

CHLORIDE PENETRATION PREDICTION IN CONCRETE THROUGH AN EMPIRICAL MODEL BASED ON CONSTANT FLUX DIFFUSION

G. de Vera¹, M. A. Climent², E. Viqueira³, C. Antón⁴, M. P. López⁵

ABSTRACT

An empirical model based on constant flux is presented for chloride transport through concrete in atmospherical exposure conditions. A continuous supply of chlorides is assumed as a constant mass flux at the exposed concrete surface. Model is applied to experimental chloride profiles obtained from a real marine structure, and results are compared with the classical error function model. The proposed model shows some advantages. It yields a better predictive capacity than the classical error function model. The previously observed chloride surface concentration increases are compatible with the proposed model. Nevertheless, the predictive capacity of the model can fail if the concrete microstructure changes with time. The model seems to be appropriate for well-maturated concretes exposed to a marine environment in atmospherical conditions.

Keywords: Chloride, Diffusion, Long-term performance, Modeling, Marine environment.

INTRODUCTION

Chloride penetration prediction is necessary or convenient during the project and exploitation phases of reinforced or pre-stressed concrete structures exposed to salt laden environments, in order to assess the full or residual service-life time of the structure in relation to steel reinforcement corrosion.

¹Departament d'Enginyeria Civil, Universitat d'Alacant, Ap. 99, 03080 Alacant, Spain. Corresponding author

²Departament d'Enginyeria Civil, Universitat d'Alacant, Ap. 99, 03080 Alacant, Spain

³Departament d'Enginyeria Civil, Universitat d'Alacant, Ap. 99, 03080 Alacant, Spain

⁴Departament d'Enginyeria Civil, Universitat d'Alacant, Ap. 99, 03080 Alacant, Spain

⁵Departament d'Enginyeria Civil, Universitat d'Alacant, Ap. 99, 03080 Alacant, Spain

22 For concrete structures exposed to marine atmospheric, splash or tidal zones, and also
 23 for deicing salt exposure conditions, concrete is not fully saturated with liquid, rather the
 24 material experiences wetting and drying periods, leading to variable and non-homogeneous
 25 degrees of water saturation of the material. There exist several physical models intending to
 26 predict the evolution of the ion penetration through materials by considering as accurately
 27 as possible the physical-chemical processes having an influence on the ionic transport. Some
 28 of them are single ion models, considering only the main deleterious species, for instance
 29 chloride (Cl^-) ion, and taking into account the different acting transport mechanisms, i.e.
 30 diffusion and convection in the case of unsaturated concrete (Saetta et al. 1993; Martín-
 31 Pérez et al. 2001; Meijers et al. 2005; Guzmán et al. 2011). Other multiionic models take
 32 into account the electrochemical couplings between the ions present in the inner electrolyte of
 33 the concrete pore solution and their effect on ionic transport and on other concrete properties
 34 (Marchand 2001; Samson et al. 2005; Nguyen et al. 2008; Baroghel-Bouny et al. 2011).

35 From a practical point of view, when dealing with chloride penetration prediction, con-
 36 crete codes (Ministerio de Fomento 2008) usually recommend simplified models based on
 37 the empirical use of Fick's second law of diffusion, with the boundary condition of constant
 38 chloride content at the surface of concrete. The chloride profile is given then by:

$$39 \quad c(x, t) = C_0 + (C_S - C_0) \operatorname{erfc} \left(\frac{x}{2\sqrt{Dt}} \right) \quad (1)$$

40 Where $c(x, t)$ is chloride concentration as a function of depth from surface x and time
 41 t , C_0 is the initial (background) chloride concentration and erfc is the complement of the
 42 error function. These models use two main parameters: the chloride diffusion coefficient, D ,
 43 and the surface chloride concentration, C_S , which are, in principle, supposed to be constant.
 44 Nevertheless, sometimes it is considered a progressive decrease of D with time (Ministerio de
 45 Fomento 2008; ACI Committee 365 2009), and a progressive build-up of C_S up to a certain
 46 age (ACI Committee 365 2009). Other researchers have also considered the annual change

47 in surface chloride concentration and its effect on the diffusion process (Cheung et al. 2009).

48 The ability to adopt appropriate values for C_S is a key issue to predict adequately the
49 future Cl^- penetration (Andrade et al. 2006). Nevertheless, it is difficult to assign values
50 for C_S since it changes with time (Uji et al. 1990; Song et al. 2008; Ann et al. 2009), and
51 because it depends not only on the environmental conditions, i.e. salt load, but also on
52 the composition of concrete (Bamforth 1999), due to the different chloride fixation capacity
53 of each binder (Yuan et al. 2009) . So, the C_S parameter cannot be deduced only from
54 environmental parameters. It is only possible to assign reliable values for C_S on the basis of
55 experimental data, i.e. Cl^- content profiles obtained with similar concrete composition and
56 in similar locations.

57 In this work the authors have used a simplified diffusion model based on the constant
58 flux boundary condition, where the main parameters are the diffusion coefficient and the
59 flux, or the amount of Cl^- ingressed through a unit surface of concrete in the unit time, J .
60 This boundary condition leads to an increase of C_S with the square root of time, which is
61 in good agreement with experimental observations in reinforced concrete aeri ally exposed to
62 marine environment (Uji et al. 1990; Costa and Appleton 1999b). Finally, another interesting
63 aspect to consider is that, on the basis of published results (Meira et al. 2007), there is
64 a possibility of finding relationships between the amount of Cl^- ingressed in concrete and
65 typical environmental parameters of the marine environment, such as the dry Cl^- deposition,
66 measured as per the wet-candle device (ASTM G140 1996) or the K3-type device (Lee and
67 Moon 2006). Nevertheless, further research is required to ascertain this possibility, and also it
68 is reasonable to suppose that these relationships will be dependent on concrete composition.

69 This model has been mentioned before and suggested to be used for the atmospheric
70 marine or de-icing salt environments (Uji et al. 1990; ACI Committee 365 2000; Ann et al.
71 2009). But, to the authors' knowledge, there has been no report on the application of the
72 model to experimental data obtained from concrete structures at different ages, which is
73 necessary to assess its capabilities regarding Cl^- penetration prediction.

74 The objective of this work is to provide some initial data on the application of the model
75 to Cl^- content profiles obtained from the aerial part of the concrete structure of a dock
76 of the Alacant harbour, in South-East Spain. The exposure conditions correspond to an
77 atmospheric Mediterranean marine environment, and the data bases correspond to ages of
78 13 and 20 years of the structure.

79 PROPOSED MODEL

80 Chloride transport is described by mass balance equation (2) and diffusion equation (3).
81 A constant D value is assumed.

$$82 \quad \frac{\partial c(x, t)}{\partial t} + \frac{\partial J(x, t)}{\partial x} = 0 \quad (2)$$

$$83 \quad J(x, t) = -D \frac{\partial c(x, t)}{\partial x} \quad (3)$$

85 Initial condition is represented by:

$$86 \quad c(x, 0) = C_0 \quad (4)$$

87 I. e., the initial (background) chloride concentration value is C_0 ($\text{kgCl}^-/\text{m}^3 \text{concrete}$). This
88 is the same initial condition as in (1).

89 Boundary conditions are given by the exposure conditions. In an atmospheric exposure,
90 chloride reaches surface in water droplets brought by wind. Considering an approximately
91 constant supply of droplets we assume that chloride flux J ($\text{kgCl}^-/((\text{m}^2 \text{surface}) \cdot \text{s})$) arriving
92 to the surface is constant. In cold weather where de-icing salts are used, this boundary
93 condition could be also acceptable (ACI Committee 365 2000). This boundary condition is
94 expressed as:

$$95 \quad -D \left. \frac{\partial c(x, t)}{\partial x} \right|_{x=0} = J \quad (5)$$

96 Where J is a constant. The second boundary condition states that far enough from the

97 surface chloride has not yet arrived, and it is expressed as:

$$98 \quad \lim_{x \rightarrow \infty} c(x, t) = C_0 \quad (6)$$

99 This boundary condition was also assumed in (1). Equations (2) and (3) with initial and
100 boundary conditions (4), (5), and (6) are solved using the Laplace transform technique (see
101 appendix) yielding the solution:

$$102 \quad \boxed{c(x, t) = C_0 + 2J\sqrt{\frac{t}{\pi D}} \exp\left(-\frac{x^2}{4Dt}\right) - \frac{Jx}{D} \operatorname{erfc}\left(\frac{x}{2\sqrt{Dt}}\right)} \quad (7)$$

103 Substituting $x = 0$ in (7) the surface concentration is obtained:

$$104 \quad C_S(t) = C_0 + 2J\sqrt{\frac{t}{\pi D}} \quad (8)$$

105 The absorbed amount of chlorides m ($kgCl^-/m^2 surface$) for the proposed model is given
106 by (see appendix):

$$107 \quad m(t) = Jt \quad (9)$$

108 Calculated profiles from both models (1) and (7) are compared in Figure 1. Selected values
109 for these plots are: $D = 10^{-12}m^2/s$, $C_S = 10kg/m^3$, $J = 10^{-9}kg/(m^2s)$, and $C_0 = 0kg/m^3$.
110 Surface concentration and absorbed amount of chlorides are also compared for both models
111 in Figure 2.

112 **EXPERIMENTAL**

113 Concrete cores extracted from the Alacant harbour were studied (Viqueira 2009). Alacant
114 is a Mediterranean city located in the south-east of Spain ($38^\circ 19' N - 0^\circ 29' W$). All samples
115 had atmospherical marine exposure conditions and were taken from the dock 17. This
116 structure was built in 1984 and samples were taken in 1997 and 2004, thus samples were taken
117 after 13 and 20 years exposure respectively. Five concrete cores were extracted in 1997 at

118 different locations in dock 17. Exposure conditions of each location are summarized in Table
119 1. Five concrete cores were extracted in 2004 in the same locations as in 1997. According
120 to the documentation, structure was fabricated with bulk concrete H-175 (MOPU 1982)
121 and cement used was ordinary Portland cement P-350 (MOPU 1975). The following tests
122 were performed following standard methods on concrete cores extracted from the studied
123 structure: compressive strength, cement content quantification (Chinchón et al. 2004), bulk
124 density, and porosity. Mean values, corresponding to at least three samples, are shown in
125 Table 2.

126 Powder samples were obtained from the concrete cores by grinding (Vennesland et al.
127 2013). This technique allows obtaining powder samples in 2mm intervals. Powder samples
128 were analysed to determine its total chloride contents. The method used was potentiometric
129 titration with end-point detection by Gran's method (Climent et al. 1999; Climent et al.
130 2004). Thus, detailed chloride profiles were obtained. Bulk density (see Table 2) was also
131 determined (ASTM C642-90) in order to refer chloride concentrations as $kgCl^-/m^3$ of
132 concrete instead of mass percentages. The mean value of density obtained from several
133 samples at both ages was $2150kg/m^3$. This value was used in all calculations involving
134 density.

135 Experimental profiles were fitted to both models, i. e. the well-known error function
136 solution of Fick's second law (1), and the proposed model (7).

137 **RESULTS AND DISCUSSION**

138 Experimental profiles are shown in Figure 3 to Figure 7. Each figure is devoted to one
139 location in dock 17. Experimental profiles obtained after 13 years of exposure time are
140 plotted with circles while the ones obtained after 20 years exposure time are plotted with
141 asterisks.

142 The presence of peaks can be observed in the experimental profiles of Figs. 3 to 7.
143 Neither model (1) nor model (7) can predict the presence of peaks in the profiles (see Figure
144 1). This is a consequence of the fact that both models are empirical, so they can not

145 take into account all the physical phenomena involved in the chloride transport process.
 146 In an atmospherical exposure, wetting and drying cycles can occur near the surface and
 147 then chloride transport due to water absorption can happen. This mechanism has not been
 148 considered in the presented models, which only consider the diffusion mechanism. It can
 149 also be seen that in general peaks move inwards with time, although a different behaviour
 150 can be observed sometimes, see Figure 5. Transport between surface and the maximum is
 151 probably determined by convection processes (Kropp 1995) and the authors refer to this
 152 zone as convective zone, while the profile from the maximum inwards is probably due to
 153 diffusion processes and the authors refer to this zone as diffusive zone. The possibility of
 154 transient wetting-drying effects, mostly affecting concrete exposed to seawater splash, can
 155 explain the differences observed in the evolution of the instantaneous peak position inside
 156 the convective zone, Figure 5. Both models presented (1) and (7) do not take into account
 157 convective processes and they fail describing the profiles in the convective zone.

158 The experimental value of the chloride mean flux \bar{J} has been calculated as:

$$159 \quad \bar{J} = \frac{1}{t} \int_0^{\infty} (c(x, t) - C_0) dx \quad (10)$$

160 The integration in last equation has been done with the trapezoidal rule and all the data
 161 points of the experimental profiles have been used. A negligible amount of chloride is ex-
 162 pected to be in the concrete when it was fabricated, and thus a value of $C_0 = 0 \text{ kg/m}^3$ has
 163 been assumed in all cases. Table 3 shows the calculated chloride mean fluxes \bar{J} . It is inter-
 164 esting to note in Table 3 that experimental values of \bar{J} increase when passing from 13 to 20
 165 years exposure time. The absorbed mass of chlorides per unit area from model (1) is given
 166 by:

$$167 \quad m(t) = 2(C_S - C_0) \sqrt{\frac{Dt}{\pi}} \quad (11)$$

168 Thus for the classical model (1) is expected a mean flux as:

$$169 \quad \bar{J}(t) = \frac{m(t)}{t} = 2(C_S - C_0)\sqrt{\frac{D}{\pi t}} \quad (12)$$

170 I. e., mean flux decreasing proportional to $t^{-1/2}$, while for the proposed model (7) a con-
171 stant J is expected. Values of Table 3 are less discordant with the proposed model (7) than
172 with the classical error function model (1). A possible explanation for the increase of J with
173 time could be the progressive modification of physical properties of concrete as its chloride
174 concentration increases with time. It has been shown before for cement mortars increasingly
175 contaminated with NaCl, and for the same water content, that the water vapour adsorption
176 isotherm is progressively shifted to lower relative humidity values (Baroghel-Bouny et al.
177 2011). This implies that for cementitious materials the presence of salts in the pore system
178 yields a decrease in the drying rate. Taking into account this, it is reasonable to suppose
179 that as concrete in locations 1 to 5 progressively increase its chloride concentration, while
180 the environmental conditions are approximately constant (considered in the long term), the
181 surface layers of the material would progressively increase their degree of water saturation.
182 This can modify the transport properties of concrete (Climent et al. 2002; de Vera et al.
183 2007), and eventually could modify the interaction between the concrete surface and the
184 marine aerosol leading to a progressive increase of chloride uptake by concrete, i.e. an incre-
185 ment of the surface chloride flux J , which is the opposite behavior to that expected by the
186 classical error-function diffusion model, see Equation (12).

187 Classical model (1) states that surface concentration C_S is constant. This point cannot
188 be checked directly from the experimental profiles due to the presence of peaks.

189 Experimental data have been fitted to models (1) and (7) in order to obtain their param-
190 eters. Parameters for model (1) are D and C_S , while parameters for model (7) are D and J .
191 The fit is done by looking for the parameters values that minimize the standard deviation
192 defined in Equation (13), i. e. minimizing the mean difference between experimental and

193 calculated profiles.

$$\sigma = \sqrt{\frac{\sum_{k=1}^n (C_k^{exp} - C_k^{calc})^2}{n}} \quad (13)$$

194
195 Here n is the number of data points in the fitting, C_k^{exp} is the experimental chloride
196 content of the data point number k , and C_k^{calc} is the corresponding value calculated with the
197 used model. Only the points of the diffusive zone (from maximum inwards) have been used
198 for the fitting, which is a common practice in profile treatment (Sandberg et al. 1998). The
199 algorithm used to find the optimum parameter values is the Nelder-Mead method, which is a
200 simplex algorithm. Calculations have been performed with MATLAB 6.1 software. Results
201 are shown in Table 4 for the classical error function model (1), and in Table 5 for the proposed
202 model (7). The correlation coefficient r used in both tables as an indicator of the goodness
203 of the fitting, is the linear regression correlation coefficient of the plot of C_k^{calc} against C_k^{exp} .

204 Good correlation coefficients are found in Table 4 and Table 5. Fitted curves for 13 years
205 exposure time are shown in Figure 3 to Figure 7. Model (1) is plotted with dotted lines
206 and model (7) is plotted with dashed lines. The zone from the maximum inwards (diffusive
207 zone), where data points have been used in the fitting, is very well represented with both
208 models. Curves from both models differ in the outer zone (convective zone), yielding model
209 (7) higher surface concentrations than model (1).

210 An interesting point observed in Table 4 and Table 5 is that diffusion coefficient for
211 both models does not change a lot when passing from 13 years exposure time to 20 years.
212 In general it remains approximately constant. The change in D observed in other cases
213 has been attributed to changes in the porous structure (Costa and Appleton 1999a). This
214 happens for concretes whose maturation is evolving. In the case here studied concrete is
215 well matured because samples were obtained after 13 and 20 years exposure time for an
216 OPC concrete. Then a change in D is not expected and this fact is confirmed in Table 4 and
217 Table 5. Nevertheless, a decrease of D in location 5 is observed. That could be attributed
218 to the fact that the exposure conditions of location 5 changed, between the first (1997) and

219 the second (2004) campaigns of core extractions, due to construction works to enlarge the
 220 harbour, resulting in an increase of the distance between the sea line and location 5.

221 An increase of surface concentration C_S with time can be observed in Table 4 when
 222 applying the classical error function model (1). This behaviour has also been reported (Uji
 223 et al. 1990; Costa and Appleton 1999a) and can be attributed to exposure conditions. In this
 224 sense it is interesting to consider equation (8). The model (7) that we propose, with specific
 225 boundary conditions for atmospherical exposure, predicts an increase of surface concentration
 226 proportional to \sqrt{t} . This is in agreement with the experimentally observed behaviour (Uji
 227 et al. 1990; Costa and Appleton 1999b) as Equation (14), see Figure 3 to Figure 7.

$$228 \quad C_S(t) = C_{S1} \left(\frac{t}{1year} \right)^m \quad (14)$$

$$D(t) = D_1 \left(\frac{1year}{t} \right)^n$$

229 Following Costa and Appleton (Costa and Appleton 1999b) experimental values of m in
 230 Equation (14) are in the range 0.54 to 0.69 when considering atmospherical exposure and in
 231 the range 0.37 to 0.69 when tidal and spray zones are also considered. This is in agreement
 232 with the value $m = \frac{1}{2}$ predicted by the proposed model (7).

233 In order to evaluate the predictive capacity of both models, they have been used to
 234 predict chloride profiles at 20 years exposure time from the measured profiles at 13 years.
 235 Parameters values obtained fitting chloride profiles at 13 years (2nd and 3rd columns in Table
 236 4 and Table 5) have been introduced into equations (1) and (7) in order to predict chloride
 237 profiles at 20 years exposure time. These predicted profiles are shown in Figure 3 to Figure 7.
 238 Profiles predicted with model (1) are plotted with dash-dot lines, and profiles predicted with
 239 model (7) are plotted with solid lines. It can be seen that model (7) yields higher chloride
 240 concentrations than model (1). Predicted profiles can be compared with the experimental
 241 profiles at 20 years (plotted with asterisks). It can be observed in Figure 3 to Figure 7

242 that both models underestimate chloride concentrations. Nevertheless, differences between
 243 experimental and predicted profiles are lower when the proposed model (7) is used.

244 In order to get a parameter representing the predictive capacity of the models, the fol-
 245 lowing has been calculated:

$$246 \quad \alpha = \frac{I}{I_0} \quad (15)$$

247 Where:

$$248 \quad \begin{aligned} I_0 &= \int_{x_1}^{x_2} (C_{exp} - C_0) dx \\ I &= \int_{x_1}^{x_2} |C_{calc} - C_{exp}| dx \end{aligned} \quad (16)$$

249 Here C_{exp} is the experimental profile at 20 years, and C_{calc} is the predicted profile with the
 250 model used. Integration limits x_1 and x_2 are selected to include as much points as possible
 251 that correspond to the diffusive zone of both profiles (13 and 20 year). Integral I_0 represents
 252 the experimental absorbed amount of chlorides (kg/m^2), while I represents the difference
 253 between experimental and predicted profiles (measured also in kg/m^2). The lower is the I
 254 value, the better is the predictive capacity of the model. In order to compare values from
 255 different profiles the relative value α (15) has been calculated. Results are shown in Table 6.

256 Table 6 shows α values in the range 18% to 58%. Thus the predictive capacity of both
 257 models is not very bad, but it can not be considered good. It is worth noting that the
 258 proposed model (7) yields slightly lower α values, i. e. the model (7) has a slightly better
 259 predictive capacity than model (1).

260 Thomas and Bamforth (Thomas and Bamforth 1999) reported chloride profiles at dif-
 261 ferent times for concretes exposed in the splash marine zone. Three concretes were used:
 262 concrete with ordinary Portland cement (PC), Portland cement with 30% fly ash replacement
 263 (P/PFA), and Portland cement with 70% slag replacement (P/GBS). Diffusion coefficient
 264 values were determined in reference (Thomas and Bamforth 1999) using model (1) and they
 265 are plotted against time in Figure 8A. Here, D values show a variation with time as described
 266 in Equation (14) for the concretes with additions. Reported values of n (see Equation (14))
 267 are 0.7 for P/PFA and 1.2 for P/GBS. This dependency could be attributed to pozzolanic

268 activity, which would decrease porosity and the chloride ingress flux should also be affected.
269 Mean flux values have been calculated with Equation (10) from profiles reported in (Thomas
270 and Bamforth 1999), and results are plotted in Figure 8B. A decrease of \bar{J} with time is
271 observed, attaining a constant value after approximately 6 years of exposure.

272 The reduction of \bar{J} and D with time observed in Figure 8 implies a limitation in the
273 predictive capacity of model (7) when concrete is not well matured. This fact is shown in
274 Figure 9. An experimental profile (Thomas and Bamforth 1999) (concrete P/PFA) at time
275 t_1 (circles in Figure 9) has been fitted to model (7) (dashed line). Parameters obtained have
276 been used to predict the profile at time t_2 (solid line). The last one can be compared with
277 the experimental profile at time t_2 (asterisks). In Figure 9A $t_1 = 0.5\text{year}$ and $t_2 = 3\text{year}$.
278 The large difference between asterisks and solid line shows a strong fail in the prediction of
279 this case. A possible explanation is that the microstructures of these concretes are probably
280 changing greatly in this time range, as indicated by strong variations of the parameters D
281 and J between 0.5 and 3 years, see Figure 8. In Figure 9B $t_1 = 6\text{year}$ and $t_2 = 8\text{year}$. In
282 this case the microstructures of the concretes do not change practically, as indicated by the
283 constancy of the parameters D and J in this time period, see Figure 8. Then, the prediction
284 observed in Figure 9B is much better than that of Figure 9A. This seems to indicate that
285 the model proposed in this work is appropriate for describing the chloride ingress and its
286 evolution of well matured concretes, whose microstructure is not evolving, exposed long
287 time to atmospheric marine environments.

288 CONCLUSIONS

289 An empirical model based on constant flux is presented for describing the chloride trans-
290 port through concrete exposed to atmospheric marine environment. Boundary conditions
291 assumed are more realistic than the ones assumed in the classical error function model. A
292 continuous supply of chlorides is assumed as a constant mass flux at the exposed concrete
293 surface. The experimental chloride flux observed, for the studied concrete structure, is less
294 discordant with the proposed model than with the classical error function model. Chloride

295 surface concentration is known to increase with time. This fact is compatible with the pro-
 296 posed model. Chloride profiles calculated at 20 years exposure time from data obtained at
 297 13 years exposure time correlate better with the experimental chloride profiles at 20 years
 298 when the proposed model is used instead of the classical error function model. The pro-
 299 posed model, as well as the classical error function model, can not be applied to the outer
 300 concrete zone if transport mechanisms different from diffusion act. The predictive capacity
 301 of the proposed model can fail if the concrete microstructure changes with time. The model
 302 seems to be appropriate for well-maturated concretes exposed to a marine environment in
 303 atmospherical conditions. Nevertheless, it would be desirable to apply the proposed model
 304 to more experimental data in order to better validate the applicability and usefulness of the
 305 proposed model.

306 **APPENDIX: MODEL DERIVATION**

307 Diffusion equation (17) must be solved. This is a mass balance equation where transport
 308 is due to diffusion with a constant diffusion coefficient, i. e. Equation (3) substituted in
 309 Equation (2).

$$310 \quad \frac{\partial c(x, t)}{\partial t} = D \frac{\partial^2 c(x, t)}{\partial x^2} \quad (17)$$

311 The following initial and boundary conditions are applied:

$$312 \quad c(x, 0) = C_0 \quad (18)$$

$$313 \quad \lim_{x \rightarrow \infty} c(x, t) = C_0 \quad (19)$$

$$314 \quad -D \left. \frac{\partial c(x, t)}{\partial x} \right|_{x=0} = J \quad (20)$$

317 These equations state that the flux is constant at surface (equation (20)), and that initial
 318 (background) concentration and concentration far enough from surface are equal to C_0 .

319 The Laplace transform method is used in order to solve (17) subjected to (18), (19), and

320 (20). Applying the Laplace transform (see Table 7) to variable t in (17):

$$321 \quad sC(x, s) - c(x, 0) = D \frac{d^2 C(x, s)}{dx^2} \quad (21)$$

322 Substituting here the initial condition (18) and rearranging:

$$323 \quad \frac{d^2 C(x, s)}{dx^2} - \frac{s}{D} C(x, s) = -\frac{C_0}{D} \quad (22)$$

324 In order to solve differential equation (22) the following homogeneous equation must be
325 solved first:

$$326 \quad \frac{d^2 C(x, s)}{dx^2} - \frac{s}{D} C(x, s) = 0 \quad (23)$$

327 Solution of (23) is:

$$328 \quad C(x, s) = k_1 \exp\left(-x\sqrt{\frac{s}{D}}\right) + k_2 \exp\left(+x\sqrt{\frac{s}{D}}\right) \quad (24)$$

329 Where k_1 and k_2 are integration constants. The solution of the inhomogeneous equation
330 (22) is then:

$$331 \quad C(x, s) = k_1 \exp\left(-x\sqrt{\frac{s}{D}}\right) + k_2 \exp\left(+x\sqrt{\frac{s}{D}}\right) + \\ 332 \quad + u_1(x) \exp\left(-x\sqrt{\frac{s}{D}}\right) + u_2(x) \exp\left(+x\sqrt{\frac{s}{D}}\right) \quad (25)$$

332 Where $u_1(x)$ and $u_2(x)$ are given by the system:

$$333 \quad \begin{bmatrix} e^{-x\sqrt{\frac{s}{D}}} & e^{+x\sqrt{\frac{s}{D}}} \\ -\sqrt{\frac{s}{D}}e^{-x\sqrt{\frac{s}{D}}} & +\sqrt{\frac{s}{D}}e^{+x\sqrt{\frac{s}{D}}} \end{bmatrix} \cdot \begin{bmatrix} \frac{du_1(x)}{dx} \\ \frac{du_2(x)}{dx} \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{-C_0}{D} \end{bmatrix} \quad (26)$$

334 Solving and integrating (26) the following expressions are obtained:

$$335 \quad u_1(x) = \frac{C_0}{2s} e^{+x\sqrt{\frac{s}{D}}} \\ u_2(x) = \frac{C_0}{2s} e^{-x\sqrt{\frac{s}{D}}} \quad (27)$$

336 Substituting (27) in (25):

$$337 \quad C(x, s) = k_1 \exp\left(-x\sqrt{\frac{s}{D}}\right) + k_2 \exp\left(+x\sqrt{\frac{s}{D}}\right) + \frac{C_0}{s} \quad (28)$$

338 Now Laplace transform is applied to boundary condition (19):

$$339 \quad \lim_{x \rightarrow \infty} C(x, s) = \frac{C_0}{s} \quad (29)$$

340 The only way to accomplish this is making $k_2 = 0$ in (28), yielding:

$$341 \quad C(x, s) = k_1 \exp\left(-x\sqrt{\frac{s}{D}}\right) + \frac{C_0}{s} \quad (30)$$

342 In order to get the other integration constant k_1 the Laplace transform is applied to the
343 last boundary condition (20):

$$344 \quad -D \left. \frac{dC(x, s)}{dx} \right|_{x=0} = \frac{J}{s} \quad (31)$$

345 The derivative of (30) is:

$$346 \quad \left. \frac{dC(x, s)}{dx} \right|_{x=0} = -k_1 \sqrt{\frac{s}{D}} \exp\left(-x\sqrt{\frac{s}{D}}\right) \Big|_{x=0} = -k_1 \sqrt{\frac{s}{D}} \quad (32)$$

347 Substituting (32) in (31) and rearranging k_1 is obtained:

$$348 \quad k_1 = \frac{J}{s^{3/2}\sqrt{D}} \quad (33)$$

349 And substituting it in (30):

$$350 \quad C(x, s) = \frac{J}{s^{3/2}\sqrt{D}} \exp\left(-x\sqrt{\frac{s}{D}}\right) + \frac{C_0}{s} \quad (34)$$

351 Finally the inverse Laplace transform is applied to get the desired expression:

$$352 \quad \boxed{c(x, t) = C_0 + 2J\sqrt{\frac{t}{\pi D}} \exp\left(-\frac{x^2}{4Dt}\right) - \frac{Jx}{D} \operatorname{erfc}\left(\frac{x}{2\sqrt{Dt}}\right)} \quad (35)$$

353 It is interesting to determine surface concentration. Making $x = 0$ in (35) surface con-
354 centration is obtained:

$$355 \quad C_S(t) = C_0 + 2J\sqrt{\frac{t}{\pi D}} \quad (36)$$

356 The absorbed amount of substance is given by:

$$357 \quad m(t) = \int_0^\infty (c(x, t) - C_0) dx \quad (37)$$

358 Applying here the Laplace transform and taking into account (34):

$$359 \quad \mathcal{L}\{m(t)\} = \int_0^\infty \left(C(x, s) - \frac{C_0}{s}\right) dx = \frac{J}{s^2} \quad (38)$$

360 Which inverse Laplace transform yields:

$$361 \quad m(t) = Jt \quad (39)$$

362 I. e. absorbed mass per surface unit is proportional to time as expected for a constant
363 flux J .

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475 **List of Tables**

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Location	Exposure condition	Wind	Seawater spray
1	Atmospheric	Protected	Low
2	Atmospheric	Protected	Low
3	Splash	Exposed	High
4	Atmospheric	Protected	Low
5	Atmospheric	Protected	Low

TABLE 1. Exposure conditions of locations.

Property	Value
Compressive strength (<i>MPa</i>)	25.6
Cement content (<i>kg/m³</i>)	220
Bulk density (<i>kg/m³</i>)	2150
Porosity (%)	15.8

TABLE 2. Properties of the studied concrete (mean values of at least three samples).

Location	\bar{J} [$mgCl^-/(cm^2 \cdot year)$]	
	13 year	20 year
1	1.1081	1.4891
2	0.5947	0.9496
3	5.8453	6.4491
4	0.4956	0.8083
5	0.4098	0.5281

TABLE 3. Experimental fluxes of chloride determined from profiles.

Location	t = 13 year			t = 20 year		
	D [$10^{-12}m^2/s$]	C_S [kg/m^3]	r [-]	D [$10^{-12}m^2/s$]	C_S [kg/m^3]	r [-]
1	2.6436	10.1675	0.9895	1.4199	30.0093	0.9948
2	1.2792	8.9620	0.9808	1.1807	18.9492	0.9953
3	1.3746	51.0054	0.9957	1.3380	45.0169	0.9728
4	1.8399	4.1488	0.9672	1.2149	15.9998	0.9144
5	5.5322	1.5748	0.9393	0.3875	51.3098	0.9390

TABLE 4. Fitting results of experimental data to model (1).

Location	t = 13 year			t = 20 year		
	D [$10^{-12}m^2/s$]	J [$mg/(cm^2year)$]	r [-]	D [$10^{-12}m^2/s$]	J [$mg/(cm^2year)$]	r [-]
1	3.4547	3.1254	0.9892	1.7594	5.6885	0.9950
2	1.6311	1.9510	0.9813	1.4886	3.2192	0.9958
3	1.7736	11.3321	0.9956	1.9262	7.5512	0.9764
4	2.4768	1.0530	0.9680	1.5455	2.7413	0.9143
5	8.0252	0.6798	0.9483	0.4654	5.2835	0.9368

TABLE 5. Fitting results of experimental data to model (7).

Location	I_0	α (%)	
	[kg/m ²]	Model (1)	Model (7)
1	0.1520	28.6	23.5
2	0.1193	48.2	35.4
3	0.6483	18.3	32.0
4	0.1006	58.6	49.3
5	0.0645	33.9	26.8

TABLE 6. Parameter α values calculated for both models. See text for details.

Function	Laplace transform
$f(t)$	$F(s) = \mathcal{L}\{f(t)\}$
$af(t) + bg(t)$	$aF(s) + bG(s)$
a	a/s
t^n	$n!/n^{s+1}$
$\frac{d^n f(t)}{dt^n}$	$s^n F(s) - \sum_{k=1}^n \left\{ \frac{d^{(k-1)} f(t)}{dt^{(k-1)}} \Big _{t=0} s^{n-k} \right\}$
$2\sqrt{\frac{t}{\pi}} \exp\left(-\frac{a^2}{4t}\right) - \operatorname{erfc}\left(\frac{a}{2\sqrt{t}}\right)$	$s^{-3/2} e^{-a\sqrt{s}}$

TABLE 7. Laplace transforms used.

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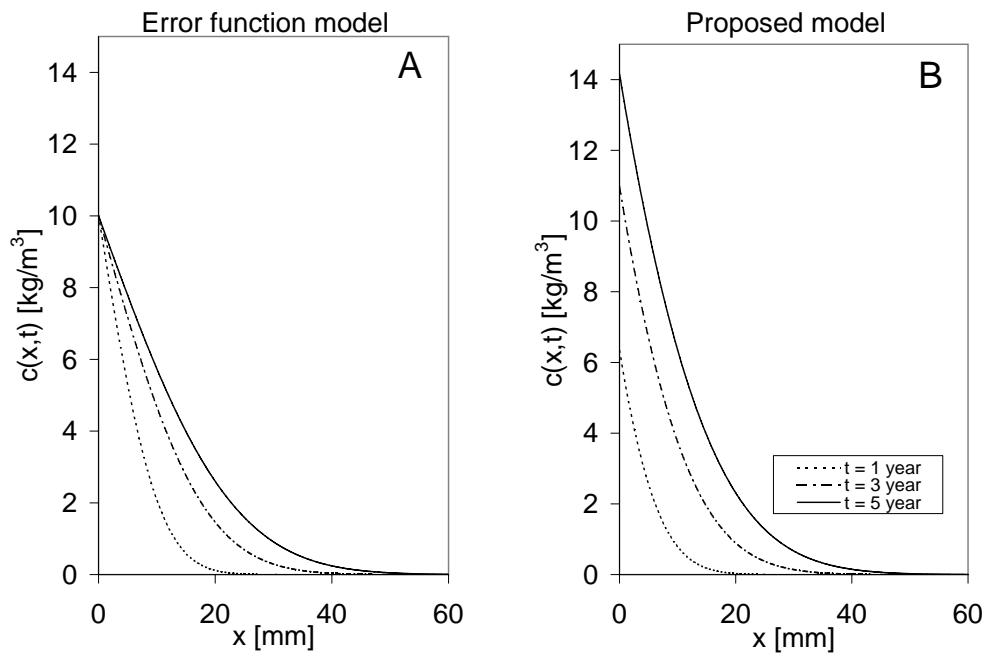


FIG. 1. Profiles calculated with (A) equation (1) and (B) equation (7).

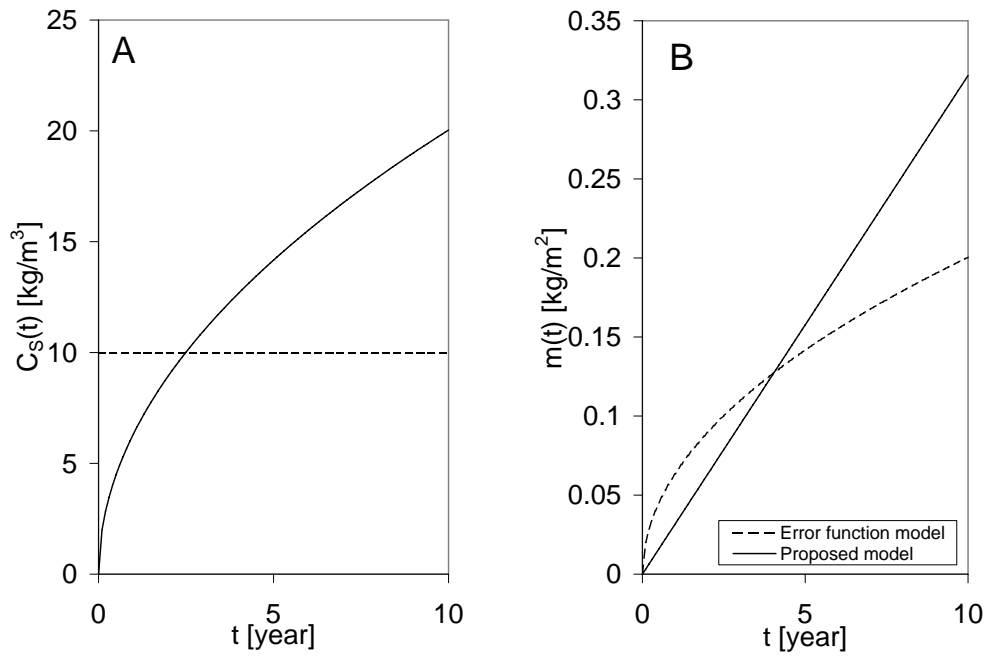


FIG. 2. (A) Surface concentration C_s , and (B) absorbed amount of chlorides m for the classical error function model (dotted line) and the proposed model (solid line).

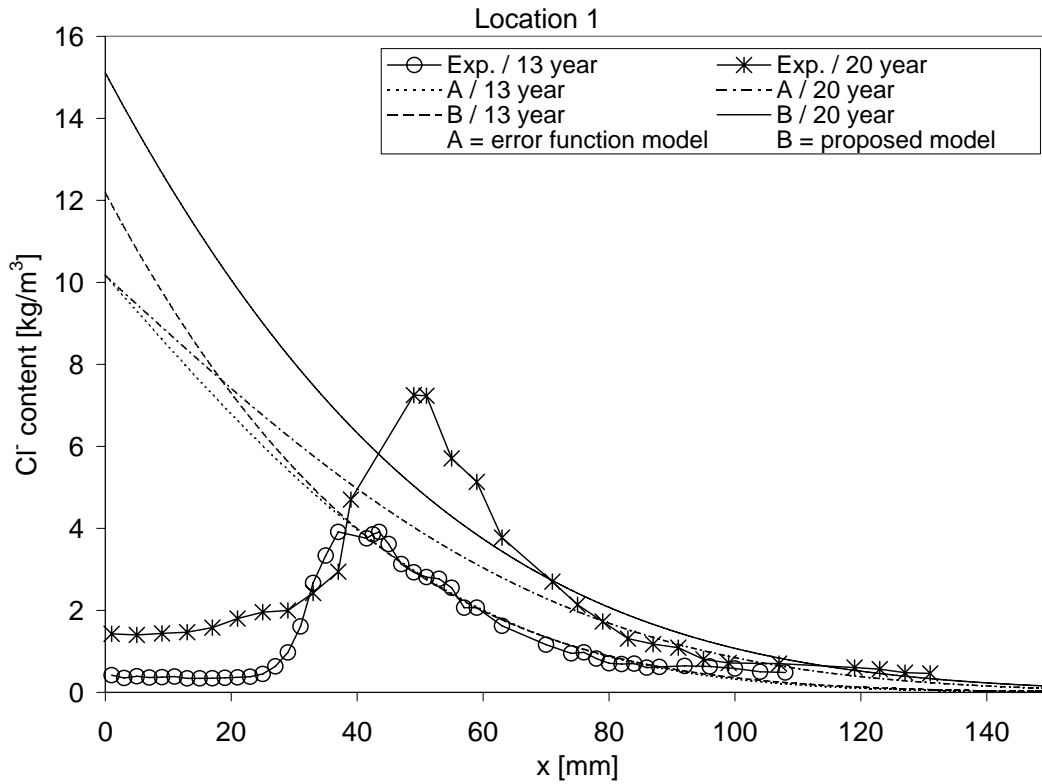


FIG. 3. Experimental and calculated chloride profiles for location number 1. A/20 year and B/20 year calculated with parameters fitted at 13 year. See text for details.

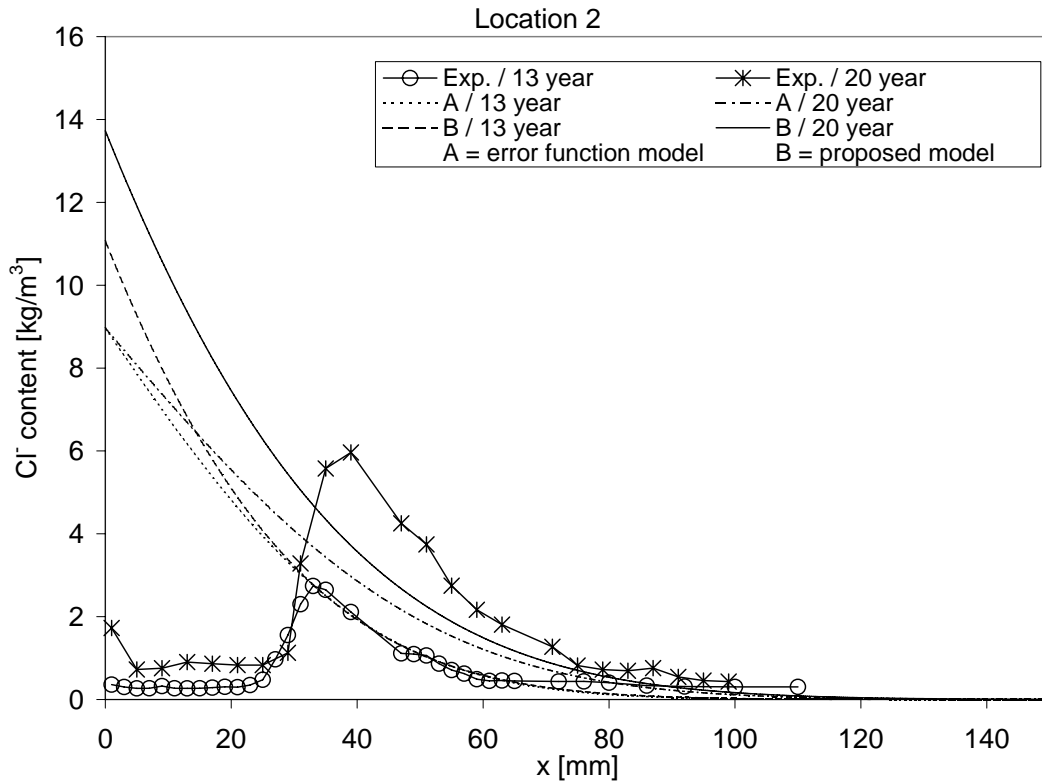


FIG. 4. Experimental and calculated chloride profiles for location number 2. A/20 year and B/20 year calculated with parameters fitted at 13 year. See text for details.

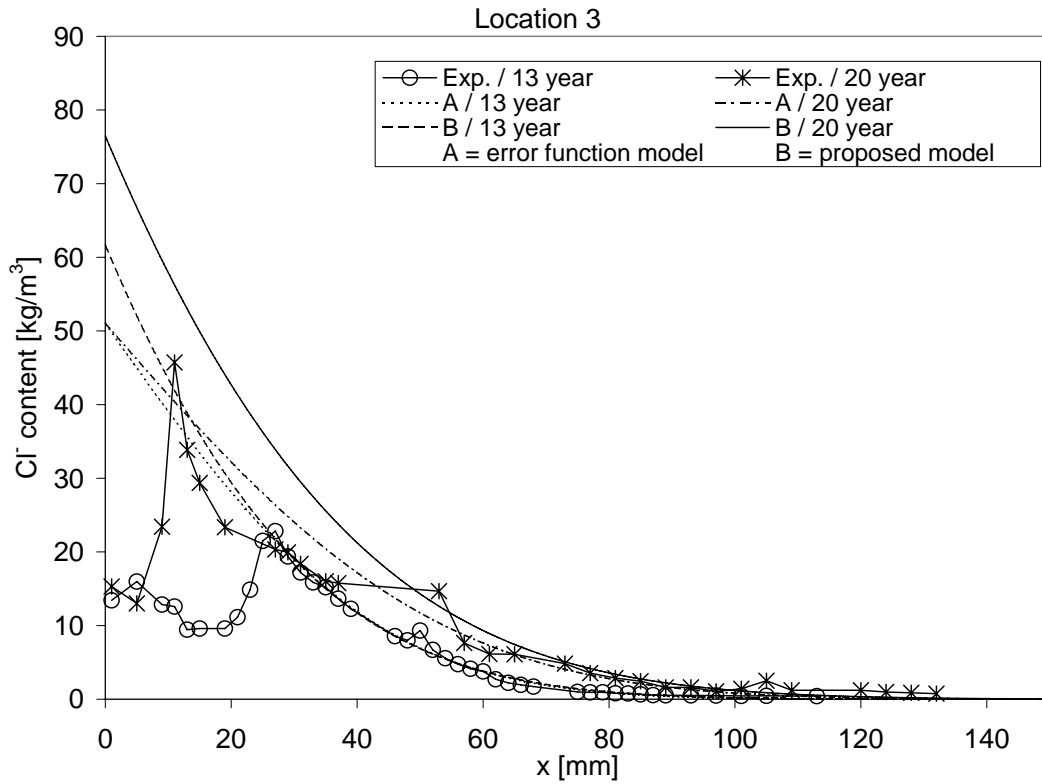


FIG. 5. Experimental and calculated chloride profiles for location number 3. A/20 year and B/20 year calculated with parameters fitted at 13 year. See text for details.

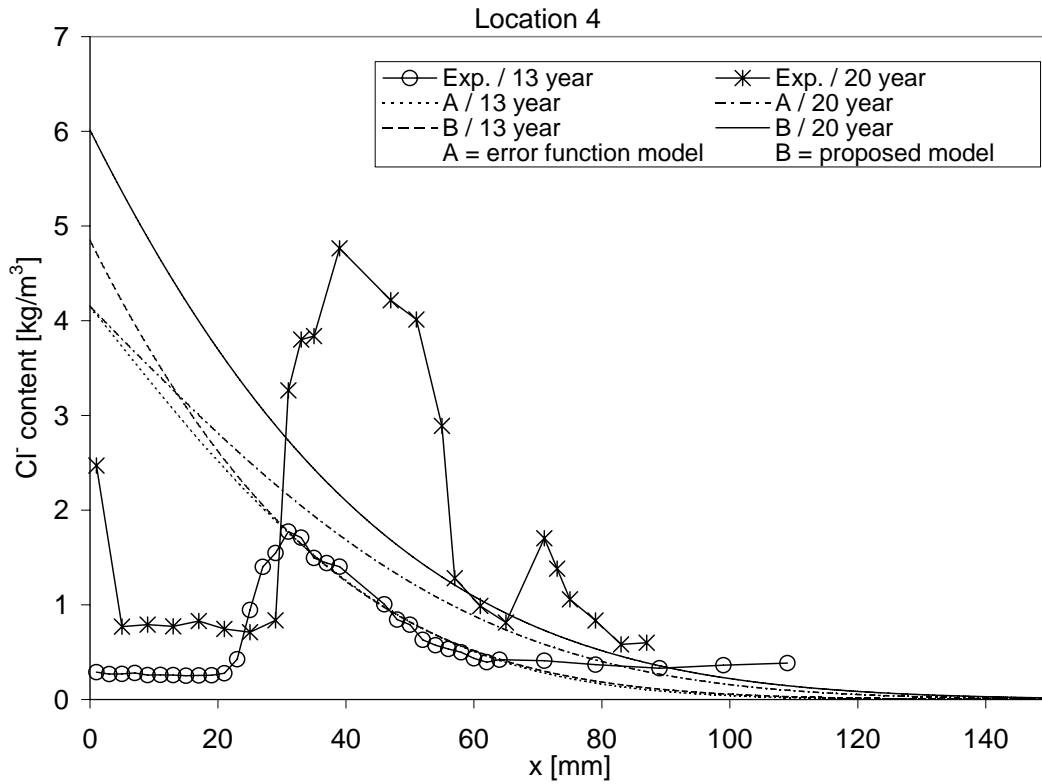


FIG. 6. Experimental and calculated chloride profiles for location number 4. A/20 year and B/20 year calculated with parameters fitted at 13 year. See text for details.

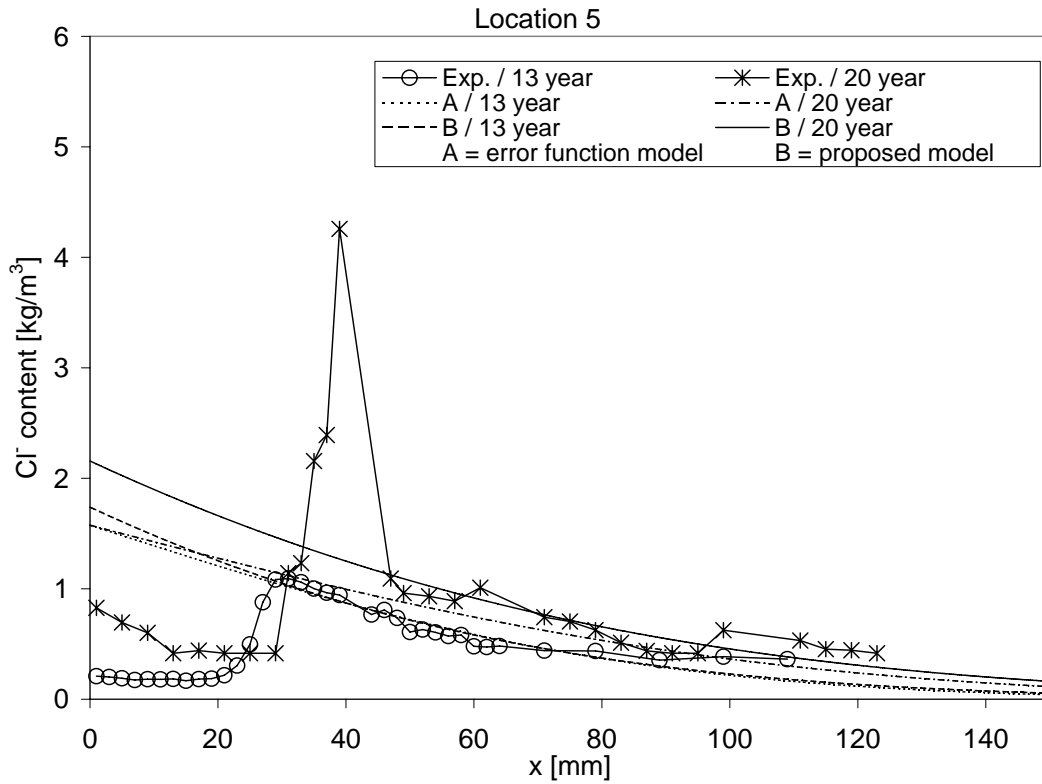


FIG. 7. Experimental and calculated chloride profiles for location number 5. A/20 year and B/20 year calculated with parameters fitted at 13 year. See text for details.

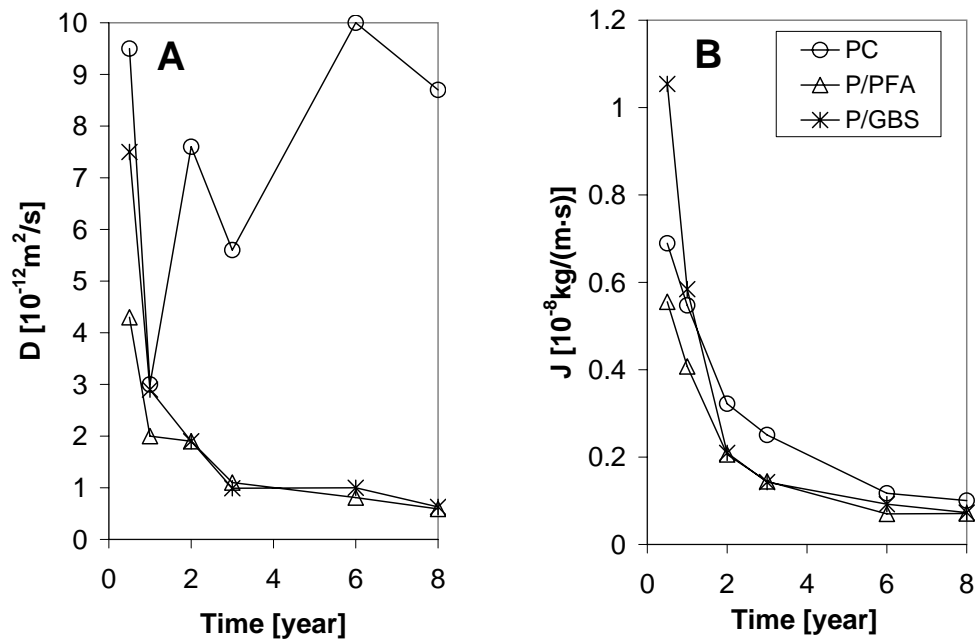


FIG. 8. (A) Diffusion coefficient against time, and (B) mean flux against time calculated from profiles. (Data from Thomas and Bamforth 1999)

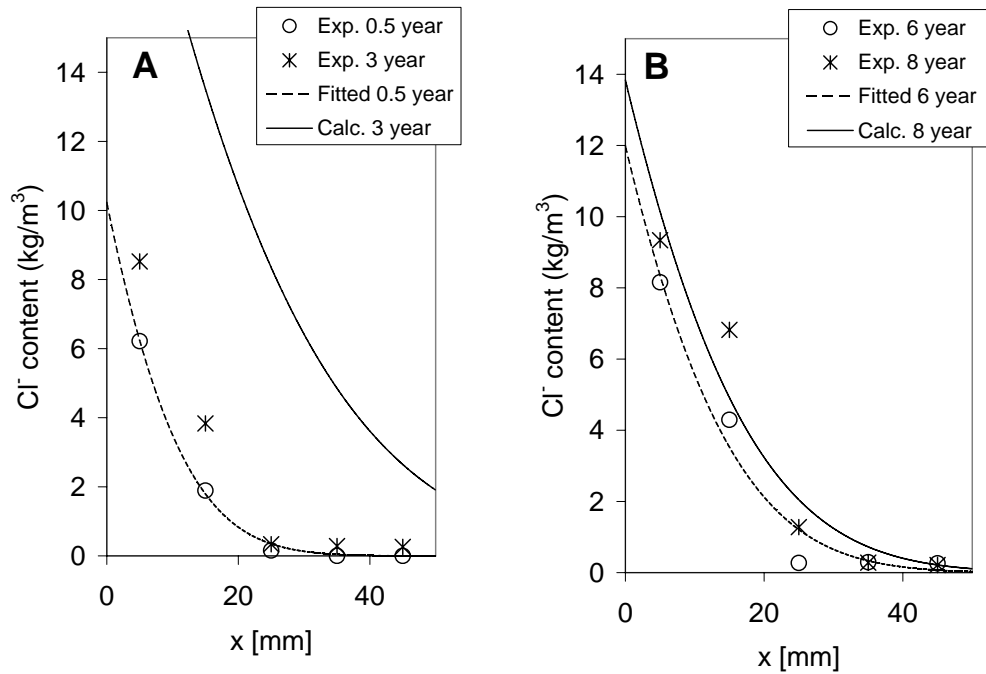


FIG. 9. Predicted profiles from a fitting compared with experimental profiles. (A) Fitted profile at 0.5 year and predicted profile at 3 year, and (B) fitted profile at 6 year and predicted profile at 8 year. (Data from Thomas and Bamforth 1999).