AIP Conference Proceedings

Generalized rabinowicz' criterion for adhesive wear for elliptic micro contacts

Cite as: AIP Conference Proceedings **1909**, 020178 (2017); https://doi.org/10.1063/1.5013859
Published Online: 01 December 2017

Valentin L. Popov

Generalized Rabinowicz' Criterion for Adhesive Wear for Elliptic Micro Contacts

Valentin L. Popov

Berlin University of Technology, Berlin, 10623 Germany

v.popov@tu-berlin.de

Abstract. This paper is devoted to an old idea suggested in 1958 by E. Rabinowicz in his paper "The effect of size on the looseness of wear fragments". Rabinowicz assumed a circular shape for two asperities coming into contact and being destroyed due to relative sliding. We generalize his analysis for the case of non-circular contacts, in particular those having elliptical shape and discuss the general case of arbitrary contact shape.

INTRODUCTION

In 1953, Archard [1] published a paper devoted to contact and rubbing of rough surfaces, which contained most important ideas of the contact mechanics of rough surfaces, including the main ideas of the theory of adhesive wear. In particular, Archard derived a simple estimation for the wear volume

$$V = k_{\text{adh}} \frac{F_N x}{\sigma_0}, \tag{1}$$

where F_N is the normal load, x is the sliding distance, σ_0 the hardness of contacting materials (or the smaller one if they are different), and k_{adh} is the coefficient of adhesive wear. This equation is often called "Archard's law". Just as Coluomb's law of friction, it is widely used in spite of general understanding that this is just a very rough zeroth-order approximation to reality. The main reasons for the popularity of Archard's law of wear are its simplicity as well as lack of any other, more elaborated wear law. Archard's equation (1) has led to the formation of a common opinion that the higher the hardness, the lower the wear, since the hardness is in the denominator of the formula. This would be true if not for the coefficient of adhesive wear. In reality, this coefficient can take values that differ by seven orders of magnitude (Fig. 1), which deprives the law of any predictive power. In fact, Kragelski [3] formulated a directly opposing condition of low wear—the principle of a positive gradient of hardness, which states that the surface layers should be softer than the deeper layers, otherwise catastrophic wear occurs. In the present paper, however, we do not consider spatial heterogeneity of surface layers and assume the contacting bodies to be homogeneous media.

The physical nature of the coefficient of adhesive wear has not been clarified yet. Archard assumed that there is some probability that an asperity contact will lead to the appearance of wear particles. However, he could not specify the detailed physics of this probability. Neither did he make any statement about the size or size distribution of wear particles. However, the amount of wear is not the only quantity that is of interest, the distribution of wear particle size may also be very important. Especially in our times, when people are actively engaged in their health, problems of emission of wear particles, for example, from brakes or car tires, attract a lot of attention. E.g. in 2017, a large project of the German Ministry of Science and Technology was started, which is devoted to the problem of the emission of particles of plastics and elastomers into the environment [4]. At present, this is an active economic and political problem.

Proceedings of the International Conference on Advanced Materials with Hierarchical Structure for New Technologies and Reliable Structures 2017 (AMHS'17)
AIP Conf. Proc. 1909, 020178-1–020178-5; https://doi.org/10.1063/1.5013859
Published by AIP Publishing. 978-0-7354-1601-7/$30.00

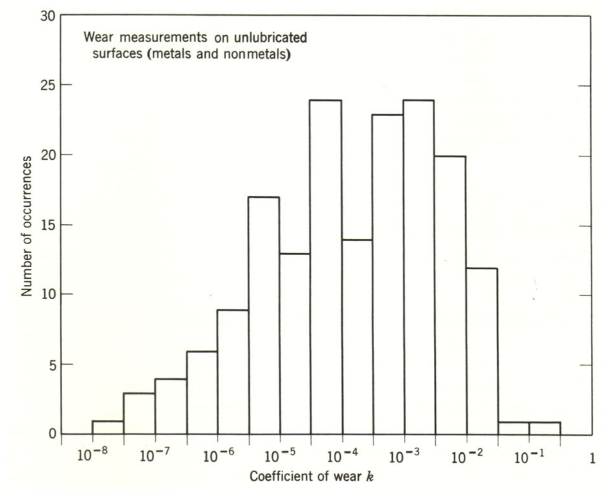

FIGURE 1. Distribution of the measured coefficients of adhesive wear according to Rabinowicz (from [2])

The key physical idea for the problem of size of wear particles (and implicitly for understanding the value of the coefficient of wear) was suggested by Ernest Rabinovich. In 1958, he wrote a short article in Wear, in which he put forward the hypothesis of a mechanism determining the size of wear particles [5].

If two micro heterogeneities collide and form a welded bridge, as suggested by Bowden and Tabor [6], (see Fig. 2) they are plastically deformed, and the maximum stress that can be achieved is of the order of the hardness σ_0 of the material. In this state, the stored elastic energy is proportional to the third power of contact size:

$$U_{\text{el}} \approx \frac{\sigma_0^2}{2G} D^3. \quad (2)$$

This energy can relax by creating a wear particle. The process of detaching a wear particle can, however, only occur if the stored elastic energy exceeds the energy

$$U_{\text{adh}} \approx \Delta w D^2 \quad (3)$$

which is needed to create new free surfaces (where Δw is the work of adhesion per unit area). Hence it follows that only particles larger than some critical particle can be detached:

$$D > \frac{2G\,\Delta w}{\sigma_0^2}. \quad (4)$$

Note that the Eq. (4) predicts only the existence of the lower bound of the size of wear particles. Thus, there also should be some mechanism suppressing the appearance of too large particles. Rabinowicz did not make any suggestions for such a mechanism. However, a possible mechanism could be very simple and follow from the same equation (4). Indeed, if some particle has size much larger than (4), it is energetically favorable for it to disintegrates into two smaller ones, assuming that detachment always occurs at the same critical stress. This process can only continue until the critical size (4) is reached. Thus, the wear particles should all have a size of the same order of magnitude as the critical length given by the Eq. (4). This was confirmed by experimental data presented by Rabinowicz [5].

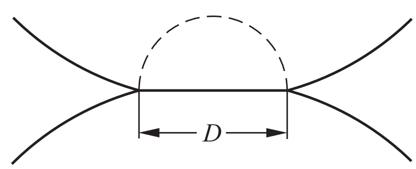

FIGURE 2. Welded joint of size *D* created due to contact and shear of two asperities

How did this idea develop in the following years? The surprising fact is that it did not develop at all! For almost 60 years, this idea was simply retold from one textbook to the other, but did not receive any essential development based on some model ideas until the 2016 the paper [7] of Aghababaei, Warner and Molinari appeared in Nature Communications. The authors of this paper considered once more the idea of Rabinowicz, but now at the level of physical mechanisms. Molinari and colleagues reproduced Rabinowicz's "thought experiment" using a mesomolecular model. They generated two media with micro heterogeneities, forced them to glide against each other and looked at what would happen. Of course, the medium studied in [7] did not correspond to any real material, but it had those parameters that only enter into Rabinowicz's formula: modulus of elasticity, surface energy and hardness. The authors of [7] used model potentials to describe interaction between the particles, where the part of the potential close to the minimum determined elastic properties and hardness, and the detachment energy could be controlled by its tail. All three parameters were ultimately determined by numerical experiments, for example, hardness by indentation.

The behavior of the system was found (or rather confirmed) to be strongly dependent on the size of colliding micro heterogeneities. For small enough initial asperity size, the main process was plastic deformation and gradual smoothing of the roughness. The process was completely different in the collision of two large asperities. In this case, from the very beginning, cracking and formation of wear particles dominated the process. Varying the size of the micro heterogeneities and the parameters of the material, Molinari and co-authors classified the systems with plastic smoothing and particle formation and placed them all on a single diagram, with axes "size of asperities" versus "critical size according to Rabinowicz" (up to a constant factor). In doing so, they brilliantly confirmed that in all cases when the size of the roughness exceeds the critical size of Rabinowicz, formation of wear particles occurs.

The paper [7] provides strong support for the hypothesis of Rabinowicz. However, it poses even more questions than it gives answers. In particular, the notion of "asperity", which is used both in the concept of Rabinowicz and simulations of Molinari and co-authors is ill-defined. It is therefore interesting to design some "toy model" that describes the interplay of the characteristic physical processes of plastic deformation and crack propagation, but in a simplified manner that would allow considering more complicated contact configurations.

A SIMPLE MODEL FOR COMPETITION OF SLIDING AND CRACKING

Since the Molinari model is quite complex and its potential is quite specific, including a rather long-range interaction, it would be interesting to come up with some simpler model in which all the components of Rabinowicz's wear process would be present, including the competition of processes of plastic deformation and fracture. It is indeed possible to design such a model. Let us consider an elastic half-space and bring a rigid cylindrical indenter of radius *a* into contact with it.

Now let us load the indenter with a force inclined under an angle of 45° to the undisturbed surface of the half-space (Fig. 3). We will assume that the local sliding between the cylinder and the half-space begins when the critical tangential stress τ_0 in the contact plane is exceeded. Applying a tangential force will lead to partial slip at the boundary of the contact, while the inner parts of the contact remain in stick. Increasing the tangential load leads to shrinking of the stick region until it completely vanishes and bulk macroscopic sliding starts. In this moment, the tangential force assumes its maximum possible value

$$F_{x,\max} = \pi a^2 \tau_0. \tag{5}$$

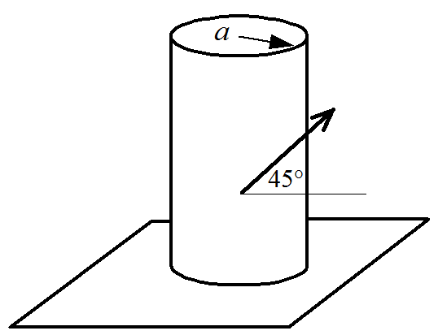

FIGURE 3. Rigid cylinder in contact with an elastic half space is loaded by an inclined force

On the other hand, the detachment of the cylinder from the half-space can occur when the normal force exceeds the critical adhesion force [8, 9]

$$F_{z,\mathrm{adh}} = \sqrt{8\pi E^{*}\Delta w a^{3}}. \tag{6}$$

Depending on what force is reached first, (5) or (6), the cylinder will show qualitatively different behavior. If (6) is larger than (5) then the condition of detaching is not fulfilled even at the moment of initiation of macroscopic sliding. Because both components of the force are assumed to be equal to each other, this means that that the indenter will slide without detaching. In the contrary case (6) smaller than (5), the condition of detaching will be fulfilled before the start of the gross slip. It is easy to see that the condition for "detaching a wear particle" is $F_{x,\max} > F_{z,\mathrm{adh}}$ which leads to the condition

$$D = 2a > \frac{16E^{*}\Delta w}{\pi\tau_0^2} \tag{7}$$

which up to a constant coefficient coincides with the Rabinowicz criterion (4).

This means that we can consider adhesive contacts loaded by an inclined force as a model for adhesive wear: the condition for detachment in such a system is the same as the condition for detachment of an adhesive particle.

This analogy opens the possibility of simulation of adhesive wear in contacts of complicated shape by considering adhesive contacts. These can be easily simulated using the boundary element method [10, 11].

GENERALIZED RABINOWICZ CRITERION FOR ELLIPTIC CONTACTS

The analogy described in the previous Section allows considering contacts of arbitrary complexity. In the present paper we confine ourselves only to a simple illustration of this analogy, applied to contacts of elliptical shape. If two asperities come together and form a welded joint in the form of an ellipse with a semi-major axis *a* and semi-minor axis *b*, then the stress distribution due to the normal force component F_z is

$$p(x, y) = -\frac{F_z}{2\pi ab}\left(1 - \frac{x^2}{a^2} - \frac{y^2}{b^2}\right)^{-1/2}. \tag{8}$$

The maximum stress intensity factor can be calculates as

$$-K_{\mathrm{I}}(a) = \frac{F_z}{2\sqrt{\pi ab}}. \tag{9}$$

The critical force of beginning of adhesive detachment can be determined by equating the stress intensity factor to its critical value $K_{\mathrm{I}} = \sqrt{2E^{*}\Delta w}$ [9], thus leading to the critical force

$$F_{z,\mathrm{cr}} = \sqrt{8E^{*}\pi ab^{2}\Delta w}. \tag{10}$$

On the other hand, the critical tangential force can be estimated as

$$F_{x,\mathrm{cr}} = \pi ab\tau_0. \tag{11}$$

The adhesive detachment will occur if (11) is larger than (10), which leads to a condition that exactly coincides with (7). This means that the condition for detaching is determined solely by the major axis of the ellipse and is not influenced at all by the size of the minor axis.

CONCLUSION

We suggested a simple "replacement model" for qualitative analysis of the process of adhesive wear. Application of this model to elliptic asperities has shown that the generalized Rabinowicz criterion for such contacts coincides exactly with the classical Rabinowicz criterion for circular contacts—if the critical radius have is replaced by the major semi- axis. This means that the process of detaching of wear particles is governed by the largest dimension of micro contacts, but not by their "aspect ratio". Analysis of the competition of plastic sliding and cracking in complicated fractal contacts [12] is still a task for the future research, which may lead to substantiation of both the wear equation (1) of Archard and the physical meaning of the coefficient of adhesive wear.

ACKNOWLEDGMENTS

The author acknowledges useful discussions with M. Popov and financial support of the Deutsche Forschungsgemeinschaft.

REFERENCES

1. J. F. Archard, J. Appl. Phys. **24**, 981 (1953).
2. E. Rabinowicz, *Friction and Wear of Materials* (John Wiley & Sons, Inc., 1995).
3. I. V. Kragelski, *Friction and Wear* (Butter Worth, 1965).
4. http://www.pressestelle.tu-berlin.de/menue/tub_medien/publikationen/medieninformationen/2017/oktober_2017/medieninformation_nr_1692017/
5. E. Rabinowicz, Wear **2**, 4–8 (1958).
6. F. P. Bowden and D. Tabor, *The Friction and Lubrication of Solids* (Clarendon Press, 2001).
7. R. Aghababaei, D. H. Warner, and J.-F. Molinari, Nat. Commun. **7**, 11816 (2016).
8. K. Kendall, J. Phys. D. Appl. Phys. **4**, 1186–1195 (1971).
9. V. Popov, M. Heß, and E. Willert, *Handbuch der Kontaktmechanik* (Springer, 2017).
10. V. L. Popov, R. Pohrt, and Q. Li, Friction **5**, 308–325 (2017).
11. R. Pohrt and V. L. Popov, Facta Univer. Mech. Eng. **13**(1), 3–10 (2015).
12. Q. Li and V. L. Popov, Phys. Mesomech. **21**(1) (2018).