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## **Down and Out or Up and In? Polarization-Based Measures of the Middle Class for Latin America**

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# Down and out or up and in?

## Polarization-based measures of the middle class for Latin America<sup>1</sup>

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### Abstract

This document presents a systematic review of empirical approaches to the identification and measurement of the middle class as the concept is used in the applied literature. It then presents an arguably less arbitrary definition of the middle class which is based on sound principles of distributional analysis and derived from income polarization measures. The document illustrates the differences between the existing approaches and the proposed methodology with a comparative analysis of the extent and evolution of the middle classes since the early 1990s in six Latin American countries. The polarization-based measurements of the middle class are shown to exhibit a greater degree of homogeneity in terms of some key socioeconomic characteristics than other measures employed in the literature.

*JEL Codes:* D3, I3, D6

*Keywords:* middle class, distribution, polarization, Latin America

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## 1 Introduction

The origins of the concept of the “middle class” can be traced to studies of social stratification. With different definitions, it has been used extensively in economic, sociological and historic analyses of modern societies. The subject has also been widely discussed in Latin America. The region’s countries, which are mostly low- and middle-income nations, are characterized by a relatively high degree of income inequality for their level of development. This aspect of the income distribution is probably what makes the issue of the middle class more salient in Latin America than in other areas of the developing world, since its potential growth would entail a reduction of the “excess” inequality in the region.

This structural factor underlies most of the studies of the middle classes in the region, which have spawned a body of literature that covers other related and potentially beneficial social effects of this group and its expansion. For instance, a larger middle class would lead to a reduction in the polarization between the rich and the poor, thus potentially enhancing social cohesion and reducing sources of conflict. The middle classes can also ease the formation of alliances that can give rise to greater redistribution and thus help to reduce poverty through the political process. Moreover, the growth of the middle class would increase the number of consumers who would be demanding goods and services above subsistence levels, thereby expanding the reach of local markets.

The abundant literature on Latin America’s middle classes reflects a common view as to their pivotal role in these and other aspects. However, as is also true for researchers looking into this issue in other regions of the world, providing a sound way to make the concept operational for applied studies is a major hurdle. While the term is undoubtedly heuristically appealing, there is therefore a lack of consensus as to the empirical definition of the concept of the middle class. The applied literature on the middle classes in Latin America and beyond is characterized by a diversity of definitions which may complement or contradict each other.

This document presents a systematic review of empirical approaches to the identification and measurement of the concept of the middle class as it is used in the applied literature. It then presents an arguably less arbitrary definition of the middle class which is based on sound principles of distributional analysis and derived from income polarization measures. The document illustrates the differences between the existing approaches and the proposed methodology with a comparative analysis of the extent and evolution of the middle classes since the early 1990s in six Latin American countries: Argentina, Brazil, Chile, El Salvador, Mexico and Uruguay. The main contributions of this paper are the systematization of existing approaches, the assessment of their relative merits in a comparative empirical exercise for Latin America, and the development of a simple definition derived from sound principles of distributional analysis.

The next section presents a review of the concept of the middle class as it has been applied in empirical work in Latin America. Section 3, in turn, discusses a definition of the middle class based on polarization measures. Section 4 presents the main empirical results. Section 5 concludes.

## **2 The middle class: A review of empirical approaches**

### ***2.1 The middle class: Sociological roots and economic approaches***

The concept of the middle class is intrinsically linked to the notion of social class and to the study of social stratification. While in some societies the strata are formally defined as, for instance, castes or other rigidly circumscribed social hierarchies, in modern western societies the concept of class is related to levels of income, wealth, educational attainment, ownership of productive assets, socioeconomic status and occupation, among others. Erikson and Goldthorpe (2002) discuss definitions based on occupational grouping and employment status in the context of intergenerational inequality. Giddens (1981) provides an in-depth discussion of the concept of social class in the sociological literature and a study of class in capitalist and socialist societies in the second half of the twentieth century. Wright (1997, 2005) presents a review of recent studies of class from contemporary Weberian and Marxian perspectives, and Bourdieu (1984) develops an analysis of class distinctions in modern life.

In contrast to these varying sociological approaches, the economic literature has generally taken a different path, starting from a given income partition of society and then studying the characteristics of the groups identified on the basis of that partition. The analysis of the homogeneity of middle class groups which is presented at the end of Section 4 discusses the relationship between socioeconomic characteristics and income levels.

### ***2.2 Definitions based on quantiles of the income distribution***

This study focuses on recent approaches developed in the field of economics and applied empirical analysis. In these approaches, a society is partitioned into groups defined by some measure of income, and the middle class is derived as a function of upper-, middle- and lower-income groups. As with poverty measurement, the key aspect of an income-based characterization of the middle class is the demarcation of the boundaries between the groups. Most of the boundaries set in the applied literature are based either on quantiles of the distribution or on measures of central tendency, with the latter being closely related to relative measures of poverty (see Deaton, 1997). The following characterization is derived from the seminal work of Foster and Wolfson (2010, first circulated in

1992), who develop an analysis of the relationship between these two types of measures of the middle class.

A first strand of applied work defines the middle class (and, by residual, the upper and lower classes) as a function of quantiles of the total or per capita income distribution. Each author adopts a particular definition, but the lower bound is usually the second or third decile of the distribution, while the upper bound is given by the top quintile or decile. The implicit rationale for this partitioning is, on the one hand, that the population in the bottom decile or quintile is poor by most absolute and relative measures and thus does not belong to the middle class. On the other hand, the rationale for defining the upper class so high up on the income ladder is based on the fact that the income distribution has a long tail, and thus only a small fraction of the population (or, at least, of the population captured by household surveys) is clearly above the highly compressed middle mass. Foster and Wolfson (2010) analyze the definition proposed by Levy (1987), who classified the middle three quintiles as belonging to middle class. In the recent applied literature, Solimano (2008) defines the middle class as those between the third and ninth deciles of per capita household income, Easterly (2001) and Barro (1999) include all households in the three middle quintiles of the distribution in this category, while, for Alesina and Perotti (1996), only the third and fourth quintiles qualify.

This approach has two main problems. On the one hand, the definition of the boundaries is arbitrary: it is hard to justify setting the lower boundary at the 15<sup>th</sup> rather than at the 20<sup>th</sup> percentile, or the upper boundary at the 85<sup>th</sup> rather than at the 90<sup>th</sup> percentile. While this arbitrariness is common to most definitions of the middle class, the second problem with this family of definitions is that, by construction, the three income groups are always of the same size: the proportion of the population between the  $x$ th and the  $y$ th percentiles is constant and equal to  $x-y$ . It is still possible to study the evolving socioeconomic characteristics of this group over time, or to study the income share of the lower, middle and upper groups in different countries or over time. However, this family of measures precludes a consideration of the question of whether the middle class is increasing or decreasing in size. A further issue documented by Foster and Wolfson (2010) is that, under any symmetric definition based on percentiles, a spreading movement in the distribution produced by transfers from individuals below the median to those above the median would result in a middle class of a similar size in terms of its share of total aggregate income.

### **2.3 Definitions based on measures of central tendency of the income distribution**

A second family of definitions of the middle class is based on measures of central tendency, such as the mean and the median. The lower bound is defined as a fraction  $x$  of mean or median income  $m$ , as in the definition of relative poverty lines, and the upper bound is defined analogously, usually as a multiple  $y$  of the same central tendency indicator. For instance, Birdsall et al. (2002)

defines the middle class as those households with per capita household incomes of between 0.75 and 1.25 times the median of the distribution, while Davis and Hudson (1992) use a wider range (0.5-1.5 of the median). Wolfson (1989) uses a range of 0.75-1.5 around the median of labor income. Finally, Foster and Wolfson (2010) discuss the definition proposed by Blackburn and Bloom (1985), which consists of a range of 0.6-2.25 around the median, and that of Thurow (1984), which coincides with the one developed by Birsdall et al. (2002).

An advantage of this family of measures is that the size of each group is sensitive to changes in the distribution of income, both in terms of growth (through the mechanical effect of changes of  $m$  on  $xm$  and  $ym$ ) and in terms of changes in the underlying dispersion of the distribution (changes in inequality will affect the size of the income groups even with a fixed  $m$ ). These definitions thus permit comparisons of the income share and of the size of each group over time and across societies.

However, even if the boundaries of the income groups and their sizes change endogenously as a function of changes in the income distribution, this family of measures still suffers from the same disadvantage as relative poverty measures: the multiplicative factors  $x$  and  $y$ , and the choice of the central tendency measure  $m$ , are arbitrary.

All previous measures can be expressed formally in terms of percentiles of the distribution by defining  $D(y)$  as the cumulative distribution of per capita income,  $p_n$  as the  $n$ th percentile, and  $y(x)$  as the income of household  $x$ . Table 1 presents these (and other authors' definitions) in terms of  $p_n$ .

Figure 1, derived from Foster and Wolfson (2010), illustrates how both types of definitions are constructed by fixing a segment of the income distribution, relying alternatively on the horizontal or the vertical axis of a cumulative distribution plot. Let  $Q=[a,b]$  be the percentiles defining the middle class under the first group of definitions and let  $R=[c,d]$  be the income segment that defines the middle class under the second group of definitions. The relative size of the middle class can be associated with two measures,  $M(R,F)$  and  $S(Q,F)$ , depending on the type of definition used.

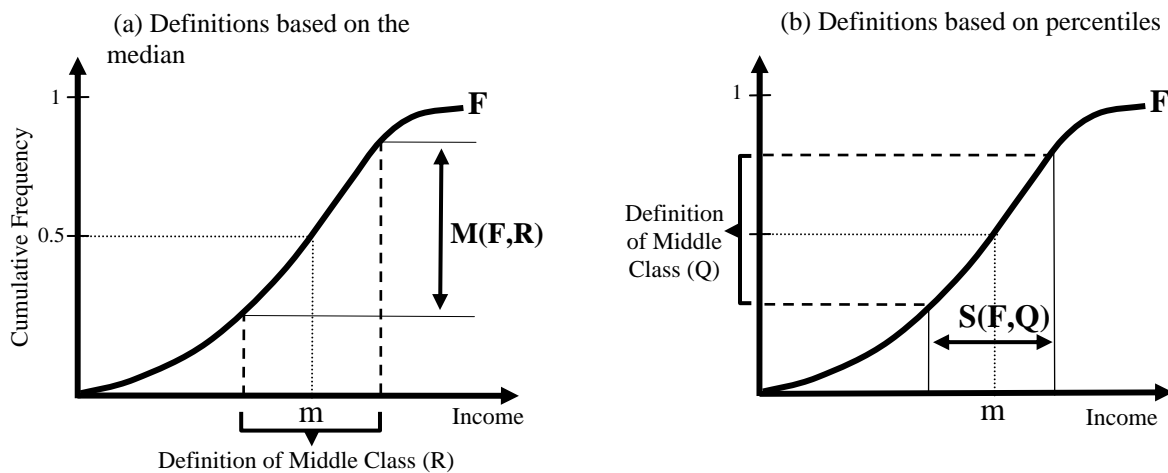
**Table 1**  
**Some Common Definitions of the Middle Class Based on Income or Consumption**

Authors		Definition as a function of cumulative distribution $D(y)$ , $n^{\text{th}}$ percentile $p_n$ , and $x$ 's household income (or expenditure) $y(x)$
Based on percentiles of the income distribution	Barro (1999) and Easterly (2001)	$x \in \text{Mid}C_3 \Leftrightarrow D^{-1}(p_{30}) \leq y(x) \leq D^{-1}(p_{80})$
	Solimano (2008)	$x \in \text{Mid}C_4 \Leftrightarrow D^{-1}(p_{30}) \leq y(x) \leq D^{-1}(p_{90})$
	Levy (1987)	$x \in \text{Mid}C_4 \Leftrightarrow D^{-1}(p_{20}) \leq y(x) \leq D^{-1}(p_{80})$
	Alesina and Perotti (1996)	$x \in \text{Mid}C_5 \Leftrightarrow D^{-1}(p_{50}) \leq y(x) \leq D^{-1}(p_{80})$
	Partridge (1997)	$x \in \text{Mid}C_6 \Leftrightarrow D^{-1}(p_{50}) \leq y(x) \leq D^{-1}(p_{60})$
Based on median income	Birdsall et al. (2000) (1)	$x \in \text{Mid}C_1 \Leftrightarrow 0.75 * D^{-1}(p_{50}) \leq y(x) \leq 1.25 * D^{-1}(p_{50})$
	Davis and Hudson (1992)	$x \in \text{Mid}C_2 \Leftrightarrow 0.5 * D^{-1}(p_{50}) \leq y(x) \leq 1.5 * D^{-1}(p_{50})$
	Wolfson (1989)	$x \in \text{Mid}C_2 \Leftrightarrow 0.75 * D^{-1}(p_{50}) \leq y(x) \leq 1.5 * D^{-1}(p_{50})$
	Blackburn and Bloom (1985)	$x \in \text{Mid}C_2 \Leftrightarrow 0.6 * D^{-1}(p_{50}) \leq y(x) \leq 2.25 * D^{-1}(p_{50})$
Based on absolute thresholds	Banerjee & Duflo (2007) (2)	$x \in \text{Mid}C_3 \Leftrightarrow 2\text{usd} \leq y(x) \leq 10\text{usd}$
	Ravallion (2009)	$x \in \text{Mid}C_3 \Leftrightarrow 2\text{usd} \leq y(x) \leq 13\text{usd}$

(1) Also used by Thurow (1984), as pointed out by Foster and Wolfson (2009).

(2) In the original framework, two alternative segments were used by the authors.

**Figure 1**  
**Relative Size of the Middle Class under Definitions based on the Median and Percentiles**



Source: Adapted from Foster and Wolfson (2010).



## 2.4 Poverty lines and other thresholds

A third family of definitions of the middle class is conceptually closer to poverty measures based on absolute thresholds. These are based on a poverty line,  $z$ , derived from the cost of a basket of basic goods and services. If an absolute poverty line such as  $z$  provides a good delimitation between lower- and middle-income groups, then the ideal boundary between the middle- and the upper-income group could be constructed as a “richness line”  $r$ , based on the same consumption and expenditure surveys from which  $z$  was derived.

The idea of a minimum basket of goods and services is relatively straightforward, even though its specific contents may be highly debatable. On the other hand, it is not clear which criteria should justify and guide the construction of a “richness line”.<sup>2</sup> It might be defined as the value of a specific basket of goods and services consumed by upper-income groups, or it could be defined as the cost of a basket of goods and services beyond basic needs, including, perhaps, “unnecessary” or conspicuous (in Veblen’s 1899 sense) consumption goods.<sup>3</sup> Because of this ambiguity, however, these thresholds are bound to be controversial.

Another strand of the literature on the middle classes borrows the conceptual toolset from international absolute poverty measures, such as those developed by the World Bank (2000). These indicators are based on poverty lines defined as some  $z$  value expressed in purchasing parity adjusted units: for instance, 1 or 2 PPP US dollars per day. Analogously, the upper and lower bound income levels that include the middle class are defined in terms of international currency units, with the lower bound usually corresponding to one of the widely used international poverty lines. The intuition is that the middle classes are the groups within each country with income levels between the per capita GDP of middle-income and rich countries (e.g., Brazil and Italy). This perspective has a series of advantages. Firstly, international comparisons are straightforward, since middle classes earn between  $x$  and  $y$  PPP USD in any country, and the measure is derived not from one society’s distribution but rather from the entire world’s income distribution. It is thus possible to study both the size and the income share of the middle class.

The World Bank (2007), for instance, defines the “global middle class” as households earning 4,000–17,000 per capita PPP USD. Banerjee’s and Duflo’s (2007) international comparative study represents another example of this strand of literature. They define the middle classes as groups with per capita consumption levels of between 2 and 4 and between 6 and 10 PPP USD per day. They also

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<sup>2</sup> Peichl et al. (2008) and Araar (2008) present a series of “richness” measures, which are basically mirrors of the standard Foster et al. (1984) family of poverty measures. However, none of these papers develops a proper “richness” line; Peichl et al. (2008), for instance, defines middle-class boundaries as 60 percent of the median equivalized income, and Araar (2008) uses 200 percent.

<sup>3</sup> It should be noted that marketing practitioners have well-defined social groups based on socioeconomic and income levels, and the ownership of certain goods or qualifications usually provides enough information to classify a household. This literature is beyond the scope of this document, however.

provide a description of a series of demographic, labor, educational and other characteristics for these middle-income groups. However, they find a notable resemblance in consumption patterns between the middle classes and the poor within countries, but with high heterogeneity between countries, which may be reflecting the problems of comparability introduced by PPP adjustments. Following a similar idea, Ravallion (2009) defines the middle class in absolute terms, arguing that, for the developing countries, the middle-class group should be defined as being composed of the individuals who are not poor in their home countries but have per capita incomes below the US poverty line (approximately 13 USD per day in PPP terms). The lower threshold suggested by Ravallion (2009) is the 2 USD poverty line. While intuitively appealing, these measures suffer from the same problems that affect international poverty comparisons, most of which are related to the reliability of PPP adjustments (Deaton, 2001).

## **2.5 Other endogenous definitions**

There have also been other attempts to define the middle classes in the economic literature. A straightforward alternative is to use the human capital stock of the household as a proxy for permanent income and vulnerability. Income levels can be supplemented with information on the educational attainment and occupational status of adult household members, as in León (2008). This line of research constitutes an update of the more traditional sociological definitions of the middle classes. Torche and López-Calva (2010), on the other hand, define the middle class according to household characteristics related to the capacity to generate income.

Yet another possibility is to “let the data talk” by performing cluster and principal-factor analyses on income (or perhaps income and education) variables. While useful for market research and other applied areas, these “black box” tools are not based on sound principles of economic analysis. The derivation of income groups through an analysis of the shape of the distribution has also been pursued in the literature. Zhu (2005) develops a non-parametric study and partition of the US personal income distribution. While sophisticated, the analysis raises some of the same “black box” concerns as those mentioned above. Some parametric alternatives have also been derived from what is known about the shape of the income distribution at different levels. D’Ambrosio, Muliere and Secchi (2002) present an example of these types of analyses, which are reviewed in detail by Olivieri (2008), who also develops an application to the Greater Buenos Aires metropolitan area of Argentina in the long run. The intuition is that different classes have different income-generating processes that result in overlapping distributions, and the cut-off points of these distributions are estimated through maximum likelihood methods based on assumptions concerning the underlying distribution functions. Massari, Pittau and Zelli (2009) develop a related non-parametric analysis. While appealing, the

results obtained from this methodology may be dependent on the parametric assumptions about these underlying functions.

Finally, another strand of research focuses on self-perceptions of class. Lora and Fajardo (2010) offer an in-depth analysis of these subjective measures for Latin America based on information from the Gallup World Survey.

The following section presents a methodology for partitioning the population into three groups that is derived from principles of distributional analysis and the polarization literature.

### 3 A definition of the middle class derived from polarization measures

#### 3.1 Identifying similar individuals in heterogeneous groups

This section develops a definition of the middle class that seeks to avoid the arbitrariness of some of the traditional measures discussed above by endogenously defining the cut-off points between income groups. This approach is based on seminal studies in the polarization literature, such as those of Esteban and Ray (1994) and Esteban, Gradín and Ray (1999), which are briefly described and then adapted to the context of the measurement of the middle class.

The concept of polarization seeks to quantify the gap between two groups which have a similar internal composition but are clearly different from each other. Society can thus be conceived of as an amalgamation of groups, where certain individuals are similar and others differ relative to some given set of attributes or observable characteristics. Esteban and Ray (1994) argue that there are necessary conditions that must be present in order for a society to have polarized groups. These criteria are based on individuals' perceptions of their own social condition and can be attributed to a certain level of income, since an individual may either identify with or feel different from any given social group based on his or her earnings.

Formally, for a level of income  $y$  with a distribution function  $F$ , Esteban and Ray (1994) define a function with two key components: (i) the *identification* of an individual with a certain group, denoted by  $I(y,F)$ ; and (ii) the *alienation* that person feels with respect to other groups, denoted by  $r(\delta(y,x))$ , where  $\delta(y,x)$  is the Euclidean distance between  $y$  and another individual's income,  $x$ .<sup>4</sup> Polarization increases when individuals identify more with their group and feel more alienated from other groups. The "effective antagonism" function joins both these concepts into one function,  $T(I,r)$ , which captures the identification of individual  $y$  with his or her own group and that

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<sup>4</sup> Both functions are continuous and increasing in their arguments. For more details on these functions, see Esteban and Ray (1994).

individual's alienation from individual  $x$ . Expanding to the entire population, polarization in any given society is the sum of all of these effective antagonisms:

$$P(F) = \iint T(I(y, F), r(\delta(y, x))) dF(x) dF(y)$$

Under the assumption that the distribution function has a bounded support function and a finite number  $n$  of social groups composed of  $\pi_i$  individuals each, this definition may be rewritten as:

$$P(\pi, y) = \sum_{i=1}^n \sum_{j=1}^n \pi_i \pi_j T(I(\pi_i), r(\delta(y_i, y_j)))$$

This general measure of polarization is narrowed down to a more manageable definition by means of a series of axioms which impose a series of restrictions on the parameters and functions found in these two equations (Esteban and Rey, 1994). The class of functions that fulfills these axioms takes the following form:

$$P(\pi, y) = k \sum_{i=1}^n \sum_{j=1}^n \pi_{i=1}^{1+\alpha} \pi_j |y_i - y_j| \text{ with } k > 0 \text{ and } \alpha \in [1, 1.6]$$

This axiomatic index makes it possible to quantify differences between groups and can be easily generalized to a number  $n$  of groups, as described in the following subsection.

### **3.2 Measures of polarization based on $N$ groups and the value of group boundaries**

Esteban, Gradín and Ray (1999) observe that the polarization measure described above requires the prior identification of a finite number of social groups within the original distribution. In order to avoid an ad hoc definition of a fixed number of groups, these authors propose an extension of the previous polarization index. If the income distribution can be associated with a density function  $f$  in a closed interval, then this function may be represented by a function with  $n$  peaks called  $\rho$ . This representation is an approximation to the original density function, and it therefore implicitly defines an error term, denoted by  $\varepsilon(f, \rho)$ . The main problem is how to go about obtaining the desired groups in an optimal way by minimizing the error term when approximating the data's real density.

Defining the original measure proposed by Esteban and Ray (1994) as  $ER(\alpha, \rho)$ , the new indicator may be rewritten as:

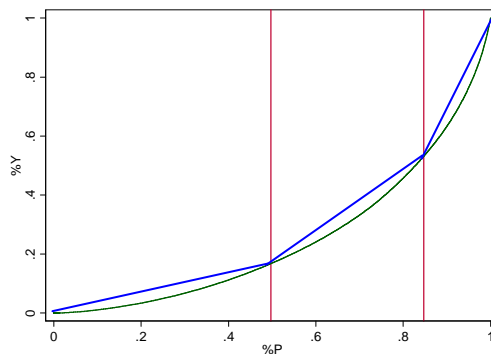
$$P(f; \alpha, \beta) = ER(\alpha, \rho) - \beta \varepsilon(f, \rho)$$

where  $\rho$  is the density function,  $\alpha$  is a parameter, and  $\beta$  is the error's weight. This method implies that all the group cut-off points are selected in such a manner as to minimize the error term. This problem may be thought of as an approximation of the Lorenz Curve using a function composed of  $n$  segments (one per group). These segments must be located in such a manner that the area between this "approximated" curve and the original is the smallest possible. Figure 2 (adapted from Gasparini, Horenstein and Olivieri, 2006) illustrates this approximation for the three-segments case, using household income data for Honduras in 2003. Any change in the cut-off points that define the three groups would produce an increase in the total area between the two curves. Esteban, Gradín and Ray (1999) show that the error term is minimized when the income cut-off point between any of the adjacent groups is exactly the same as the mean income when only those two groups are taken.

This definition of groups, based on the polarization literature, can be adopted to provide, as a by-product, a methodology to define low-, middle- and high-income groups. The middle class shown in Figure 2 corresponds to the second (or middle) segment of the distribution.

The main advantages of this definition are that the cut-off points are determined endogenously by the shape of the whole income distribution and that the resulting groups are derived from precisely defined concepts such as identification, alienation and effective antagonisms. For a fixed number of groups (in this case, three), these measures provide the highest possible level of inter-group polarization.

**Figure 2**  
**Lorenz Curve and Polarization: Determination of Income Groups for n=3:**  
**Honduras (2003)**



Source: Gasparini, Horenstein, Olivieri (2006).

## 4 Empirical results

### 4.1 Data

This section compares the performance of the measures of the middle class presented in Sections 2 and 3. These empirical results are based on household survey microdata for six countries in Latin America and the Caribbean (LAC): Argentina, Brazil, Chile, El Salvador, Mexico and Uruguay. Since this study seeks to analyze trends, each country has a first observation in the early 1990s and three additional cross-section surveys,<sup>5</sup> with the last observation in the mid-2000s. The surveys originate in the *Socio-Economic Database for Latin America and the Caribbean-SEDLAC* project (CEDLAS and World Bank, 2010). The information has been homogenized using the same criteria, allowing maximum comparability between countries and years,<sup>6</sup> a characteristic which is particularly important for the main variable of interest in this paper, i.e., individual per capita income.<sup>7</sup>

### 4.2 The size of the middle class in Latin America

Tables 2-7 present the main results by country for some of the definitions of the middle class presented in Table 1: Birdsall et al. (2000) represent the definitions based on the measures of central tendency; Barro and Easterly (2001) correspond to those based on quantiles; and Ravallion (2009) and Banerjee and Duflo (2007)<sup>8</sup> are used to illustrate recent measures based on absolute thresholds.<sup>9</sup> Finally, the results are also presented for the definition proposed in the previous section based on the EGR3 tripolarization group. The first four rows in each of the tables give the absolute size of the middle group and show how it has evolved over time.

The first remarkable result from Table 2 is that these definitions imply widely varying sizes of the middle class within each country. Moreover, the changes over time in the relative size of the middle-class population are also heterogeneous. Absolute measures appear to display erratic and volatile behavior, particularly in Argentina, Brazil, Chile and Uruguay. Measures based on polarization indices tend to be more stable across years. Finally, those based on percentiles or the median also seem relatively stable, although by definition the former are less affected by

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<sup>5</sup> Specific years are summarized in Table A.1 in Appendix 1.

<sup>6</sup> See CEDLAS and World Bank (2010) for the methodology of this homogenization process.

<sup>7</sup> Additionally, to improve comparability, the comparisons are over the same geographic regions, and non-responses and incoherent income responses are dealt with accordingly. For Argentina, only the 15 main cities surveyed in the 1991 household survey are included in the estimations.

<sup>8</sup> For these absolute measures, the lower threshold for the middle class is defined as the 2 PPP USD international poverty line, while the upper threshold is 10 PPP USD for Banerjee and Duflo (2007) and 13 PPP USD for Ravallion (2009). These values have been slightly adapted, and the results for both measures are based on the update to the 2 USD PPP poverty line, which was recently revised by the World Bank to 2.5 USD PPP.

<sup>9</sup> Banerjee & Duflo (2007) use expenditures instead of income and consider two alternative intervals: [2,4] and [6,10] PPP USD. The lack of systematic expenditure or consumption information in Latin America implies that the estimations are based on the combination of these two intervals and that per capita income is the relevant dimension. Conconi and Ham (2009) present a study of the Banerjee and Duflo (2007) measures for Argentine expenditure data.

distributional changes.<sup>10</sup> Figure 3 depicts these trends. While the results presented here refer only to the middle class, some of these trends can be attributed to the variation in either the upper or lower groups. For instance, the increase in the middle class in the last period for Argentina and Uruguay (using the polarization definition) is due to a reduction in the size of the lower-income group, as depicted in Tables A.8 and A.13 in Appendix 1.

The changes between periods for relative and polarization measures are fairly small – usually less than one percentage point. This indicates that the growth/reduction in the group’s size follows a smooth trajectory and that social mobility (in either direction), if happening at all, is spread out over a long period. The changes in the size of the middle class that are seen when each country is observed separately are small. Brazil, Chile and El Salvador show stable patterns across time, with little or no change in each observed period. Argentina and Mexico stand out as being the most sensitive to changes in the definition, although the variation is still relatively small. Finally, Uruguay is the only country where the number of households belonging to the middle class appears to have fallen, although this group is nonetheless the largest when compared to the rest of the countries in the sample.

In contrast to these patterns, the measures based on Banerjee’s and Duflo’s (2007) and Ravallion’s (2009) absolute definitions point to large fluctuations in the size of the middle class, even between consecutive periods. This is especially true for Argentina and Uruguay, which exhibit large changes for the period following the 2001-2002 crisis. Chile and Brazil, on the other hand, display a decreasing pattern in the size of middle class, while the results for El Salvador show a significant increase across the whole period. Mexico is the only country in the sample where these absolute measures exhibit a level of stability comparable to that observed with relative measures. The description of these patterns can be complemented by the trends in the size of upper- and lower-income groups. The results shown in Tables A.8 and A.13 indicate that, in Argentina and Uruguay, the lower class expanded considerably in 2000-2003, while the upper-income group in Argentina increased in size during the 2003-2006 post-crisis recovery. Moreover, the patterns of relative growth and stability in Brazil and Chile over the period are reflected in the shrinkage of the lower-income group as defined by the absolute measures, and the observed reduction in the middle class is correlated with a sharp increase in the size of the upper class (see Tables A.9 and A.10). In contrast, the size of the middle class in El Salvador increases because the reduction in the size of the lower class is larger than the expansion of the upper class. Finally, the stability of the Mexican middle class across time is not correlated with a stable pattern in the other income groups; instead, the lower- and higher-income groups have changed in size in opposite directions, thereby offsetting each other, with

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<sup>10</sup> While the quantiles are defined in terms of the distribution of per capita household income for all individuals, the size of the middle class is defined as the proportion of households in this group. The changes in the size of quantile-based measures reflect underlying differences in household composition by income group.

the result that the middle class has remained constant. Finally, the results from the absolute measures also indicate that lower-income households represent a very small proportion of the total — much lower than with other measures. This is due to the low levels of extreme poverty that exist in the region when using the 2 PPP USD international poverty line (Gasparini, Cruces and Tornarolli, 2011). The size of the lower-income group is substantially larger for the polarization measure, while only a small percentage of households are classified as belonging to the highest-income group.

### **4.3 The middle-class income share**

The results discussed in the previous section indicate that, with some exceptions, the size of the middle class as a proportion of total households has been relatively stable over time. Another relevant dimension for the analysis of the middle class is the portion of national income appropriated by this income group. This information is presented in the fifth row of Tables 2-7 and is plotted in Figure 4 for each country and year in the sample.

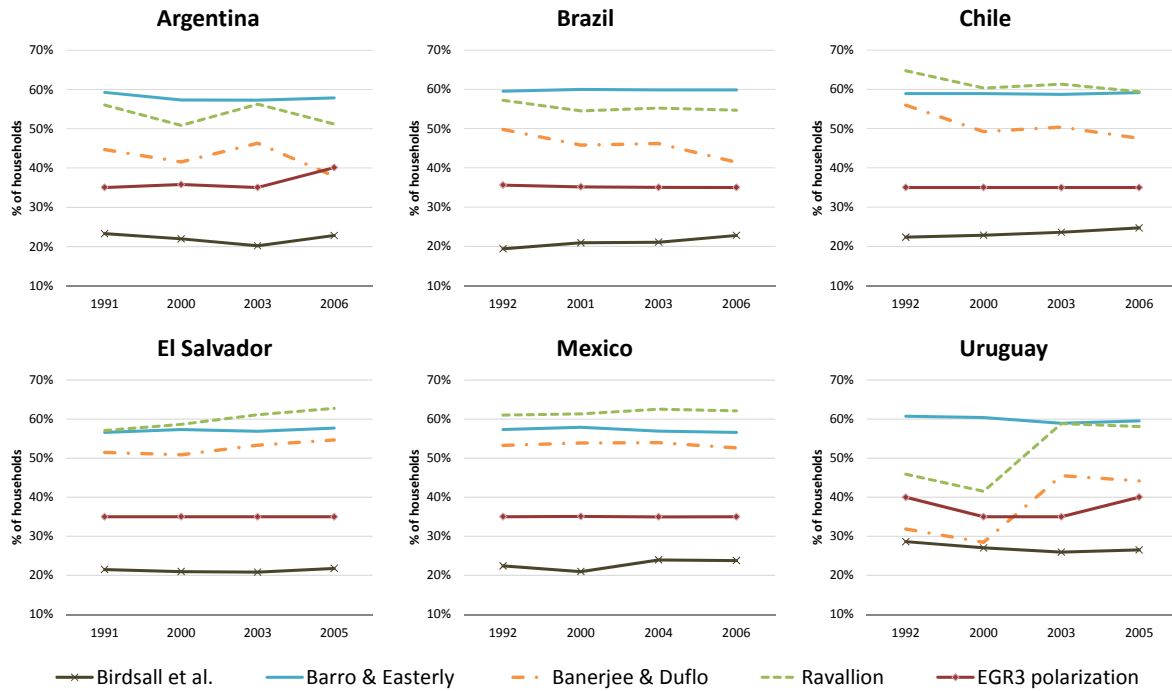
Focusing solely on the EGR3 polarization-based measure, the share of income accruing to the middle class appears to be relatively stable at approximately 30 to 40 percent. However, this proportion is decreasing over time in the cases of Brazil, Mexico and Uruguay, while only Argentina and Chile exhibit modest increases in this income share in the period under analysis.

Definitions of the middle class based on absolute thresholds exhibit a far greater volatility in the income share, with all countries except Uruguay displaying a reduction over the entire period under analysis. Brazil and Chile are the countries with the largest decreases in the absolute-threshold middle-class income share, and the evidence in the appendix indicates that this pattern is mainly accounted for by a drastic increase in the upper-income group's share of total income (the same pattern for the upper-income group is observed in most of the countries in the sample). In contrast, the increase in the middle class's share in Uruguay is offset by a substantial fall in the upper class's income share. It should also be noted that, while the size of the middle class appears to fluctuate substantially in Argentina, its share of total income has remained relatively stable when compared to the other countries in the sample.

Finally, in contrast with the definitions of the middle class based on absolute thresholds, the income proportions derived on the basis of the relative definitions are significantly more stable. This result, however, is an artifact of the underlying definitions, which use income shares and quantiles as their main inputs.

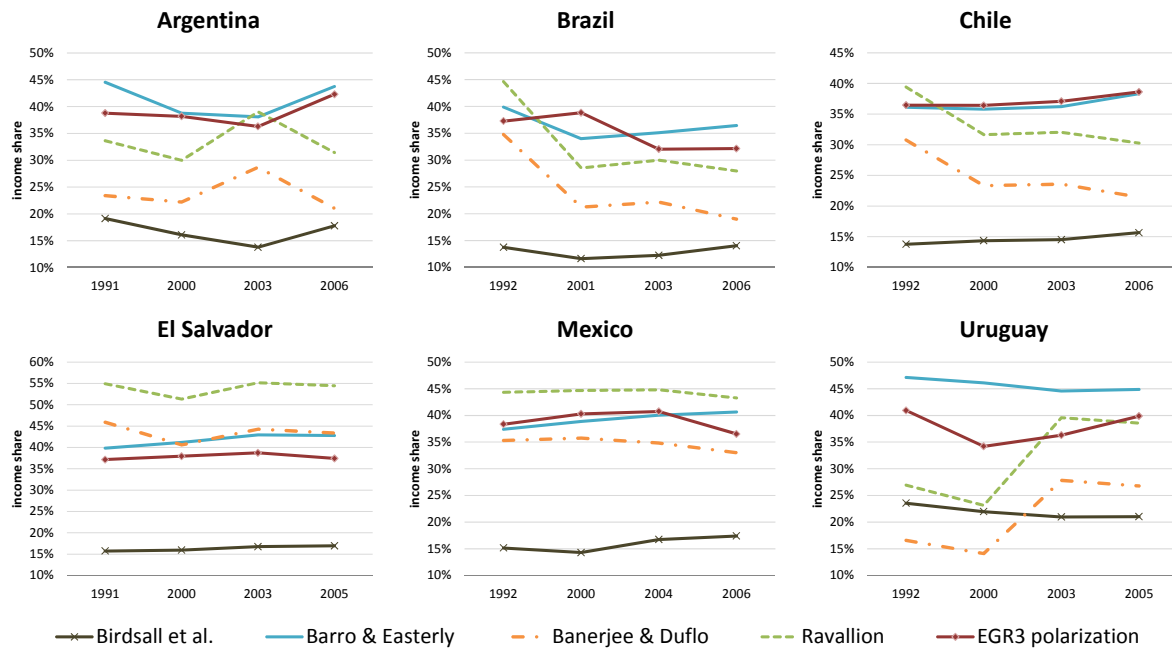


**Figure 3**  
**Changes in the Size of the Middle Class, by Definition**  
**(Percentage of Households)**



Source: Own calculations based on household survey results.

**Figure 4**  
**Share of the Middle Class in Total Income**



Source: Own calculations based on household survey results.

#### **4.4 Characteristics of middle-class households**

Until now, the analysis has focused primarily on the size of the middle classes and the income share accruing to them according to different definitions. This section builds on the poverty profile literature and presents a brief overview of middle-class household characteristics. These results are presented in rows 12-15 of Tables 2-7, which correspond to four main aspects of the household: (i) the household head's years of formal education, (ii) the percentage of householders that have completed at least their secondary schooling, (iii) the proportion of household heads that have completed their university education, and (iv) household size.<sup>11</sup>

The findings indicate that Argentina and Chile are clearly the cases where the middle-class household heads' average level of education is the highest for all definitions. However, Figure 5 indicates that, over time, the education level of middle-class heads of household in all countries in the sample has increased. The results are similar when considering alternative education indicators, such as the proportion of household heads with at least a secondary education (Argentina and Chile once again exhibit the highest levels). With the exception of Uruguay, education levels rose in all cases during the first decade under analysis, with large overall increases in all of the countries by the end of the period. The situation is similar for the proportion of household heads with a university education, but the proportion is much smaller for the middle class, as heads of household with this level of education are concentrated in the upper-income group for all definitions employed in the analysis.

Finally, following broader demographic patterns, average household size has fallen in the middle class, with a few exceptions for certain countries and definitions.

To sum up, middle-class households have substantially higher stocks of human capital than those in the lower-income group, but are still lagging behind their upper-class counterparts, especially in terms of tertiary education. Nevertheless, the average education level has been rising over time for most definitions and countries under analysis, and additional evidence indicates that educational attainment has been climbing at a faster rate for this group than for the lower-income group.

The information in the appendix sheds some further light on the differences between these groups in terms of other characteristics. Based on the latest available year for each country, Table A.2 indicates that in Argentina, Chile and Uruguay the differences in house ownership and infrastructure indicators are larger between the middle and the lower class than the differences between the middle and upper classes. In the other three countries (Brazil, El Salvador and Mexico), the middle and lower classes appear to be more homogeneous in terms of house ownership. This conclusion is true for all

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<sup>11</sup> More detailed profiles are presented in Appendix 1, Tables A.2-A.6. These profiles are computed for a larger number of dimensions, including housing and household infrastructure, education, labor and income structure. The tables provide data only for the last available year in the sample for each country.

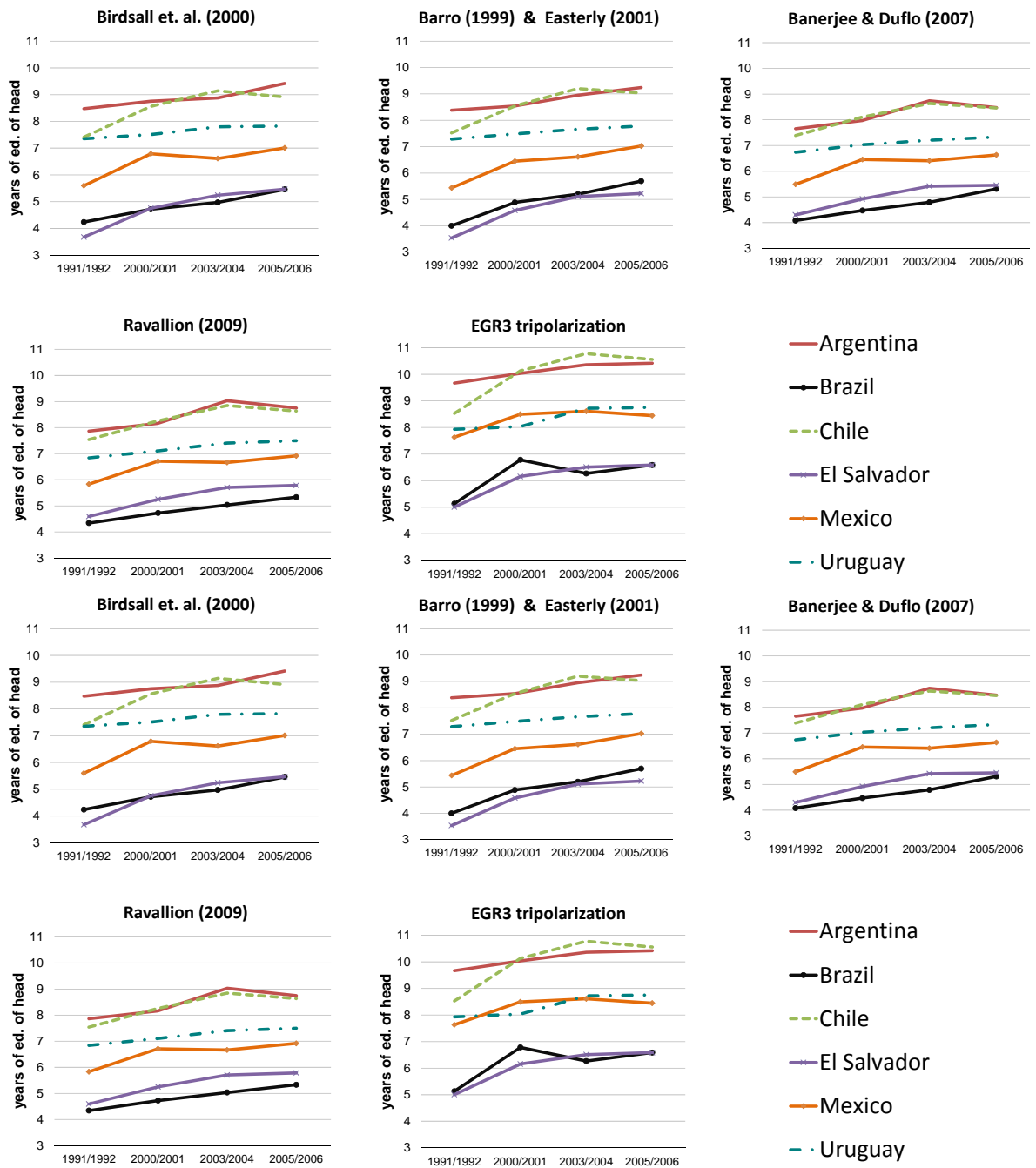
but the polarization definition, which yields a smaller gap between the middle- and upper-class housing indicators for all countries under analysis.

Similar observations can be made regarding the educational outcomes presented in Table A.3: the gaps in all the educational variables obtained with the EGR3-based definitions are larger than those obtained with the alternative measures. Labor-market outcomes by class are presented in Tables A.4 and A.5, which reflect a similar pattern for all the countries and definitions: activity and occupation rates are higher for the middle and upper classes, and unemployment rates are higher for the lower classes. Labor formality – defined as contributions to social security (not reported) – also rises by income group for all definitions. Unlike the educational and housing variables, the values of the labor variables are almost equidistant between classes, indicating that the labor dimension is an important underlying factor in the income-based identification process. The tables also show that entrepreneurs and salaried workers are concentrated in the higher classes and that there is more self-employment in the lowest class for all definitions. Finally, the evidence in Table A.6 indicates that, the higher the income group, the larger the share of total household income represented by labor income, with the exception of the results for the EGR3-based definition, for which the proportion of total income accounted for by labor income for the lowest-income group is similar to or slightly larger than it is for the middle class in almost all of the countries.

#### **4.5 Poverty and the middle class**

The discussion of the definition of lower-, middle- and upper-class income boundaries presented in Section 2 indicated that only some of the absolute-threshold-based definitions incorporated an explicit consideration of the relationship between class and poverty measures. For all other measures, there might be poor households in any of the income groups, since the poverty line can be located anywhere with respect to the lower-middle and middle-upper thresholds. An illustration of this point is provided by Figure A.1 in Appendix 1, which depicts the poverty line with respect to the cut-off points for each definition of lower, middle and upper classes in the case of Argentina in 2000 and 2003. The location of the poverty line varies significantly depending on the definition used, due primarily to the proximity of the poverty line and the cut-off point for the low and middle classes. While the poverty line is close to the lower threshold, depending on the year and the definition, in some cases a large portion of middle-class households appear to be poor.

**Figure 5**  
**Middle-Class Household Head's Average Years of Formal Education**



Source: Own calculations based on household survey results.

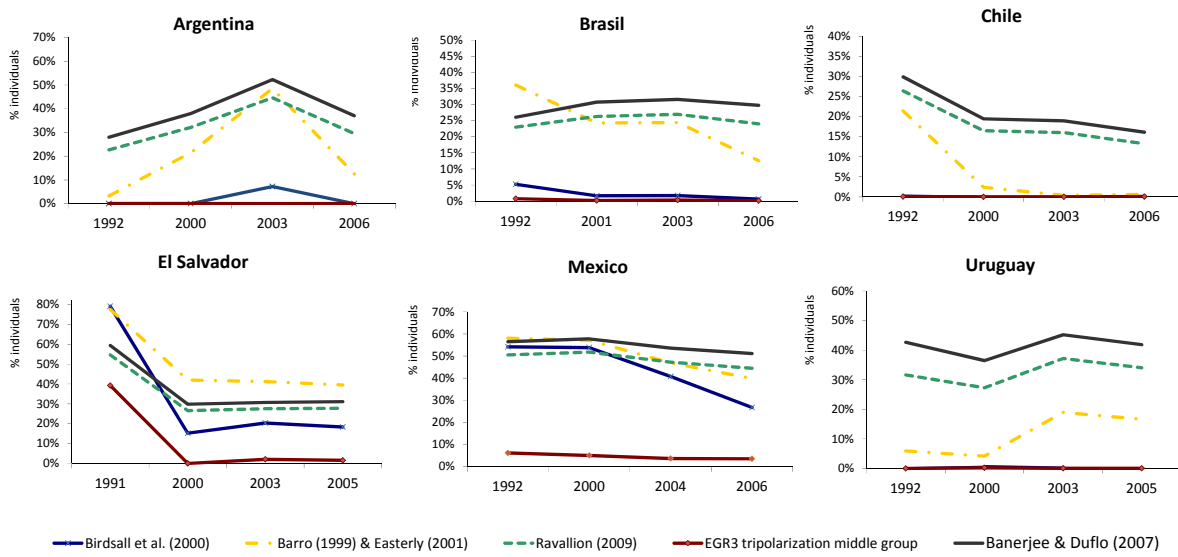
This phenomenon is not restricted to Argentina. Figure 6 depicts the poverty headcount rates for official moderate poverty lines estimated for each definition of the middle class for all the countries and years in the sample. The figure indicates that poverty rates are substantially volatile across years. Moreover, there is no clear pattern across definitions in any country, with the exception of Mexico for the relative threshold definitions (Barro and Easterly and Birdsall et al.), which tend to be higher. The EGR3-based definition of the middle class tends to yield the lowest poverty rates and the most stable patterns in this indicator. Poverty rates for absolute-threshold-based definitions of the poverty line also exhibit a relative degree of stability over time, although poverty rates are always positive and high for the middle class. This is not surprising, since there is a high correlation between the lower bounds of absolute measures (the 2 PPP USD poverty line) and the moderate poverty lines defined by each country (Gasparini et al., 2011). The four bottom rows in Tables 2-7 give middle-class poverty rates based on alternative poverty lines ; these rates are zero in a number of cases, since the 1 PPP USD and the official extreme poverty lines tend to be substantially lower than the lower/middle threshold for most definitions and countries.

Finally, the results indicate that, for all countries except Argentina and Uruguay, there is a decreasing pattern over time in the middle-class moderate poverty headcount for all definitions. In the case of Argentina and Uruguay, the 2003 surveys exhibit a steep increase in poverty rates due to the severe economic crises that affected both countries in 2002.

#### **4.6 *Income-based definitions of the middle class and intra-group homogeneity***

The analysis presented in the previous sections described the changes occurring in the size, income share, socioeconomic characteristics and poverty levels of the middle class for countries in Latin America based on a series of alternative definitions of this group. While the levels and trends given by different definitions can be compared, the evidence presented so far allows only a partial assessment of which of the alternatives captures the underlying concept of the middle class the best. For instance, definitions associated with high volatility in terms of the middle class's size and income shares, such as those based on absolute thresholds, are probably capturing some spurious variation, such as changes in the underlying PPP values of the thresholds at the national level.

**Figure 6**  
**Changes in the Poverty Headcount for the Middle Class**  
**(Official Moderate Poverty Lines)**



Source: Own calculations based on household survey results.

The lack of a consensus about the notion of the middle class implies that there is no proper metric to compare the performance of alternative empirical implementations. However, the idea of dividing society into groups implies that these groups should have some characteristics in common, and it is possible to assess the degree of within-group (or class) homogeneity and between-group heterogeneity along a series of socioeconomic indicators. In a sense, this type of analysis reverses the direction of the analysis conducted by the sociological and earlier traditions, which based the notion of class not on income, but on common aspects such as labor-market participation, occupational status or employment type, among others. The salience of these labor-related characteristics and the importance of labor-market outcomes as determinants of household income in Latin America (Gasparini et al., 2011) motivate a homogeneity-heterogeneity analysis focused on labor-related indicators. To capture the degree of within-group separation and the multiple dimensions of labor-market outcomes, this exercise relies on linear discriminant analysis techniques, which are described in detail in Appendix 2. The results are presented in terms of two measures of separation or discrimination between groups:

- *Wilk's Lambda*, defined as  $\Lambda = 1/(\lambda_1 + 1)(\lambda_2 + 1)$ , where  $\lambda_1, \lambda_2$  are the eigenvalues associated with each discriminant function. The lower the value of Wilk's Lambda, the better is the discriminant power of the functions, which indicates a higher degree of separation between groups given the set of discriminant variables.

- *Mahalanobis distance between centroids*, defined as  $D_m(x, y) = \sqrt{(x - y)'M^{-1}(x - y)}$ , where  $x, y$  are the centroids of any pair of groups and  $M$  is the correlation matrix between variables  $x$  and  $y$ .

While applications of discriminant analysis usually evaluate the discriminant power of different variables, the results in this section evaluate the degree of separation of different group definitions based on a fixed set of labor-market-related variables. The set of variables is composed of: years of formal education of the household head ; the weekly number of hours worked by the head of household ; the highest hourly wage among employed members of the household ; a set of dummies indicating if the head is unemployed, employed or inactive ; a set of dummies indicating if the head of household is salaried, self-employed or the boss in his/her activity ; and, finally, a set of dummies indicating whether the head of household or his/her spouse have the following job-related characteristics: a job with pension rights, a job with health-care coverage, full-time employment and affiliation with labor unions.<sup>12</sup>

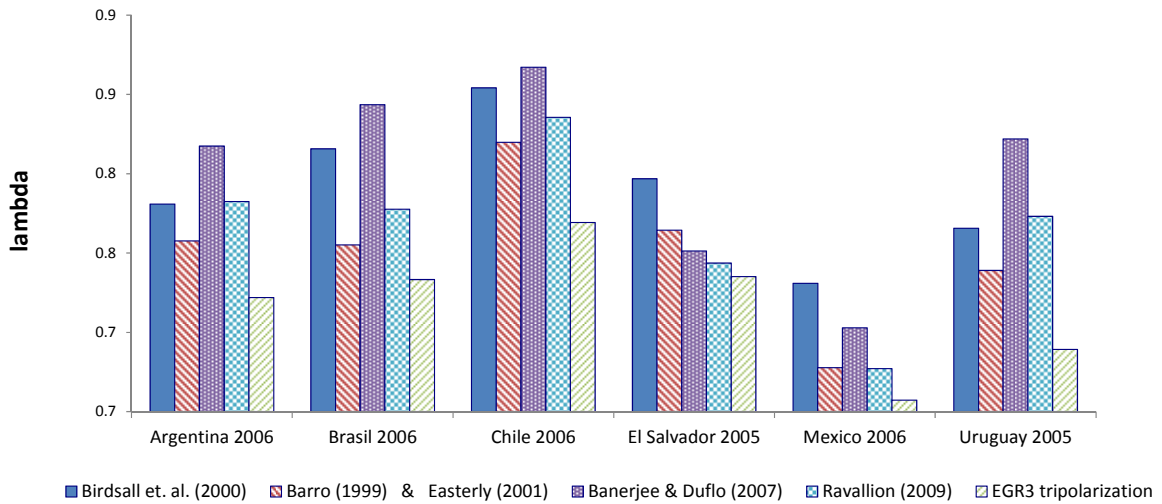
The results for all years and countries are presented in Table A.7 in Appendix 1. The first row in the table presents Wilk's Lambda, the second row gives the Mahalanobis distance between the lower and the middle class, and the third row shows the Mahalanobis distance between the middle and the upper class. The analysis of Wilk's Lambda indicates, in most cases, that the EGR3-polarization-based definition of the middle class generates higher degrees of separation between social classes in terms of labor variables. Definitions based on absolute thresholds result in a high degree of separation between the lower and the middle class, but perform relatively poorly in separating middle- and upper-class groups, notably in Chile and Brazil. Definitions based on relative thresholds exhibit the worst performance of the group in terms of Mahalanobis distance, while the higher values of Wilk's Lambda indicate a relatively worse performance in terms of separations between classes.

Figures 7 and 8 plot both measures of distance for the last survey in each country in the sample. In all cases, the lowest Wilk's Lambda is given by the EGR3-polarization-based definition of the middle class, followed by Barro's (1999) and Easterly's (2001). Figure 8 indicates that in all cases the Mahalanobis distance between the middle and the upper class is highest for the EGR3-polarization definition (with the exception of El Salvador, where it is the second-highest), while this measure also has the best relative performance in terms of the distance between the lower and the middle class.

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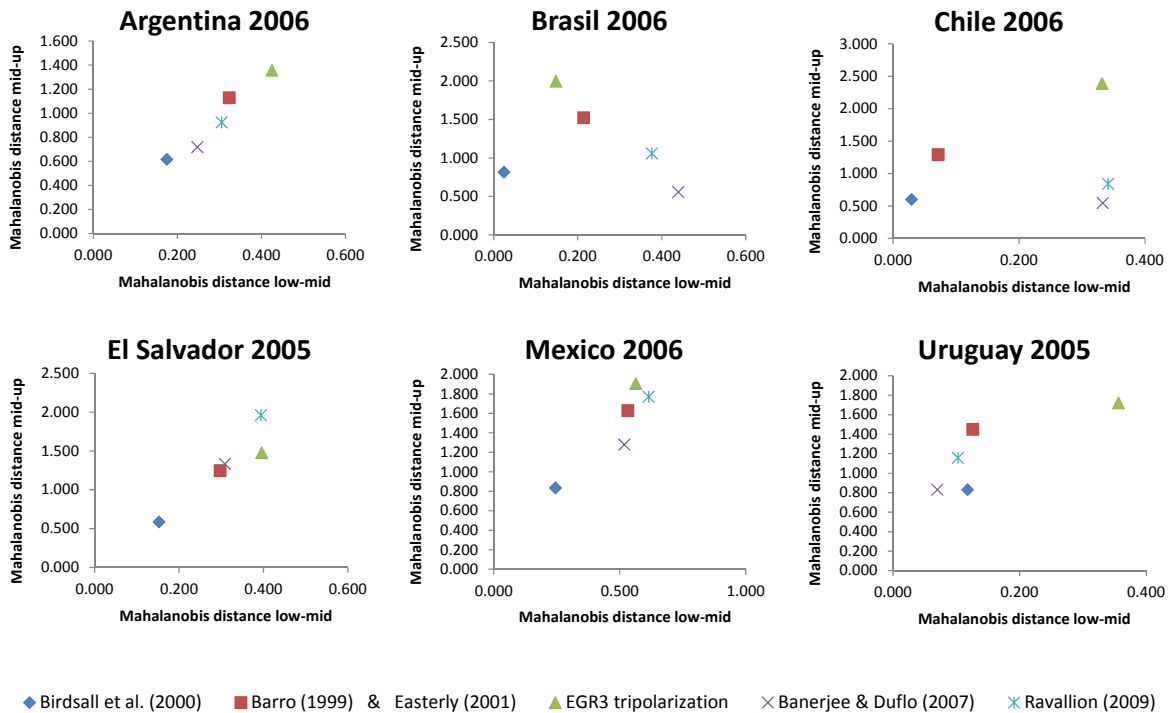
<sup>12</sup> To avoid some range and colinearity problems associated with binary variables, the whole set of dummies is first aggregated into an index created by means of principal component analysis.

**Figure 7**  
**Linear Discriminant Analysis: Wilk's Lambda**



Source: Own calculations based on household survey results.

**Figure 8**  
**Linear Discriminant Analysis: Mahalanobis squared distance between classes**



Source: Own calculations based on household survey results.



## 5 Conclusions

This study has reviewed and compared some of the most frequently used income-based definitions of the middle class in the empirical literature. The relative arbitrariness of these approaches motivated the implementation of an alternative methodology based on polarization indices which has a series of advantages. The concepts of alienation and identification embedded in the polarization-based definition have strong theoretical underpinnings for the partition of the income distribution into different groups. Percentiles and other relative measures, while with some intuitive appeal, do not have a solid theoretical grounding for defining the thresholds, while the definition of arbitrary thresholds affects the comparability of the analysis, since, for different countries and years, the groups might not be located in the same part of the income distribution. Moreover, an analysis of the way in which the middle class has evolved over time requires a measure that is sensitive to changes in the income distribution. Some measures suffer from inherent insensitivity because they “move” along with the distribution. On the other hand, while measures based on absolute thresholds might not have this problem, their rigidity may influence the results: absolute thresholds combined with short-term movements of the income distribution around those levels could induce excessive volatility. This is most notable in periods of economic crisis, during which a viable definition of the middle class should distinguish structural change in the size of this group from the transient impoverishment of households in the middle and upper sections of the income distribution. For instance, the results indicate that measures based on relative and absolute thresholds depict large (and thus implausible) fluctuations in the size of this group before, during and after the 2001-2002 crisis in Argentina and Uruguay.

The empirical results for relative-threshold definitions indicate that the size of the middle class has been fairly stable in most countries, although with some important outliers, such as the case of Argentina during the crisis of 2001-2002, while definitions based on absolute thresholds display a greater degree of volatility in terms of the size of the middle class as they track the cyclical movements of aggregate income. Both families of measures also exhibit a relatively high level of variability in terms of the different social classes' shares of total income and in terms of poverty levels in the middle class.

The empirical results when the polarization-based definition is used show a stable pattern for the six countries under analysis in terms of the size of the middle class. Despite some fluctuations in the early 2000s, this group represents about a third of the households in all the countries in the sample. This group's share of total income, however, differs by country. Brazil and Chile both experienced relatively stable growth with falling poverty levels, but the income share of the middle class shrank steadily in Brazil, while it increased over the period as a whole in Chile. The income share fell from the early to the mid-2000s in El Salvador and Mexico, but it increased in Argentina and Uruguay, reflecting in part the recovery from the 2001-2002 economic crisis in both countries.

Besides a lower degree of volatility in middle-class size and income shares, the definitions based on polarization measures have a number of additional advantages over the other alternatives presented in this study. For instance, they result in more stable poverty patterns for the middle class for all countries. With respect to socioeconomic characteristics, for all the definitions members of middle-class households have significantly higher levels of education than those in the lower class but substantially lower levels than those in the upper class. Labor-market participation and occupation rates are higher for the upper classes, as are social-security contribution rates, while unemployment is more prevalent among the middle and lower classes. However, the partition resulting from the polarization definition yields more homogeneous levels of household characteristics such as education and labor-market outcomes within the groups and in larger differences in the averages of these variables between groups. These descriptive results were confirmed by a systematic analysis of between- and within-group variations in labor-market outcomes, which also indicates that the polarization-based measure results in greater homogeneity within groups and larger differences between groups.

The evidence presented in this document suggests that definitions of the middle class based on sound principles of distributional analysis result in more coherent income-based groups than those obtained using alternative definitions. The results also point to some avenues for further research. On the one hand, the availability of new internationally comparable data sources paves the way for the study of the relationship between middle-class definitions and self-perceptions in different societies. Such data could also be used to test the robustness of some political economy predictions with respect to class and political attitudes. On the other hand, future studies could assess options for refining income-based definitions of the middle class through the incorporation of dimensions other than income, as in Goldthorpe and McKnight (2004), where the notion of economic security is used as a differentiating factor among social classes. The results presented in this study on the effects of the economic crisis of the early 2000s in some Latin American countries confirm the salience of the relative vulnerability of the middle class as an issue to be considered in the relevant academic and policy debate.

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## Tables

### Table 2

#### Definitions of the Middle Class. Argentina

	Birdsall et al. (2000)				Barro (1999) & Easterly (2001)				Banerjee & Duflo (2006)				Ravallion (2009)				EGR3 tripolarization Middlele groUpper			
<b>Argentina</b>	1992	2000	2003	2006	1992	2000	2003	2006	1992	2000	2003	2006	1992	2000	2003	2006	1992	2000	2003	2006
No. of persons (millions)	3.04	3.18	2.57	3.08	7.37	9.03	8.08	8.66	6.10	7.21	6.68	6.60	7.53	8.53	7.82	8.27	3.97	4.57	3.94	4.98
% persons	24.8%	21.0%	19.1%	21.2%	60.0%	59.7%	60.2%	59.6%	49.7%	47.7%	49.8%	45.4%	61.4%	56.4%	58.3%	56.9%	32.4%	30.2%	29.4%	34.2%
No. of households (millions)	0.84	0.98	0.82	1.02	2.13	2.55	2.32	2.58	1.60	1.85	1.88	1.69	2.01	2.26	2.28	2.28	1.26	1.59	1.42	1.79
% households	23.3%	22.0%	20.2%	22.8%	59.3%	57.3%	57.3%	57.9%	44.7%	41.6%	46.3%	37.9%	56.1%	50.9%	56.3%	51.2%	35.1%	35.8%	35.1%	40.1%
Share of income (per capita)	0.19	0.16	0.14	0.18	0.45	0.39	0.38	0.44	0.23	0.22	0.29	0.21	0.34	0.30	0.39	0.31	0.39	0.38	0.36	0.42
Mean income (USD PPP 2005)	311	293	206	321	298	248	180	282	189	178	164	178	220	203	191	212	482	482	353	474
Min. income (USD PPP 2005)	237	227	161	247	141	96	66	116	76	76	76	76	76	76	76	76	315	300	212	294
Max. income (USD PPP 2005)	396	379	268	411	558	495	379	547	304	304	304	304	395	395	395	395	784	818	603	803
Years of education (head of household)	8.5	8.8	8.9	9.4	8.4	8.5	9.0	9.2	7.7	8.0	8.7	8.5	7.9	8.2	9.0	8.7	9.7	10.0	10.4	10.4
% of hh. heads with complete sec. ed.	24.7%	29.8%	29.3%	39.5%	23.9%	27.7%	28.2%	36.3%	16.7%	20.9%	26.0%	27.7%	18.5%	23.3%	29.1%	30.9%	36.8%	45.0%	43.8%	49.5%
% of hh. heads with complete tertiary ed.	4.9%	4.9%	5.9%	7.9%	4.5%	5.1%	5.8%	7.4%	2.5%	3.3%	4.9%	4.0%	3.1%	3.5%	6.1%	5.2%	8.9%	11.6%	13.5%	14.0%
Size of household	3.6	3.3	3.1	3.0	3.5	3.5	3.5	3.4	3.8	3.9	3.6	3.9	3.7	3.8	3.4	3.6	3.2	2.9	2.8	2.8
Poverty 1USD (headcount)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Poverty 2USD (headcount)	0.00	0.00	0.00	0.00	0.00	0.00	0.05	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Extreme poverty (headcount)	0.00	0.00	0.00	0.00	0.00	0.00	0.04	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00
Moderate poverty (headcount)	0.00	0.00	0.07	0.00	0.03	0.21	0.49	0.13	0.28	0.38	0.52	0.37	0.23	0.32	0.45	0.30	0.00	0.00	0.00	0.00

### Table 3

#### Definitions of the Middle Class. Brazil

	Birdsall et al. (2000)				Barro (1999) & Easterly (2001)				Banerjee & Duflo (2006)				Ravallion (2009)				EGR3 tripolarization Middlele groUpper			
<b>Brazil</b>	1992	2001	2003	2006	1992	2001	2003	2006	1992	2001	2003	2006	1992	2001	2003	2006	1992	2001	2003	2006
No. of persons (millions)	26.96	32.49	33.52	39.25	82.83	99.72	102.80	110.70	67.66	78.46	81.84	85.61	76.78	91.75	96.05	106.20	44.23	52.42	52.66	56.34
% persons	19.5%	19.6%	19.6%	21.3%	60.0%	60.0%	60.0%	60.0%	49.0%	47.2%	47.8%	46.4%	55.6%	55.2%	56.0%	57.6%	32.0%	31.5%	30.7%	30.5%
N° households (millions)	7.19	10.41	11.06	13.32	22.09	29.85	31.42	34.99	18.47	22.78	24.25	24.22	21.22	27.10	29.00	31.96	13.21	17.50	18.39	20.47
% households	19.4%	20.9%	21.1%	22.8%	59.5%	60.0%	59.9%	59.9%	49.8%	45.8%	46.2%	41.4%	57.2%	54.5%	55.2%	54.7%	35.6%	35.2%	35.0%	35.0%
Share of income (per capita)	0.14	0.12	0.12	0.14	0.40	0.34	0.35	0.36	0.35	0.21	0.22	0.19	0.45	0.29	0.30	0.28	0.37	0.39	0.32	0.32
Mean income (USD PPP 2005)	162	225	230	281	153	214	215	258	164	170	171	174	185	195	196	207	268	465	383	448
Min. income (USD PPP 2005)	124	172	175	214	55	76	77	101	76	76	76	76	76	76	76	76	164	253	233	281
Max. income (USD PPP 2005)	206	287	292	356	324	473	467	539	304	304	304	304	395	395	395	395	461	958	678	766
Years of education (head of household)	4.2	4.7	5.0	5.5	4.0	4.9	5.2	5.7	4.1	4.5	4.8	5.3	4.3	4.7	5.0	5.3	5.1	6.8	6.3	6.6
% of hh. heads with complete sec. ed.	9.5%	13.7%	16.5%	21.3%	8.9%	14.4%	17.2%	22.0%	9.3%	11.3%	13.9%	17.5%	11.1%	13.2%	15.9%	18.6%	16.6%	30.1%	27.4%	31.1%
% of hh. heads with complete tertiary ed.	1.0%	1.0%	1.1%	1.6%	1.1%	1.2%	1.4%	1.8%	1.1%	0.7%	0.7%	0.9%	1.5%	1.0%	1.1%	1.1%	3.0%	5.7%	3.8%	4.5%
Size of household	3.7	3.1	3.0	2.9	3.7	3.3	3.3	3.2	3.7	3.4	3.4	3.5	3.6	3.4	3.3	3.3	3.3	3.0	2.9	2.8
Poverty 1USD (headcount)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Poverty 2USD (headcount)	0.00	0.00	0.00	0.00	0.15	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Extreme poverty (headcount)	0.00	0.00	0.00	0.00	0.03	0.01	0.01	0.00	0.00	0.01	0.01	0.01	0.00	0.01	0.01	0.01	0.00	0.00	0.00	0.00
Moderate poverty (headcount)	0.05	0.02	0.02	0.01	0.36	0.24	0.24	0.13	0.26	0.31	0.32	0.30	0.23	0.26	0.27	0.24	0.01	0.00	0.00	0.00

**Table 4**  
**Definitions of the Middle Class**  
**Chile**

	Birdsall et al. (2000)				Barro (1999) & Easterly (2001)				Banerjee & Duflo (2006)				Ravallion (2009)				EGR3 tripolarization Middlele groUpper			
<b>Chile</b>	1992	2000	2003	2006	1992	2000	2003	2006	1992	2000	2003	2006	1992	2000	2003	2006	1992	2000	2003	2006
No. of persons (millions)	2.96	3.32	3.56	3.86	7.93	8.68	9.07	9.40	7.74	7.90	8.34	8.18	8.78	9.32	9.87	9.96	4.08	4.31	4.71	5.01
% persons	22.4%	23.1%	23.5%	24.6%	60.0%	60.4%	60.0%	60.0%	58.6%	54.9%	55.2%	52.2%	66.4%	64.8%	65.2%	63.6%	30.9%	29.9%	31.2%	32.0%
N° households (millions)	0.76	0.86	0.95	1.05	1.99	2.21	2.38	2.52	1.89	1.85	2.04	2.02	2.19	2.27	2.48	2.53	1.18	1.32	1.42	1.49
% households	22.3%	22.8%	23.6%	24.7%	58.9%	58.9%	58.7%	59.2%	56.0%	49.3%	50.4%	47.5%	64.8%	60.3%	61.3%	59.4%	35.0%	35.0%	35.0%	35.0%
Share of income (per capita)	0.14	0.14	0.15	0.16	0.36	0.36	0.36	0.38	0.31	0.23	0.24	0.21	0.39	0.32	0.32	0.30	0.36	0.36	0.37	0.39
Mean income (USD PPP 2005)	191	253	254	284	187	242	249	285	164	173	176	184	185	199	203	213	367	495	491	540
Min. income (USD PPP 2005)	148	40	196	220	88	27	118	137	76	21	76	76	76	21	76	76	219	48	290	323
Max. income (USD PPP 2005)	246	329	327	366	373	490	488	550	304	304	304	304	395	395	395	395	707	972	958	1,029
Years of education (head of household)	7.4	8.6	9.1	8.9	7.5	8.5	9.2	9.0	7.4	8.1	8.6	8.5	7.5	8.3	8.8	8.6	8.5	10.1	10.8	10.6
% of hh. heads with complete sec. ed.	22.2%	35.2%	37.2%	34.9%	22.7%	34.8%	37.0%	35.7%	21.3%	29.2%	30.4%	28.7%	22.6%	31.2%	32.9%	30.8%	30.3%	50.7%	53.8%	51.7%
% of hh. heads with complete tertiary ed.	3.9%	6.7%	6.5%	6.4%	3.7%	6.6%	7.9%	7.0%	3.3%	3.9%	4.3%	3.9%	3.8%	4.8%	5.5%	4.8%	6.7%	17.5%	19.4%	17.4%
Size of household	3.9	3.9	3.7	3.7	4.0	3.9	3.8	3.7	4.1	4.3	4.1	4.0	4.0	4.1	4.0	3.9	3.5	3.3	3.3	3.4
Poverty 1USD (headcount)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Poverty 2USD (headcount)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Extreme poverty (headcount)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Moderate poverty (headcount)	0.00	0.00	0.00	0.00	0.21	0.02	0.00	0.01	0.30	0.19	0.19	0.16	0.26	0.17	0.16	0.13	0.00	0.00	0.00	0.00

**Table 5**  
**Definitions of the Middle Class**  
**El Salvador**

	Birdsall et al. (2000)				Barro (1999) & Easterly (2001)				Banerjee & Duflo (2006)				Ravallion (2009)				EGR3 tripolarization Middlele groUpper			
<b>El Salvador</b>	1991	2000	2003	2005	1991	2000	2003	2005	1991	2000	2003	2005	1991	2000	2003	2005	1991	2000	2003	2005
No. of persons (millions)	1.13	1.35	1.45	1.51	3.04	3.71	3.96	4.10	2.57	3.22	3.59	3.76	2.79	3.61	4.01	4.21	1.64	1.97	2.12	2.14
% persons	22.4%	21.8%	22.0%	22.1%	60.0%	60.0%	60.0%	60.0%	50.7%	52.0%	54.4%	55.0%	55.1%	58.4%	60.7%	61.6%	32.4%	31.9%	32.0%	31.3%
N° households (millions)	0.23	0.30	0.33	0.36	0.61	0.82	0.90	0.96	0.55	0.73	0.85	0.91	0.61	0.84	0.97	1.05	0.37	0.50	0.56	0.58
% households	21.5%	21.0%	20.8%	21.8%	56.6%	57.3%	56.9%	57.7%	51.4%	50.9%	53.3%	54.7%	57.1%	58.6%	61.1%	62.8%	35.0%	35.0%	35.0%	35.0%
Share of income (per capita)	0.16	0.16	0.17	0.17	0.40	0.41	0.43	0.43	0.46	0.41	0.44	0.43	0.55	0.51	0.55	0.54	0.37	0.38	0.39	0.37
Mean income (USD PPP 2005)	118	149	151	157	112	140	142	146	152	159	161	162	168	180	180	181	193	243	240	246
Min. income (USD PPP 2005)	91	115	116	121	43	53	56	60	76	76	76	76	76	76	76	76	121	152	153	161
Max. income (USD PPP 2005)	151	191	193	202	225	284	276	282	304	304	304	304	395	395	395	395	327	410	392	396
Years of education (head of household)	3.7	4.8	5.2	5.5	3.5	4.6	5.1	5.2	4.3	4.9	5.4	5.4	4.6	5.2	5.7	5.8	5.0	6.2	6.5	6.6
% of hh. heads with complete sec. ed.	1.9%	10.7%	13.1%	15.3%	2.2%	10.3%	13.2%	14.2%	3.6%	12.0%	14.8%	15.7%	4.7%	14.4%	17.0%	18.2%	5.5%	19.6%	22.6%	23.8%
% of hh. heads with complete tertiary ed.	0.3%	1.2%	1.7%	2.1%	0.3%	1.6%	2.5%	2.7%	0.6%	1.8%	2.9%	3.0%	1.0%	2.5%	3.6%	4.1%	1.1%	4.0%	5.4%	6.3%
Size of household	4.9	4.5	4.4	4.1	5.0	4.5	4.4	4.3	4.7	4.4	4.2	4.1	4.6	4.3	4.1	4.0	4.4	3.9	3.8	3.7
Poverty 1USD (headcount)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Poverty 2USD (headcount)	0.00	0.00	0.00	0.00	0.29	0.16	0.14	0.12	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Extreme poverty (headcount)	0.09	0.00	0.00	0.00	0.25	0.04	0.05	0.03	0.08	0.00	0.01	0.01	0.08	0.00	0.01	0.01	0.00	0.00	0.00	0.00
Moderate poverty (headcount)	0.79	0.15	0.20	0.18	0.78	0.42	0.41	0.39	0.59	0.30	0.31	0.31	0.55	0.27	0.28	0.28	0.39	0.00	0.02	0.01

**Table 6**  
**Definitions of the Middle Class**  
**Mexico**

	Birdsall et al. (2000)				Barro (1999) & Easterly (2001)				Banerjee & Duflo (2006)				Ravallion (2009)				EGR3 tripolarization Middlele groUpper			
<b>Mexico</b>	1992	2000	2004	2006	1992	2000	2004	2006	1992	2000	2004	2006	1992	2000	2004	2006	1992	2000	2004	2006
No. of persons (millions)	19.05	20.34	25.81	26.09	49.19	57.53	60.92	62.03	45.47	53.64	58.90	59.83	50.87	60.02	67.08	69.06	25.13	30.23	32.57	34.04
% persons	23.5%	21.4%	25.4%	25.2%	60.5%	60.5%	60.0%	60.0%	56.0%	56.4%	58.0%	57.9%	62.6%	63.1%	66.1%	66.8%	30.9%	31.8%	32.1%	32.9%
N° households (millions)	3.85	4.81	6.04	6.22	9.85	13.27	14.35	14.81	9.15	12.35	13.60	13.76	10.48	14.05	15.76	16.24	6.02	8.04	8.81	9.16
% households	22.4%	21.0%	24.0%	23.8%	57.3%	57.9%	57.0%	56.6%	53.3%	53.9%	54.0%	52.6%	61.0%	61.3%	62.5%	62.1%	35.0%	35.1%	35.0%	35.0%
Share of income (per capita)	0.15	0.14	0.17	0.17	0.37	0.39	0.40	0.41	0.35	0.36	0.35	0.33	0.44	0.45	0.45	0.43	0.38	0.40	0.41	0.37
Mean income (USD PPP 2005)	164	174	183	207	157	167	185	203	160	165	166	171	180	184	188	194	315	329	352	332
Min. income (USD PPP 2005)	127	134	141	160	67	70	85	94	76	76	76	76	76	76	76	76	188	198	211	213
Max. income (USD PPP 2005)	211	222	235	267	313	328	355	383	304	304	304	304	395	395	395	395	613	631	665	566
Years of education (head of household)	5.6	6.8	6.6	7.0	5.4	6.4	6.6	7.0	5.5	6.5	6.4	6.6	5.8	6.7	6.7	6.9	7.6	8.5	8.6	8.4
% of hh. heads with complete sec. ed.	9.6%	17.9%	14.8%	16.7%	9.7%	15.1%	14.7%	17.1%	9.9%	14.8%	13.1%	14.0%	12.4%	17.1%	15.2%	16.5%	25.6%	30.5%	30.7%	29.0%
% of hh. heads with complete tertiary ed.	1.8%	4.3%	3.9%	4.6%	2.2%	4.0%	4.3%	4.9%	2.2%	3.9%	3.8%	3.4%	3.2%	4.8%	4.5%	4.7%	8.6%	12.1%	13.1%	11.2%
Size of household	4.9	4.2	4.3	4.2	5.0	4.3	4.2	4.2	5.0	4.3	4.3	4.3	4.9	4.3	4.3	4.3	4.2	3.8	3.7	3.7
Poverty 1USD (headcount)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Poverty 2USD (headcount)	0.00	0.00	0.00	0.00	0.06	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Extreme poverty (headcount)	0.01	0.01	0.01	0.00	0.11	0.11	0.05	0.03	0.08	0.10	0.08	0.07	0.07	0.09	0.07	0.06	0.00	0.00	0.00	0.00
Moderate poverty (headcount)	0.54	0.54	0.41	0.27	0.58	0.57	0.47	0.40	0.56	0.58	0.54	0.51	0.51	0.52	0.47	0.44	0.06	0.05	0.04	0.04

**Table 7**  
**Definitions of the Middle Class**  
**Uruguay**

	Birdsall et al. (2000)				Barro (1999) & Easterly (2001)				Banerjee & Duflo (2006)				Ravallion (2009)				EGR3 tripolarization Middlele groUpper			
<b>Uruguay</b>	1992	2000	2003	2005	1992	2000	2003	2005	1992	2000	2003	2005	1992	2000	2003	2005	1992	2000	2003	2005
No. of persons (millions)	0.76	0.58	0.57	0.56	1.67	1.36	1.40	1.37	1.10	0.85	1.28	1.20	1.48	1.15	1.56	1.48	0.97	0.68	0.67	0.75
% persons	27.3%	25.6%	24.5%	24.6%	60.0%	60.0%	60.0%	60.0%	39.5%	37.3%	54.8%	52.6%	53.4%	50.5%	66.6%	64.7%	34.8%	30.1%	28.4%	32.9%
N° households (millions)	0.25	0.21	0.20	0.21	0.53	0.48	0.45	0.46	0.28	0.22	0.35	0.34	0.40	0.33	0.45	0.45	0.35	0.28	0.27	0.31
% households	28.6%	27.0%	26.0%	26.5%	60.7%	60.4%	58.9%	59.5%	31.9%	28.5%	45.5%	44.2%	45.9%	41.5%	58.8%	58.1%	40.0%	35.0%	35.0%	40.0%
Share of income (per capita)	0.24	0.22	0.21	0.21	0.47	0.46	0.45	0.45	0.17	0.14	0.28	0.27	0.27	0.23	0.40	0.39	0.41	0.34	0.36	0.40
Mean income (USD PPP 2005)	401	437	299	303	365	390	260	265	195	192	178	181	235	233	208	211	546	576	448	430
Min. income (USD PPP 2005)	307	336	231	234	179	181	124	123	76	76	76	76	76	76	76	76	375	406	306	283
Max. income (USD PPP 2005)	512	560	385	390	640	709	482	494	304	304	304	304	395	395	395	395	843	847	694	697
Years of education (head of household)	7.4	7.5	7.8	7.8	7.3	7.5	7.7	7.8	6.7	7.0	7.2	7.3	6.8	7.1	7.4	7.5	7.9	8.0	8.7	8.8
% of hh. heads with complete sec. ed.	18.3%	16.2%	19.0%	20.5%	17.4%	16.0%	17.4%	19.3%	13.4%	12.6%	12.3%	13.4%	14.1%	13.4%	14.5%	15.8%	21.9%	19.4%	28.3%	30.1%
% of hh. heads with complete tertiary ed.	3.4%	3.9%	4.6%	3.8%	2.7%	3.3%	4.0%	3.8%	1.1%	1.1%	2.1%	1.7%	1.4%	1.6%	2.9%	2.4%	4.5%	5.6%	8.5%	8.1%
Size of household	3.0	2.7	2.9	2.7	3.2	2.9	3.1	3.0	4.0	3.8	3.7	3.5	3.7	3.5	3.4	3.3	2.8	2.5	2.5	2.4
Poverty 1USD (headcount)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Poverty 2USD (headcount)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Extreme poverty (headcount)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Moderate poverty (headcount)	0.00	0.01	0.00	0.00	0.06	0.04	0.19	0.17	0.43	0.36	0.45	0.42	0.32	0.27	0.37	0.34	0.00	0.00	0.00	0.00

## Appendix 1

**Table A.1**  
**Surveys from SEDLAC (CEDLAS and World Bank, 2010) Used in the Study**

<b>Country</b>	<b>Survey</b>	<b>Years</b>			
<b>Argentina</b>	Encuesta permanente de Hogares	1992	2000	2003	2006
<b>Brazil</b>	Pesquisa Nacional por Amostra de Domicilios	1992	2001	2003	2006
<b>Chile</b>	Encuesta de Caracterización Socioeconómica Nacional	1992	2000	2003	2006
<b>El Salvador</b>	Encuesta de Hogares de Propósitos Múltiples	1991	2000	2003	2005
<b>Mexico</b>	Encuesta Nacional de Ingresos y Gastos de los Hogares	1992	2000	2004	2006
<b>Uruguay</b>	Encuesta Continua de Hogares	1992	2000	2003	2005



**Table A.2**  
**Social Class Profiles**  
**Housing**

	Birdsall et al. (2000)			Barro (1999) & Easterly (2001)			Banerjee & Duflo (2007)			Ravallion (2009)			EGR3 tripolarization		
	Lower	Middle	Upper	Lower	Middle	Upper	Lower	Middle	Upper	Lower	Middle	Upper	Lower	Middle	Upper
<b>Argentina 2006</b>															
House ownership	58.1	67.0	69.3	51.2	65.8	69.3	52.1	60.3	69.5	52.1	62.8	69.3	59.0	68.4	71.1
Number of rooms	2.6	2.9	3.1	2.4	2.9	3.2	2.3	2.7	3.1	2.3	2.8	3.1	2.7	3.0	3.2
Persons per room	1.9	1.2	0.9	2.3	1.3	0.8	2.4	1.6	0.9	2.4	1.5	0.9	1.8	1.0	0.7
Poor housing	5.5	2.9	0.8	7.7	2.9	0.8	9.0	4.4	1.2	9.0	3.8	0.9	5.2	1.6	0.6
Lower-quality material	4.3	1.3	0.4	7.4	1.5	0.4	9.4	2.7	0.5	9.4	2.2	0.4	3.9	0.6	0.4
Water	97.2	99.4	99.9	94.5	99.3	100.0	95.0	98.2	99.8	95.0	98.5	99.9	97.6	99.7	100.0
Hygienic restrooms	71.4	90.9	97.9	59.7	87.4	98.4	55.0	78.5	96.5	55.0	82.2	97.8	74.1	95.1	99.4
Sewerage	34.2	58.6	77.4	27.0	53.1	80.9	23.2	40.3	73.6	23.2	46.2	77.0	36.8	68.1	86.6
Electricity	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
<b>Brazil 2006</b>															
House ownership	64.3	70.7	74.3	62.5	69.0	75.7	63.7	64.5	75.3	63.7	67.6	74.5	64.1	74.5	76.2
Number of rooms	5.2	5.6	6.7	5.0	5.5	7.1	5.1	5.3	6.5	5.1	5.4	6.8	5.3	5.9	7.6
Persons per room	0.8	0.6	0.4	0.9	0.6	0.4	0.9	0.7	0.4	0.9	0.7	0.4	0.8	0.5	0.4
Poor housing	0.7	0.5	0.4	0.9	0.5	0.3	1.0	0.5	0.4	1.0	0.5	0.3	0.7	0.4	0.2
Lower-quality material	4.3	1.4	0.3	7.0	1.6	0.1	7.9	2.4	0.5	7.9	2.1	0.3	3.7	0.8	0.0
Water	83.1	92.8	98.3	74.7	92.2	99.2	72.7	89.0	97.2	72.7	90.1	98.6	85.2	95.9	99.6
Hygienic restrooms	55.4	69.7	85.8	46.5	68.9	89.5	44.9	62.8	82.6	44.9	65.0	86.7	58.4	77.7	92.2
Sewerage	40.8	55.5	74.3	32.7	54.6	78.8	31.6	47.9	70.6	31.6	50.4	75.3	43.8	64.7	82.4
Electricity	95.7	98.3	99.5	93.2	98.1	99.8	92.4	97.4	99.2	92.4	97.6	99.6	96.3	98.9	99.9
<b>Chile 2006</b>															
House ownership	60.3	68.6	70.1	54.2	68.2	69.5	47.9	63.9	70.1	47.9	65.1	70.2	63.1	70.3	68.9
Number of rooms	4.9	5.4	6.1	4.7	5.4	6.4	4.5	5.1	6.0	4.5	5.2	6.2	5.0	5.8	6.8
Persons per room	1.0	0.7	0.6	1.1	0.8	0.5	1.2	0.9	0.6	1.2	0.9	0.5	0.9	0.6	0.4
Poor housing	2.7	1.3	0.9	3.7	1.5	0.8	5.7	2.0	1.0	5.7	1.9	0.9	2.2	1.1	0.6
Lower-quality material	14.9	9.5	5.2	17.4	10.0	3.8	21.1	12.8	5.7	21.1	11.9	4.9	13.2	6.3	2.7
Water	94.5	96.6	98.2	93.6	96.5	98.5	91.5	95.4	98.0	91.5	95.8	98.2	95.2	97.9	98.7
Hygienic restrooms	85.2	92.3	96.8	81.8	91.7	97.9	77.8	88.1	96.2	77.8	89.2	97.1	87.5	95.8	98.5
Sewerage	74.7	82.7	90.4	72.0	82.2	92.1	69.2	77.7	89.3	69.2	79.0	90.9	77.3	88.6	93.2
Electricity	99.0	99.5	99.7	98.7	99.5	99.8	97.6	99.3	99.7	97.6	99.3	99.7	99.2	99.7	99.8
<b>El Salvador 2005</b>															
House ownership	66.7	68.1	73.1	69.6	67.1	75.1	69.4	66.9	76.2	69.4	67.6	78.1	66.7	69.9	78.2
Number of rooms	1.9	2.4	3.0	1.7	2.3	3.2	1.8	2.4	3.3	1.8	2.4	3.5	2.0	2.7	3.5
Persons per room	3.3	2.2	1.4	3.7	2.4	1.2	3.6	2.2	1.1	3.6	2.1	1.0	3.1	1.7	1.0
Poor housing	6.7	5.8	4.0	6.6	5.9	3.6	6.5	6.0	3.0	6.5	5.8	2.3	6.6	5.1	2.3
Lower-quality material	42.7	24.8	13.6	49.4	28.2	10.6	47.8	26.3	9.4	47.8	24.5	7.8	38.9	18.6	7.8
Water	44.9	62.3	76.4	37.9	59.4	80.5	40.1	61.0	82.3	40.1	63.0	85.4	48.7	69.0	85.4
Hygienic restrooms	16.3	35.0	57.5	11.1	31.3	65.7	11.9	33.7	68.2	11.9	36.9	73.2	20.0	45.3	73.2
Sewerage	14.3	31.3	52.6	9.9	27.9	60.4	10.7	30.0	62.9	10.7	32.8	68.8	17.7	40.4	68.7
Electricity	75.9	91.8	96.1	68.4	88.6	97.2	70.7	90.0	97.6	70.7	90.8	98.3	79.5	94.2	98.3
<b>Mexico 2006</b>															
House ownership	66.4	68.1	72.0	68.4	68.4	71.1	70.0	67.1	71.8	70.0	68.1	71.0	66.9	71.8	69.8
Number of rooms	4.2	4.9	6.0	4.1	4.8	6.4	4.1	4.6	6.1	4.1	4.8	6.4	4.4	5.3	6.8
Persons per room	1.2	1.0	0.6	1.3	1.0	0.5	1.3	1.1	0.6	1.3	1.0	0.5	1.2	0.8	0.5
Poor housing	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
Lower-quality material	49.3	32.0	16.9	57.9	32.7	13.7	58.4	36.6	15.8	58.4	34.4	13.6	45.3	22.6	11.8
Water	83.4	92.3	95.5	79.0	91.2	96.2	78.5	89.7	95.7	78.5	90.4	96.3	85.7	94.2	96.6
Hygienic restrooms	41.2	65.7	85.8	33.2	63.1	90.6	33.1	57.8	87.6	33.1	61.1	90.7	47.1	77.6	93.4
Sewerage	35.0	59.1	79.7	27.6	56.4	84.8	27.3	51.3	81.4	27.3	54.4	85.2	40.7	71.0	88.1
Electricity	98.2	99.8	99.7	97.1	99.5	99.9	96.3	99.5	99.7	96.3	99.5	99.9	98.6	99.6	99.9
<b>Uruguay 2005</b>															
House ownership	47.1	67.3	77.2	32.8	63.5	79.4	18.8	55.3	75.2	18.8	58.7	77.4	51.0	71.7	81.7
Number of rooms	3.0	3.2	3.6	2.9	3.2	3.6	2.7	3.1	3.5	2.7	3.2	3.6	3.1	3.3	3.8
Persons per room	1.4	0.9	0.6	1.8	1.0	0.6	2.1	1.2	0.7	2.1	1.1	0.6	1.3	0.7	0.6
Poor housing	2.4	1.8	1.0	3.3	1.8	0.9	3.2	2.3	1.0	3.2	2.1	0.9	2.4	1.4	0.4
Lower-quality material	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
Water	98.1	98.9	99.5	97.8	98.7	99.6	96.8	98.4	99.4	96.8	98.6	99.5	98.3	99.1	99.7
Hygienic restrooms	86.0	97.4	99.5	76.6	95.4	99.8	67.6	91.0	99.2	67.6	92.8	99.5	88.1	98.8	99.9
Sewerage	46.4	65.5	84.9	37.1	62.2	88.2	31.5	52.5	80.8	31.5	56.5	85.2	49.3	75.0	92.3
Electricity	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A





**Table A.6**  
**Social Class Profiles**  
**Income Structure**

	Birdsall et al. (2000)			Barro (1999) & Easterly (2001)			Banerjee & Duflo (2007)			Ravallion (2009)			EGR3 tripolarization		
	Lower	Middle	Upper	Lower	Middle	Upper	Lower	Middle	Upper	Lower	Middle	Upper	Lower	Middle	Upper
<b>Argentina 2006</b>															
Labor	80.0	78.3	82.6	74.5	79.7	83.4	69.0	81.3	81.6	69.0	79.2	82.7	80.4	79.4	84.4
Non-labor	20.0	21.7	17.4	25.5	20.3	16.6	31.0	18.7	18.4	31.0	20.8	17.3	19.6	20.6	15.6
Total	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
<b>Brazil 2006</b>															
Labor	76.9	73.5	76.2	74.1	75.8	76.1	70.3	78.0	75.5	70.3	75.6	76.1	77.3	74.5	76.3
Non-labor	23.1	26.5	23.8	25.9	24.2	23.9	29.7	22.0	24.5	29.7	24.4	23.9	22.7	25.5	23.7
Total	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
<b>Chile 2006</b>															
Labor	80.5	82.9	85.9	77.9	83.3	86.2	67.1	81.9	85.6	67.1	82.2	86.0	81.6	84.4	87.0
Non-labor	19.5	17.1	14.1	22.1	16.7	13.8	32.9	18.1	14.4	32.9	17.8	14.0	18.4	15.6	13.0
Total	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
<b>El Salvador 2005</b>															
Labor	78.5	79.7	83.1	76.4	79.2	84.2	76.8	79.2	84.6	76.8	79.7	85.5	79.1	79.7	85.5
Non-labor	21.5	20.3	16.9	23.6	20.8	15.8	23.2	20.8	15.4	23.2	20.3	14.5	20.9	20.3	14.5
Total	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
<b>Mexico 2006</b>															
Labor	88.0	89.9	88.3	85.4	89.8	87.9	83.2	89.8	88.1	83.2	89.7	87.9	89.0	89.7	87.4
Non-labor	12.0	10.1	11.7	14.6	10.2	12.1	16.8	10.2	11.9	16.8	10.3	12.1	11.0	10.3	12.6
Total	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
<b>Uruguay 2005</b>															
Labor	70.1	64.8	62.6	69.4	66.0	62.6	66.5	68.8	62.8	66.5	67.3	62.6	68.9	63.6	62.2
Non-labor	29.9	35.2	37.4	30.6	34.0	37.4	33.5	31.2	37.2	33.5	32.7	37.4	31.1	36.4	37.8
Total	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100

**Table A.7**  
**Linear Discriminant Analysis using Labor Variables**  
**Measures of Separation and Discrimination Power**

	Birdsall et al. (2000)				Barro (1999) & Easterly (2001)				EGR3 tripolarization				Banerjee & Duflo (2007)				Ravallion (2009)			
	1992	2000	2003	2006	1992	2000	2003	2006	1992	2000	2003	2006	1992	2000	2003	2006	1992	2000	2003	2006
<b>Argentina (15 main cities)</b>																				
Lambda Wilks	0.786	0.777	0.861	0.781	0.771	0.751	0.856	0.758	0.738	0.687	0.841	0.722	0.818	0.795	0.853	0.817	0.797	0.772	0.855	0.783
Mahalanobis distance lower-middle	0.202	0.161	0.079	0.175	0.252	0.276	0.184	0.324	0.470	0.491	0.329	0.426	0.199	0.227	0.142	0.248	0.257	0.276	0.179	0.306
Mahalanobis distance middle-upper	0.576	0.677	0.373	0.615	1.302	1.318	0.619	1.128	1.493	2.053	0.421	1.356	0.761	0.833	0.560	0.716	0.970	1.031	0.611	0.924
<b>Brazil</b>																				
Lambda Wilks	0.715	0.781	0.807	0.816	0.607	0.704	0.744	0.755	0.568	0.653	0.724	0.733	0.616	0.758	0.785	0.844	0.589	0.727	0.757	0.778
Mahalanobis distance lower-middle	0.162	0.042	0.029	0.024	0.317	0.276	0.243	0.214	0.524	0.506	0.196	0.148	0.234	0.224	0.210	0.439	0.335	0.257	0.238	0.377
Mahalanobis distance middle-upper	1.032	0.927	0.827	0.814	2.955	1.968	1.607	1.520	3.380	3.143	1.957	1.995	2.563	1.135	0.972	0.557	3.556	1.531	1.320	1.057
<b>Chile</b>																				
Lambda Wilks	0.857	0.873	0.859	0.854	0.810	0.818	0.823	0.820	0.729	0.747	0.784	0.769	0.832	0.871	0.849	0.867	0.805	0.837	0.824	0.836
Mahalanobis distance lower-middle	0.044	0.026	0.038	0.030	0.074	0.050	0.103	0.072	0.294	0.251	0.320	0.332	0.060	0.215	0.361	0.333	0.085	0.195	0.372	0.342
Mahalanobis distance middle-upper	0.524	0.512	0.557	0.599	1.383	1.415	1.230	1.291	3.985	4.148	2.272	2.388	0.978	0.564	0.624	0.543	1.591	0.956	0.950	0.840
<b>El Salvador</b>																				
Lambda Wilks	0.781	0.726	0.788	0.797	0.749	0.679	0.745	0.764	0.724	0.631	0.702	0.735	0.722	0.664	0.731	0.751	0.721	0.645	0.711	0.744
Mahalanobis distance lower-middle	0.261	0.232	0.154	0.153	0.525	0.412	0.267	0.297	0.490	0.614	0.403	0.396	0.504	0.423	0.284	0.309	0.587	0.543	0.364	0.394
Mahalanobis distance middle-upper	0.535	0.852	0.618	0.581	1.087	1.967	1.413	1.244	1.274	2.571	1.871	1.474	1.450	1.997	1.550	1.330	1.995	3.061	2.434	1.957
<b>Mexico</b>																				
Lambda Wilks	0.784	0.802	0.744	0.731	0.724	0.759	0.685	0.678	0.701	0.767	0.654	0.657	0.726	0.762	0.700	0.703	0.718	0.759	0.674	0.677
Mahalanobis distance lower-middle	0.195	0.203	0.259	0.245	0.409	0.512	0.569	0.532	0.558	0.580	0.673	0.564	0.366	0.478	0.578	0.518	0.444	0.544	0.678	0.615
Mahalanobis distance middle-upper	0.603	0.508	0.745	0.831	1.486	1.104	1.488	1.627	2.409	1.246	2.242	1.903	1.378	0.965	1.259	1.275	1.836	1.246	1.755	1.767
<b>Uruguay</b>																				
Lambda Wilks	0.861	0.818	0.796	0.766	0.841	0.793	0.768	0.739	0.804	0.765	0.721	0.689	0.918	0.905	0.840	0.822	0.893	0.873	0.801	0.773
Mahalanobis distance lower-middle	0.095	0.111	0.089	0.118	0.155	0.170	0.105	0.126	0.184	0.198	0.313	0.357	0.073	0.096	0.052	0.070	0.095	0.111	0.085	0.103
Mahalanobis distance middle-upper	0.405	0.584	0.702	0.829	0.752	1.032	1.234	1.448	1.030	0.999	1.395	1.719	0.390	0.498	0.733	0.828	0.460	0.580	0.989	1.154







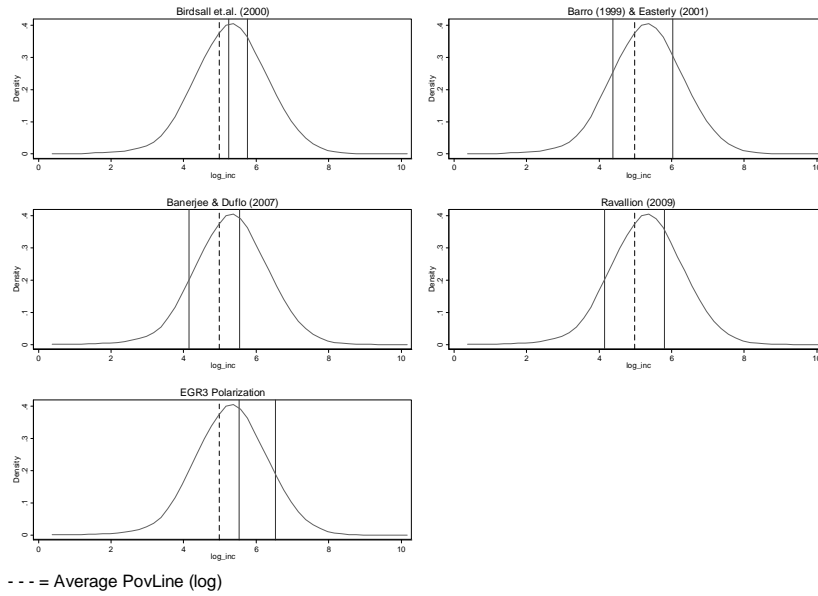




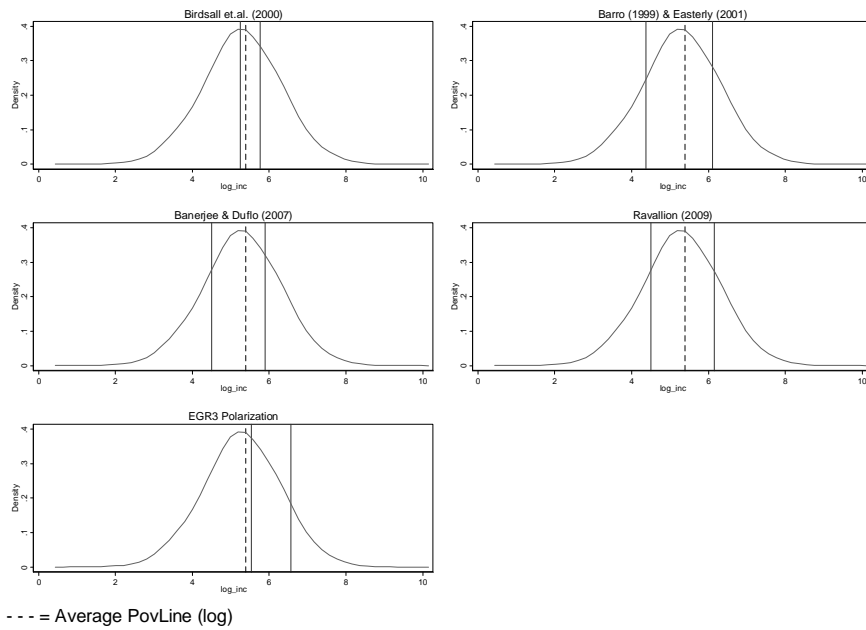




**Figure A.1**  
**Distribution of Per Capita Income, Poverty Lines and Middle/Upper-Class Cut-off Points**  
**(in Logarithms)**  
**Argentina 2000**



**Argentina 2003**



Source: Own calculations based on household survey results.

## Appendix 2

### Linear Discriminant Analysis

The main idea behind the methodology is that the multidimensional problem of separation between groups can be reduced to a single variable, called the discriminant punctuation. Given a set of  $G$  groups with  $n_g$  observations or individuals in each group and a set of covariates  $y = (y_1, y_2, \dots, y_p)$ , called “discriminant variables”, the objective is the estimation of a series of linear “discriminant functions” of the following form:

$$d : R^p \rightarrow R / d(y) = a_1 y_1 + a_2 y_2 + \dots + a_p y_p \quad (1)$$

where  $a_1, a_2, \dots, a_p$  are unknown coefficients. The value taken by the function  $d(y)$  for every observation is the “discriminant punctuation”. Formally, if  $d_{gk}$  is the discriminant punctuation of individual  $k$ , who belongs to group  $g$ ,  $\bar{d}_g$  is the mean discriminant punctuation within group  $g$ , and  $\bar{d}$  is the mean discriminant punctuation for the whole sample, then coefficients  $a_1, a_2, \dots, a_p$  are selected using the procedure outlined by Fisher (1936):

$$\underset{a=(a_1, \dots, a_p)}{\text{Max}} \left( \frac{\text{Between Variance}}{\text{Within Variance}} \right) = \frac{\sum_{g=1}^G n_g (\bar{d}_g - \bar{d})^2 / (G-1)}{\sum_{g=1}^G \sum_{k=1}^{n_g} (d_{gk} - \bar{d}_g)^2 / (n-G)} \quad (2)$$

The former expression is equal to  $\frac{u' B u}{u' W u} \cdot \frac{n-G}{G-1}$ , where:

$$B = \sum_{g=1}^G n_g (\bar{y}_g - \bar{y}) \cdot (\bar{y}_g - \bar{y})' \text{ and } W = \sum_{g=1}^G \sum_{k=1}^{n_g} (y_{gk} - \bar{y}_g) (y_{gk} - \bar{y}_g)', \text{ with the additional condition that } u'$$

$W u = 1$ . It can be shown that if  $\lambda_1$  is the highest eigenvalue of the matrix  $W^{-1} B$ , then the associated eigenvector  $u_1$  is a solution of (2). The discriminant power of variable  $p$  (or the importance of that variable in the classification of individuals into groups) can be measured<sup>13</sup> by the coefficient  $a_p$ .

In general, it is possible to obtain as many discriminant functions as eigenvalues,<sup>14</sup> and these functions can be ordered according to their discriminant power based on the absolute value of the

<sup>13</sup> The comparison between variables should be carried out by means of standardized coefficients.

<sup>14</sup> The maximum number of discriminant functions is given by the range of the matrix  $W^{-1} B$  equal to  $\min[G-1, p]$ .

associated eigenvalue.<sup>15</sup> The number of significant functions can be tested by the sequential hypothesis that  $\lambda_{k+1} = \dots = \lambda_{\min\{G-1, p\}} = 0$  using an exact F test.

### **Separation and discrimination measures**

The power of a group of variables to classify individuals into given groups can be gauged using different statistics and measures. Wilks' lambda ( $\Lambda$ ) is one of the most frequently used statistics for this purpose. It measures the importance of the within-group variance in total:

$$\Lambda = \frac{|W|}{|W + B|} = \frac{1}{\prod_{i=1}^{\min\{G-1, p\}} (\lambda_i + 1)} \quad (3)$$

Wilks' lambda takes values between 0 and 1, converging to 0 when discrimination is perfect. Lower values of  $\Lambda$  indicate a better degree of separation between groups in terms of discriminant variables.

Another way of determining the separation between given groups is to measure the separation of the "centroids" of every group, which are defined as the vector containing the mean discriminant punctuation within the group. The separation of any pair of centroids  $x, y$  with correlation matrix  $M$  is computed using the Mahalanobis distance, defined as:

$$D_m(x, y) = \sqrt{(x - y)' M^{-1} (x - y)} \quad (4)$$

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<sup>15</sup> By construction, the functions are orthogonal.