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Pseudo-Ptolemy *De Speculis*

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Introduction

The Text.

The present article provides a new edition of William of Moerbeke's Latin translation of a lost Greek work on mirror optics, together with an English translation that attempts so far as possible to recover the sense of the original Greek text, which was presumably entitled κατοπτρικὰ or περὶ κατόπτρων, "On Mirrors."[1] It has been printed before: first at Venice in 1518 (**ed**), with a reprinting the following year;[2] secondly by Valentin Rose in 1870; and most recently by Wilhelm Schmidt in the second volume (1900) of the Teubner edition of Hero of Alexandria. Largely on account of the manner of its inclusion there, *De Speculis* is now commonly cited as Hero's *Catoptrics*.

The text as given below is transcribed from William's autograph, *Ottobonianus Lat.* 1850 (**O**). All previous editions depend on a somewhat debased version of the text. It was Rose himself who rediscovered the collection of Latin translations of Archimedes and other scientific authors in **O**, but this was in 1884, long after his publication of *De Speculis* from the fourteenth-century Erfurt manuscript *Amplonianus Qu.* 387 (**A**).[3] By the time that Schmidt came to reedit the work, Heiberg had not only reinforced Rose's suggestion that the translations in **O** (or most of them, at any rate) were by William, but also had pointed out clear signs that **O** was William's autograph.[4] Schmidt, however, seems not to have been familiar with Heiberg's article, and his knowledge of **O** was at first only through the medium of a

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[1] I am grateful to Marshall Clagett, G. J. Toomer, and Elaheh Kheirandish, as well as the Biblioteca Apostolica Vaticana, for supplying me with photographs of manuscripts used in this article. I also thank Aven McMaster for helping to weed out errors in the edition.

[2] *Sphera mundi noviter recognita cum commentariis et authoribus* (Venice, dated June 30, 1518); *Sphera cum commentis in hoc volumine contentis* (Venice, dated January 19, 1518 [i.e. 1519 modern reckoning]). Schmidt (1900, 307–308) describes the June issue as the reprint, not realizing that in Venice the new year began with March.

[3] Rose 1884 (*non vidi*).

[4] Heiberg 1890, 3–10.

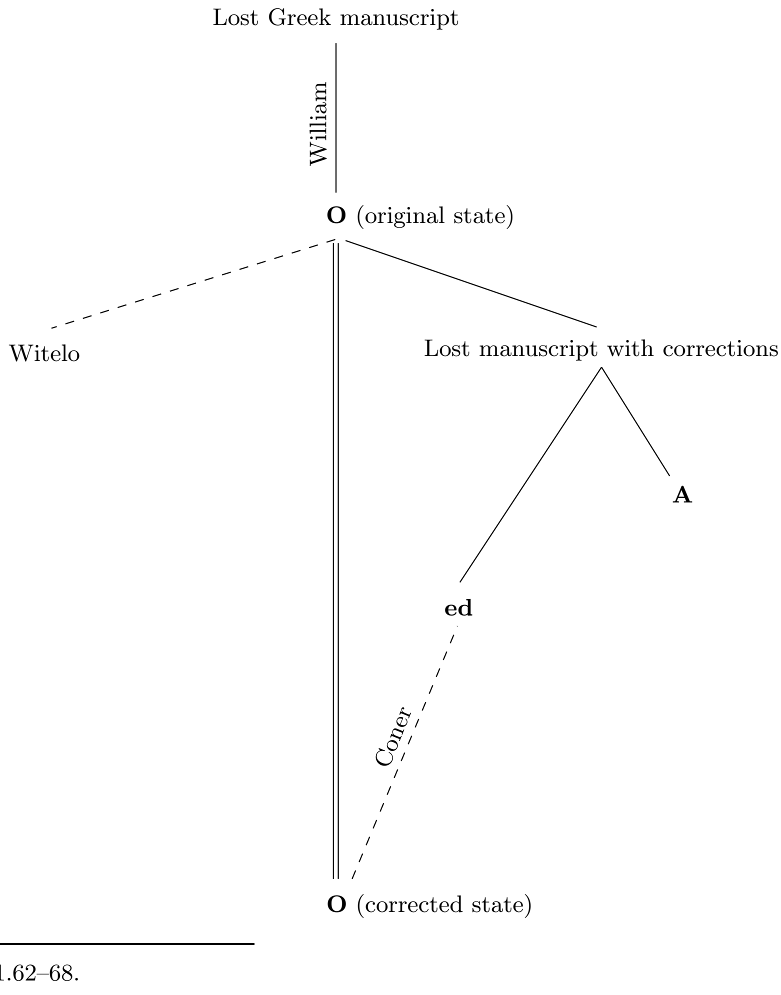
collation by Giuseppe Arsenio, which appears to have been singularly bad. Schmidt suspected at this stage that both **A** and the Venice edition (**ed**) depended on **O**, but this did not hinder him from frequently preferring variant readings in **A** and **ed**, or from adopting numerous conjectural emendations in the text. Schmidt's edition was already set in type, and indeed furnished with a first set of *Nachträge*, when in October, 1900 he had the opportunity (it is not clear whether in person or only through photographs) of checking Arsenio's collation. Seven dense pages of corrigenda added to the end of the volume attest to how much Schmidt discovered that he had missed, including the precious marginalia in Greek that provide clues to many cruxes and lucunae in William's translation. But even these corrections (which most readers probably never discover in any case) do not entirely suffice to make Schmidt's text an accurate representation of William's translation. For one thing, Schmidt, though he no longer questioned that **O** was the sole independent authority for the text, still did not realize that it was an autograph, and hence that its text could not be subjected to extensive emendation. Secondly, Schmidt did not always succeed in distinguishing the second hand that from time to time has altered **O**'s text. Thus he altered passages that we have directly from William's hand, while allowing words to stand that a later corrector substituted for William's.

Editing an autograph is not necessarily a mere matter of faithful transcription; and William's translation of *De Speculis* in **O** confronts us with some interesting textual problems. In the first place, the manuscript does not present a single state of completion of the translation. It is not clear whether William made drafts of his translation before writing it out in **O**. One or two obscurities could be explained as resulting from words missed in recopying of the Latin: should an editor restore such words? William had many second thoughts while he was writing the copy we have, and also when he went back over it. In particular, he often left blank spaces in the translation where the rendering was uncertain, writing the problematic Greek words or other remarks in the margin. Sometimes the blanks were subsequently filled and the marginal notes erased. It is often difficult or impossible to be sure of the exact sequence of William's revisions, or (even with the help of an ultraviolet lamp) to read what has been erased.

We have no evidence that William produced a further fair copy of the translations in **O** or that he made any subsequent alterations beyond those that stand in **O**. One might hope to find traces of such changes in the extensive borrowings from *De Speculis* in the *Perspectiva* of Witelo, who may indeed have been the intended user of the translations in **O**; if a fair copy was made, he (if anyone) would have used it. But Witelo recasts his sources so thoroughly that fine questions of textual detail cannot be settled on the basis of his testimony. On the other hand, **A** and **ed** present a version of the text that often diverges, and diverges intentionally, from what William wrote in **O**. But these are not authoritative changes; on the contrary, they often show themselves to be attempts to correct William's translation without reference to a Greek text and on the erroneous hypothesis that the Latin was corrupt where it was only reflecting the corruptions or obscurities of the Greek.

These variants in **A** and **ed** often coincide with the corrections of the second hand in **O**. Clagett has identified this as the hand of the German scholar Andreas Coner, who owned **O** in the early sixteenth century and made revisions in several of the translations contained in it.[5] Comparison of Coner's readings with the collations reported by Schmidt suggest that Coner corrected **O** against a text resembling **ed**. Where Coner has obliterated William's original writing — and this is, alas, frequent — **A** and **ed** can help us so long as the impulse to correct was Coner's. But where Coner is merely following the printed text, we must either attempt to decipher what he erased, or, where this is not possible, resort to conjecture. Coner also redrew all the figures, but the traces of William's original drawings, which likely imitate his Greek exemplar closely, can usually be seen, and these form the basis of the diagrams in this edition.

The presumed relations between the various states of the text are displayed in the following stemma:



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[5] Clagett 1976, 1.62–68.

Plan of the work.

The text as it is handed down to us is divided into two rather brief "books" (a feature that is suppressed in Schmidt's edition), but according to subject matter the work falls more naturally into four parts: an introduction, a physico-metaphysico-mathematical discussion of reflection and the equal-angle law, a series of geometrical theorems concerning simple optical properties of plane and circular mirrors (this section straddles the division between the two "books"), and finally a collection of problems exhibiting the construction of mirror arrangements to achieve certain practical or thaumaturgical effects.

The introduction begins by distinguishing the two senses of hearing and sight as those that lead to knowledge (the author surprisingly cites Plato as his authority), and he declares there is a science pertaining to each of these senses. Harmonic science is described in neo-Pythagorean terms, with a crude account of the music of the spheres. Then the subject of sight is, without further ado, partitioned into the three divisions of optics proper, dioptrics, and catoptrics. Optics, we are told, has been treated by Aristotle among others, and dioptrics by our author himself in some other work; but he also finds merit in the study of catoptrics. For does catoptrics not show us how to make mirrors that reverse the viewer, or show him with three eyes and two noses, or that let him spy on his neighbours? The author therefore thinks it desirable to record the contributions of his predecessors.

The next, theoretical, part of *De Speculis* sets out to explain what "practically all who have written on dioptrics and optics have wondered about," namely why lines of sight are reflected at equal angles by mirrors. The first point to be established is that lines of sight are straight. For whatever is forced to move with "continuous" speed, for example an arrow, is compelled by the "transmitting force" to travel in the shortest possible path, a straight line; and our sight must travel at "unlimited" speed, since we can see the heavenly bodies immediately when we have opened our eyes. (As has often been remarked, this argument is an early version of "Fermat's Principle" that light always travels by the easiest possible path.) Secondly, smooth surfaces or polished bodies reflect lines of sight, whereas unpolished bodies have tiny voids on their surfaces that fail to repel the incident rays. Thirdly, the author shows geometrically that the shortest of all inflections of a straight line with given endpoints on a straight line or circle (so long as the endpoints are outside the circle) is the one that makes equal angles at the point of inflection. By the "Fermat" principle the line of sight will therefore follow such a reflected path to the seen object.

The third part continues the application of geometry to the theory of mirrors with a proposition that the object of sight will not be visible in a plane mirror if we cover up the point on the mirror where reflections make equal angles. (This theorem has a curious relation to a notorious axiom in Euclid's *Catoptrics* that states that an object will not be seen if we cover up the point on the mirror where the *normal* from the object falls upon the mirror.) Then follow four theorems lifted with little change from Euclid concerning the divergence or concurrence of lines of sight reflected in plane and circular mirrors.

The eight practical problems that make up the greater part of "Book 2" follow no obvious plan, and mix constructions of single mirrors of curved surface with arrangements of several plane mirrors. No use is made here of the contents of the previous sections, except to the extent that the equal-angle law is assumed. First comes a mirror of double curvature that, among other things, reflects the viewer's face with right and left reversed but otherwise in more or less correct proportion. Secondly, there is an arrangement of two plane mirrors joined by a hinge, to produce various kaleidoscopic effects. Thirdly, a "mocking" mirror turns out to be simply a convex cylindrical surface. The fourth, "theatrical" mirror consists of numerous plane mirrors in a concave polyhedral arrangement such that the viewer will see his face (or part of it) reflected in each. The fifth arrangement sets two mirrors on a ceiling so that the viewer will see a figure (actually himself) flying through the air. Then come periscope mirrors, which allow one to snoop on the passers-by in the street. The seventh construction is a perversion of one from Euclid's *Catoptrics*, which showed how to arrange a train of mirrors so that the line of sight is reflected by each of them in turn until it reaches the object; but in *De Speculis* the eye is trivially made to see itself reflected separately in each mirror. Lastly, another periscope arrangement is described by which an image other than the viewer himself is beheld in a large mirror. The work ends abruptly with this proposition.

Author.

De Speculis is ascribed to Ptolemy in **O**, where it immediately precedes Ptolemy's *Analemma*.[6] The Greek manuscript from which William of Moerbeke undoubtedly translated both these works was inventoried in 1295 and again in 1311 as part of the papal library of Greek manuscripts, as follows:[7]

> (1295) *liber Tholomei de resumptione.*
> (1311) *item undecim quaternos mediocris forme, scriptos de lictera greca in cartis pecudinis, in quibus est liber Tholomei de resumptione, perspectiua ipsius, perspectiua Euclidis, et quedam figure Arcimenidis.*

The second description suggests the possibility that this codex (which was apparently not bound when the 1311 inventory was made) had its contents ordered

[6]For detailed description of **O** see Clagett 1976, 1.60–68.

[7]Jones 1986, 19–20. On the lost Greek manuscript see Clagett 1976, 1.54–60.

differently from the sequence of William's translations, beginning with Ptolemy's *Analemma* and following this with *De Speculis* and Euclid's *Catoptrics* (a work that William presumably did not translate, since a Latin version was already available). If so, then it is also possible that the ascription of *De Speculis* to Ptolemy was a conjecture, either by William or already present in the Greek manuscript, on the assumption that an unattributed work immediately following one by Ptolemy was the work of the same author.

Whatever the origin of this attribution, it is unquestionably false, and has been held so by all scholars who have considered the matter since Venturi.[8] Ptolemy's seriousness of purpose, manifested in all his writings, is incompatible with the frivolities of the mirror constructions in *De Speculis*, while the philosophical meanderings of the opening pages are far below his competence. Neither the cosmology nor the concept of the harmony of the spheres in section 1 have any relation to Ptolemy's beliefs as expressed, for example, in the *Planetary Hypotheses* and the *Harmonics*, just as the material visual rays of sections 3–6 have nothing to do with Ptolemy's visual flux in the *Optics*.

There are, on the other hand, arguments for associating the contents of *De Speculis* with Hero of Alexandria, as was first suggested by Venturi.[9] Hero wrote a work called κατοπτρικά: it is cited by the optical compilator Damianus, though not explicity by any other ancient author. According to Damianus, Hero proved that a straight line inflected at equal angles on a "homeomeric" line (i.e. on a straight line or circle) is the shortest of all inflected straight lines sharing the same endpoints, and he further remarked that "if nature does not mean to lead our visual ray about pointlessly, it will reflect it at equal angles." As we have seen, Pseudo-Ptolemy furnishes such a proof in the second part of *De Speculis*. Moreover, there is an obvious similarity of format between *De Speculis* and Hero's *Pneumatics*, with an introduction discussing physical principles followed by a sequence of practical constructions of varied devices. Pseudo-Ptolemy shares with Hero the assumption that matter is composed of particles interspersed with small pockets of void, a theory that can be traced back at least in its fundamentals to Strato. A degree of expertise in both mathematics and handicraft also fits Hero.

Some caution is, however, in order. Pseudo-Ptolemy and Hero may well share general characteristics of style and competence, not because they are one man, but because they belonged to a single intellectual tradition; mechanical engineers in antiquity would likely have read the same books and modelled their own works on them. The specific coincidence that both Hero and Pseudo-Ptolemy derived the equal-angle law from minimal paths also fails to prove that their texts were one and

[8]Venturi 1813 and 1814 (*non vidi*); anticipated by Edward Bernard 1704 (*non vidi*, but reprinted in Fabricius, *Bibliotheca graeca* 2.583).

[9]See also Martin 1854, esp. 52–88; Rose 1864, 290–296; Schmidt 1900, 303–306.

the same, since the mechanical and optical literatures were particularly susceptible to successive borrowings and adaptations of material. It is perhaps significant in the present instance that Pseudo-Ptolemy does not appeal to the metaphysical argument about nature doing nothing in vain. Pseudo-Ptolemy claims in section 2 to have written *de dioptrico*, and at first glance it is tempting to identify this work as Hero's *Dioptra* (περὶ διοπτρᾶς) — but Hero's subject is a surveying instrument named after the sighting apparatus with which it is equipped, whereas Pseudo-Ptolemy is apparently referring to a division, it is not entirely clear which one, of the science of vision. A phrase in the enunciation of section 22, echoed in the introduction (2.8), can only have been written by a Christian or at least an author late enough to be influenced by the diffusion of Christian literature. Lastly, *De Speculis* is suspiciously short, so that the closest relationship it can likely have to Hero's lost book is as a selection or abridgement; this would explain why the introduction (section 2) lists mirror devices that include some that are not described in *De Speculis* as we have it.

Relation to other optical texts.

De Speculis is part of a tradition of technical writings in which borrowing and adaptation are normal expedients of composition. I am aware of the following texts that contain material overlapping or closely related to parts of *De Speculis*:

Euclid, *Catoptrics*. This is the Greek treatise on mirror optics attributed to Euclid. On its disputed authorship see most recently Knorr 1994. I take it to be representative in the main of the state of catoptrics in the third century B.C., and I believe that it is the source of *De Speculis* sections 11–12 and 15–16, and the less direct inspiration of sections 10 and 23.

"Anthemius," *On Burning Mirrors and Other Mirrors*. This brief text, which is not the same as Anthemius' well-known *On Paradoxical Devices*, is not extant in Greek, and I know of it only from the Arabic adaptation by ʿUṭārid ibn Muḥammad in the manuscript Istabul *Laleli* 2759 (See Jones 1987, 4). The attribution to Anthemius is credible but not certain. The fourth and fifth problems in this work are substantially the same as *De Speculis* 24 and 17, while the third and seventh concern mirror devices mentioned in *De Speculis* 2. I believe that this text drew on the same source material as Pseudo-Ptolemy rather than on *De Speculis* itself.

Pseudo-Euclid, *On Mirrors*. The Latin text of this short work (which has nothing directly to do with Euclid's *Catoptrics*), published in Björnbo and Vogl 1912, 97–119, is at least in part a translation of an Arabic text which survives in a rather different version (unpublished) in the manuscript Florence Laur. Or. 152 (see Sabra 1977, 283). Like "Anthemius," it contains versions of *De Speculis* 24 and 17, as well as the rear-viewing mirror arrangement alluded to in *De Speculis* 2. The relationship between Pseudo-Euclid and "Anthemius" must be close, but it is not clear to me which, if either, copies from the other.

Witelo, *Perspectiva*. Witelo made extensive use of William's translations of mathematical and scientific writings in **O**. The following is a concordance of the sections of *De Speculis* adapted (sometimes quite creatively) by Witelo and the relevant propositions in the *Perspectiva*.

Pseudo-Ptolemy	*Witelo*
7	1.17
8	1.18
11	5.47
17	9.35
20	5.58
21	5.59
22	5.57
23	5.61
24	5.56

Abbreviations used in the Apparatus

O	*Ottobonianus Lat.* 1850
Coner	Hand of Andreas Coner in **O**
A	*Amplonianus Qu.* 387
ed	Venice edition, June 30, 1518

Notes written in the margins of **O** are transcribed between the text and the apparatus. Not all notes in Coner's hand are reported.

Latin Text

[0] |[1]Claudii Ptolọmei de speculis. f. 60r c. 2

[2]Incipit liber primus.

[1] [1]Duobus sensibus existentibus per quos fit uia ad sapientiam secundum Platonem, auditu scilicet et uisu, amborum speculatio. [2]de hiis que auditus musica consistit, symfoniarum et armoniarum scientia et, ut summatim dicatur, melodiose et armonizate nature speculatio. [3]de eo enim quod est coordinatum esse mundum secundum musicam armoniam, multa et uaria prodiit ratiocinatio. [4]distributo enim toto celo in speras octo numero, uidelicet septem planetarum et in continentem omnes et ferentem non erraticas, accidit in ipsis processum astrorum melodiosum et armonizatum existere propter conformem uigorem motuum inter ipsa, sicut et in instrumento lyre melodizant corde. [5]sonos enim quosdam intelligere oportet ex processu astrorum per aerem, et hos quidem grauiores ipsorum, hos autem magis acutos, sicut hec quidem tardiorem, hec autem celeriorem faciunt motum. [6]quo enim modo aiunt pulsa corda fluctuantem intelligimus aerem, ita et astris per zodiacum delatis cogitare oportet alteratum et transmutantem continue aerem bonam contemperantiam nobis exhibere.

[2] [1]negotium autem quod circa uisus diuiditur in opticam, id est uisiuam, et dioptricam, id est perspectiuam, et katoptricum, id est inspectiuum negotium. [2]et opticum quidem oportune ab hiis qui ante nos descriptum est et maxime ab Aristotele. [3]de dioptrico autem a nobis in aliis dictum est copiose quanta uidebantur. [4]uidentes autem et katoptricum negotium esse dignum studio — habet enim quandam admirabilem speculationem. [5]per ipsum enim construuntur specula ostendentia dextra dextra et sinistra similiter sinistra, communibus speculis contrapatenti[a] nature et contraria ostendentia. [6]est autem per ipsa uidere posterius apparentes et se inuersos et supercapitales habentesque tres oculos et duos nasos et luctus instar dispersis partibus faciei. [7]non autem ad speculationem utilis existit, sed et ad oportunitates necessarias. [8]quomodo enim non bene utile quis existimabit degentes in habitatione auersa uidere, si contingat, presentes in rymis quot sint et quid agentes existant? [9]aut quomodo non utique mirabile existimabit alias considerare

0.1 (*in upper margin of page, erased, not in William's hand*) claudii ptolemei de speculis incipit liber primus

0.1 (Ptol)o(mei): *sic A, ed*: e *Coner in ras.* || 1.2 <h>armoniarum *add. Coner* | <h>armonizate *add. Coner* || 1.3 <h>armoniam *add. Coner* || 1.4 sp<h>eras *add. Coner* | <in> septem *add. A, ed, Coner* | <h>armonizatum *add. Coner* || 2.5 (contrapatienti)a: -bus *A, ed, Coner in ras.* || 2.7 existit <tantum> *add. ed, Coner* || 2.8 a<d>uersa *add. ed, Coner* | (rym)i(s) *supra O* per speculum nocte et die instantem horam per apparentia ydola? [10]quot enim nocte aut die existunt hore, tot et ydola apparent, et etiam si pars diei extiterit, et ydoloapparebit. [11]quomodo autem et non mirabile existimabit quis per speculum neque se ipsum neque alium uidere, solum autem quodcumque quis elegerit? [12]tali igitur existente negotio, puto necessarium existere accepta ab hiis qui ante nos descriptione dignificari, ut in nullo deficiat negotium.

[3] [1]dubitatum est itaque fere ab omnibus qui de dioptrico et optico scripserunt negotio propter quam causam in speculis radii a nobis incidentes refringuntur et refractiones in angulis equalibus faciunt. [2]quod autem secundum effusiones rectarum a uisu uideamus, sic consideretur. [3]omnia enim quecumque feruntur continua uelocitate, hec in recta linea feruntur, sicut uidemus sagittas emissas ab arcubus. [4]propter uiolentiam enim emittentem conatur quod fertur ferri linea breuissima in distantia, non habens tempus tarditatis, ut et feratur linea maiori in distantia, non sinente uiolentia transmittente. [5]propter quod utique, propter uelocitatem, conatur breuissima ferri. [6]recta autem est minima linearum habentium eadem ultima.

[4] [1]quod autem et radii emissi a nobis |uelocitate infinita ferantur, hinc est addiscere. [2]quando enim post clausuram oculorum respexerimus ad celum, non fit aliqua distantia temporis pertingentie ipsorum ad celum. [3]simul enim cum aspicere uidemus astra, cum tamen, ut est dicere, sit distantia infinita. [4]et si ergo maior utique esset hec distantia, idem accideret utique, ut ex hoc palam sit quod uelocitate infinita emittuntur emissi radii. [5]propter quod utique interruptionem non habent neque circuitionem neque fractionem accipient aliquam, minima autem, scilicet recta, ferentur. f. 60v c. 1

[5] [1]quod quidem igitur secundum rectam uideamus, sufficienter dictum est. [2]quod autem radii incidentes speculis, adhuc autem et aquis et omnibus planis corporibus refringuntur, nunc ostendemus. [3]politorum enim corporum natura existit in superficies ipsorum spissas esse. [4]specula igitur ante politionem quidem habebant aliquas raritates, quibus radii incidentes non poterant repelli. [5]poliuntur autem a tritione quatinus loca rara impleantur a subtili substantia. [6]deinde sic incidentes radii spisso corpori repelluntur. [7]sicut enim lapis emissus cum uiolentia et appulsus spisso corpori resultat, puta ligno alicui aut muro, molli autem ut lane aut alii tali quies<cit>, quia uis emittens assequitur et in duro quidem cedere non potens adhuc prosequi et mouere emissum, molli autem incidens iacet et abscedit ab emisso, eodem modo et radii a nobis uelocitate multa delati, ut demonstratum est, et

4.5 (*at* circuitionem) *illegible Greek? word, erased* || 5.7 (*at* quia uis, *erased, reading uncertain*) nota

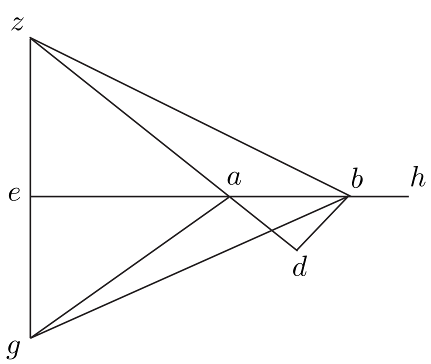
2.9 y(dola): i *Coner* || 2.10 y(dola): i *Coner* | y(dol)o: i(dol)um *ed, Coner* || 3.3 *post* feruntur *rasura in O* (sic consideratur *legi potest*) || 3.4 (transmi)t(tente) *supra O* || 3.5 propter: patet *ed, Coner* || 5.7 (quies)cit *Coner in ras.*

appulsi spisso corpori refringuntur. [8]in aquis autem et uitris omnes refringuntur, quia habent utrasque substantias raritates componunturque ex subtilium partium rebus et solidis corporibus. [9]per uitrum enim et per aquas uidemus nos ipsos et ultra iacentia. [10]in palustris enim aquis que in fundo uidemus et per uitra et que ultra iacent. [11]quicumque enim radii solidis corporibus incidunt ipsi repulsi refringuntur, quicumque autem per rara corpora penetrant ipsi ultra iacentia uident. [12]propter quod utique in talibus non perfecte uidentur que representantur, quia non omnes radii ad ipsa refringuntur, sed quidam, ut dictum est, per raritates exterminantur.

[6] [1]quod quidem igitur incidentes politis corporibus refringantur, sufficienter demonstratum esse putamus. [2]quod autem et refractiones faciant in angulis equalibus in speculis planis et circularibus, per eadem demonstrabimus. [3]celeritati enim incidentie et refractionis necessarium est rursum [et] per ipsas minimas rectas conari. [4]dico igitur quod omnium incidentium et refractorum in idem radiorum minimi sunt qui secundum equales angulos in speculis planis et circularibus. [5]si autem hoc, rationabiliter in angulis equalibus refringuntur.

[7] [1]sit enim speculum planum •ab•, uisus autem signum •g•, uisum autem •d•, et incidat ipsi quẹ •ga•. [2]et copuletur que •ad•, et sit equalis angulus qui sub •eag• angulo qui sub •bad•. [3]et alius radius similiter incidat qui •gb•, et copuletur qui •bd•. [4]dico quod minores sunt qui •ga•, •ad• quam •gb•, •bd•.

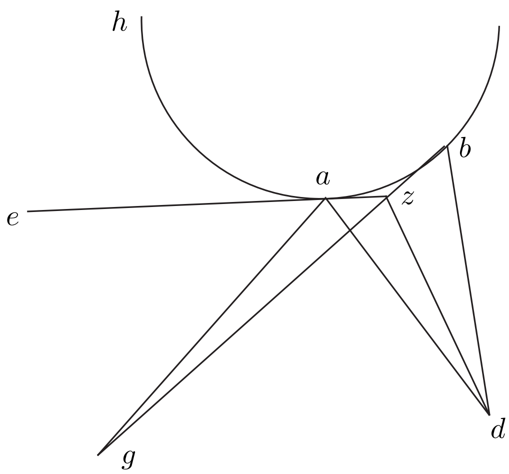
[5]ducatur enim a •g• super •ab• perpendicularis que •ge•, et educantur que •ge•, •da• ad •z•, et copuletur que •zb•. [6]quoniam equalis est qui sub •bad•, hoc est qui sub •zae•, ei qui sub •eag•, sed et recti qui apud •e•, equalis ergo que quidem •za• ipsi •ag•, que autem •zb• ipsi •bg•. [7]quoniam igitur minor est que •zd• quam •zb•, •bd•, equalis autem que quidem •za• ipsi •ag•, que autem •zb• ipsi •bg•, minores ergo sunt que •ga•, •ad• quam •gb•, •bd•. [8]quia enim equalis est qui sub •eag• ei qui sub •bad•, sed angulo quidem qui sub •eag• est minor qui sub •ebg•, angulo autem qui sub •bad• est maior qui sub •hbd•, multo ergo maior qui sub •hbd• quam qui sub •ebg•.



5.10 palustris: perlust^{i}s *O* || 5.12 exterminantur: ext^{r}iantur *O* || 7.1 ipsi <radius> *add. A, ed, Coner* | que: qui *Coner in ras.* || 7.2 que: qui *Coner*

[8] [1]sit etiam speculum circulare, cuius periferia sit que •ab•, uisus autem •g•, uisum autem •d•, et incidant in equalibus quidem angulis que •ga•, •ad•, in inequalibus autem que •gb•, •bd•. [2]dico quod minores sunt que •ga•, •ad• quam •gb•, •bd•.

[3]ducatur enim |contingens que •eaz•. [4]equalis ergo est qui sub •hae• angulus f. 60$^{\mathrm{v}}$ c. 2
ei qui sub •baz•. [5]et reliquus qui sub •eag• est equalis ei qui sub •zad•. [6]si ergo copuletur que •zd•, propter prius demonstratum minores sunt que •ga•, •ad• quam •gz•, •zd•, que autem •gz•, •zd• sunt minores quam •gb•, •bd•. [7]que ergo •ga•, •ad• sunt minores quam •gb•, •bd•.

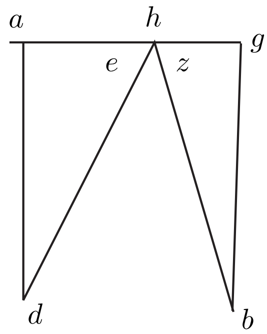


[9] [1]uniuersaliter igitur in speculis etsi non in angulis equalibus refringi possunt radii incidentes, oportet considerari in speculo signum ut radius a uisu incidens et refractus ad id quod uidetur faciat simul utrumque, scilicet incidentem et refractum, minorem omnibus similiter incidentibus et refractis.

[10] [1]in planis speculis est aliquis locus quo apprehenso non adhuc uidetur ydolum. [2]sit enim speculum planum quod •ag• aut in recta sibi, oculus autem •b•, uisibile autem •d•, et perpendiculares ducantur ad speculum que •ad•, •bg•, et secetur que •ag• penes •h•, ita ut sit ut que •ad• ad •bg• que •ah• ad •hg•. [3]dico itaque quod apprehenso loco •h• non adhuc uidetur •d•.

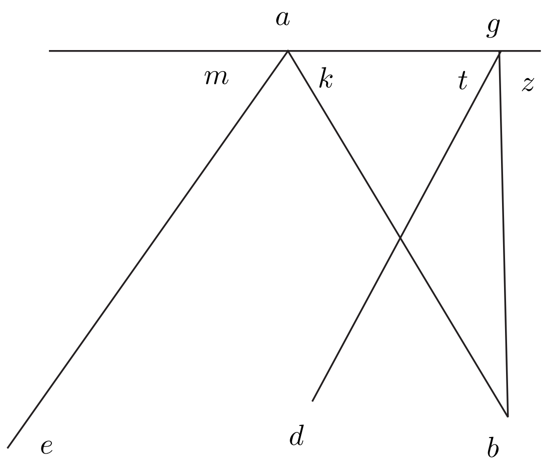
[4]copulentur enim que •bh•, •hd•. [5]propter proportionem itaque similia erunt trigona. [6]equalis enim est angulus •e• angulo •z•, quare per signum •h• apparebit •d•. [7]apprehenso ergo loco cera uel aliquo alio non adhuc uidebitur •d•. [8]si autem signum •h• excidat a speculo, apparebit ydolum in speculo. [9]omnes enim radii incidentes speculo in angulis equalibus refringentur.

8.6 *post* •gb• *rasura in O* || 9.1 *post* uidetur *rasura in O* (minorẹṃ *legi potest*) || 10.1 adhuc: amplius *Coner* | y(dolum): i *Coner* || 10.2 *post* ut que: •ag• *deletum in O* || 10.3 adhuc: amplius *Coner* || 10.7 adhuc: amplius *Coner* || 10.8 y(dolum): i *Coner*



[11] [1]in speculis planis uisus refracti neque concurrent inuicem neque equedistantes sunt.

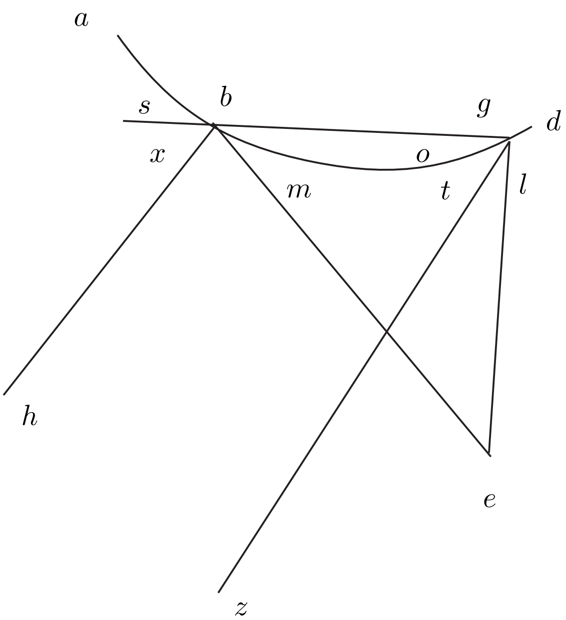
[2]sit enim speculum planum •ag•, uisus autem •b•, et incidant que •gd•, •ae•. [3]equales ergo sunt anguli •z•, •t•; maior autem est angulus •z• angulo •k•, hoc est angulo •m•. [4]maior ergo est angulus •t• quam •m•. [5]quẹ ergo •gd•, •ae• neque equedistantes sunt neque concurrunt ex parte uersus •d•, •e•.



[12] [1]in speculis conuexis uisus refracti neque concidunt inuicem neque equedistantes sunt.

[2]sit enim speculum conuexum •abgd•, uisus autem •e•, et incidant radii qui •eg•, •eb•; refringantur etiam que •gz•, •bh•. [3]equalis ergo est angulus quidem •t• angulo •l•, et angulus •m• angulo •x•. [4]propter hoc itaque maior est angulus •ot• quam •sx•. [5]que ergo •gz•, •bh• neque equedistantes sunt neque concidunt ex parte •z•, •h•.

11.2 que: qui *Coner* || 11.5 que: qui *Coner* || 12.2 que: qui *Coner* || 12.4 •sx•: sit •x• *ed, Coner*

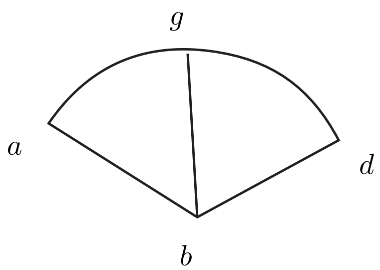


[13] [1]Explicit primus.

[14] [1]Incipit secundus.

[15] [1]in speculis concauis quando oculus super centrum positus fuerit, uisus refracti ad oculum refringentur.

[2]sit speculum concauum quod •agd•, cuius centrum •b•. [3]apud •b• autem iaceat oculus, et incidant radii qui •ba•, •bg•. [4]equales ergo {sunt refractiones ergo} facient angulos apud periferiam, quia anguli semicirculorum equales sunt. [5]refractiones ergo [cum?] ipsis •ba•, •bg•, •bd• erunt. [6]apud signum ergo •b• concurrent, hoc est apud oculum. [7]ex hoc autem manifestum quod, si fiat speculum concauum uelut spericum, in centro autem spere oculus positus fuerit, [nichil] aliud quam oculus in speculo apparebit.

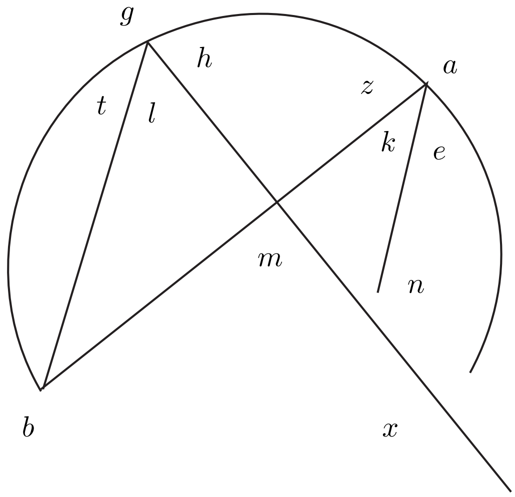


[16] [1]in speculis concauis, quando in circumferentia oculus positus fuerit, refracti radii inuicem concurrent.

15.4 sunt refractiones ergo *deleui* || 15.5 (ergo) in (ipsis) *A, ed, Coner in ras.* || 15.7 sp<h>ericum *add. Coner* | nichil *conieci*: nihil *A, ed, Coner in ras.*

[2]sit speculum concauum •bga•, uisus autem •b•. [3]et incidant radii •bg•, •ba•, refringantur autem •gx•, •an•. [4]dico quod que •gx•, •an• concurrent uersus •n•, •x•.

[5]quoniam enim maior est que •ba• quam •bg•, maior ergo est angulus •z• angulo •t•. [6]sic et qui •e• quam •h•. [7]reliquus ergo qui •l• maior angulo •k•. [8]angulo autem •l• maior qui •m•. [9]maior ergo est angulus •m• quam •k•. [10]que ergo •gx•, •an• concurrent ex parte •n•, •x•.



[17] [1]speculum dextrum construere.

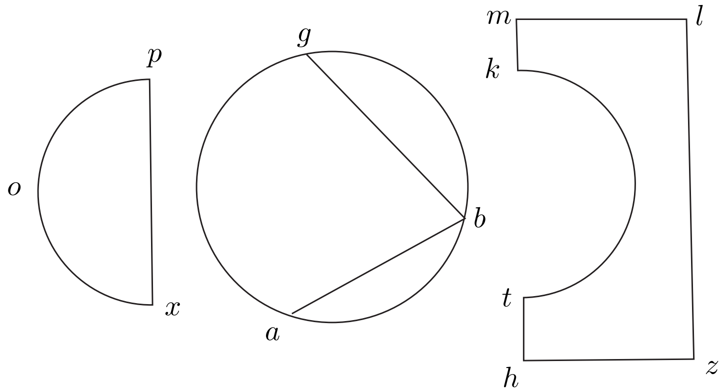
[2]exponatur circulus qui •abg• in magnitudine qua uolumus construere speculum. [3]ed inscribatur in ipsum latus quidem pentagoni quod •ab•, exagoni autem quod •bg•, et secentur [apud?] apsides •aeb•, •bzg• abscisas a |rectis •ab•, •bg• f. 61r c. 1
ex circulo (*lacuna*) eorum qui quidem altitudinis ad apsidem •aeb• suspensus sit concauus qualis qui •zhtklm•, latitudinis autem (*lacuna*) qui ad apsidem •bzg• sit conuexus, qualis qui •xop•. [4]et preparetur speculum de achario rectangulum altitudinem quidem habens equalem recte •ab•, latitudinem autem equalem ipsi •bg•, superficierum autem eam quidem que longitudinis conuexam adoperatam ad concauum superficiem (*lacuna*) •aeb•, eam autem que latitudinis concauam adoperatam ad conuexaṃ [(*illegible word*) per]iferiam (*lacuna*) •bzg•.

17.3 (*at lacuna following* circulo) ἐμβολεῖς quasi iniectae (*above this Coner has written* id est limae immissoriae) | (*at lacuna following* latitudinis autem) ἐμβολεύς || 17.4 (*at first occurrence of* adoperatam) ἀπειργασμένην | (*at lacuna following* superficiem) ἐμβολέως | (*at lacuna following* periferiam) ἐμβολέως

16.10 •n•, •x•: •nx• *O* || 17.3 apsides: abscides *ed, Coner* | •aeb•, •bzg• *mg. O* | eorum: horum *A, ed, Coner* | apsidem (*bis*): abscidem *ed, Coner* | •zht<f>klm• *add. Coner* | qui ad: qui sit ad *O, sed* sit *expunctum* (*restat tamen in A, ed*) || 17.4 (conuexa)m per(iferiam) *A, Coner in ras. compendiis spretis* (connexam pariferiam *ed*)

[5]apparent autem dextra dextra et sinistra similiter. [6]et distante quasi duobus cubitis apparet [y]dolum commensuratum et simile uero. [7]magis autem distante uidebitur apparentis [y]dolum in anterius protendi; propius autem accedente uisu ut ad conuexam superficiem speculi, fit informe [y]dolum apparentis, et magis accedente adhuc magis. [8]conuerso etiam eo quod speculatur, ex contrariis adhuc accedente prolixius [y]dolum apparet, et facies consimilis speciei equi fit. [9]et semper magis inclinato speculo et [y]dolum inclinatum apparet. [10]propter quod et oportunum est ipsi preparare sedem uolubilem in qua conseruatur speculum, ut apparens [y]dolum quandoque quidem habeat capud sursum, quandoque autem deorsum, pedes autem sursum.

[11]si autem duarum facierum fiat speculum, hoc est ex posterioribus et anterioribus partibus, dextra dextra apparebunt, ex posterioribus autem supercapitales demonstrabit sicut antipodas.



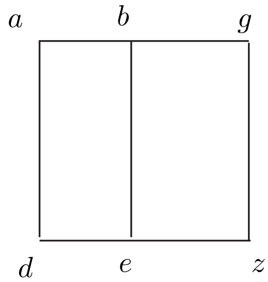
[18] [1]speculum construere quod dicitur polytheoron, id est multiụịdum. [2]facit autem dextra dextra apparere, adhuc autem et (*lacuna*) motum facit apparere, (*lacuna*) attestatur quia Pallas genita fuit ex uertice Iouis, multas facies [(*illegible word*)], unum digitum facit multos, deinde (*lacuna*) distracta boum capita manifestat.

[3]sint duo specula erea rectangula plana ad regulam operata secum inuicem iacentia que •aeg• super eandem basem existentia scilicet •dz•, ita ut latus •be• sit

17.8 (*at* speculatur) ἐνοπτριζομένου || 17.10 (*at* sedem uolubilem) στυλον χήσιον (*below this Coner has written* columpnam fusilem) || 18.2 (*at lacuna following* autem et) ~~Δ~~ nescio ΔανΔ (*two Latin letters crossed out*) τριкαρον tricapitum | (*at lacuna following* apparere) χόρευουσας νείκας ἀποτελεῖ cedentes uictorias | (*at lacuna following* deinde) δία σπώμενα βουκέφαλα φαίνει

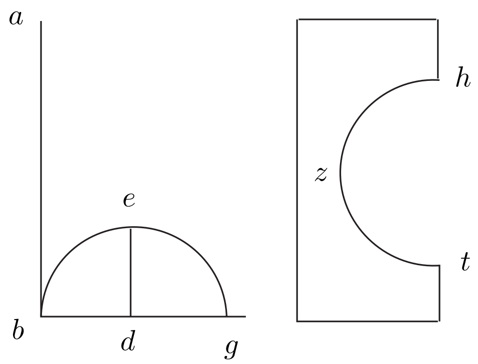
17.6 y(dolum): i *Coner in ras.* || 17.7 y(dolum): i *Coner in ras.* (*bis*) || 17.8 y(dolum): i *Coner in ras.* || 17.9 y(dolum): i *Coner in ras.* || 17.10 y(dolum): i *Coner in ras.* || 18.1 multiuidum: multitudinum *A* multinidum *ed* multitudum *Coner* || 18.2 *ante* motum *rasura in O* | (facies) manifestat (unum) *A, ed, Coner in ras.*

commune amborum. [4]habeant autem specula altitudinem •be• duplam latitudinis •ab•. [5]placet autem quibusdam facere altitudinem emioliam latitudinis. [6][nichil] autem differt gratia bone proportionis facere quamcumque mensuram quis uoluerit. [7]ut igitur aperiantur et claudantur specula, reuoluantur secundum commune ipsorum latus •be•, secundum nihil uariantia [y]dolis (*lacuna*) esse. [8]et erit factum.



[19] [1]speculum construere quod dicitur mokeion.

[2]exponantur due recte que •ab•, •bg•, et sit que •ab• dupla ipsius •bg•, uel proportionem aliam habeat quamcumque uoluerit. [3]et sit que quidem •ab• altitudo speculi, que autem •bg• latitudo. [4]et centro quidem extremitatibus latitudinis, distantia autem ipsa •bg•, periferie descripte secent inuicem penes •d•, et rursum centro quidem •d•, distantia autem utracumque ipsarum •db•, •dg•, periferia describatur concaua que •beg•. [5]et sit factus |ad eam que in recta •beg• periferiam f. 61r c. 2
•beg• concauus (*lacuna*) qui •zht•. [6]et preparetur speculum ereum rectangulum habens altitudinem equalem ipsi •bag•, latitudinem autem equalem ipsi •beg• recte, superficierum autem eam quidem que altitudinis rectilineam, eam autem que latitudinis conuexam ad concauum embolea •zht• operatam. [7]et erit facta cylindri sectio, figura conuexe superficiei.

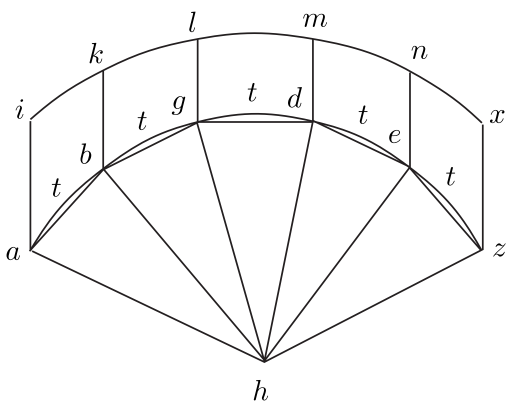


18.7 (*at lacuna following* ydolis) ἀσταραποδίστων nescio credo tamen quasi non impeditis || 19.5 (*at lacuna following* concauus) immissorum pro lima dicitur ἐμβολεύς

18.5 <h>emioliam *add. Coner* || 18.6 nichil *conieci*: nihil *A, ed, Coner in ras.* || 18.7 y(dolis): i *Coner in ras.* || 19.4 autem[2]: -em *in ras. in O* || 19.5 •beg•[1]: e *expunctum in O* (*Coner?*) || 19.6 •bag•: a *expunctum in O* (*Coner?*) | •beg• e *expunctum in O* (*Coner?*)

[20] [1]speculum construere quod dicitur theatrale.

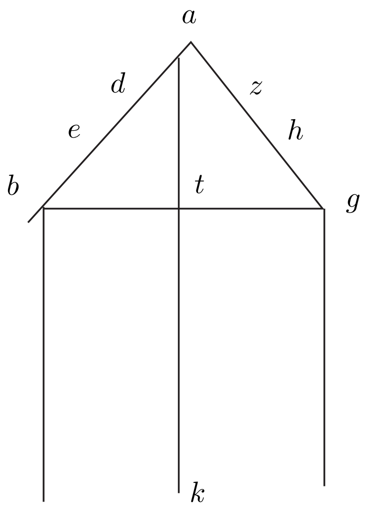
[2]exponatur circuli periferia contingens que •abgd•, centrum autem ipsius sit •h•, et sit diuisa que •abgdez• in partes equales quinque, scilicet •atb•, •btg•, •gtd•, •dte•, •etz•, et copulentur subtendentes periferias recte que •ab•, •bg•, •gd•, •de•, •ez•. [3]et intelligantur a centro ad signa •a•, •b•, •g•, •d•, •e•, •z• copulate recte que •ha•, •hb•, •hg•, •hd•, •he•, •hz•. [4]et ablatis hiis que super •ab•, •bg•, •gd•, •de•, •ez• uadunt periferiis, scilicet •atb•, •btg•, •gtd•, •dte•, •etz•, super rectas •ab•, •bg•, •gd•, •de•, •ez• erigantur specula erea suspensa, figura quidem tetragona, superficiebus autem plana, equedistantia ipsis •ai•, •bk•, •gl•, •dm•, •en•, •zx•, tangentia inuicem, ita ut sint communia ipsorum latera que •kb•, •lg•, •md•, •ne•, inclinata autem ita ut anguli contenti ab •ai• •ik•, •bk• •kl•, •gl• •lm•, •dm• •me•, •en• •nx• sint equales angulis contentis ab •ha• •ab•, •hb• •bg•, •hg• •gd•, •hd• •de•, •he• •ez• rectis, et ut sint que quidem per •abgdez• plana [(*illegible word*)] supposito plano, latera autem •ik•, •kl•, •lm•, •mn•, •nx• stantium speculorum eleuatorum in quibus planum iaceant equedistantia plano quod per signa •ab•, •bg•, •gd•, •de•, •ez•. [5]et erit factum. [6]specula enim super rectas •ab•, •bg•, •gd•, •de•, •ez• iacentia erunt nuentia ad centrum •h•.



[21] [1][alipe?]dem preparare oportunum.

[2]esto trigonum rectangulum •abg•, et in duo equa secetur que •bg• penes •t•, et super lineam quidem •ag• planum •zh• speculum sit •me•, quod autem super •ag• quod •de• planum speculum. [3]et sit qui quidem intuetur •tk•, oculus autem ipsius signum •t•, intuens in utrumcumque uoluerit speculorum. [4]et erit factum. [5]iacente autem altero speculo, dico autem adnuente et abnuente existente retro ueniet radius usque ad signum quod est in calcaneo intuentis in speculo, et putabit uolare.

20.2 •abgd<ez>• *add. Coner* || 20.3 •hg• *supra in O* || 20.4 uadunt: firmantur *uult Coner in mg.* | •etz• *supra in O* | tangentia inuicem *supra in O* | (plan)a: -o *ed, Coner* | in (supposito) *A, ed, Coner in ras.* || 21.1 alipe(dem) *conieci*: aliter i(dem) *A, ed, Coner in ras.* || 21.2 *post* speculum[1] *rasura in O* (autem *legi potest*) | •me• *expunctum in O* (*Coner?*)



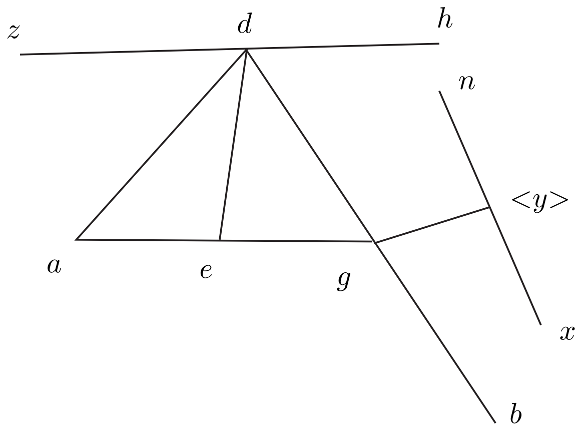
[22] [1]in aliqua domo fenestra existente, oportunum sit ponere in domo speculum per quod apparebunt qui in auerso uenientes siue in rymis siue in plateis conuersantes, uidentes in aliquo dato loco, in domo tamen.

[2]sit qui quidem in domo locus •a•, quod autem uolumus apparere •b•, fenestra autem •g•, et copulata que •bg• educatur et incidat in pariete domus et planiciei secundum •d•, et copuletur que •ad•. [3]oportebit ergo per •ad• radium quendam procedentem a uisu et speculo incidentem secundum •d• in angulo equali refringi ad •b•. [4]iaceat igitur speculum •zh• rectum ad planum quod per •ad•, •db•. [5]equales ergo erunt anguli qui sub •zda•, •hdb•. [6]secetur itaque in duo equa angulus qui sub •adb• per rectam •de•. [7]que ergo •de• ad rectos est speculo •zh•. [8]quoniam igitur datum est utrumque ipsorum •bge•, positione ergo radius ipsorum •bgd•; positione autem et cui incidit muro. [9]datum ergo •d•. [10]sed et •a•. [11]positione ergo que •ad•. [12]datus est ergo angulus qui sub •adb•. [13]et in duo equa secatur per rectam •de•. [14]positione ergo que •de•. [15]et a dato •d• ad rectos producta est super •zh•. [16]positione ergo et planum, hoc est speculum.

[17]componetur itaque sic. [18]iaceat apud signum •g• (*lacuna*) •nygx•, et moueatur circa •d•, donec utique per ipsum uideantur signum •b•. |[19]consideretur signum f. 61^{v} c. 1
aliquod planorum continentium domum. [20]et consideratum sit •d•, et copuletur que •ad•, et in duo equa secetur angulus qui sub •adg• per rectam •de•. [21]secabitur itaque sic, si copulata que •ag• recta secetur penes •e•, ita ut sit ut que •ad• ad •dg•. [22]utraque enim ipsarum <data>; data itaque •ae• ad •eg•. [23]construatur

22.2 (*at* planiciei) τη ὀρόφη tecto || 22.18 (*at lacuna following* •g•) δϊοπτρα instrumentum quo per uisus iudiciatur distantia uel quantitas

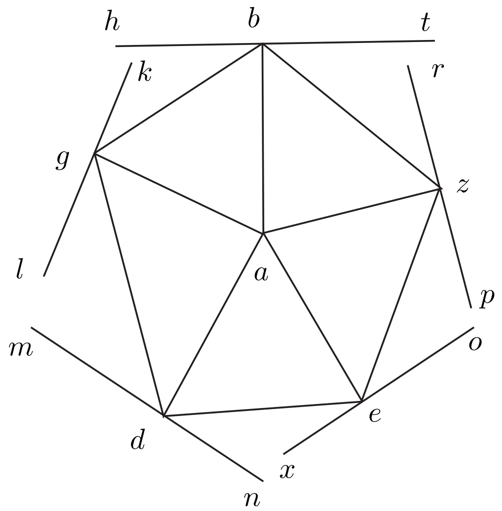
22.2 *post* autem[1] *rasura in O* || 22.3 •ad•: •adg• *prius scripsit O, sed* g *rasit* || 22.8 •bge•: e *rasum in O* (*Coner?*) || 22.18 dioptra *in lacunam inseruit Coner* | •d• *expunctum et* g *supra Coner* itaque et speculum planum, et iaceat ad angulos rectos ipsi •de•, ita ut medium ipsius sit signum •d•, et ita apud signum •d• uisiones habens uidebit que apud •b• posita qualicumque exstiterint et que in ante.



[23] [1]in pluribus speculis positis in ordine aliquo possibile est idem ydolum uideri.

[2]sit quod uolumus per plura specula uideri •a•, [(*illegible word*)] quotcumque fuerint specula equilatera multiangula uel equiangula consistant que •b•, •g•, •d•, •e•, •z•, quorum medium sit •a• centrum circuli comprehendentis ipsa. [3]et copulentur que •ab•, •ag•, •ad•, •ae•, •az•, et hiis ad rectos angulos ducantur que •ht•, •kl•, •mn•, •xo•, •pr•, et in hiis iaceant specula recta ad planum •bgdez•. [4]dico quod uisus incidentes speculis reflectuntur ad •a•.

[5]incidentes enim facient angulos rectos ad specula. [6]refractiones ergo habebunt in se ipsos. [7]reflectuntur ergo ad •a•.



23.1 y(dolum): i *Coner* || 23.2 *post* •a•: et *A, ed, Coner in ras.* | *post* specula *add.* tot laterum figura *Coner in mg.* | m(ultiangula): a *supra Coner* | uel *expunctum et* et *supra Coner* | quorum *expunctum et* cuius *supra Coner* | ipsa *in* ipsam *uertit Coner*

[24] [1]speculum in dato loco ponere, ita ut omnis accedens neque se ipsum neque alium aliquem uideat, solam autem ỵmaginem quamcumque quis preelegerit.

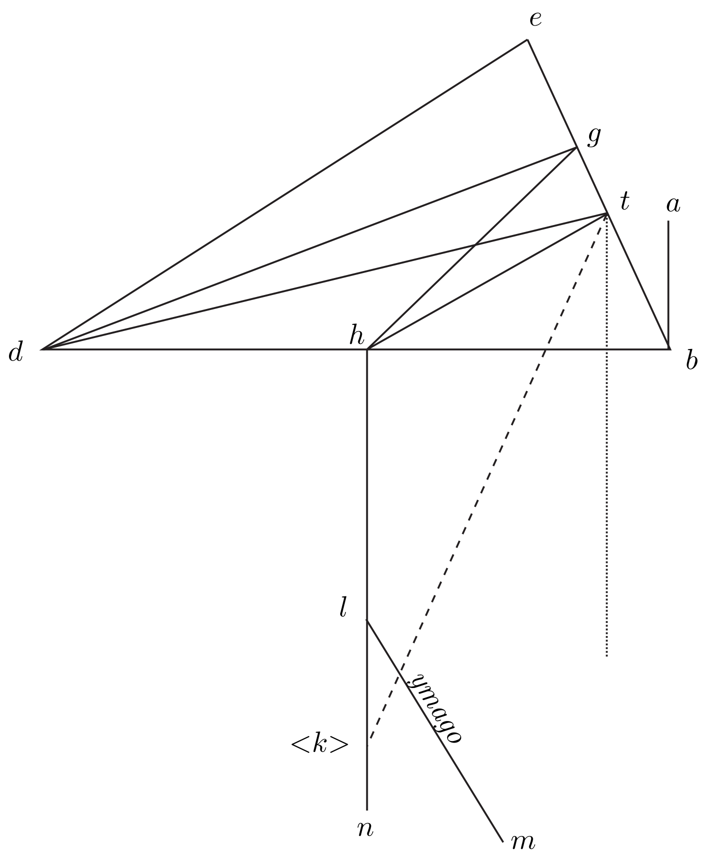
[2]sit enim murus, in quo oportet speculum poni, •ab•, speculum autem sit inclinatum ad ipsum in angulo aliquo. [3]comensurate autem utique habeat ac si fieret angulus tercie partis recti. [4]et sit superficies speculi que •bg•, et a •b• ipsi •ab• ad rectos angulos intelligatur que •bd•, in qua iaceat signum uisus •d•, ita ut perpendicularis ab ipso producta ad speculum •bg• extra ipsum cadat. [5]sit autem. [6]et a •d• ad extremitatem speculi ipsum •r• copuletur que •dg•, et angulo qui sub •edg• equalis consistat qui sub •hgd•. [7]si ergo incidat aliquis radius a •d• uisu termino speculi •g•, reflectetur ad •h•. [8]ducatur igitur ab •h• ipsi •db• ad rectos angulos que •hn•. [9]et incidat alius radius qui •dt•, et copuletur que •ht•. [10]maior ergo est angulus qui sub •bth• quam qui sub •etd•. [11]consistat igitur ei qui sub •gtd• equalis qui sub •btk•. [12]secat ergo [(*illegible phrase*)] •hn•. [13]similiter et omnes incidentes speculo radii reflexi [(*illegible phrase*)] •hn•. [14]ducatur igitur ipsi •gb• speculo planum equedistans quod •lm• iacens intra •hn• et sectum a radio reflexo. [15]quare manifestum quod [nichil] aliud uidebit oculus nisi quecumque iacent intra •hn•. [16]quamcumque igitur ỵmaginem uoluerimus ponamus apud planum •lm•, et accedentium quidem neque unus apparebit, sola autem dicta ỵmago.

[17]quare oportebit, sicut intrapositum esse ipsam •hn•, ut dicta ỵmago interiaceat in plano equedistante speculo. [18]oportebit igitur in aliquo plano protrahere rectam ipsam •ab• lineam et constituere angulum qui sub •abg• existentem terciam partem recti, et ponere altitudini speculi equalem ipsam •bg•, et educere ad •e•; et ipsi •ab• ad rectos angulos producere ipsam •bd• et accipere signum aliquod •e•, ita ut ab •e• ad rectos producta que •eb• cadat extra •m•. [19]sit igitur acceptum, et sit •e•, et ipsi •eb• ad rectos que •ed•, et copuletur que •dg•. [20]et angulo qui sub •edg• equalis consistat qui |sub •[g]ḥd•. [21]et ad rectos ipsi •db• ducatur que •hn•. f. 61^{v} c. 2
[22]inclinato igitur speculo, ut dictum est, distare oportet a muro per equalem ipsi •bh• et obstructorium rectum stare archam apertam ex superiori parte altitudinem uiri habentem et intraponere planum •lm• equedistans speculo in quo dicta ponetur ỵmago. [23]uisum autem stare oportet apud •d•, prohibitorio aliquo existente ad non

24.12 (*at lacuna following* secat ergo) *illegible erasure* || 24.13 (*at lacuna following* reflexi) Δ (*to the right of this, illegible erasure*) || 24.14 *illegible erasure* || 24.17 (*at* intrapositum) φραγμα || 24.18 (*at* protrahere, *erased*) Δ

24.1 y(maginem): i *Coner in ras.* || 24.6 *post* speculi *rasura in O* (•r• *legi potest*) | •r•: •e• *A, ed, Coner* || 24.12 (ergo) que •tk• ipsam (•hn•) *Coner in ras.* (ipsam *A, ed*) || 24.13 et *iterauit O incipiente uersu* | (reflexi) secant ipsam (•hn•) *Coner in ras.* (secant *A, ed*) || 24.15 nichil *conieci*: nihil *Coner in ras.* || 24.16 y(maginem): i *Coner in ras.* | y(mago): i *Coner in ras.* || 24.17 y(mago): i *Coner in ras.* || 24.20 •gh(d•): *sic ed*; hg *Coner in ras.* || 24.22 y(mago): i *Coner in ras.*

interius cedere. [24]sic enim incidentes speculo radii non excident extra intersticium, sed intra, in quo loco est ỵmago. [25]de ea autem que extra comprehenditur dispositione non adieci admonere. [26]oportet enim unumquodque ornare et disponere, ut utique locus et preparantis electio patiuntur. [27]ipsum tamen speculum in templo aliquo ligneo congruit poni inplens non totum locum, templum autem ornatum esse adiacente loco, et prominentiis autem ỵmaginem occultatam, ut non palam uideatur, habere autem et speculum lumen ex aere ipsum continente, ỵmaginem autem ex posteriori parte fenestra existente ex lateribus. [28]non enim potest uideri in tenebris iacens, quoniam neque aliorum aliquid eorum et que sine speculo iacens in tenebris uidetur.



[25] [1]Explicit liber Ptolomei de speculis. [2]completa fuit eius translatio ultima die decembris anno Christi 1269.

24.24 (*at* intersticium) φραγμα

24.24 y(mago): i *Coner in ras.* || 24.27 (congru)it: -e *ed, Coner* | y(maginem): i *Coner in ras.* (*bis*) || 24.28 *super uerba* eorum, et, que, sine, speculo, iacens, in, uidetur *scripsit litteras* a, e, b, f, g, c, d, h *Coner*

English Translation

[0] [1]Claudius Ptolemy on Mirrors.

[2]Book I.

[1] [1]There are two senses by which a road is made to knowledge, according to Plato: hearing and sight; and both have a theory. [2]Of these, the theory of hearing is Music, the science of concords and modalities and, in short, the theory of melodious and tuned nature. [3]For there is a manifold and various argument concerning the fact that the world is organized according to musical modality. [4]The whole of the heavens is distributed into eight spheres, seven for the planets and one that contains them all and bears the fixed stars, and in these spheres occurs a melodious and tuned progress of the stars, because of the matching force of motions among them, just as the strings in a lyre make melody. [5]For one should perceive certain sounds from the progress of the stars through the air, some of them deeper, others sharper, just as some stars make a slower, and others a faster motion. [6]For in the same way as, they say, we perceive the air undulating when a string is struck, so too when the stars are borne through the zodiac, one should think that the changed and continuously transforming air presents us with a good concert.

[2] [1]The study concerning sight is divided into optics, dioptrics, and catoptrics. [2]Optics has been satisfactorily discussed by our predecessors, and especially by Aristotle. [3]Concerning dioptrics we have said as much as we saw fit elsewhere at length. [4]But since we see that the study of catoptrics too deserves attention — for it possesses a certain wonderful theory. [5]Through it mirrors are fashioned that show right as right and left likewise as left, behaving contrary to usual mirrors, which (?) show the opposite of nature. [6]It is possible by means of these mirrors to see those who are coming behind (*or* to see oneself from behind?), and to see oneself upside-down and topsy-turvy, and as having three eyes and two noses, and a likeness of grief with the parts of the face in disarray. [7]But catoptrics is useful not only for theory, but for practical applications as well. [8]For how could one not think it useful to see, if one can, how many people are in the street and what they are doing, while remaining in a house across? [9]Or how could one not think it wonderful under other circumstances to behold through a mirror, night and day, the present hour by means of images that appear? [10]For as many images appear as there are hours in the night or day, and moreover if a fraction of a day has elapsed, it will be apparent by an image. [11]And how can one not think it wonderful when one sees by a mirror neither oneself nor someone else, but only whatever someone has chosen in advance? [12]Since, then, the subject is of this kind, I think that the things that have been received from our predecessors ought to be furnished with an exposition, so that the subject will lack in nothing.

[3] [1]Well then, it is wondered by nearly all who have written about dioptrics and optics, why rays that fall from us upon mirrors are reflected and make their reflections at equal angles. [2]Let the fact that we see along emissions of straight lines from the sight be established as follows. [3]Everything that travels with continuous speed travels in a straight line, just as we see arrows sent from bows. [4]For because of the propelling force, what travels tries to travel in the shortest line in distance, because it has no time for slowness so as to travel in a longer line in distance, since the transmitting force does not allow it. [5]Therefore because of the speed it tries to travel the shortest way. [6]But a straight line is the shortest of lines that have the same ends.

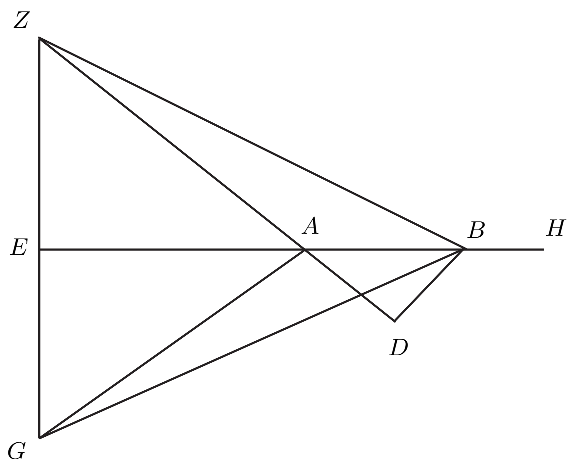
[4] [1]The fact that moreover the rays that are emitted by us travel with infinite speed can be learned from the following. [2]When we shut our eyes and then look again at the sky, there is no interval of time for them to reach the sky. [3]For the moment we look, we see the stars, although the distance is, so to speak, infinite. [4]Even if this distance were greater, the same thing would happen; so that from this it is clear that the emitted rays are emitted with infinite speed. [5]They are therefore not interrupted and undergo no curvature or inflection, but rather travel the shortest way, that is a straight line.

[5] [1]Enough has been said to the fact that we see along a straight line. [2]We shall now show that rays are reflected when they fall upon mirrors, and also upon water and all plane bodies. [3]The nature of polished bodies is to be dense in their surfaces. [4]Hence mirrors, before they are polished, have some gaps, and when rays fall upon these they cannot be repulsed. [5]But then they are polished by a filing until the empty places are filled with fine material. [6]Then the rays, thus falling upon a dense body, are repelled. [7]For just as a stone emitted with force and driven to a dense body bounces back, say against some wood or wall, but it comes to rest against something soft, such as wool or another such, because the emitting force follows and, not being able to give way in something hard, it continues to follow and move the emitted thing, but when it falls upon something soft it falls and departs from the emitted thing: in the same way too, rays borne from us with great speed, as has been proved above, and driven to a dense body, are reflected. [8]But on water and glass <not> all are reflected, because both materials have gaps and are made up of things with fine parts and of solid bodies. [9]For in glass and water we see [both] ourselves and the things that lie beyond. [10]For in the water of a pool we see the things on the bottom, and in glass we see the things beyond. [11]For whatever rays fall upon solid bodies are repelled and reflected, while all those that penetrate through the empty bodies see the things that lie beyond. [12]Therefore the things that are displayed in such are not seen perfectly, because not all the rays are reflected to them, but some, as has been said, pierce through the gaps.

[6] [1]We think that we have adequately shown that rays that fall on polished bodies are reflected. [2]We shall show by the same arguments that they also make their reflections at equal angles on plane and circular mirrors. [3]Because of the speed of incidence and reflection, again they have to try to travel by the shortest straight lines. [4]I say therefore that the shortest of all rays that fall upon and are reflected on the same thing are those at equal angles, on plane and circular mirrors. [5]But if this is so, then logically they are reflected at equal angles.

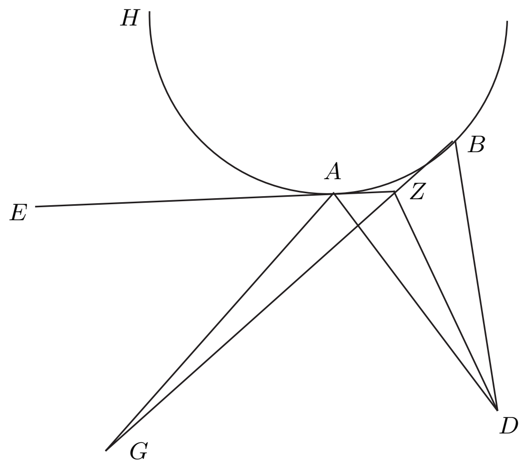
[7] [1]For let there be a plane mirror AB, sight point G, D the thing seen; and let line GA fall on the mirror. [2]And let line AD be joined, and let angle EAG equal angle BAD. [3]And let another ray GB likewise be incident, and let BD be joined. [4]I say that lines GA, AD are shorter than GB, BD.

[5]For let perpendicular GE be drawn from G upon AB, and let GE and DA be produced to Z, and let ZB be joined. [6]Since angle BAD, that is angle ZAE, equals angle EAG, but also the angles at E are right, therefore ZA equals AG, and ZB equals BG. [7]Hence since ZD is less than ZB and BD, and ZA equals AG, and ZB equals BG, therefore GA and AD are less than GB and BD. [8]For since angle EAG is equal to angle BAD, but angle EBG is less than angle EAG, and angle HBD is greater than angle BAD, therefore angle HBD is much greater than angle EBG.



[8] [1]Let there also be a circular mirror, and let its circumference be AB, the sight G, the seen thing D, and let GA and AD be incident at equal angles, and GB and BD at unequal angles. [2]I say that GA and AD are less than GB and BD.

[3]For let tangent EAZ be drawn. [4]Then angle HAE equals angle BAZ. [5]And the remainder angle EAG equals angle ZAD. [6]Hence if ZD is joined, because of what has been proved above, GA and AD are less than GZ and ZD; but GZ and ZD are less than GB and BD. [7]Therefore GA and AD are less than GB and BD.

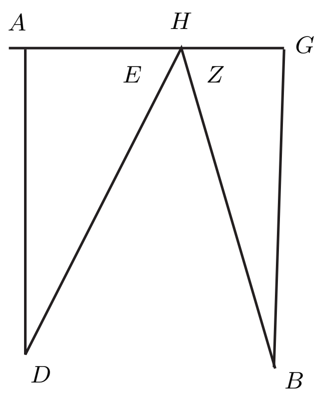


[9] [1]Hence in general, even if rays cannot be reflected at equal angles on mirrors, a point has to be conceived on the mirror such that an incident ray from the sight, reflected to that which is seen, will make both together, that is the incident and reflected ray, less than all such incident and reflected rays.

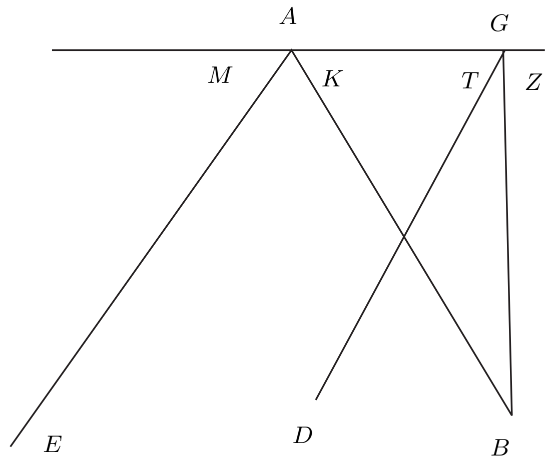
[10] [1]In plane mirrors there is some place such that if it is occupied, the image is no longer seen.

[2]For let there be a plane mirror AG, or collinear with it, and B the eye, and D the thing seen, and let perpendiculars AD, BG be drawn to the mirror, and let AG be divided at H so that AH is to HG as AD is to BG. [3]Then I say that if the place of H is occupied, D is no longer seen.

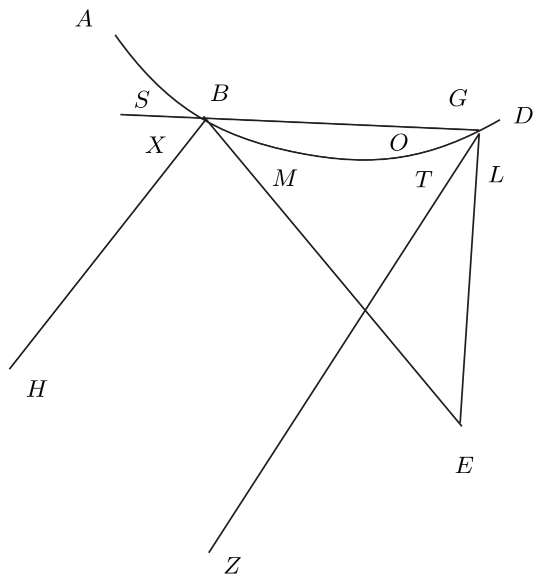
[4]For let BH and HD be joined. [5]Then because of the proportionality the triangles will be similar. [6]So angle E equals angle Z, and hence D will appear through point H. [7]Therefore if the place is occupied by wax or something else, D will no longer be seen. [8]But if point H falls off the mirror, the image will appear in the mirror. [9]For all rays that fall upon the mirror will be reflected at equal angles.



[11] [1]Lines of sight reflected on plane mirrors neither intersect nor are parallel. [2]For let there be a plane mirror AG, the sight B, and let <BA and BG> be incident, <and let> GD and AE <be reflected>. [3]Then angles Z and T are equal; but angle Z is greater than angle K, that is angle M. [4]Therefore angle T is greater than angle M. [5]Hence GD and AE neither are parallel nor intersect in the direction of D and E.



[12] [1]Lines of sight reflected on convex mirrors neither intersect nor are parallel. [2]For let there be a convex mirror $ABGD$, and E the sight, and let rays EG and EB be incident; and also let GZ and BH be reflected. [3]Then angle T equals angle L, and angle M equals angle X. [4]Therefore angle $O+T$ is greater than angle $S+X$. [5]GZ and BH thus neither are parallel nor intersect in the direction of Z and H.

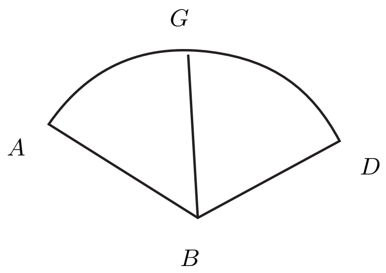


[13] [1]End of Book I.

[14] [1]Book II.

[15] [1]In concave mirrors, when the eye is placed at the centre, the reflected lines of sight will be reflected to the eye.

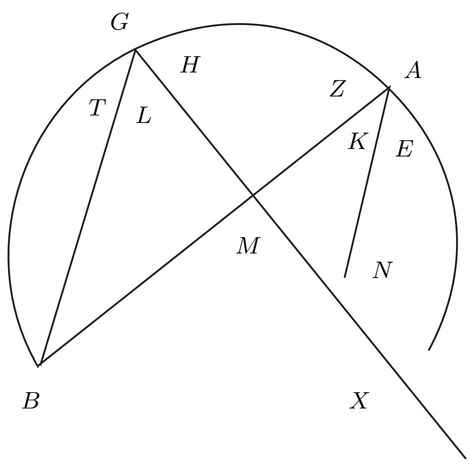
[2]Let there be a concave mirror AGD, and let its centre be B. [3]Let the eye lie at B, and let rays BA, BG, <and BD> be incident. [4]Then they will make equal angles at the arc, because the angles of semicircles are equal. [5]Hence the reflections will be along BA, BG, and BD. [6]They will therefore intersect at point B, that is at the eye. [7]From this it is evident that if there is a concave mirror, such as a spherical one, and the eye is placed in the sphere's centre, nothing other than the eye will appear in the mirror.



[16] [1]In concave mirrors, when the eye is placed on the circumference, the reflected rays will intersect.

[2]Let there be a concave mirror BGA, and let B be the sight. [3]And let rays BG and BA be incident, and let GX and AN be reflected. [4]I say that GX and AN will intersect in the direction of N and X.

[5]For since BA is greater than BG, therefore angle Z is greater than angle T. [6]Likewise angle E is greater than angle H. [7]Therefore the remainder angle L is greater than angle K. [8]But angle M is greater than angle L. [9]Therefore angle M is greater than angle K. [10]Thus GX and AN will intersect in the direction of N and X.

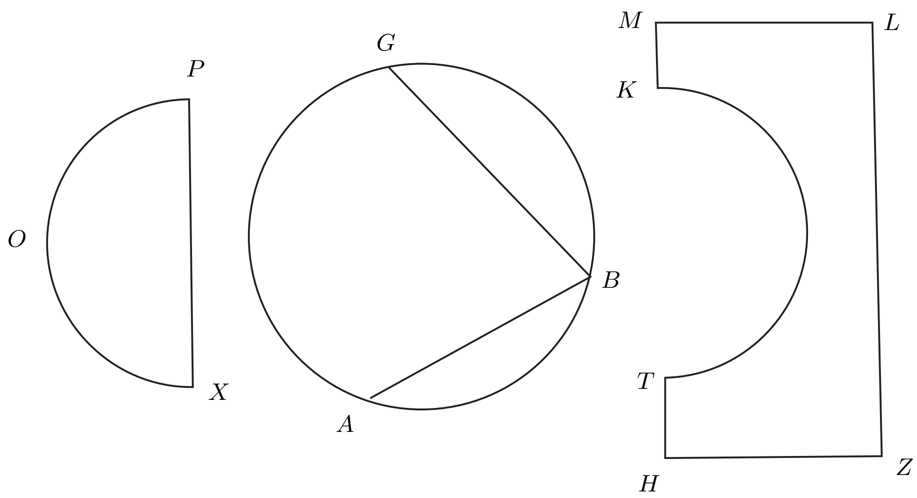


[17] [1]To fashion a dextral mirror.

[2]Let circle ABG be described in the size in which we want to fashion the mirror. [3]And let there be inscribed in it the side AB of a pentagon and BG that of a hexagon, and let templates be cut conforming to arcs AEB and BZG which are cut off from the circle by lines AB and BG: let the template for the height be made concave conforming to arc AEB, as $ZHTKLM$; and let the template for the breadth be convex, conforming to arc BZG, as XOP. [4]And let a rectangular mirror on a base be prepared, having height equal to line AB, and breadth equal to BG, and let its vertical surface be convex, worked against the concave surface of template AEB, and its horizontal surface concave, worked against the convex (*illegible word*) arc of template BZG.

[5]Right will appear as right, and left likewise (as left). [6]And when (the sight) is about two cubits away, the image will appear in proper proportion and realistic. [7]But when (the sight) is farther away, the image of the person who is seen will seem to stretch backwards; while as the sight approaches closer towards the convex surface of the mirror, the image of the person who is seen becomes monstrous, the more so the closer it gets. [8]And the mirrored person will be reversed, and contrariwise, as the sight still approaches the image will appear farther away, and the face becomes like a form of a horse. [9]And as the mirror is progressively tilted, the image will appear tilted. [10]One should therefore prepare a stand with a universal joint for it, on which the mirror is kept, so that the image that is seen will sometimes have its head up, sometimes down and feet up.

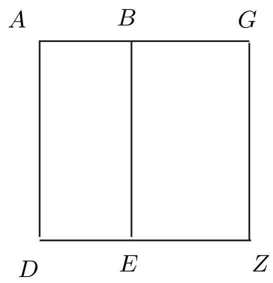
[11]If the mirror is made with two faces, that is on the back and front, then right will appear as right, but from the rear it will exhibit people topsy-turvy like antipodeans.



[18] [1]To fashion a mirror which is called multiview. [2]It makes right appear as right, also three-headed Zeus, it makes motion appear, it effects dancing Victories,

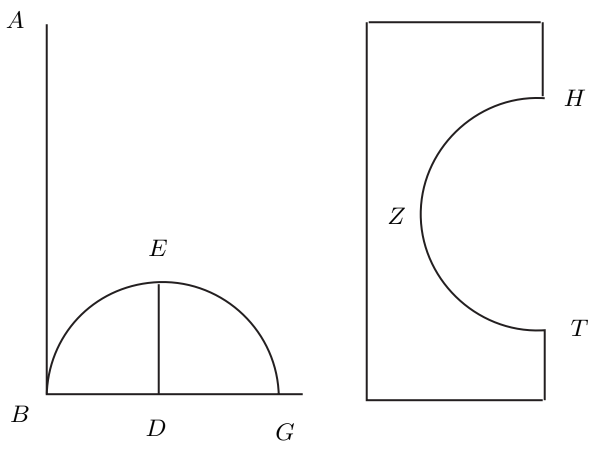
it attests that Pallas was born from the brow of Zeus, it shows many faces, it makes one finger many, and lastly it shows distorted bulls' heads.

[3]Let there be two bronze rectangular plane mirrors worked against a ruler, and adjacent to one another, namely AEG, standing on the same base DZ, so that side BE is common to both. [4]Let the mirrors have height BE twice the breadth AB. [5]But some choose to make the height one and a half times the breadth. [6]It makes no difference if one makes it whatever measure one wants for the sake of good proportions. [7]Then so that the mirrors can open and close, let them revolve about their common side BE, without wobbling at all and with the images unobstructed. [8]And it will be accomplished.



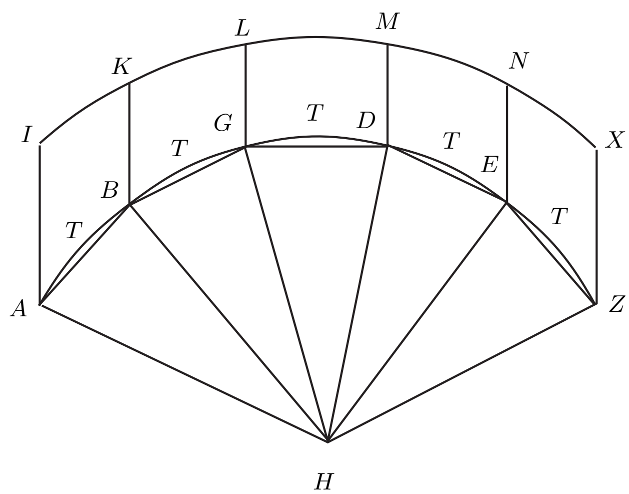
[19] [1]To fashion a mirror which is called a mocker.

[2]Let two lines AB and BG be drawn, and let AB be twice BG, or let it have whatever other ratio one wants. [3]And let AB be the mirror's height, and BG its breadth. [4]And with the endpoints of the breadth for centre and BG for radius, let arcs be described and intersect at D, and again with centre D and radius either DB or DG, let a concave arc BEG be described. [5]And let a concave template ZHT be made conforming to arc BEG on line BG. [6]And let a bronze rectangular mirror be prepared, having height equal to BA, and breadth equal to line BG, and its vertical surface rectilinear, its horizontal convex, worked against concave form ZHT. [7]And a section of a cylinder will be made, a shape of convex surface.



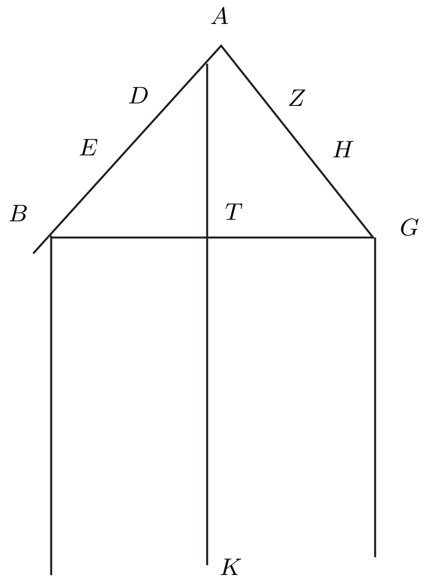
[20] [1]To fashion a mirror which is called theatrical.

[2]Let an arc of a circle be described, passing through $ABGDEZ$, and let its centre be H, and let $ABGDEZ$ be divided into five equal parts ATB, BTG, GTD, DTE, ETZ, and let straight lines AB, BG, GD, DE, EZ be joined subtending the arcs. [3]And let lines be conceived as joined from the centre to points A, B, G, D, E, Z, namely HA, HB, HG, HD, HE, HZ. [4]And after removing the arcs ATB, BTG, GTD, DTE, ETZ that go over AB, BG, GD, DE, EZ, let elevated bronze mirrors be erected on lines AB, BG, GD, DE, EZ, square in shape, plane in surface, parallel to AI, BK, GL, DM, EN, ZX, touching one another, so that their common sides are KB, LG, MD, NE, and so inclined that the angles contained by AI and IK, BK and KL, GL and LM, DM and MN, EN and NX are equal to the angles contained by HA and AB, HB and BG, HG and GD, HD and DE, HE and EZ, and so that the plane through $ABGDEZ$ is in the plane of reference, and sides IK, KL, LM, MN, NX of the standing elevated mirrors lie in a plane parallel to the plane through AB, BG, GD, DE, EZ. [5]And it will be accomplished. [6]For the mirrors lying on lines AB, BG, GD, DE, EZ will be pointing to centre H.



[21] [1]It is required to prepare a winged-foot.

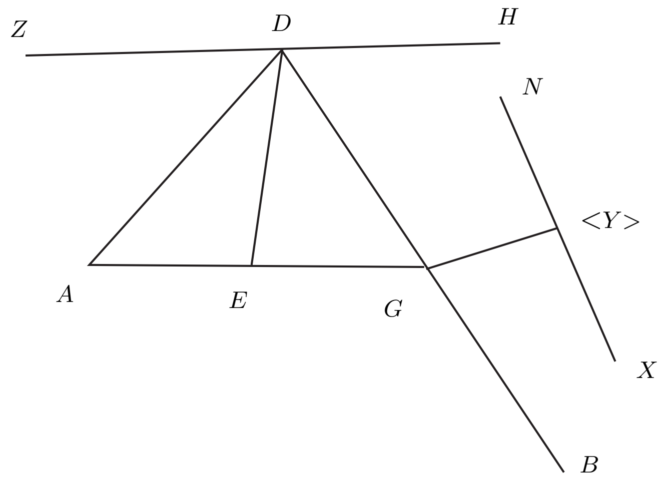
[2]Let there be a right-angled triangle ABG, and let BG be bisected at T, and let plane mirror ZH be on line AG, and plane mirror DE on line AG. [3]And let the viewer be TK, point T his eye, looking at whichever mirror he wants. [4]And it will be accomplished. [5]With the other mirror stationary, I say that as he leans forward and backward a ray will come back to a point that is in the heel of the viewer in the mirror, and he will think he is flying.



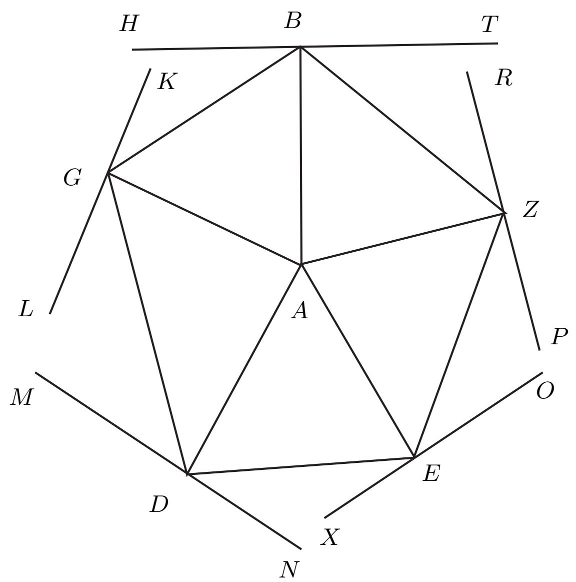
[22] [1]Let it be required to put in a house in which there is a window, a mirror in which will appear the people coming on the other side, and circulating in the streets or lanes, if one views in a certain place, but one in the house.

[2]Let the place in the house be A, what we want to have appear B, the window G, and let BG be joined and produced, and let it meet the house's wall and ceiling in D, and let AD be joined. [3]Then some ray going from the sight along AD and falling upon the mirror at D will have to be reflected at equal angles to B. [4]So let mirror ZH be situated at right angles to the plane through AD and DB. [5]Then angles ZDA and HDB will be equal. [6]Then let angle ADB be bisected by line DE. [7]Hence DE is at right angles to mirror ZH. [8]Since therefore both B and G are given, their ray BGD is given in position; but also the wall upon which it falls is given in position. [9]Hence D is given. [10]But A too is given. [11]Therefore AD is given in position. [12]Hence angle ADB is given. [13]And it has been bisected by line DE. [14]Therefore DE is given in position. [15]And it has been produced at right angles to ZH from given point D. [16]Thus the plane too is given in position, that is the mirror.

[17]The synthesis will be made as follows. [18]Let diopter $NYGX$ be placed at point G, and revolved about G until point B is seen through it. [19]Let some point of the planes that contain the house be sighted. [20]And let D be sighted, and let AD be joined, and let angle ADG be bisected by line DE. [21]It will be so divided, if line AG is joined and divided at E so that <AE is to EG> as AD is to DG. [22]For both (AD and DG) are given; and thus the ratio AE to EG is given. [23]So let a plane mirror be fashioned, and let it be situated at right angles to DE, so that its middle is point D, and so the viewer at point D will see whatever is put at B and behind it.



[23] ¹It is possible to see the same image in many mirrors placed in some order. ²Let A be what we want to see in many mirrors, and let B, G, D, E, Z be any number of equilateral and equiangular polygonal mirrors, and let A be their middle, the centre of the circle that circumscribes them. ³And let AB, AG, AD, AE, AZ be joined, and let HT, KL, MN, XO, PR be drawn at right angles to them, and let mirrors be situated on these at right angles to plane $BGDEZ$. ⁴I say that lines of sight falling upon the mirrors will be reflected to A.



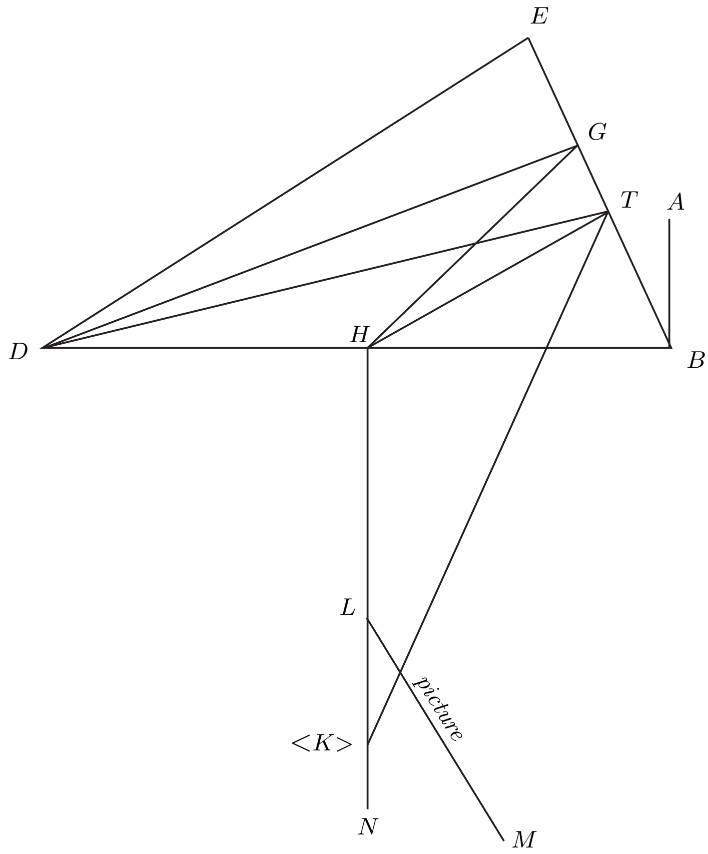
⁵For as they fall they will make right angles with the mirrors. ⁶Hence they will have their reflections in themselves. ⁷Thus they will be reflected to A.

[24] [1]To put a mirror in a given place, so that everyone who approaches will see neither himself nor someone else, but only whatever picture someone has chosen in advance.

[2]Let AB be the wall on which the mirror is to be put, and let the mirror be inclined at some angle to it. [3]It will be suitably proportioned if the angle is one third of a right angle. [4]And let BG be the mirror's surface, and let BD be conceived at right angles to AB from B, and let the point of sight D so lie in BD that a perpendicular produced from it to the mirror BG will fall outside it. [5]Let it be <DE>. [6]And let DG be joined from D to the edge G of the mirror, and let angle HGD (?) be made equal to angle EDG (?). [7]Then if some ray falls from sight D upon the edge G of the mirror, it will be reflected to H. [8]So Let HN be drawn from H at right angles to DB. [9]And let another ray DT be incident, and let HT be joined. [10]Then angle BTH is greater than angle BTD. [11]So let angle BTK be made equal to angle GTD. [12]Hence TK will cut HN. [13]Likewise all rays that fall upon the mirror will be reflected and cut HN. [14]Then let plane LM be drawn parallel to mirror GB, and let it lie between H and N, cut by the reflected ray. [15]Hence (?) it is obvious that the eye will see nothing other than whatever lies between H and N. [16]Let us therefore put whatever picture we want in plane LM, and no one who approaches will appear, but only the picture mentioned.

[17]Hence HN should be a sort of screen, so that the mentioned picture will lie in a plane parallel to the mirror. [18]Line AB should therefore be produced in some plane, and angle ABG should be made one third of a right angle, and BG should be made equal to the height of the mirror, and it should be produced to E; and BD should be produced at right angles to AB, and some point E chosen so that ED produced at right angles will fall outside H (?). [19]Let it be chosen, and let it be E, and let ED be at right angles to EB, and let DG be joined. [20]And let angle HGD (?) be made equal to angle EDG (?). [21]And let HN be drawn at right angles to DB. [22]Then with the mirror inclined, as has been said above, one should stand back from the wall by a distance equal to BH, and an upright obstacle should stand there, a coffer open at the top and as tall as a man, and plane LM should be inserted parallel to the mirror, and the mentioned picture should be put in it. [23]The sight should stand at D, with something there to block him from moving closer. [24]For in this way rays that fall upon the mirror will not land outside the screen, but within it, where the picture is. [25]I have not added remarks about the external arrangement. [26]For everything should be arranged as the place and the fabricator's purpose allow. [27]But it is appropriate to put the mirror in some wooden coffer, not filling the entire space, and the coffer should be furnished with space around it, and the picture should be hidden by protrusions so that it cannot openly be seen, and the mirror should have light from the air containing it, and the picture from behind, by having a window on the sides. [28]For a thing situated in darkness cannot be seen, since nothing else, even without a mirror, can be seen when it is situated

in darkness.



[25] [1]End of Ptolemy's book on mirrors. [2]The translation of it was finished on the last day of December, A.D. 1269.

Notes

1.1 Plato singles out sight and hearing (most explicitly at *Phaedo* 65a-b) as the senses that would have the best claim to providing knowledge if (as he denies) any sense could.

1.4 With this rather crude conception of the harmony of the spheres may be compared (among numerous other passages) Aristotle, *De Caelo* 290b12ff, and Nicomachus, *Enchiridion* 3. Pseudo-Ptolemy's belief that the heavenly bodies are in contact with air (rather than aether) is unexpected in a late text.

2.1 William glosses the Greek terms ὀπτική, διοπτρική, and κατοπτρικὴ (πραγματεῖα) with Latin equivalents. Just what Pseudo-Ptolemy meant by these divisions of the science of vision is not clear, except that κατοπτρικὴ obviously signifies the study of reflection. I would guess that ὀπτικὴ refers to the philosophical investigation of the mechanics of visual perception, the allusion in 2.2 being to Aristotle, *De Anima* and *De Sensu*, whereas διοπτρικὴ refers to geometrical optics founded on the visual ray hypothesis, as in Euclid's *Optics*.

2.3 The anacoluthon is resolved, if at all, only in 2.12.

2.5 The Greek text of the end of 2.5 seems to have been corrupt, since notwithstanding the conflicting case endings it is the ordinary mirrors that exhibit *contraria*, i.e., right as left. The promise of handedness-preserving mirrors is fulfilled in sections 17 and 18.

2.6 The dextral mirror of section 17, when rotated, shows the viewer head over heels. (A head-over-heels mirror is mentioned by Olympiodorus, *In Meteor.* ed. Stüve (*CAG* 12.2) 264. The display of multiple eyes or noses is not accomplished by any of the arrangements in *De Speculis*, but are strongly reminiscent of a mirror to show one's head with multiple eyes in the Arabic text *On Burning Mirrors* attributed to Anthemius. The latter work also has an arrangement to show the viewer his own back, which, as Schmidt (319 n. 2) points out, is probably what lies behind Pseudo-Ptolemy's *uidere posterius apparentes*. See Jones 1987, 14–15. (Olympiodorus, 211 and 264, names such a mirror arrangement ὀπισθοφανές.) Like the mirror clock in 2.9–10, these references to mirror constructions not presented in *De Speculis* suggest that the work as we have it is either an abridgement or a selection from a larger body of similar material.

2.8 This apparatus is described in section 22. For *rymis* see the note to 22.1.

2.10 *et etiam si pars diei extiterit*: the meaning is obscure.

2.11 For this, see section 24.

3.3 *continua* (translating συνεχές) would literally mean "unremitting," but the author evidently links the idea to great speed.

3.4 The appeal is not to nature "doing nothing in vain," as in Damianus' paraphrase of Hero's *Catoptrics* or the evidently related argument in Olympiodorus *In Meteor.* 3.2 (ed. Stüve, *CAG* 12.2, 212–213), but less metaphysically, to the hurry of the propelled object to arrive at its destination in the least possible time. The visual ray is clearly thought of as a material rod propelled out of the eye.

4.4 The point of the first half of this sentence is unclear.

5.3 Pseudo-Ptolemy may derive his conception of matter as a composition of particles and voids (which in transparent bodies form channels) from Hero; cf. the introduction to Book 1 of Hero's *Pneumatics*, esp. sections 151–152, ed. Schmidt p. 26, where the issue is how solar rays (not visual rays) selectively penetrate the surface of water. For the Peripatetic background of this passage see Jones 1994.

5.7 The correction *quiescit* makes good sense, and likely William's lost original reading was ungrammatical or corrupt.

7.2 The "equal angles" of reflection are with respect to the surface of reflection, as is customary in Greek catoptrics. Cf. note to 8.1.

7.8 An afterthought: the author, or a reviser, wishes to prove that if GA is reflected along AD at equal angles, then GB cannot also be reflected along BD at equal angles. Similar proof in Ptolemy, *Optics* 3.68–69, ed. Lejeune 120–121.

8.1 As the proof makes clear, reflection "at equal angles" for a curved mirror is understood as meaning that the rays contain equal angles with the curve of the mirror, not the tangent, at the point of reflection. Note that Pseudo-Ptolemy wisely does not attempt to prove the theorem for concave mirrors, since the point of reflection at equal angles can in this situation be the point determining the *longest* reflected path for the ray.

8.4 The "horn angles" contained by an arc and tangent (or secant) seem to have been often invoked in Greek catoptrics; cf. sections 12 and 16 as well as Euclid, *Catoptrics* 1.

9.1 Pseudo-Ptolemy appears to state that a reflection will occur at the point on a mirror that determines the minimum path from eye to mirror to object, even if the two parts of the ray do not contain equal angles with the surface of the mirror. This, a logical consequence of his physical justification of the equal-angle law, is obviously nonsense, and is contradicted by section 10.

10.1 This proposition is clearly related in inspiration to the fourth postulate of Euclid's *Catoptrics*, that "in plane mirrors when the place is occupied on which the perpendicular from the thing seen falls, the thing seen is no longer seen," and Euclid's fifth and sixth postulates which make analogous assertions for convex and concave circular mirrors. In Euclid the postulates are only valid (and indeed seem only to be employed in the text) in the situation where the viewer and the object of vision are the same point. Pseudo-Ptolemy's point is a banal reassertion of the equal-angle law, whereas Euclid's postulates are the basis for his location of reflected images.

10.8 Obviously meaning, "if the object at point H falls off the mirror."

11.1 An abbreviated version of the first part of Euclid, *Catoptrics* 4. The figure in Heiberg's edition of Euclid (which is in other respects identical to Pseudo-Ptolemy's) labels the angle contained by BA and AE as L (lambda) and one can infer from the order of letters that the angle contained by BG and GD should have been labelled H (eta). Neither angle is referred to in Euclid's or Pseudo-Ptolemy's proof, and neither label can be seen in the diagrams in manuscript **O**.

12.1 The proof is close to that of the second part of Euclid, *Catoptrics* 4, but the figure, though similar, is differently lettered.

13.1 The break between the two "books" is awkwardly placed, separating propositions that are closely related. A division in the logical place, after section 16, would also have made the two parts of more nearly equal length. It is in any case perverse to divide such a short work into "books."

15.1 The proposition closely follows the first part of Euclid, *Catoptrics* 5.

15.3 Mention of the third ray, BD, has dropped out of the text.

15.4 The deleted words, which resulted from eyeskip, are a strong indication that the text in manuscript **O** is William's transcription from a draft copy of his translation.

15.7 This sentence is not present in Euclid.

16.1 This is the second part of Euclid, *Catoptrics* 5. The figure is similar to the one in Euclid, but not identically lettered.

17.1 The use of δεξιὸν (*dextrum*) to mean "right-hand-preserving" seems not to be elsewhere attested; Olympiodorus, *In Meteor.* ed. Stüve (*CAG* 12.2) 264 mentions a δεξιοφανὲς ἔνοπτρον. The mirror is a saddle surface, with convex circular vertical arcs for its cross sections and concave circular arcs for its horizontal cross sections. On the adaptations (and mangling) of this proposition in later texts ("Anthemius" *On Burning Mirrors*, Pseudo-Euclid *On Mirrors*, and Witelo) see Jones 1987, 11–14.

17.3 The choice of pentagon and hexagon only affects the relative dimensions of the breadth and height of the mirror, not the curvature. The term ἐμβολεὺς for "template," which William did not translate but records in the margins wherever it occurred in the Greek text, is used elsewhere in connection with mirror construction only, so far as I know, by Anthemius, *On Paradoxical Devices*, and the Bobbio Mathematical Fragment. In none of these texts is it stated whether the mirror is shaped by hammering against the template or whether its curvature, formed in some other way, is merely checked against the template. Cf., however, Philo, *Belopoeica* 70 for a wooden ἐμβολεὺς against which bronze plates are hammered into a curved shape. (In Hero, *Pneum.* 1.28, bronze cylinders are turned [κατατετορνευμέναι] to fit an ἐμβολεὺς, but here the term refers to a piston, not a template.) For a mirror of double curvature made using different templates for the vertical and horizontal sections, the templates would presumably be narrow objects laid on the surface to test its curvature.

17.4 I accept Schmidt's guess (pp. 410–411) that William's incomprehensible *achario* represents a corruption in the Greek of ἐσχάριον, "a base or platform."

17.5 Ptolemy, *Optics* 4.161 (ed. Lejeune, 209–210) has similar but less figurative remarks about saddle-surface mirrors. Ptolemy considers in turn:

1. object of vision is at a distance from mirror such that its image is behind the mirror, and
   a. convex cross sections of mirror are vertical: vertical dimension of image will appear diminished but not inverted, horizontal dimension will appear enlarged but not inverted.
   b. convex cross sections of mirror are horizontal: horizontal dimension of image will appear diminished but not inverted, vertical dimension will appear enlarged but not inverted.
2. object of vision is at a distance from mirror such that its image is in front of the mirror, and
   a. convex cross sections of mirror are vertical: vertical dimension of image will appear diminished but not inverted, horizontal dimension may appear diminished or enlarged or neither, but always inverted.
   b. convex cross sections of mirror are horizontal: horizontal dimension of image will appear diminished but not inverted, horizontal dimension may appear diminished or enlarged or neither, but always inverted.

17.6 The distance of two cubits implies that the mirror is not very large.

17.9 This apparently refers to rotating the mirror about an axis perpendicular to the centre of its face.

17.10 In the margin William notes στυλον χήσιον. Schmidt proposes (pp. 411–412) that this was a corruption of στῦλον χαλκήσιον, a universal joint. This word is only attested in Hero, *Belop.* 88, and Marsden 1971, 51 plausibly emends it there to the normal form of the word, καρχήσιον. Presumably καρχήσιον also was Pseudo-Ptolemy's word.

18.1 *id est multiuidum* is a translator's gloss.

18.2 William had particular difficulties with parts of this sentence listing possible applications of the "multiview" mirror, a simple arrangement of two hinged plane mirrors. Schmidt (p. 412) succeeded in making sense of the marginal jottings as indicating that the Greek text had the words Δία τρικάρανον, Zeus with the attribute of three heads (I know of no iconographic example of this), and χορεύουσας Νίκας ἀποτελεῖ, "effects dancing Victories." Some of these displays are obviously temple knicknacks; I have no notion of what the bulls' heads are for.

19.1 The *ei* in *mokeion* is written by William as the conventional Greek ligature for epsilon-iota. μωκειον is not attested Greek, but it is presumably a corruption of something like μωκῶν, "mocking." The figure as it appears in **O** (reproduced here) does not conform to the text, since point D should be below, not on, BG.

20.2 The last two letters of the first occurrence of $ABGDEZ$ are missing from the Latin (and hence also from the Greek?).

21.1 The original first word (or two words) of William's text of this proposition have been obliterated except for the last three letters *dem*, which have been made into the termination of Coner's *aliter idem*, "the same thing another way." (*aliter idem* also appears in the other sources for the text.) This proposition is however not a second treatment of the foregoing problem; so quite likely the lost word(s) was something puzzling to an early reader of William's translation, who took it for a corruption. My guess is that the Greek text had πτερόποδα, "wing-foot," indicating that the viewer would interpret his own elevated image in the ceiling mirrors as a flying god.

21.2 The superfluous •*me*• in the Latin is probably a textual error in William's Greek exemplar.

22.1 The distinctive Greco-Latin phrase *in rymis siue in plateis* (ἐν ῥύμαις ἢ ἐν πλατείαις, "in the streets or lanes") marks the author, or redactor, of this problem as Christian and therefore probably well after Hero's time. The words ῥῦμαι and πλατεῖαι appear in proximity to each other only in Christian authors recalling the phrase εἰς τὰς πλατείας καὶ ῥύμας τῆς πόλεως in Luke 14.21 (which in turn seems to echo Septuagint *Is.* 15.3 ἐν ταῖς πλατείαις αὐτῆς περιζώσασθε σάκκους καὶ κόπτεσθε, ἐπὶ τῶν δωμάτων αὐτῆς και ἐν ταῖς ῥύμαις αὐτῆς πάντες ὀλολύζετε μετὰ κλαυθμοῦ). Since the introduction to *De Speculis* (2.8) picks up *in rymis* from this proposition, it too must be part of the late material. Christian phraseology does not seem to sit well alongside the pagan temple embellishments of section 18 (and perhaps 21), but this is a characteristic of the magpie composition of our text.

22.2 The passage 22.2–16 is, unusually in a mechanical problem, an analysis (in the Greek geometrical sense), which is followed by a synthesis explicitly introduced in 22.17.

22.8 "B and G" is required be the sense, in place of the text's •*bge*•. Other errors in the Latin apparently reproducing corruptions in the Greek text available to William in this proposition include 22.17 •*d*• for the second occurrence of G, and in 22.21 omission of •*ae*• *ad* •*eg*•.

22.17 The diopter required here would be a simple sighting tube. First one looks through it in the direction of B to establish the direction of the line of vision, and then one looks through it the other way to determine the location of D on the wall opposite.

23.1 Pseudo-Ptolemy's proposition is a perversion of Euclid, *Catoptrics* 14, in which a similar polygonal arrangement of mirrors (again illustrated by vertices of a pentagon!) is employed so that a viewer at one of the vertices will see an object at an adjacent one by looking in the mirror on his other side (the visual ray is reflected on all the mirrors in turn). See Jones 1987, 6–8.

24.1 Versions of this construction are found in "Anthemius" *On Burning Mirrors*, Pseudo-Euclid *On Mirrors*, and Witelo; see Jones 1987, 8–11. The Greek text appears to have been fairly corrupt. Errors in the Latin corrected in the English translation include 24.5 omission of •*de*• (from misaccenting of ΔE as δέ?); 24.6 •*r*• for •*g*•, and •*gd*• for •*hgd*•; 24.10 •*etd*• for •*btd*•; 24.20 •*ghd*• for •*hgd*•. In 24.18 the last part of the sentence, following "and some point E chosen," if translated as it stands would read "so that EB produced at right angles will fall outside M." The entire sentence 24.15 seems to belong before 24.14. Erasures in manuscript **O** in 24.12 and 24.13 show that something was wrong here, although the sense can be recovered. In the figure in **O**, instead of the oblique line TK (here added as a broken line in the diagram accompanying the Latin text) William has drawn a line from T perpendicular to DB (dotted line in the text diagram here).

24.6 Comparison with 24.20 shows that the text, before corruptions, really did stipulate that angles HGD and EDG should be made equal. This would make GH a normal to the mirror, not a reflection of DG. Perhaps this questionable construction arose from corruption of an earlier version that made angles HGB and EGD equal. "Anthemius" and Pseudo-Euclid correctly construct the reflected ray.

24.16 With LM parallel to the mirror, its image will have the same apparent tilt as LM rather than being upright. It is not clear whether the author intended this effect.

24.17 This last part of the proposition seems to be an afterthought; in Jones, 1987, 9 I characterized the two passages 24.2–16 and 24.17–28 as loose analysis and synthesis, but the former part is not truly analytic in approach.

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