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Measuring the effective focal length and shape factor of a thick lens using a microscope

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Abstract

The optical power of a thick spherical lens and its Coddington shape factor are essential magnitudes that characterize its image quality. Here, we propose an experimental procedure and apparatus that allow accurate determination of those magnitudes for any spherical lens from geometrical measurements. The performance of the technique and the used instruments are simple since it only requires a microscope and an optical mouse. The propose overcomes the drawbacks of other devices that need of the refractive index or may damage the lens surfaces, like spherometers, and provides similar results to those from commercial lensmeters.

Keywords: Thick lens; Effective focal length; Shape factor

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1. Introduction

The optical power and the shape factor are essential magnitudes that characterize the image quality of any lens, including ophthalmic, contact or intraocular lenses. In fact, optical aberrations are highly dependent on the shape and the index of refraction of the lens [1-3]. Despite the determination of the effective focal length (EFL) of a lens is an old topic in optical metrology, new works are still published [4-6]. Some methods, like interferometry or Moiré deflectometry, provide very accurate measurements of EFL [7-9]; however, they need of sophisticated material with complex experimental setups.

Spherical lenses are the most common ones since they are simpler and cheaper to manufacture. The optical power and the shape factor of these lenses are usually computed from the measurement of the curvature radii of its surfaces through a spherometer [10]. However, this simple procedure cannot be translated to delicate lenses since the spherometer may produce scratches on the optical surface and damage the treatment layers.

The general spherical thick lens consists of two spherical surfaces in air with their centres aligned. The optical power of a thick lens, P, the equivalent EFL, and the Coddington shape factor can be respectively obtained as:

$$P = (n-1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right) + \frac{(n-1)^2}{n}\frac{d}{r_1 \cdot r_2} = \frac{1}{EFL}$$

$$q = \frac{r_2 + r_1}{r_2 - r_1} \qquad (1)$$

where r_1 and r_2 are the curvature radii, n is the refractive index of the media and d stands for the central thickness of the lens. Geometrical Optics [11] determines the optical power of a thick lens as a function of the back vertex power, P_b, as

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$$P = P_b\left(1 - d\,\frac{n-1}{nr_1}\right) \tag{2}$$

In this work, we present a simple method to obtain the EFL and shape of a lens just through measuring the location of some points on its surfaces. We mark several dots on the surface of the lens and determine the position of each dot in the cloud just with the aid of a microscope and a gauge. With these points, we calculate the curvature radii of both surfaces and the Coddington shape factor in (1). Then, through the calculation of lens thickness, we can also obtain the final EFL. Since very precise location of the spots on the surface is needed, the experimental errors, although small, play a fundamental role here. Thus, the sample size and convergence of results are also discussed.

The manuscript is structured as follows: In the Methods section, we develop the steps that allow determining the curvature radii of the surfaces and the central thickness. Later, we make some considerations about the condition number of the equation system to be solved and discuss about the sample size. Next, the Results, where we apply the technique to a spherical surface to test the method and compare the curvature radius; and to a meniscus contact lens and compare the obtained power to that provided by a commercial lensmeter. Finally, we discuss the conclusions of the work.

2. Method

A spherical surface centred at $C = \left(x_c, y_c, z_c\right)$ and curvature radii r can be represented by the equation:

$$\left(x - x_c\right)^2 + \left(y - y_c\right)^2 + \left(z - z_c\right)^2 = r^2 \tag{3}$$

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Let us consider n points located on a spherical shell at the Cartesian coordinates $P_p = (x_p, y_p, z_p)$, with $p = 0, \ldots, (n-1)$; and let us suppose that the origin of the spatial reference system is set on one of these points, P_0. The remaining $(n-1)$ points with respect to P_0 can be expressed as:

$$x_p = x_0 + \Delta x_p; \quad y_p = y_0 + \Delta y_p; \quad z_p = z_0 + \Delta z_p \tag{4}$$

From expressions (2) and (3), we get that the n considered points fulfill:

$$(\Delta x_p)^2 + 2(x_0 - x_c)\Delta x_p + (\Delta y_p)^2 + 2(y_0 - y_c)\Delta y_p + (\Delta z_p)^2 + 2(z_0 - z_c)\Delta z_p = 0 \tag{5}$$

Now, if in (5), we make the changes $x'_0 = (x_0 - x_c); y'_0 = (y_0 - y_c)$ and $z'_0 = (z_1 - z_c)$, it results:

$$(\Delta x_p)^2 + 2x'_0\,\Delta x_p + (\Delta y_p)^2 + 2y'_0\,\Delta y_p + (\Delta z_p)^2 + 2z'_0\,\Delta z_p = 0; \tag{6}$$

which is a system of $(n-1)$ equations with three unknowns (x'_0, y'_0, z'_0), that are the coordinates of the origin of the new spatial reference system with respect to the centre of the sphere.

A system of equations has solution if it is compatible and determinate, i.e., in our case, it should have, at least, 3 equations. In order to fulfill this condition, the minimum number of points in the surface must be n =4. Therefore, from (6), we get:

$$x'_0\,\Delta x_p + y'_0\,\Delta y_p + z'_0\,\Delta z_p = -\frac{1}{2}\left[(\Delta x_p)^2 + (\Delta y_p)^2 + (\Delta z_p)^2\right]; \qquad p = 1,2,3; \tag{7}$$

that, in matrix form, can be written as:

$$\begin{pmatrix} \Delta x_1 & \Delta y_1 & \Delta z_1 \\ \Delta x_2 & \Delta y_2 & \Delta z_2 \\ \Delta x_3 & \Delta y_3 & \Delta z_3 \end{pmatrix} \begin{pmatrix} x'_0 \\ y'_0 \\ z'_0 \end{pmatrix} = -\frac{1}{2} \begin{pmatrix} (\Delta x_1)^2 + (\Delta y_1)^2 + (\Delta z_1)^2 \\ (\Delta x_2)^2 + (\Delta y_2)^2 + (\Delta z_2)^2 \\ (\Delta x_3)^2 + (\Delta y_3)^2 + (\Delta z_3)^2 \end{pmatrix} \;\rightarrow\; AX'_0 = B \tag{8}$$

We just must solve $X'_0 = A^{-1}B$ and finally obtain the curvature radius of the sphere as:

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$$x'^2_0 + y'^2_0 + z'^2_0 = r^2 \tag{9}$$

Therefore, if we manage to measure three points over a shell with respect to a fourth one, it is possible to determine the curvature radius of the lens. To this end, we select several points on one surface of a spherical lens, just by marking them with black ink. Then, we place the lens on a microscope plate, with the marks oriented towards the objective (Fig. 1). We have used a microscope (Alphaphot-2 microscope from Nikon with a 10x objective and NA=0.25) with a plate that can be moved in the horizontal plane (X and Y directions) and vertically (Z axis). Two gauges of 0.1 mm sensitivity provide the measurement of position in the horizontal XY plane, while a 0.0022 mm sensitivity micrometer measure vertical movements. In order to increase the sensitivity in the XY plane, we have attached an optical mouse to a platform, which is mounted attached to the vertical displacement of the microscope. The optical mouse shows the horizontal movement on a computer screen (1280x1024 px). The variation of the cursor location (in pixels) is directly related to the horizontal movement through a calibration factor of 0.021 mm/px, so the sensitivity is improved in 5 times.

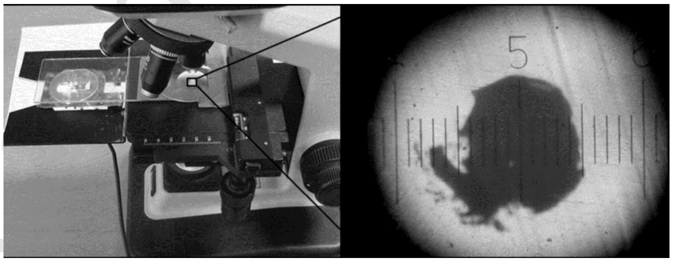

Fig. 1. a) Experimental setup consisting of a microscope and an optical mouse. b) Image of a mark on the surface

The measuring process begins by setting the origin P_0. So, we first simply focus one of the marks on and, then, focus the 3 remaining points on and measure the displacements $(\Delta x_p, \Delta y_p, \Delta z_p)$ with respect to the origin point. Hence, we can solve the system in (8) and, consequently, obtain the curvature radius (9) for each surface.

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Curvature radii help us to obtain the central thickness of the lens. In the scheme in Fig. 2, we start focusing P_0 on and then we look for focusing $V\left(0,0,r\right)$ on, just by moving the plate of the microscope. The movement of the plate consists of a horizontal displacement, x'_0 and y'_0 above obtained from (8), and a vertical shift until we reach the vertex.

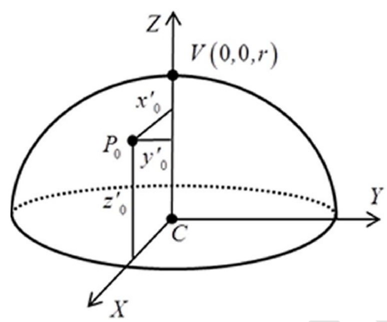

Fig. 2. Location of P_0, the origin of a spatial reference system, on a spherical surface.

Once reached V_1, the determination of the central thickness is different depending on the shape of the lens. In Fig. 3, we represent three different types of thick lenses. In cases 3(a) and 3(b), central thickness is obtained just by looking for the point K, which is directly marked on the supporting plate. The lens is removed from the microscope and the plate is focused. The vertical movement of the microscope is directly the central thickness of the lens.

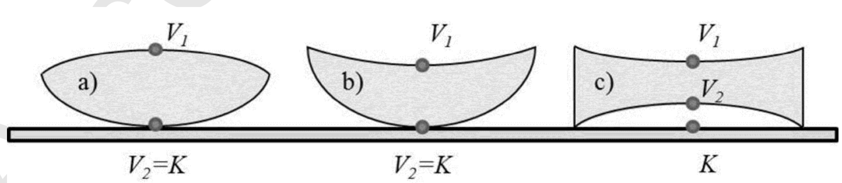

Fig. 3. (a) Biconvex lens. (b) Meniscus. (c) Biconcave lens

The process is more complex in the case 3(c). One possibility would be as follows. If we look at Fig. 4, where different distances are defined, it can be deduced that, if we manage to measure distances *e* and *g*, we can obtain the thickness. From V_1, we can horizontally displace

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the plate and look for the point A, just in the border of the lens. That displacement corresponds to the distance e. Then, we go back to V_1 and remove the lens in order to focus on the plate base. The vertical displacement from V_1 to the base is the distance g. Therefore, the central thickness of the lens can be calculated as:

$$\left.\begin{array}{c} h = r_2 - \sqrt{r_2^2 - e^2} \\ d = g - h \end{array}\right\} \longrightarrow d = g + \sqrt{r_2^2 - e^2} - r_2 \qquad (10)$$

Last magnitude we need to obtain before computing the EFL in expression (1) is the refractive index. Again, we follow the scheme in Fig. 4 and start focusing on V_1. Then, without removing the lens, we focus the opposite vertex on, by displacing the plate an amount $|s'_2|$. Since we are looking at V_2 through the lens, what we see is the image V'_2. This image is given by the upper surface, with curvature radius r_1, that separates a medium of refractive index n (index of the lens) from air.

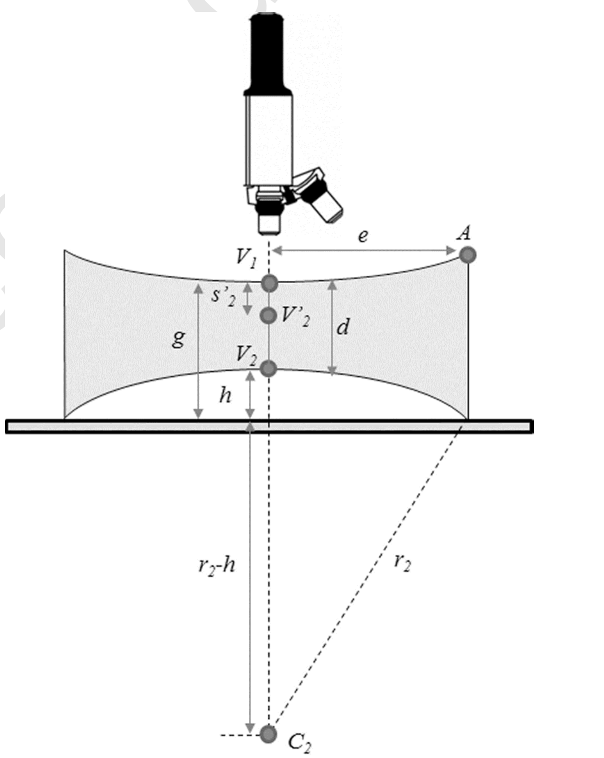

Fig. 4. Scheme that shows distances needed to obtain the central thickness and the refractive index

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From Geometrical Optics, and taking into account the sign convention for the curvature radii (biconvex: $r_1>0$, $r_2<0$; meniscus: $r_1>0$, $r_2>0$, biconcave: $r_1<0$, $r_2>0$) and for distances ($s'_2<0$ in all cases), we can deduce that the refractive index is

$$n = \frac{d\left(r_1 - |s'_2|\right)}{s'_2\left(r_1 - d\right)} \tag{11}$$

Finally, from expressions (1), (10) and (11), the optical power of a thick lens can be obtained as a function of the measured variables as:

$$P = \frac{r_2 + |s'_2| - g - \sqrt{r_2^2 - e^2}}{r_2 |s'_2| \left(g + \sqrt{r_2^2 - e^2} - r_2 - r_1\right)} \left\{ r_2 + r_1 \frac{r_2 + r_1 - g - \sqrt{r_2^2 - e^2}}{|s'_2| - r_1} \right\} \tag{12}$$

3. Experimental considerations: Stability

Although the presented method is clear, we notice that equation system in (8) is very sensitive to small changes of the measurements, even within the experimental error. This happens because the matrix A is often ill conditioned, i.e. the condition number is too high [12]. Consequently, a small variation in some of the elements of the array causes the results to be completely different. As shown in [13], a condition number higher than 30 implies an unstable system of equations. Therefore, in order to avoid the ill conditioning of A, it becomes necessary to increase the number of sampled points in the spherical shell and, accordingly, to obtain a new system of equations. Now, instead of expression (8), the system is an overdetermined linear system of m equations with 3 unknowns:

$$A_{m\times 3} X'_0 = B_{m\times 1} \tag{13}$$

Overdetermined linear systems are generally incompatible since the vector B does not belong to the subspace generated by the columns of the matrix A. However, there are several approximated solutions to an overdetermined linear system. Matlab offers a tool named *lscov* to

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provide a least square solution that minimizes the sum of squared errors $\left(\left\|A_{m\times3}X'_0 - B_{m\times1}\right\|_2\right)^2$ [14], where the vector $X'_0 \in \mathbb{R}^m$ is the approximate and unique solution. We have applied this script on equation (12), $X'_0 = lscov\left[(A)_{m\times3},(B)_{m\times1}\right]$ to determine both the solution and the standard deviation of each one of the elements of the matrix solution (see more details in the Appendix). First, we have evaluated the suitability of the amount of samples selected. An analysis of the condition number provides information about the numerical stability of the solution. As we will show in an example, the condition number tends to stabilize after a number of samples.

Besides the stability of the solution, another issue to consider is the error in obtaining of the coordinates (x'_0, y'_0, z'_0). We consider the set of solutions corresponding to a stable condition number and take as result the mean of those solutions. Therefore, the absolute error assigned to the final result (the curvature radius) is:

$$\Delta r = \frac{\left|x'_0\,\sigma(x'_0)\right| + \left|y'_0\,\sigma(y'_0)\right| + \left|z'_0\,\sigma(z'_0)\right|}{r}; \tag{14}$$

where $(\sigma(x'_0), \sigma(y'_0), \sigma(z'_0))$ are the standard deviation of each coordinate.

4. Results

Initially, and in order to test our method, we measured curvature radii of a spherical steel ball of diameter $d = 12.00 \pm 0.01$ mm. As we stated in the method, we mark several points on the surface and measure their location using a microscope. We compute the coordinates of the origin solving the system of equation (13). Fig. 5a show the evolution of the condition number with the number of sampled points on the sphere. It is observed that the condition number tends to

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stabilize after a certain amount of samples. Furthermore, if we represent the coordinates of the origin as a function of the number of points (Fig. 5b), we also find that stabilization.

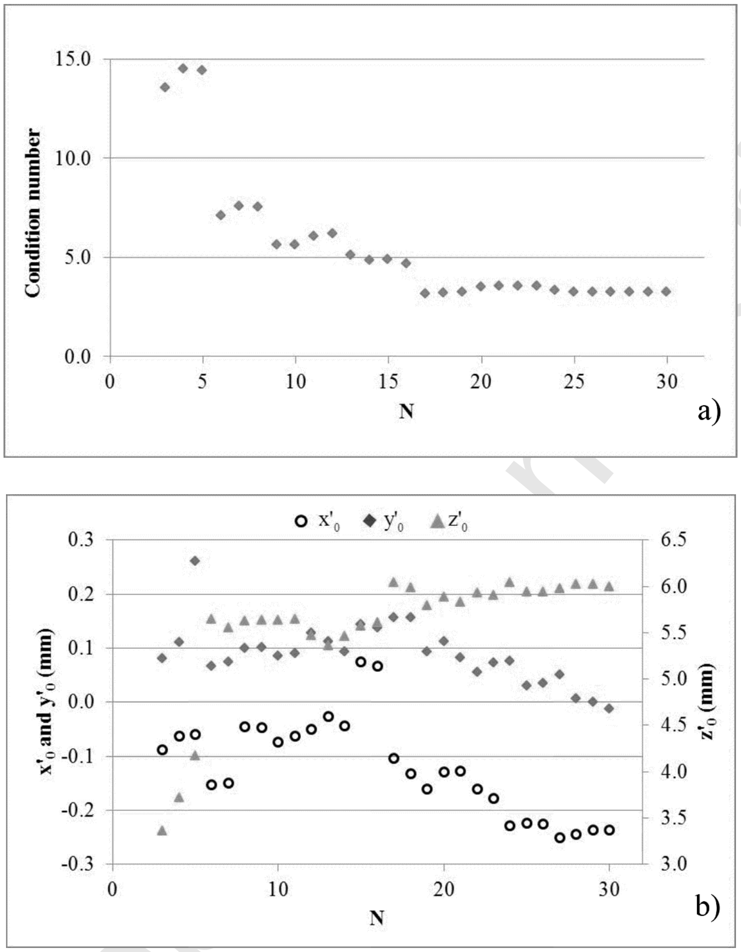

Fig. 5. Evolution of the computed a) condition number and b) coordinates of the origin for different sampling. Note that, after 17 samples the solution (curvature radius), stabilizes.

We establish that a result is reliable if it is located at the range of stability of the condition number ($N \geq 17$, in our case) and assign the mean of the results at this range as final value. Therefore, the origin results $(x'_0, y'_0, z'_0) = (-0.19 \pm 0.05,\ 0.07 \pm 0.05,\ 5.95 \pm 0.08)\,\text{mm}$ and the curvature radius $r = 5.96 \pm 0.08\,\text{mm}$. The deviation of the curvature radius respect to the real value only is 0.66% so the method is useful to accurately obtain small curvature radii.

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After this consideration, we apply the technique to measure the shape factor and power of an ophthalmic meniscus contact lens. First, we have measured the back vertex power) of the lens through a lensmeter (Nidek LM-770) and it results $P_b = 4.00 \pm 0.12\, D$. Next, we have sampled different points at the two surfaces of the lens and we have determined the condition number of the system in (13). In Fig. 6, we show the variation of the condition number with the number of sampled points for both surfaces. As can be seen, the system tends to stabilize after 9 and 13 samples for the concave and convex surface respectively.

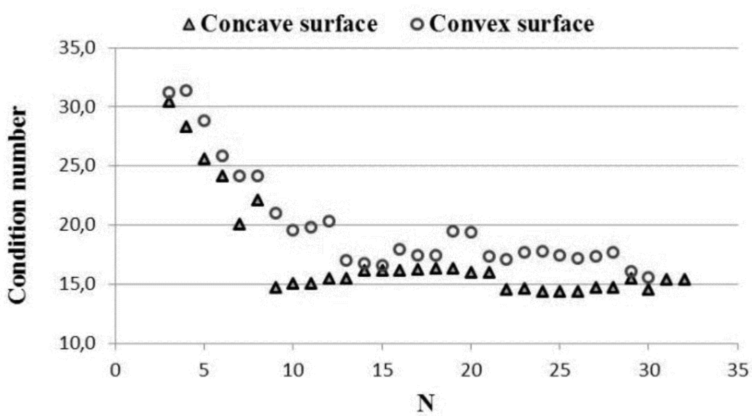

Fig. 6. Variation of the condition number with the number of sampled points for both surfaces

We have also computed the coordinates of the origins and their errors as above for both surfaces and using different number of sampled points. Obtained parameters are in Table 1. Parameters in Table 1 lead us to conclude that the curvature radii are 68.09±0.08 mm and 151.9±0.1 mm, for convex and the concave sides of the lens, respectively.

Table 1. Parameters obtained for each radius and its error

	$x'_0\,(mm)$	$y'_0\,(mm)$	$z'_0\,(mm)$
Convex surface	-14.001±0.004	-14.96±0.02	64.94±0.08
Concave surface	-10.08±0.01	-8.28±0.01	151.4±0.1

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Finally, we have completed the measurements by estimating *e*, *g* and s'_2. They result $e = 22.49 \pm 0.03\,mm$, $g = 6.982 \pm 0.003\,mm$ and $s'_2 = -3.693 \pm 0.007\,mm$, so the optical power and the shape factor are $P = 3.92 \pm 0.06\,D$ and $q = 2.625 \pm 0.005$, respectively. If we apply equation (2), we get $P_b = 4.02 \pm 0.06\,D$, which implies a relative deviation below 0.5% compared to the value measured using the commercial lensmeter.

5. Conclusions

We have proposed a method for measuring the geometry and optical power of a spherical lens. The principal advantages in front of other methods are that the experimental setup is simple, and allows controlling the accuracy of the obtained magnitudes. Moreover, it does not need of contact devices, like spherometers or callipers that can spoil the optical surface and it is able to provide the curvature radii and power of lenses of any size. Applying spherometers is not always possible if the lens is too small. We just have to accurately measure the location of different points on the surfaces. In principle, this can be achieved simply by changing the microscope. At last, but not least, the method does not need of the refractive index, whereas other instruments used to measuring optical power or curvature radii do.

In this work, we have just presented a meniscus lens case. However, the procedure can be extended to any shape of lens, just by modifying the technique, and be applied to astigmatic or toric lenses. In this case, the solution must provide the coordinates of the reference point and the two principal curvature radii. The system of equations would not be lineal anymore and, then, the calculation method would have to be different.

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Appendix

Let us consider an overdetermined linear system of *m* equations with 3 unknowns:

$$A_{m\times 3} X'_0 = B_{m\times 1} \tag{A.1}$$

A usual way to solve the system is to minimize the norm of the residual $\Upsilon = A_{m\times 3} X'_0 - B_{m\times 1}$. If $A_{m\times 3}$ is full rank, the approximate and unique solution will be the vector $X'_0 \in \mathbb{R}^m$ that minimizes the value $\left(\left\| A_{m\times 3} X'_0 - B_{m\times 1} \right\|_2\right)^2$, i.e. the Euclidean norm of Υ. This approach applied to (A.1) leads to the following system of normal equations:

$$\left[A_{m\times 3}\right]^T A_{m\times 3} X'_0 = \left[A_{m\times 3}\right]^T B_{m\times 1} \quad \rightarrow \quad X'_0 = \left[A_{m\times 3}\right]^+ B_{m\times 1}; \tag{A.2}$$

where $\left[A_{m\times 3}\right]^+ = \left(\left[A_{m\times 3}\right]^T A_{m\times 3}\right)^{-1} \left[A_{m\times 3}\right]^T$ is the Moore-Penrose pseudo-inverse matrix of $A_{m\times 3}$ [11]. Unfortunately, the use of normal equations often presents a problem of numerical

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stability since the condition number of $\left[A_{m\times3}\right]^T A_{m\times3}$ is the square of that of $A_{m\times3}$. Thence, the solutions of the normal equations are extremely sensitive to possible disturbances in the measurements.

The most reliable methods to solve the problem with equation (A.2) involve the reduction of the $A_{m\times3}$ matrix to a canonical form by using orthogonal transformations. Among these methods, the most commonly used is the QR factorization. In this case, the matrix $A_{m\times3} \in \mathbb{R}^{m\times3}$ is decomposed into the product $A_{m\times3} = QR$, where Q is orthogonal and $R \in \mathbb{R}^{m\times3}$ is upper triangular. Then, after calculating $Y = Q^T B_{m\times1}$, where $Y \in \mathbb{R}^{m\times1}$, one can resolve the upper triangular system $R_1 X'_0 = Y_1$; where $R_1 \in \mathbb{R}^{3\times3}$ and $Y_1 \in \mathbb{R}^{3\times1}$; and $R = \begin{pmatrix} R_1 & 0 \end{pmatrix}^T$ and $Y = \begin{pmatrix} Y_1 & Y_2 \end{pmatrix}^T$. This process is the one followed in Matlab to solve equation through the tool *lscov* used in the text.

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