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A MODEL FOR INDUCTION MOTORS WITH NON-UNIFORM AIR-GAP

G. BOSSIO[†], C. DE ANGELO[†], J. SOLSONA[‡], G.O. GARCÍA[†] and M.I. VALLA[#]

†*Grupo de Electrónica Aplicada, Facultad de Ingeniería, Univ. Nac. de Río Cuarto, Ruta Nac. #36 Km. 601, X5804BYA Río Cuarto, Argentina*
gbossio@ing.unrc.edu.ar; cdeangelo@ieee.org; g.garcia@ieee.org

‡ *Instituto de Investigaciones en Ingeniería Eléctrica "Alfredo Desages", Dto. de Ingeniería Eléctrica y de Computadoras, Univ. Nac. del Sur, Avda. Alem 1253, B8000CPB Bahía Blanca, Argentina*
jsolsona@uns.edu.ar

[#] *Laboratorio de Electrónica Industrial, Control e Instrumentación (LEICI), Univ. Nac. de La Plata, CC 91, 1900 La Plata, Argentina*
m.i.valla@ieee.org

***Abstract*—Equations to calculate inductances of induction motors, considering non-uniform air-gap, are proposed. The analyzed air-gap variations are static and dynamic eccentricity and stator slots. The equations for inductance calculation, obtained from the modified winding functions and the energy stored in the air-gap, allow considering the effect of rotor bar skewing. Experimental results that validate the proposed method are presented.**

***Keywords*— Air-Gap eccentricity, Induction Motors, Modeling, Winding Function Approach.**

I. INTRODUCTION

The conventional *d-q* model of an induction motor (IM) is based on the assumption that the stator phase windings and the squirrel cage can be modeled as three-phase winding sets, sinusoidally distributed. This implies that the harmonics of the winding distribution are neglected in the analysis of the machine. A model based on the geometry and winding distribution, having no restrictions regarding its symmetry, is more suitable for the motor analysis and simulation under asymmetry and fault conditions.

In Toliyat *et al.* (1991), a multiple-coupled circuit machine model and a method to calculate the mutual inductances, known as "*Winding Function Approach*" (WFA), was presented. This model was used for the analysis of concentrated winding IM in adjustable speed drive applications. A detailed depiction of the procedure, needed to implement such model, together with simulation results for an IM, was presented in Luo *et al.* (1995). By means of this model, all the harmonics of the spatial winding distribution are taken into account, with no restrictions concerning the symmetry of both the stator windings and the rotor bars. Hence, this model has been applied in the analysis of asymmetry and fault conditions in IM. The WFA has also been used in the analysis of a five-phase reluctance motor Toliyat *et al.* (1992).

In Luo *et al.* (1995), Toliyat and Lipo (1995), Milimonfared *et al.* (1999), Joksimovic and Penman (2000) and Nandi and Toliyat (2002), the WFA was used to analyze IM faults such as shorting, opening and abnormal connections of the stator phase circuits, as well as broken rotor bars and cracked rotor end rings. The analysis of static and dynamic eccentricity effects using the cited model is presented in Toliyat *et al.* (1996) and Joksimovic *et al.* (2000). The authors of these works calculate the inductances using the equations presented in Luo *et al.* (1995), which do not take air-gap variations into account. From this analysis, the mutual inductances between stator phase and rotor loops (L_{sr}) are different from those between the rotor loops and stator phase (L_{rs}), and it is difficult to find a physical meaning for this asymmetry. In Al-Nuaim and Toliyat (1998), a modification of this method, considering air-gap eccentricity, is proposed and used in the analysis of dynamic eccentricity in a synchronous machine. The new method was called "*Modified Winding Function Approach*" (MWFA), and it has been applied to analyze static, dynamic, and mixed eccentricity in IM by Nandi *et al.* (1997), Nandi *et al.* (2001) and Nandi *et al.* (2002).

In the previous works, based on WFA or MWFA, the machine analysis is carried out assuming uniformity down the axial length of the motor. That is, without skew and with uniform air-gap along the rotor. In Joksimovic *et al.* (1999), the skew effect on the inductances is analyzed using the equations developed

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for the analysis of the machine with axial uniformity. This extension, however, does not allow the analysis of radial and axial air-gap non-uniformity effects.

In Barakat *et al.* (2001), the effect of stator and rotor slots, as well as static eccentricity, is analyzed. The permeance function, which includes slot effects, is derived from simulation using finite elements. The equations for inductance calculation allow considering radial and axial non-uniformity effects of both air-gap and windings. However, the effects of such non-uniformities on the magnetomotive-force distribution are not considered.

In Bossio *et al.* (2002), the equations for the calculation of the machine winding inductances, considering winding and air-gap radial and axial non-uniformities, are developed. These equations are obtained from linked flux and applied to the analysis of static air-gap eccentricity effects. In this work, experimental results and those obtained from the model are similar except for the rotor slot effects, not modeled.

In the present work, the equations for IM inductance calculation are derived from the stored magnetic energy. These equations are applied to the analysis of air-gap eccentricity effects considering skew and stator slots. The stator slot modeling allows obtaining a great similitude with the experimental results.

II. INDUCTION MOTOR MODEL

Considering an m-stator-circuit, n-rotor-bar IM, the cage can be viewed as n identical and equally spaced rotor loops (Luo *et al.,* 1995). Voltage equations for the IM can be written in vector-matrix form as follows:

$$\mathbf{V}_s = \mathbf{R}_s\,\mathbf{I}_s + \frac{d\boldsymbol{\lambda}_s}{dt}, \tag{1}$$

$$\mathbf{V}_r = \mathbf{R}_r\,\mathbf{I}_r + \frac{d\boldsymbol{\lambda}_r}{dt}, \tag{2}$$

where,

$$\mathbf{V}_s = \begin{bmatrix} v_1^s & v_2^s & \ldots & v_m^s \end{bmatrix}^T, \tag{3}$$

$$\mathbf{V}_r = \begin{bmatrix} 0 & 0 & \ldots & 0 \end{bmatrix}^T, \tag{4}$$

$$\mathbf{I}_s = \begin{bmatrix} i_1^s & i_2^s & \ldots & i_m^s \end{bmatrix}^T, \tag{5}$$

$$\mathbf{I}_r = \begin{bmatrix} i_1^r & i_2^r & \ldots & i_n^r \end{bmatrix}^T, \tag{6}$$

and the stator and rotor flux linkages are given by,

$$\boldsymbol{\lambda}_s = \mathbf{L}_{ss}\,\mathbf{I}_s + \mathbf{L}_{sr}\,\mathbf{I}_r, \tag{7}$$

$$\boldsymbol{\lambda}_r = \mathbf{L}_{rs}\,\mathbf{I}_s + \mathbf{L}_{rr}\,\mathbf{I}_r, \tag{8}$$

where $\mathbf{L}_{ss}$ is an $m*m$ matrix containing the stator self and mutual inductances, $\mathbf{L}_{rr}$ is an $n*n$ matrix also containing the rotor self and mutual inductances, $\mathbf{L}_{sr}$ is an $m*n$ matrix composed of the mutual inductances between the stator phases and the rotor loops, $\mathbf{L}_{rs}$ is an $n*m$ matrix composed of the mutual inductances between the rotor loops and the stator phases, and $\mathbf{L}_{sr} = \mathbf{L}_{rs}^T$.

The mechanical equations for the machine are,

$$\frac{d\omega}{dt} = \frac{1}{J_{rl}}\left(T_e - T_l\right), \tag{9}$$

$$\frac{d\theta_r}{dt} = \omega, \tag{10}$$

where θ_r is the rotor position, ω is the angular speed, J_{rl} is the rotor-load inertia and T_l is the load torque. Speed and position dependant viscosity can be included into the load torque.

The machine electromagnetic torque, T_e, can be obtained from,

$$T_e = \left[\frac{\partial W_{co}}{\partial \theta_r}\right]_{\left(I_s, I_r \text{ constant}\right)}. \tag{11}$$

When saturation is neglected, the magnetic co-energy can be expressed as the energy stored in the magnetic circuits,

$$W_{co} = \frac{1}{2}\mathbf{I}_s^T\mathbf{L}_{ss}\mathbf{I}_s + \frac{1}{2}\mathbf{I}_s^T\mathbf{L}_{sr}\mathbf{I}_r + \frac{1}{2}\mathbf{I}_r^T\mathbf{L}_{rs}\mathbf{I}_s + \frac{1}{2}\mathbf{I}_r^T\mathbf{L}_{rr}\mathbf{I}_r. \tag{12}$$

The precise knowledge of the inductances making up the matrices in (7) and (8) is essential for the analysis and simulation of the IM. The following sections present a method for the calculation of these inductances, considering air-gap non-uniformities due to rotor and stator slots and air-gap eccentricity. Skew effects can also be included in inductance calculation with this method.

III. INDUCTANCE EQUATIONS DERIVED FROM STORED ENERGY

The MMF in the air-gap, $F_x(\phi, z, \theta_r)$, produced by a current i_x flowing in any coil x, is given by (Bossio *et al.*, 2002),

$$F_x(\phi, z, \theta_r) = N_x(\phi, z, \theta_r)\, i_x, \tag{13}$$

where $N_x(\phi, z, \theta_r)$ is the Modified Winding Function (MWF), and can be obtained as,

$$N_x(\phi, z, \theta_r) = n_x(\phi, z, \theta_r) - \frac{1}{2\pi L\left\langle g^{-1}(\phi, z, \theta_r)\right\rangle}\int_0^{2\pi}\int_0^{L} n_x(\phi, z, \theta_r)\, g^{-1}(\phi, z, \theta_r)\, dz\, d\phi, \tag{14}$$

where θ_r is the rotor position, ϕ and z are the angular and axial position of an air-gap arbitrary point, respectively. $n(\phi, z, \theta_r)$ is the "*Winding Spatial Distribution*", $g^{-1}(\phi, z, \theta_r)$ is the inverse of the air-gap function and $\langle g^{-1}(\phi, z, \theta_r)\rangle$ is their average value.

The MWF proposed in Bossio *et al.* (2002) allows considering a non-uniform geometric distribution of the windings and rotors bars down the motor axial length, *e.g.* skew. It is also possible to model the rotor eccentricity by means of the air-gap function, $g(\phi, z, \theta_r)$, which has no restrictions about axial non-uniformity. For the induction machine, the MWF can be defined for each stator winding and each rotor loop composed of two consecutive bars.

Writing the magnetomotive force in the air-gap as a function of the MWF, the equations for the motor inductance calculation can be derived from the stored magnetic energy as follows.

The energy stored in the air-gap by two windings A and B, is given by (Lipo, 1996):

$$W_{AB} = \frac{1}{2} L_{AB}\, i_A\, i_B = \frac{1}{2}\int_v \mathbf{B}_A \bullet \mathbf{H}_B \, dv\,, \tag{15}$$

where $\mathbf{B}_A$ is the magnetic flux density produced by a current I_A flowing in any coil A, $\mathbf{H}_B$ is the magnetic field intensity produced by a current I_B flowing in any coil B and v is the air-gap volume.

Writing the volume integral in cylindrical coordinates and considering that $\mathbf{B}_A$ and $\mathbf{H}_B$ are in radial direction (radial flux), yields

$$W_{AB} = \frac{1}{2}\int_0^L \int_0^{2\pi} \int_{r_i(\phi,z,\theta_r)}^{r_e(\phi,z,\theta_r)} r\, B_A(\phi,z,\theta_r) H_B(\phi,z,\theta_r)\, dr\, d\phi\, dz\,. \tag{16}$$

If $B_A(\phi, z, \theta_r)$ and $H_B(\phi, z, \theta_r)$ are constant in the radial direction,

$$W_{AB} = \frac{1}{2}\int_0^L \int_0^{2\pi} r_m\, g(\phi,z,\theta_r) B_A(\phi,z,\theta_r) H_B(\phi,z,\theta_r)\, d\phi\, dz\,, \tag{17}$$

where

$$g(\phi,z,\theta_r) = r_e - r_i\,, \tag{18}$$

$$r_m = \frac{r_e + r_i}{2} \cong cte. \tag{19}$$

Then substituting

$$H_B(\phi,z,\theta_r) = \frac{F_B(\phi,z,\theta_r)}{g(\phi,z,\theta_r)}\,, \tag{20}$$

and

$$B_A(\phi,z,\theta_r) = \mu_0 \frac{F_A(\phi,z,\theta_r)}{g(\phi,z,\theta_r)}\,, \tag{21}$$

in (17) yields

$$W_{AB} = \frac{1}{2}\int_0^L \int_0^{2\pi} r_m\, \mu_0 \frac{F_A(\phi,z,\theta_r) F_B(\phi,z,\theta_r)}{g(\phi,z,\theta_r)} d\phi\, dz\,. \tag{22}$$

Replacing, in the previous equation, F_A and F_B by Eq. (13)

$$W_{AB} = \frac{1}{2}\int_0^L \int_0^{2\pi} r_m\, \mu_0 \frac{N_A(\phi,z,\theta_r) N_B(\phi,z,\theta_r)}{g(\phi,z,\theta_r)} i_A i_B\, d\phi\, dz\,, \tag{23}$$

then

$$L_{AB} = \int_0^L \int_0^{2\pi} r_m\, \mu_0 \frac{N_A(\phi,z,\theta_r) N_B(\phi,z,\theta_r)}{g(\phi,z,\theta_r)}\, d\phi\, dz\,. \tag{24}$$

From this equation it is possible to calculate the stator coil and rotor loop mutual and self inductances. This is a function of winding distribution and air-gap geometry. Furthermore, the equation shows that using the proposed MWF and the obtained equations for the inductance calculation, the inductance L_{BA} results equal to the inductance L_{AB}. This equality does not depend on either the winding distribution or the air-gap. The same equations for the inductance calculation can be obtained by using linked flux as shown in Bossio *et al.* (2002).

IV. INDUCTANCE CALCULATIONS

The motor inductance calculation, using the equations obtained in the previous section, is presented. Bar skewing effects, as well as those from rotor slots and static air-gap eccentricity, are considered in inductance calculation. As an example, the mutual inductances between a stator phase and a rotor loop for the motor, whose parameters appear in the appendix, are shown.

A. Skew

Rotor bar skewing is modeled in the rotor loop spatial distribution function, $n(\phi, z, \theta_r)$. If the air-gap variation due to rotor slots is not included in the air-gap function, $g^{-1}(\phi, z, \theta_r)$, then the average effect can be included in the Carter coefficient (Lipo, 1996). Figure 1 (a) shows the inductance for no skew whereas Fig. 1 (b) shows the corresponding inductance when a skewing of one stator slot period is considered.

From the obtained results, no changes in the mutual inductance magnitude are observed. However, its shape gets smoother in presence of skew, mainly in the region where the inductance varies. In the case of air-gap uniformity, the equations used for inductance calculation can be reduced to the equations presented in Joksimovic *et al.* (1999).

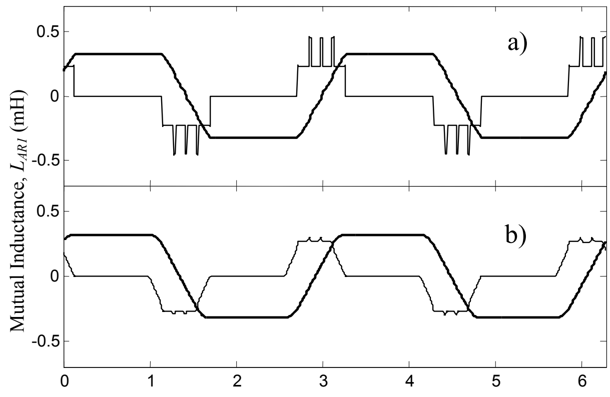

Figure 1. Calculated mutual inductance (thick line) and its derivative (thin line) of stator phase *A* and rotor loop 1, L_{AR1}. a) without skew, b) with skew.

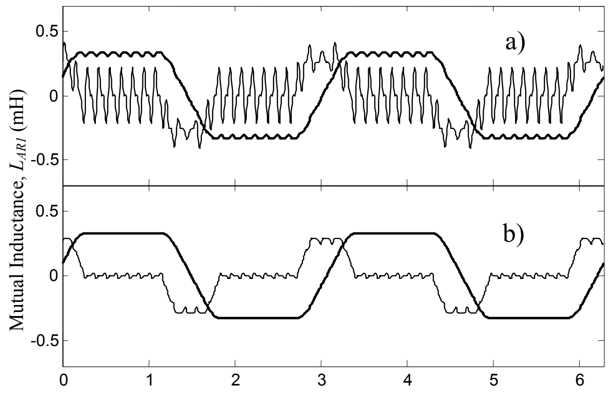

Figure 3. Calculated mutual inductance (thick line) and its derivative (thin line) of stator phase *A* and rotor loop 1, L_{AR1}, including the effect of stator slots. a) without skew, b) with skew.

B. Stator Slot Modeling

The air-gap variation, due to the stator slots, was modeled considering the distribution of flux lines on the slots, as proposed in Lipo (1996). Figure 2 a) shows this flux line distribution on the slots. Regarding this flux lines distribution, the air-gap in a slot rises linearly to the center of the slot and then drops up to its nominal value, g_0, at the other slot side, as shown in Fig. 2 b).

Figure 3 shows the effect of the stator slots on the mutual inductance between the rotor loop and the stator phase *A*. Figure 3 a) shows the inductance when skew is not present, whereas Fig. 3 b) shows the corresponding inductance when skew bars are considered. The effect of stator slots is bigger for the not skewed motor and it seems better to be appreciated in the inductance derivative. The rotor-slot skewing produces a significant reduction of this harmonics as it can be seen in Fig. 3 b). The results for the no-skewed motor are similar to the obtained, using finite elements, in Nandi *et al.* (2002).

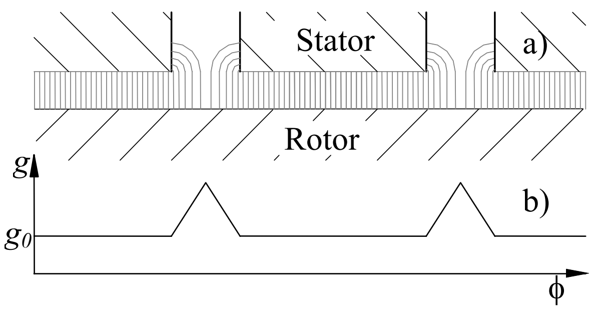

Figure 2. Flux lines distribution on the air-gap a), air-gap function b).

C. Air-Gap Eccentricity Modeling

As it was stated in Nandi *et al.* (2002), there are two types of air-gap eccentricity: static and dynamic. In practice, these eccentricities appear as a mixed form.

The air-gap function for mixed eccentricity can be represented by,

$$g(\phi,\theta_r) = g_0\left(1 - e_s\cos\phi - e_d\cos(\phi-\theta_r)\right), \qquad (25)$$

where e_s and e_d are static and dynamic eccentricity amounts, respectively.

This equation is used in (14) and (24) for the IM self and mutual inductance calculation. The combination of air-gap eccentricity and stator slot effects can also be analyzed. For this purpose, the increase of the air-gap function due to rotor slots is added to the air-gap function (25).

For static eccentricity, self and mutual inductances of the stator windings, $\mathbf{L}_{ss}$, are constant whereas those of the rotor loops, $\mathbf{L}_{rr}$, change with rotor position. For dynamic eccentricity, on the other hand, the stator self and mutual inductances are rotor position functions. Unlike them, self and mutual inductances of the rotor loops are constant, since they do not experiment any air-gap change when the rotor turns. As an example, the mutual inductance of stator phase *A* and rotor loop 1, L_{AR1} , with a 50% static eccentricity, without skew and with skew, is shown in Fig. 4 (a) and in Fig. 4 (b), respectively. This figures shows that the static eccentricity produces a significant change in the mutual inductance due to the periodic variation of the air-gap in front of the rotor loop. For the motor without skew, Fig. 4 (a), the eccentricity increases the harmonics produced by the stator slots mainly when the rotor loop is in the minimal air-gap region. The skew smoothes this inductance variation due to static eccentricity, reducing the stator slots harmonics amplitude. This is evident in the inductance derivative (thin line).

For static, dynamic and mixed air-gap eccentricities, uniform down the axial length and without rotor bar and stator winding skew, the equations used for the inductance calculation can be reduced to the ones used in Nandi *et al.* (2001) and Nandi *et al.* (2002). This is the case shown in Fig. 4 (a), where the result obtained from the proposed method is the same as that obtained from the MWFA. In addition, the proposed method allows considering skew and air-gap eccentricity combined effects, as shown in Fig. 4 (b). This proposal also allows modeling the non-uniform air-gap eccentricity down the axial length of the IM.

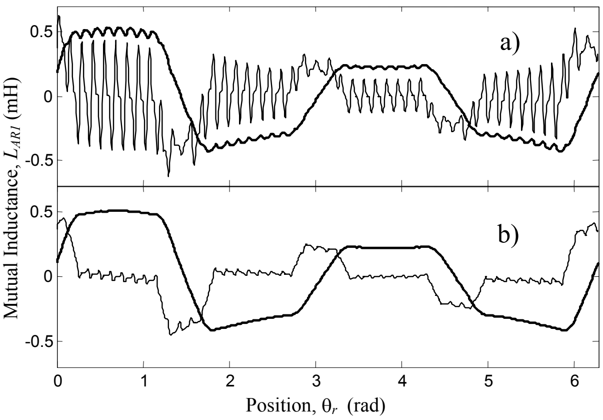

Figure 4. Calculated mutual inductance (thick line) and its derivative (thin line) of stator phase *A* and rotor loop 1, L_{AR1} as a function of rotor position, θ_r. a) with 50% of static eccentricity and without skew, b) with 50% static eccentricity and skew.

V. EXPERIMENTAL RESULTS

In order to validate the proposed method, the mutual inductance between a rotor loop and a stator phase was measured on an IM consisting of a standard stator and two special rotors. The standard stator corresponds to a 5.5 KW-380 V-Frame 132S machine with all its phase coils in series connection. The IM parameters were used to calculate the inductances in Section IV. Two special no-bar rotors were constructed, one with and the other with no skew. A special winding was placed along two consecutive rotor slots.

A 10-Hz sinusoidal voltage was applied in a stator phase to calculate the inductance between this phase and the rotor winding. The voltage induced in the rotor winding was measured as a rotor position function. The inductance was calculated from the ratio between the induced and the applied voltage,

$$L_{AR_1}(\theta_r) = \frac{V_{R_1}(\theta_r)}{V_A} L_{AA}, \tag{26}$$

where L_{AA} is the stator phase self inductance, previously calculated by means of stator voltage and current measurements.

The obtained inductance, referred to one turn on the rotor, is shown in Fig. 5. Figure 5 (a) shows the measured mutual inductance for the non-skewed rotor (thick line) and its calculated derivative (thin line). This inductance is very similar, in amplitude and shape, to that calculated using the proposed method, shown in Fig. 3 (a). As for the measured and the calculated using the proposed method inductances, a small 48-cycle-per-rotor-revolution harmonic component appears, which can clearly be seen in the inductance derivative. This harmonic is generated by the air-gap variation produced by the stator slots.

Including the stator slots in the model, not only allows to have a good approximation in the inductance calculation but also in its derivative, improving the authors previous proposal (Bossio *et al.*, 2002).

Figure 5 (b) shows the mutual inductance measured in a machine with skewed-rotor bars (thick line) and its derivative (thin line). The harmonic component, produced by the stator slots, is reduced due to skew.

As for inductance calculation using the proposed method, experimental results show that the mutual inductance magnitude does not change but its shape gets smoother with skew, mainly in the region where the inductance varies. This effect is better appreciated in the inductance derivative.

VI. CONCLUSIONS

In this paper, a method to calculate the inductances of an induction motor with non-uniform air-gap was proposed. This method allows taking radial and axial asymmetries such as skew and air-gap non-uniform eccentricity down the machine axial length into account. As an example, skew, air-gap eccentricity and stator slot effects on the mutual inductances between stator phases and rotor loops were shown.

Experimental results that validate the proposed method were also presented.

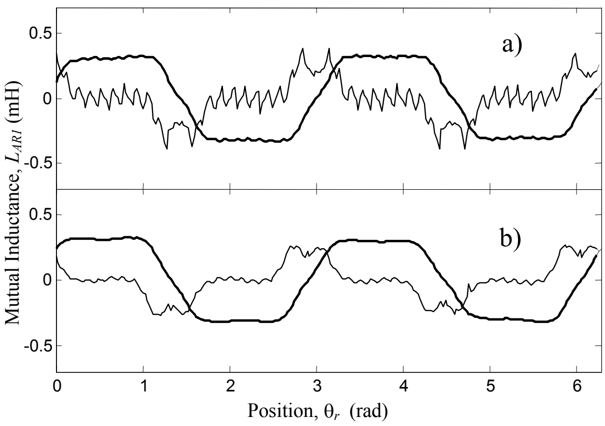

Figure 5. Measured mutual inductance (thick line) and its derivative (thin line) of stator phase *A* and rotor loop 1, L_{AR1} as a function of rotor position, θ_r. a) without skew, b) with skew.

APPENDIX

Motor data:

Nominal power: 5.5 Kw.
Pair of poles: 2.
Stator winding: 67 turns per coil, 2 coil per group, 4 group per phase, series connection, step 1:10:12.
Number of stator slots: 48.
Number of rotor slots: 40.
Air-gap (g_0): 0.45 mm.
Stator length (L): 0.11 m.
Air-gap average radius (r_m): 0.075 m.

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