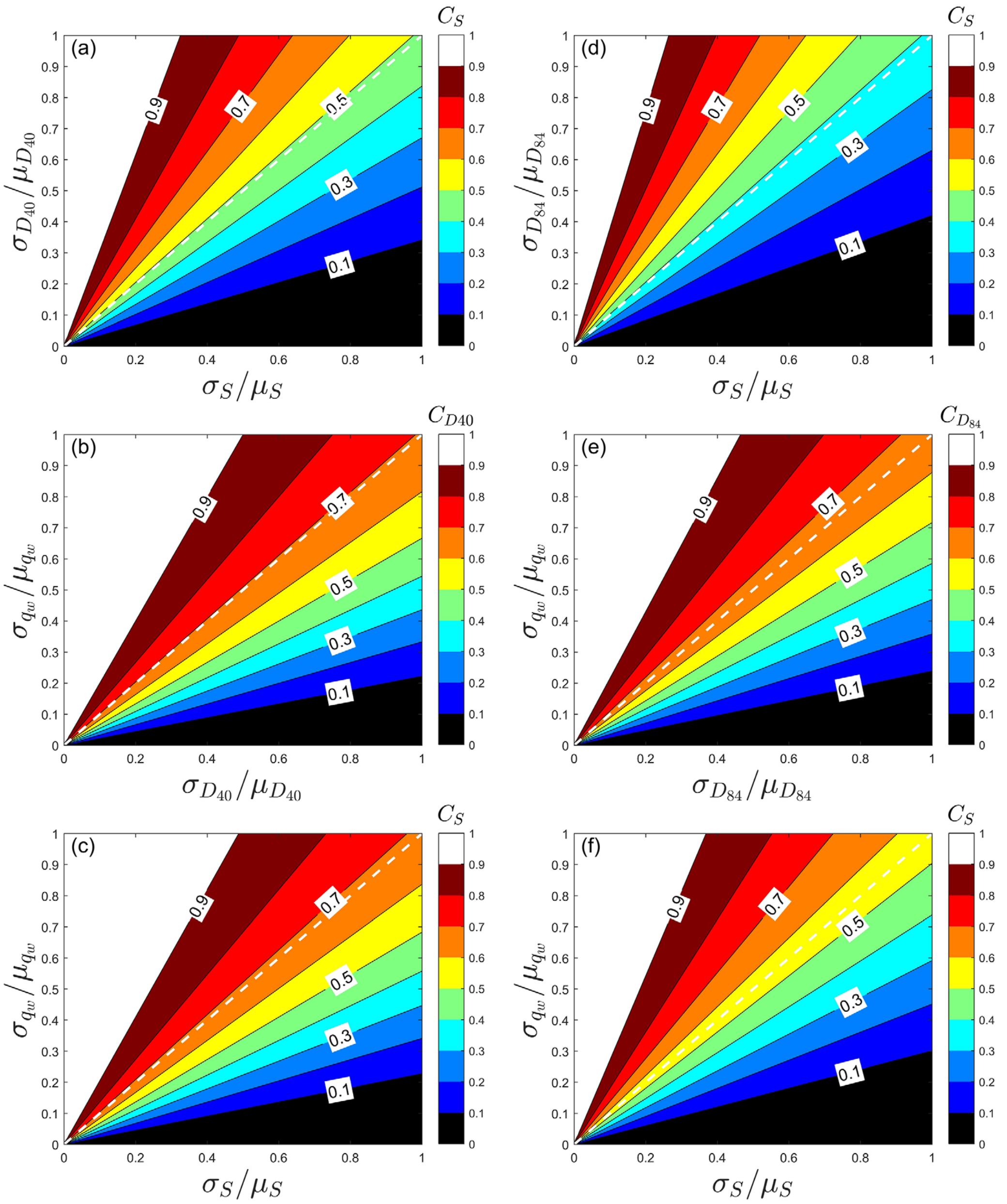

Figure 4. Contribution of input-variable variance to total variance of the (a–c) *Schoklitsch* [1962] and (d–f) *Recking* [2013b] sediment transport relations. Colors represent the contribution of the input variable on the horizontal axis (also indicated above the colorbar). The complement values are equal to the contribution of the variable on the vertical axis. (a) C_S for $0 < (\sigma_{D_{40}}/\mu_{D_{40}},\ \sigma_S/\mu_S) \le 1$ and $\sigma_{q_w}=0$; (b) $C_{D_{40}}$ for $0 < (\sigma_{q_w}/\mu_{q_w},\ \sigma_{D_{40}}/\mu_{D_{40}}) \le 1$ and $\sigma_{q_w}=0$; (c) C_S for $0 < (\sigma_{q_w}/\mu_{q_w},\ \sigma_S/\mu_S) \le 1$ and $\sigma_{D_{40}}=0$; (d) C_S for $0 < (\sigma_{D_{84}}/\mu_{D_{84}},\ \sigma_S/\mu_S) \le 1$ and $\sigma_{q_w}=0$; (e) $C_{D_{84}}$ for $0 < (\sigma_{q_w}/\mu_{q_w},\ \sigma_{D_{84}}/\mu_{D_{84}}) \le 1$ and $\sigma_{q_w}=0$; and (f) C_S for $0 < (\sigma_{q_w}/\mu_{q_w},\ \sigma_S/\mu_S) \le 1$ and $\sigma_{D_{84}}=0$.

total variance. The results obtained for the *Recking* [2013b] relation are not easily inferred by simple inspection of (18) as could be done with the *Schoklitsch* [1962] relation (6). The grain size for which 84% of the distribution is smaller appears both in the numerator and denominator with different exponents. The slope also appears in the numerator and denominator as a factor and as an exponent. The use of the MVFOSM allows assessing the sensitivity of the equation to each input variable.

3.3. Comparison of the MVFOSM Method With Results From Monte Carlo Simulations

One thousand Monte Carlo simulations were conducted to determine the variability of the Einstein dimensionless number obtained with the *Schoklitsch* [1962] and *Recking* [2013b] sediment transport relations

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Table 3. Contributions of Each Input Variable in the *Schoklitsch* [1962] Equation to the Total Variance Under the Assumption That All Variables Deviate an Equal Percentage From Their Mean Value

Input Variable		Mean μ_{x_i}	Variation σ_{x_i}/μ_{x_i} (%)	Standard Deviation σ_{x_i}	Variance $\sigma^2_{x_i}$	Contribution C_{x_i} (%)
q_w	m^2/s	6.7	10	0.67	0.45	18.5
D_{40}	mm	39	10	3.9	15.2	41.7
S	%	1.30	10	0.13	1.69E-6	39.7

under different input-variable variabilities. Specific discharge, characteristic grain size (D_{40} and D_{84}, respectively) and slope were considered for the analysis. First, each variable was allowed to have a standard deviation equal to 10%, 20%, and 30% of the mean while the other two had zero variability. Second, all variables were allowed to deviate 10%, 20%, and 30% simultaneously from the mean. In the first set of simulations, it was assumed that the input variables had normal probability distributions whereas in the second set, the assumption was that they have lognormal probability distributions. The standard deviations of the result (σ_{q^*}) for all combinations were computed and the percent variation of the result (σ_{q^*}/μ_{q^*}) was determined. For the set of simulations with the lognormal distributions, the mean and standard deviation values used were those of the equivalent normal distribution. Results for the *Schoklitsch* [1962] are shown in Table 7 and results for the *Recking* [2013b] are shown in Table 8.

The percent variation (column 3 in Tables 7 and 8) obtained with the MVFOSM method and the Monte Carlo simulations are, in most cases, very similar. This indicates that the hierarchy of input-variable contributions to the overall variability of the transport estimates obtained with the MVFOSM method is valid. Regarding the values themselves, in the case of the *Schoklitsch* [1962] relation, the absolute differences between Monte Carlo with normal probability distributions (MC_{norm}) and the MVFOSM method are less than 5% except for three cases: (1) the case in which D_{40} has a variation from the mean ($\sigma_{D_{40}}/\mu_{D_{40}}$) equal to 30%; and the cases in which all variables have percent variations from mean equal to (2) 20% and (3) 30%. In the case of Monte Carlo with lognormal probability distributions (MC_{log}), all cases have absolute differences below 2.5%. In the case of the *Recking* [2013b] relation, the absolute differences between MC_{norm} and MVFOSM are less than 5% for four cases. Three are the same as with the *Schoklitsch* [1962] relation and the additional one is the case for which the characteristic grain size has a variation from the mean ($\sigma_{D_{84}}/\mu_{D_{84}}$) equal to 20%. All absolute differences between the percent variations of MC_{log} and MVFOSM are below 5%.

In regard to the hierarchy of input-variable sensitivity, the MVFOSM method suggests that in the case of the *Schoklitsch* [1962] relation, D_{40} and S were the most important variables, with the characteristic diameter slightly more important in the case of the example presented here. Results obtained with Monte Carlo simulations confirm the result. Taking, for example, the case for which each variable has a variation from mean of 10%, MVFOSM ranks the contribution of each variable to overall variance in the same way as the Monte Carlo cases (see Table 7). Variability of the Einstein number associated with D_{40}, S, and q_w are $(16.9, 17.8, 17.0)$, $(16.4, 16.3, 16.6)$, and $(11.3, 11.2, 11.3)$ respectively. In the case of the *Recking* [2013b] relation, results of Monte Carlo simulations also confirm input-variable hierarchy determined with MVFOSM, namely D_{84} is more important than S which in turn is more important than q_w. Although these results are not tabulated, the Monte Carlo analysis also confirmed that the variability in the Einstein number associated with D_{50} and W are very small (for $\sigma_{D_{50}}/\mu_{D_{50}}=10\%$, $\sigma_{q^*}/\mu_{q^*}=5\%$ and for $\sigma_W/\mu_W=10\%$, $\sigma_{q^*}/\mu_{q^*}=3\%$ for both normal and lognormal distributions).

4. Discussion

The MVFOSM method presented here provides an estimate of the relative importance that each input variable has in a sediment transport equation. The method is a first-order estimate of sensitivity which uses the first two moments (mean and standard deviation) and assumes that (i) the input variables are independent of each other and follow a normal distribution and (ii) to determine the contribution of each input variable, the transport relation may be

Table 4. Contributions of Each Input Variable in the *Schoklitsch* [1962] Equation to the Total Variance for Specific Conditions in the Reventazón River, Costa Rica

Input Variable		Mean μ_{x_i}	Variation σ_{x_i}/μ_{x_i} (%)	Standard Deviation σ_{x_i}	Variance $\sigma^2_{x_i}$	Contribution C_{x_i} (%)
q_w	m^2/s	6.7	0	0.00	0.00	0.0
D_{40}	mm	39	30	11.7	136.9	90.4
S	%	1.30	10	0.13	1.69E-6	9.6

Table 5. Contributions of Each Input Variable in the *Recking* [2013b] Equation to the Total Variance Under the Assumption That All Variables Deviate an Equal Percentage From Their Mean Value

Input Variable		Mean μ_{x_i}	Variation σ_{x_i}/μ_{x_i} (%)	Standard Deviation σ_{x_i}	Variance $\sigma^2_{x_i}$	Contribution C_{x_i} (%)
q_w	m²/s	6.7	10	0.67	0.45	23.3
D_{84}	mm	192	10	19.2	368.6	45.4
D_{50}	mm	52	10	5.20	27.0	2.0
S	%	1.30	10	0.13	1.69E-6	28.5
W	m	27	10	2.70	7.29	0.8

linearized around a base value given by the river's characteristics. The following sections explore the effects of such assumptions on the results of the MVFOSM method in order to assess its applicability and limitations.

4.1. Probability Distribution Choice and Effect of Linearization of Transport Relations

The use of a normal distribution with the MVFOSM is acceptable insofar the standard deviations are not so high as to cause input-variable values that are unrealistic. For example, cases in which the possible values associated with the input variables yield negative values will fall in this category. The cases in Tables 7 and 8 where there is no agreement between the MC_{norm} and the MVFOSM results are due to unrealistic variables in the Monte Carlo simulations. These were corrected with the use of lognormal distributions which are representative of hydraulic variables [*Parker*, 2008] and have the advantage of always being positive. The fact that the percent variations of the MVFOSM method results compared very well with the MC_{log} results also indicates the reliability of the method but care should be taken when conducting analyses with large variations (>20–30%) of the input variables about mean values. The MVFOSM is a local sensitivity analysis method and as such, only perturbations about a base state should be considered. Large variations are not recommended due to the following two aspects: (1) unrealistic input variables could end up being implicitly considered and (2) sediment transport relations are nonlinear but, the MVFOSM method use of Taylor series expansion about the mean, assumes that the behavior is locally linear.

4.2. Variable Independence

The relation for the MVFOSM method presented in (1) assumes that the variables in question are independent of each other. The assumption is made for the purpose of the analysis but it does not mean that in nature they are not related. The relationship between the variables is usually presented with Lane's balance diagram [*Lane*, 1955; *Rosgen*, 1996] where the product of the sediment load and the sediment size is proportional to the product of the stream slope and the stream discharge ($q_s \times D \propto S \times q_w$). For instance, the expression for the critical flow discharge in the *Schoklitsch* [1962] formula (4) includes the diameter and the slope. The *Schoklitsch* [1962] formula is indeed a version of the Lane relationship. Considering the relation between variables would require use of the variation of the MVFOSM method presented in (25) which includes the covariance between variables. Determination of this covariance is something that cannot be achieved under the data constraints that justified the implementation of the MVFOSM method for transport relations in the first place.

$$\sigma_f^2 = \sum_{i=1}^{n} \sum_{j=1}^{n} \frac{\partial f}{\partial x_i} \frac{\partial f}{\partial x_j} Cov(x_i, x_j) \tag{25}$$

4.3. Site or Transport Relation Dependence of the MVFOSM Method

Alluvial rivers tend to a state of dynamic equilibrium by adjusting roughness, planform shape, cross-sectional morphology, and bed slope [*Nanson and Huang*, 2017]. Quasi-universal relations describing the bankfull geometry of sand-bed and single-thread gravel bed rivers have been developed by *Wilkerson and Parker* [2011] and *Parker et al.* [2007], respectively. These quasi-universal relations and tendency to dynamic equilibrium might suggest that the MVFOSM method would produce similar results for a given transport relation at different sites. However, this remains a hypothesis to be confirmed by conducting the analysis at different sites and with

Table 6. Contributions of Each Input Variable in the *Recking* [2013b] Equation to the Total Variance for Specific Conditions in the Reventazón River, Costa Rica

Input Variable		Mean μ_{x_i}	Variation σ_{x_i}/μ_{x_i} (%)	Standard Deviation σ_{x_i}	Variance $\sigma^2_{x_i}$	Contribution C_{x_i} (%)
q_w	m²/s	6.7	0	0.00	0.00	0.0
D_{84}	mm	192	30	57.6	3317.8	89.8
D_{50}	mm	52	30	15.6	243.4	4.0
S	%	1.30	10	0.13	1.69E-6	6.3
W	m	27	0	0.00	0.0	0.0

Table 7. Comparison of Einstein Number $\left(q^*=q_s/\sqrt{gRD_{40}^3}\right)$ Standard Deviations and Percent Variations Obtained for the *Schoklitsch* [1962] Relation With the Mean Value First Order Second Moment Method and Monte Carlo Simulations Using Normal and Lognormal Probability Distributions for Different Input-Variable Variability Conditions[a]

Variation From Mean (%) σ_{x_i}/μ_{x_i} (q_w, D_{40}, S)	Standard Deviation σ_{q^*} (MVFOSMb, MC^c_{norm}, MC^d_{log})	Percent Variation (%) σ_{q^*}/μ_{q^*} (MVFOSMb, MC^c_{norm}, MC^d_{log})
(10, 0, 0)	(3.04E-2, 2.68E-1, 2.70E-1)	(11.3, 11.2, 11.3)
(0, 10, 0)	(4.55E-2, 2.73E-1, 2.68E-1)	(16.9, 17.8, 17.0)
(0, 0, 10)	(4.42E-2, 2.69E-1, 2.73E-1)	(16.4, 16.3, 16.6)
(20, 0, 0)	(6.06E-2, 2.69E-1, 2.79E-1)	(22.5, 21.5, 23.3)
(0, 20, 0)	(9.08E-2, 2.88E-1, 2.73E-1)	(33.7, 37.4, 33.4)
(0, 0, 20)	(8.85E-2, 2.73E-1, 2.88E-1)	(32.9, 31.5, 33.4)
(30, 0, 0)	(9.08E-2, 2.69E-1, 2.93E-1)	(33.7, 33.2, 34.1)
(0, 30, 0)	(1.36E-1, 3.30E-1, 2.67E-1)	(50.6, 83.8, 52.7)
(0, 0, 30)	(1.33E-1, 2.78E-1, 3.14E-1)	(49.3, 45.4, 47.0)
(10, 10, 10)	(7.02E-2, 2.75E-1, 2.76E-1)	(26.1, 26.2, 26.3)
(20, 20, 20)	(1.40E-1, 2.93E-1, 2.96E-1)	(52.2, 58.4, 52.4)
(30, 30, 30)	(2.11E-1, 3.49E-1, 3.53E-1)	(78.2, 97.8, 80.6)

[a]Mean Einstein number and sediment transport per unit width values are $\mu_{q^*}=2.69$E-1 and $\mu_{q_s}=8.34$E-3 m^2/s. Note that D_{40} is used for the conversion.
[b]MVFOSM, Mean Value First Order Second Moment Method.
[c]MC_{norm}, Monte Carlo with normal probability distributions.
[d]MC_{log}, Monte Carlo with lognormal probability distributions.

different transport relations. In the absence of such analyses, the MVFOSM method is considered site dependent. The relations used herein were chosen both for pedagogical purposes and their tested applicability in the tropical mountain rivers in Costa Rica. However, a next step would include bed load, suspended-load, and total-load transport relations developed for sand-bed rivers. It must be noted that even if the method is found to be transport relation dependent, it will always be useful at a given site if data are available. Data availability will allow assessing the specific contribution of each input variable to the overall variation under the specific conditions encountered at any given site.

4.4. Use of the MVFOSM Method Where Data Are Available

Use of the MVFOSM method where data are available and both the mean and standard deviations can be readily calculated, is only recommended to quantify the percent contribution of each input variable to the overall variance. Results for standard deviations obtained with MVFOSM do not compare to those obtained with Monte Carlo simulations (column 2 in Tables 7 and 8), likely due to the effect of linearizing the transport relation and neglecting all higher order terms. Similar methods involving second order terms and third

Table 8. Comparison of Einstein Number $\left(q^*=q_s/\sqrt{gRD_{40}^3}\right)$ Standard Deviations and Percent Variations Obtained for the *Recking* [2013b] Relation With the Mean Value First Order Second Moment Method and Monte Carlo Simulations Using Normal and Lognormal Probability Distributions for Different Input-Variable Variability Conditions[a]

Variation From Mean (%) $\sigma_{x_i}/\mu_{x_i}(q_w, D_{84}, S)$	Standard Deviation σ_{q^*} (MVFOSM[b], MC^c_{norm}, MC^d_{log})	Percent Variation σ_{q^*}/μ_{q^*} (MVFOSM[b], MC^c_{norm}, MC^d_{log})
(10, 0, 0)	(1.48E-3, 9.20E-3, 9.29E-3)	(16.0, 15.8, 15.5)
(0, 10, 0)	(2.06E-3, 9.44E-3, 9.27E-3)	(22.3, 24.2, 22.1)
(0, 0, 10)	(1.63E-3, 9.24E-3, 9.46E-3)	(17.7, 18.0, 17.0)
(20, 0, 0)	(2.96E-3, 9.18E-3, 9.72E-3)	(32.0, 31.5, 31.6)
(0, 20, 0)	(4.12E-3, 1.07E-2, 9.40E-3)	(44.7, 57.0, 44.4)
(0, 0, 20)	(3.27E-3, 9.26E-3, 9.92E-3)	(35.4, 34.8, 35.5)
(30, 0, 0)	(4.43E-3, 9.28E-3, 1.05E-2)	(48.1, 45.0, 45.5)
(0, 30, 0)	(6.18E-3, 1.50E-2, 9.93E-3)	(67.0, 133.4, 71.6)
(0, 0, 30)	(4.90E-3, 9.55E-3, 1.10E-2)	(53.1, 52.0, 51.1)
(10, 10, 10)	(3.02E-3, 9.42E-3, 9.57E-3)	(32.7, 33.4, 32.9)
(20, 20, 20)	(6.03E-3, 1.10E-2, 1.15E-2)	(65.4, 74.7, 70.4)
(30, 30, 30)	(9.05E-3, 1.60E-2, 1.23E-2)	(98.1, 161.4, 97.5

[a]Mean Einstein number and sediment transport per unit width values are $\mu_{q^*}=9.23$E-3 and $\mu_{q_s}=3.12$E-3 m^2/s. Note that D_{84} is used for the conversion.
[b]MVFOSM, Mean Value First Order Second Moment Method.
[c]MC_{norm}, Monte Carlo with normal probability distributions.
[d]MC_{log}, Monte Carlo with lognormal probability distributions.

and fourth moments have been developed for structural engineering applications [*Kriegesmann*, 2012; *Hong et al*., 1999] and could potentially be adapted to the case of sediment transport relations. In the absence of such approaches, the authors recommend that the MVFOSM presented herein be used to assess input-variable sensitivity only.

5. Conclusions

A simple, quick-to-implement method for assessing input-variable sensitivity of sediment transport relations has been presented and the results obtained were confirmed with the help of Monte Carlo simulations to assess the reliability of the method. The value of the MVFOSM method for determining which variables of a given sediment transport formula are most important to measure due to the effect they have on the overall variance of the transport estimates has been shown. The importance of the method is twofold since it not only indicates to which variable a given transport relation is most sensitive but also highlights the importance of conducting adequate and representative measurements in the field. In the case of the sediment transport relations tested, precise knowledge of sediment size distribution is more important than knowledge of other input variables. The methodology proposed can be easily extended to other sediment transport formulations.

Appendix A

The derivatives for the *Recking* [2013b] transport equation are presented in this section due to the extent of the relations.

$$\begin{aligned}
\frac{\partial q^*}{\partial D_{84}} &= -\frac{14a_3^{2.5}}{a_2^2}\left[(a_2-1)\left(\frac{4}{D_{84}}+\frac{a_5\left(\frac{D_{84}}{D_{50}}\right)^{-1}}{D_{50}}+\frac{296a_1p^{2.6}(gS)^p(3p-1)}{a_6q_w^{2p}}\right)\right]\\
&\quad-\frac{35a_3^{1.5}}{a_2}\left[a_3\left(\frac{1}{D_{84}}+\frac{74a_1p^{2.6}(gS)^p(3p-1)}{q_w^{2p}a_6}\right)\right]\\
a_1 &= D_{84}^{3p-2}\\
a_2 &= \frac{D_{84}^4R^4a_6^4\left(\frac{D_{84}}{D_{50}}\right)^{a_5}a_4}{S^4}+1\\
a_3 &= \frac{S}{D_{84}Ra_6}\\
a_4 &= (5S+0.06)^4\\
a_5 &= 17.6\sqrt{S}-6\\
a_6 &= \frac{2}{W}+\frac{74D_{84}^{3p-1}p^{2.6}(gS)^p}{q_w^{2p}}
\end{aligned} \tag{A1}$$

$$\begin{aligned}
\frac{\partial q^*}{\partial S} &= -\frac{14b_3^{2.5}}{b_2^2}\left[(b_2-1)\left(\frac{20}{(5S+0.06)}-\frac{4}{S}+\frac{8.8\ln\left(\frac{D_{84}}{D_{50}}\right)}{S^{1/2}}+\frac{296b_7b_1gp^{3.6}}{b_6q_w^{2p}}\right)\right]\\
&\quad+\frac{35b_3^{1.5}}{b_2}\left[\frac{b_3}{S}\left(1-\frac{74b_7p^{3.6}gSb_1}{q_w^{2p}b_6}\right)\right]\\
b_1 &= (gS)^{p-1}\\
b_2 &= \frac{D_{84}^4R^4b_6^4b_5b_4}{S^4}+1\\
b_3 &= \frac{S}{D_{84}Rb_6}
\end{aligned}$$

$$b_4 = \left(\frac{D_{84}}{D_{50}}\right)^{17.6\sqrt{S}-6}$$
$$b_5 = (5S+0.06)^4$$
$$b_6 = \frac{2}{W} + \frac{74 b_7 p^{2.6} (gS)^p}{q_w^{2p}} \tag{A2}$$
$$b_7 = D_{84}^{3p-1}$$

$$\frac{\partial q^*}{\partial q_w} = \frac{c_6 p^{3.6} (gS)^p \left(\frac{S}{D_{84}Rc_3}\right)^{1.5}}{c_1 c_2 c_3}\left[\frac{5180S}{D_{84}Rc_3} + \frac{8288(c_2-1)\left(\frac{S}{D_{84}Rc_3}\right)}{c_2}\right]$$
$$c_1 = q_W^{2p-1}$$
$$c_2 = \frac{D_{84}^4 R^4 c_3^4 c_4 c_5}{S^4} + 1$$
$$c_3 = \frac{2}{W} + \frac{74 c_6 p^{2.6} (gS)^p}{q_w^{2p}} \tag{A3}$$
$$c_4 = \left(\frac{D_{84}}{D_{50}}\right)^{17.6\sqrt{S}-6}$$
$$c_5 = (5S+0.06)^4$$
$$c_6 = D_{84}^{3p-1}$$

$$\frac{\partial q^*}{\partial W} = \frac{\left(\frac{S}{D_{84}Rd_2}\right)^{1.5}}{W^2 d_1 d_2}\left[\frac{70S}{D_{84}Rd_2} + \frac{112(d_2-1)\left(\frac{S}{D_{84}Rd_2}\right)}{d_1}\right]$$
$$d_1 = \frac{D_{84}^4 R^4 d_2^4 \left(\frac{D_{84}}{D_{50}}\right)^{17.6\sqrt{S}-6} (5S+0.06)^4}{S^4} + 1 \tag{A4}$$
$$d_2 = \left(\frac{D_{84}}{D_{50}}\right)^{17.6\sqrt{S}-6}$$

$$\frac{\partial q^*}{\partial D_{50}} = \frac{14 D_{84}^5 R^4 e_1^4 \left(\frac{D_{84}}{D_{50}}\right)^{17.6\sqrt{S}-7} (17.6\sqrt{S}-6)\left(\frac{S}{D_{84}Re_1}\right)^{2.5}}{\left(\frac{D_{50}^2 S^4 \left(D_{84}^4 R^4 e_1^4\right)\left(\frac{D_{84}}{D_{50}}\right)^{17.6\sqrt{S}-6e_2}}{S^4} + 1\right)^2} \tag{A5}$$
$$e_1 = \frac{2}{W} + \frac{74 D_{84}^{3p-1} p^{2.6} (gS)^p}{q_w^{2p}}$$
$$e_2 = (5S+0.06)^4$$

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