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CONCRETE GRAVITY DAMS FE MODEL PARAMETER UPDATING USING AMBIENT VIBRATIONS

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Abstract

Most of the dams around the world were designed before the introduction of seismic regulations and without concerns about their dynamic behavior. The failure of a large gravity dam might have catastrophic effects putting at risk a large number of human lives, not counting the considerable economic consequences. Since there are no case histories of concrete gravity dams failed after seismic events, numerical models assume great importance for the evaluation of the seismic performance of such structures or to control them within a SHM framework. Several different sources of uncertainty are involved in numerical models of concrete gravity dams, their effects can be reduced by exploiting all available information about the structure. Ambient vibrations are an important source of information because they can be used to characterize the dynamic behavior of the structure. In this paper, a procedure, defined in the Bayesian framework, which allows calibrating the dynamic model parameters using ambient vibration is presented. Ambient vibrations are used to determine the modal characteristics of the system, by applying the Operational Modal Analysis (OMA), which are used in the updating process. The use of meta models based on the general Polynomial Chaos Expansion (gPCE) and a modified version of Markov Chain Monte Carlo (MCMC) allows both considering the SSI in the numerical model of the dam and solving the problem of coherence between experimental and numerical modes. Finally, the proposed procedure is applied to the case of an Italian dam showing the applicability to real cases.

Keywords: concrete dams, gPCE, OMA, Bayesian Updating, MCMC, UQ.

1 INTRODUCTION

Concrete dams are fundamental infrastructures due to their use for energy production, floods control and industrial supply. However, the largest part of existing concrete dams located in developed countries have been designed by following only static concepts. In light of the revaluation of some areas as seismic and the higher reliability levels required by the community, nowadays a large number of existing concrete dams are outdated [1]. Therefore, the evaluation and the mitigation of the seismic risk of concrete dams is a task of primary importance for our society [2].

Structural Health Monitoring (SHM) is a powerful tool both to control the structural behaviour (Diagnosis phase) and to predict the remaining life expectancy of the dam (Prognosis phase) [3]. In this context, one or more Quantities of Interest (QI) of the dam are monitored in order to detect abnormal behaviour of the structure. Predictive models, which reproduce the selected QI, must be defined in order to forecast the dam behaviour considering the effects of the uncertainties and those of the errors [4]. Numerical models are the only way to investigate the dynamic behaviour of concrete dams. Indeed, as discussed by Hall [5] there are no case histories on concrete dams failed after seismic events.

Numerical models commonly used in dam engineering field are particularly complex due to the presence of three different interacting domains, dam, basin and soil. In this context, De Falco et al. [6–8] showed the influence of modelling strategy on the solution of dynamic analyses of concrete dams. Once a deterministic model has been defined, the uncertainties related to the model parameters lead to a biased result, which must be quantified (UQ) and reduced as much as possible in order to perform a reliable numerical model of the dam. All available information must be used for this purpose. In particular, the observations recorded by the monitoring system can be used to calibrate the model parameters and not only to control the health state of the structure.

The largest part of existing concrete dams is equipped by static monitoring systems, which record the displacements of few points on the structure and the environmental conditions, i.e. reservoir level, air and water temperatures. De Falco et al. [9] showed how to use information coming from the static monitoring system to update the mechanical parameters of the model materials in a Bayesian framework. Despite static SHM can provide useful information both for the structural control during normal operations and for the calibration of the model parameters, dynamic monitoring systems seem to be more appropriate when the seismic behaviour of a structure is investigated. With the aim to perform a permanent dynamic SHM system the only practicable choice is the registrations of ambient vibrations [10]. The observations recorded by a dynamic monitoring system based on ambient vibrations can be directly used in the updating process or elaborated through Operational Modal Analysis (OMA) [11] in order to obtain the modal characteristics of the system, i.e. frequencies and mode shapes. In this latter case, the updating process is defined with regard to the modal characteristics of the system, which are then the QI of the problem. The use of modal characteristics as QI is the commonly adopted approach in civil engineering field, because it leads to a simplification from the numerical point of view. Indeed, in this way, modal analyses are used within the updating process, instead of transient ones needed in the case of the direct use of ambient vibrations.

In dam engineering field dynamic SHM systems are very rare, even though some applications are available in the literature [12]. Most of the available research works aim to verify the feasibility of the installation of dynamic monitoring systems on concrete dams, but they do not discus the use of the observations for structural control or model calibration purposes. The numerical complications in the modal analysis of concrete dams, related to the SSI, has led to a broader use of static SHMs rather than dynamic ones.

In this paper, the effects of the uncertainties related to the mechanical parameters of the materials on the dynamic behaviour of concrete dams are investigated and discussed. Subsequently, the hierarchical Bayesian procedure for the updating of dynamic model parameters, proposed by Sevieri et al. [13], is applied in order to verify its effect on real cases. The numerical problems, discussed next, are solved by using a modified version of MCMC which allows both selecting and reordering the numerical mode shapes. A large concrete gravity dam, located in the centre of Italy, is used to investigate these two topics.

2 DAM DESCRIPTION

In this work, a large Italian concrete gravity dam is used as benchmark for the quantification of the effects of the epistemic uncertainty on the modal behaviour, and for the application of the hierarchical procedure to reduce them. The dam, showed in Figure 1, is composed by 26 monoliths for a total crest length around 450 m, and a maximum height of 65 m. The monoliths are connected each other through vertical contraction joints, which show an opening-closing movement during the year. This behaviour, recorded by the static monitoring system, is related to the variation of the environmental conditions, and in particular that of temperatures. Despite this movement has quasi-static nature, and then it cannot be directly used as source of information for dynamic properties updating, it must be considered in the updating procedure.

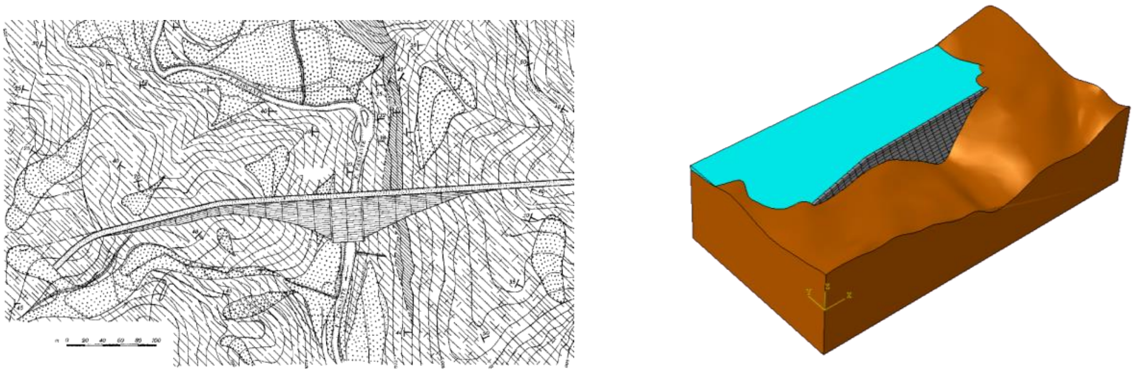


Figure 1: Dam drawing and FE model.

The mechanical parameters of the materials have been deduced from the experimental campaigns conducted in the past. The values of specific weight ρ, Young modulus E, Poisson's ratio υ, compressive and tensile strength, f_t and f_c, of the concrete and the soil (subscript C and S, respectively) are reported in Table 1.

	ρ_C [kg/m^3]	E_C [MPa]	ν_C	$f_{t,C}$ [MPa]	$f_{c,C}$ [MPa]	ρ_S [kg/m^3]	E_S [MPa]	ν_S	$f_{t,S}$ [MPa]	$f_{c,S}$ [MPa]
mean	2500.0	25000.0	0.20	1.85	15.3	2600.0	15000.0	0.22	1.7	51.2
s. d.	87.5	5875.0	0.069	0.629	3.443	725.4	7185.0	0.105	0.613	19.661

Table 1: Mechanical parameters of the materials.

3 UQ IN THE MODAL ANALYSIS OF A CONCRETE DAM

In this section the effect of the uncertainties related to the mechanical parameters of the materials on the modal characteristics of the dam are investigated. There are only few research works on the Uncertainty Quantification (UQ) in dam engineering field [14,15], but none of them addresses the problem of the concrete gravity dam modal characteristics considering the SSI and the FSI. FE models which consider SSI and FSI are characterized by a high computational burden, so the use of probabilistic procedure for UQ could be prohibitive. In this application, the computational burden is strongly reduced by using the general Polynomial Chaos Expansion (gPCE) [16,17] to approximate both numerical frequencies $\boldsymbol{f}^{\mathrm{FEM}}$ and mode shapes $\boldsymbol{\Phi}^{\mathrm{FEM}}$. Only the uncertainty related to the elastic parameters are considered in this application due to the elastic nature of the modal analysis. The elastic parameters of the materials are collected in $\boldsymbol{\theta}_{\mathrm{el}}$, while deterministic measurable variables, e.g. the basin level, are collected in $\mathbf{x}$, that is $\boldsymbol{f}^{\mathrm{FEM}}(\mathbf{x},\boldsymbol{\theta}_{\mathrm{el}})$ and $\boldsymbol{\Phi}^{\mathrm{FEM}}(\mathbf{x},\boldsymbol{\theta}_{\mathrm{el}})$. The uncertain output of the FEA can be described in a probabilistic space defined by the triplet $(\Omega,\mathsf{F},\mathrm{P})$: where Ω is the space of all events, F is the σ-algebra and P the probability measure. Assuming that $\boldsymbol{f}^{\mathrm{FEM}}(\mathbf{x},\boldsymbol{\theta}_{\mathrm{el}})$ and $\boldsymbol{\Phi}^{\mathrm{FEM}}(\mathbf{x},\boldsymbol{\theta}_{\mathrm{el}})$ are smooth enough to be represented in terms of simple random variables $\boldsymbol{\theta}_{\mathrm{el}}(\omega)$ corresponding to the Askey scheme [18], they can be approximated through the gPCE,

$$
\begin{aligned}
\hat{\boldsymbol{f}}\left(\mathbf{x},\boldsymbol{\theta}_{\mathrm{el}}\right) &= \sum_{\alpha\in\mathbf{I}} f^{(\alpha)}\psi_{(\alpha)}\left(\mathbf{x},\boldsymbol{\theta}_{\mathrm{el}}\right) \\
\hat{\boldsymbol{\Phi}}\left(\mathbf{x},\boldsymbol{\theta}_{\mathrm{el}}\right) &= \sum_{\beta\in\mathbf{J}} \phi^{(\beta)}\gamma_{(\beta)}\left(\mathbf{x},\boldsymbol{\theta}_{\mathrm{el}}\right).
\end{aligned}
\tag{1}
$$

In the previous equations $\hat{\boldsymbol{f}}(\mathbf{x},\boldsymbol{\theta}_{\mathrm{el}})$ and $\hat{\boldsymbol{\Phi}}(\mathbf{x},\boldsymbol{\theta}_{\mathrm{el}})$ are the gPCE approximations, $\psi_{(\alpha)}$ and $\gamma_{(\beta)}$ are the multivariate orthogonal polynomials with finite multi-index sets $\mathbf{I}$ and $\mathbf{J}$, while $f^{(\alpha)}$ and $\phi^{(\beta)}$ are the polynomials coefficients. Assuming that the dam concrete and the foundation soil are isotropic and heterogeneous, their elastic tensors □ are fully described by the bulk modulus K and the shear modulus G [19]. A parametrization of the forward problem in K and G is more convenient than the use of Young modulus E and Poisson's ratio υ, because □ is linear in K and G. The prior distributions of K and G are defined starting from Table 1 by assuming them log-normally distributed as reported in Table 2.

	K_C [MPa]	G_C [MPa]	K_S [MPa]	G_S [MPa]
distribution	LN	LN	LN	LN
mean	14880.0	10424.0	19210.0	10446.0
s. d.	5824.3	2520.5	19590.0	5203.0

Table 2: Prior distributions of $\boldsymbol{\theta}_{\mathrm{el}}$.

The only measurable variable in this application is the basin level, which oscillates between 29 m and 63 m, which are respectively the minimum and maximum regulation level. The opening-closing behavior of the vertical contraction joints can be investigated in modal analysis by considering two limit cases:

- the vertical joints are completely closed, then the contacts between monoliths are modeled as "bonded";
- the vertical joints are decompressed and by assuming that the monoliths can have relative displacements, the contacts are modeled as "frictionless".

In both cases the contacts are defined as relationship among two surfaces, then the two FE models have the same numbers of elements and nodes. Starting from the orography of the soil and the structural drawings of the dam, a 3D model of the system dam-soil-basin was imported from a CAD program to ABAQUS® v 6.14 [20]. The FE model (Figure 2) is composed by 40638 quadratic tetrahedral mechanical elements C3D10 for the soil, 14397 quadratic tetrahedral mechanical elements C3D10 for the dam body, 28707 linear tetrahedral acoustic elements AC3D4 for the basin and 1550 linear hexahedral one-way infinite elements as boundary conditions for the soil domain. An acoustic impedance is placed at the end of the reservoir in order to avoid the reflection of incident waves [8].

The solutions are calculated for different sets of the mechanical parameters, whose values are sampled from the prior distributions. They are used to train the gPCE of the "bonded" case, i.e. $\hat{\boldsymbol{f}}_{\mathrm{b}}(\mathbf{x},\boldsymbol{\theta}_{\mathrm{el}})$ and $\hat{\boldsymbol{\Phi}}_{\mathrm{b}}(\mathbf{x},\boldsymbol{\theta}_{\mathrm{el}})$, and the "frictionless" one, i.e. $\hat{\boldsymbol{f}}_{\mathrm{f}}(\mathbf{x},\boldsymbol{\theta}_{\mathrm{el}})$ and $\hat{\boldsymbol{\Phi}}_{\mathrm{f}}(\mathbf{x},\boldsymbol{\theta}_{\mathrm{el}})$.

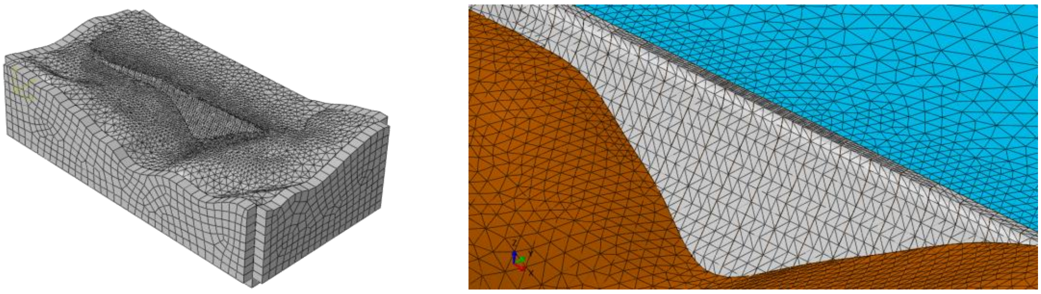


Figure 2: Mesh of the FE model.

The outputs of 350 analyses, for each model, are used to determine the gPCE coefficients, by using the approach proposed by Rosić and Matthies [16], while the polynomial expansion degrees are selected in order to minimize the errors in terms of mean and variance. The attention has been focused on the frequency range 2-20 Hz, where the fundamental modes of the system can be found. The presence of the SSI leads to a large number of numerical modes related to the soil mass. In this work, we refer to this issue as "coherence problem of the numerical modes", and in the context of the forward problem the order of numerical modes could change, due to the variation of the mechanical parameters set.

In this paper, the coherence problem has been tackled by reordering three fundamental numerical modes before they are used in the gPCE coefficients calculation. The Modal Assurance Criterion (MAC) [21] is used for this purpose. Let's consider two mode shapes $\boldsymbol{\phi}_{\mathrm{i}}$ and $\boldsymbol{\phi}_{\mathrm{j}}$, the MAC coefficient which allows measuring the difference between them is defined as

$$\mathrm{MAC}(i,j)=\frac{\left|\boldsymbol{\phi}_i^T\boldsymbol{\phi}_j^*\right|^2}{\left(\boldsymbol{\phi}_i^T\boldsymbol{\phi}_i^*\right)\left(\boldsymbol{\phi}_j^T\boldsymbol{\phi}_j^*\right)}, \tag{2}$$

where T and * indicate the transposed vector and the complex conjugated vector respectively. The MAC coefficient is always a real number, ranging from 0, in the case of no correlation, to 1 in the case of full correlation.

Only the modes related to the dam body are significant from the updating point of view, because experimental modes are usually recorded on the structure. In this paper, the 3 first modes which mobilize the largest amount of dam mass (Figure 3) are chosen as reference for the forward problem.

Hermite polynomials are used as basis functions, the relative relationships between errors and expansion degree are shown in Figure 4. In the end, a 5th order expansion degree is chosen for both frequencies and mode shapes, in order to have a small error both in terms of mean values and variances.

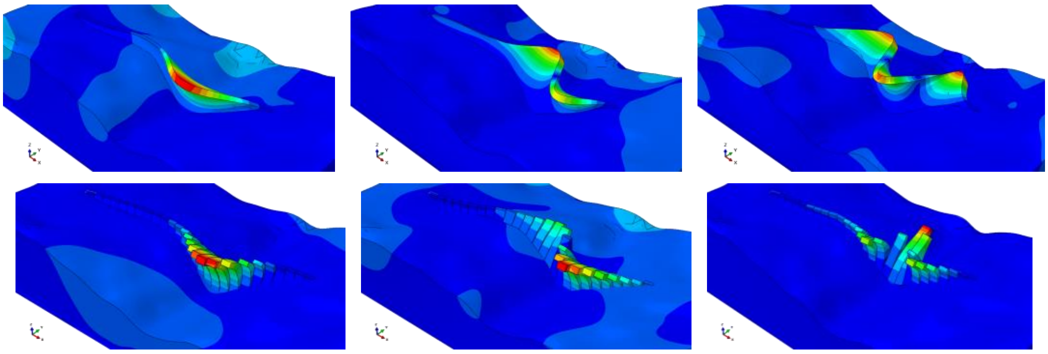
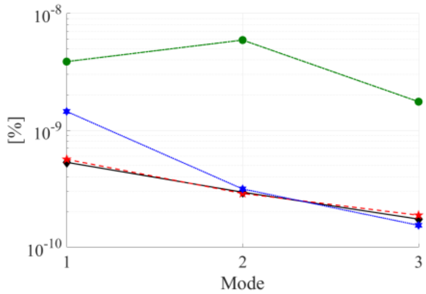
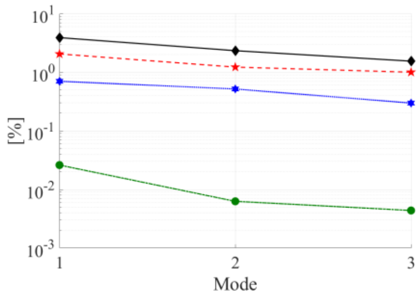


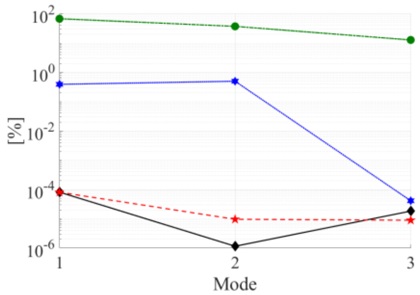
Figure 3: Reference modes for the UQ of bonded model (first line) and frictionless model (second line).



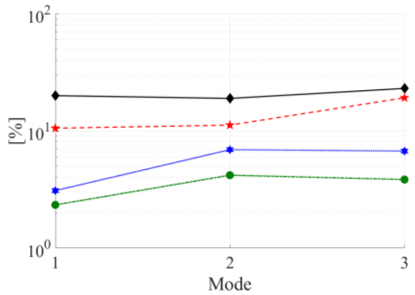
a) “B”/Frequencies: error of the mean values



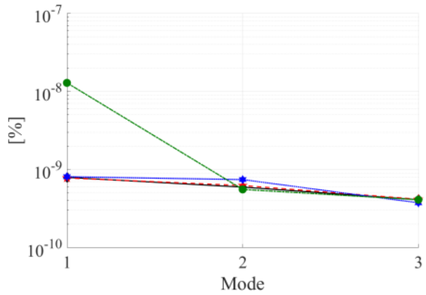
b) “B”/Frequencies: error of the sd



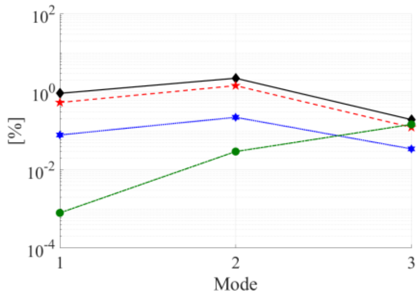
c) “B”/Mode shape: max error of the mean values



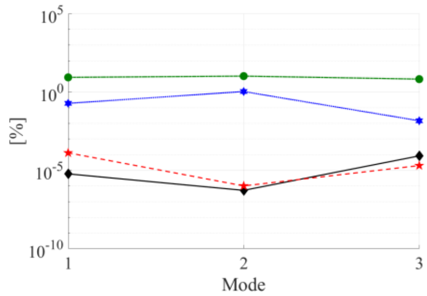
d) “B”/ Mode shape: max error of the sd



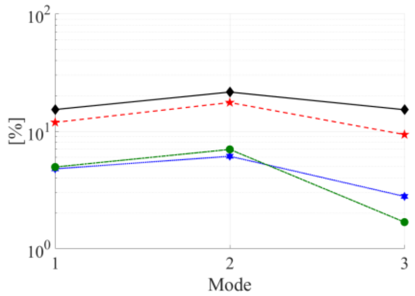
e) “F”/ Frequencies: error of the mean values



f) “F”/ Frequencies: error of the sd



g) “F”/Mode shape: max error of the mean values

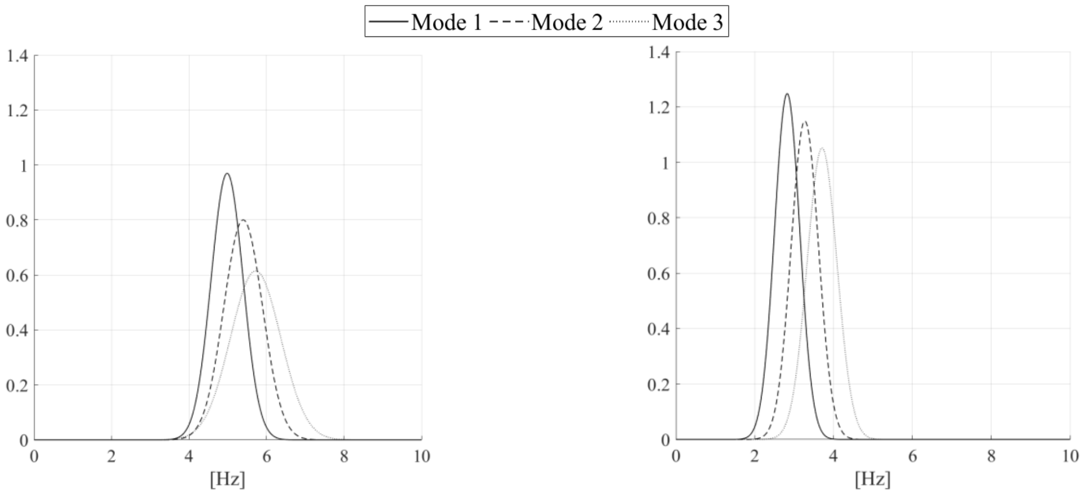


h) “F”/Mode shape: max error of the sd

Figure 4: Meta model errors. B = bonded model; F = frictionless model.

Figure 5 shows the distributions of the frequencies of both models. In the "bonded" case the mean values of the first three frequencies are in the range 5-6 Hz, while in the "frictionless" one they are in the range 3-4 Hz. The lack of stiffness of the "frictionless" model, due to the absence of the interaction among monoliths in U-D direction, leads to smaller frequencies values. In both cases the standard deviations increase toward higher frequency and they are relatively smaller in the "frictionless" case.

The gPCE based meta models $\hat{\boldsymbol{f}}_{\mathrm{b}}(\mathbf{x},\boldsymbol{\theta}_{\mathrm{el}})$, $\hat{\boldsymbol{\Phi}}_{\mathrm{b}}(\mathbf{x},\boldsymbol{\theta}_{\mathrm{el}})$, $\hat{\boldsymbol{f}}_{\mathrm{f}}(\mathbf{x},\boldsymbol{\theta}_{\mathrm{el}})$ and $\hat{\boldsymbol{\Phi}}_{\mathrm{f}}(\mathbf{x},\boldsymbol{\theta}_{\mathrm{el}})$ are used instead of the FE models to solve the inverse problem. In this way, the computational burden is strongly reduced, thus making possible the solution of the inverse problem without needing High Performance Computing (HPC).



a) "Bonded" model b) "Frictionless" model

Figure 5: Distributions of the first three frequencies.

4 HIERARCHICAL BAYESIAN PROCEDURE FOR DYNAMIC MODEL PARAMETERS UPDATING

In this Section, the procedure proposed by Sevieri et al. [13] for the dynamic model parameters updating is applied to the case of study. The procedure, defined in a hierarchical Bayesian framework, allows solving the inverse problem by using experimental modal characteristics of the system determined through the elaboration of ambient vibrations with OMA.

Let's consider the meta models previously defined (Section 3), the relationships between the *i*-th observation of the *k*-th experimental frequency $f_{\mathrm{k,i}}$ and the corresponding numerical prediction $\hat{f}_{k,i}$ is expressed by a multi-variate additive probabilistic model [22]

$$\ln\left(f_{k,i}\left(\mathbf{x},\boldsymbol{\theta}_{\mathrm{el}},\boldsymbol{\Sigma}_f\right)\right)=\ln\left(\hat{f}_{k,i}\left(\mathbf{x},\boldsymbol{\theta}_{\mathrm{el}}\right)\right)+\sigma_{f_k}\varepsilon_{f_{k,i}}. \tag{3}$$

All entries in Equation 3 have been already defined except for $\sigma_{f_k}\varepsilon_{f_{k,i}}$ which is the error term composed by random variables normally distributed $\varepsilon_{f_{k,i}}$ and their standard deviations σ_{f_k}, while $\boldsymbol{\Sigma}_f$ is the covariance matrix. The logarithmic function is used to stabilize the variance and to satisfy the homoskedasticity assumption [23].

Let's consider *q* modes of a system characterized by *m* dynamic d.o.f. ($q \le m$). The corresponding mode shapes matrix $\boldsymbol{\Phi}=\left[\boldsymbol{\phi}_1,\ldots,\boldsymbol{\phi}_k,\ldots,\boldsymbol{\phi}_q\right]^{\mathrm{T}}$ (*m* x *q* dimension), can be reorganized in only one column vector, thus obtaining $\boldsymbol{\phi}^{\mathrm{total}}$ with dimension $m\cdot q$ x *1*. Finally, by defining a global index h such that $1 \le h \le m \cdot q$, the relationship between the i-th observation of the h-th component of $\boldsymbol{\phi}^{\text{total}}$, i.e. $\phi_{h,i}$, and its corresponding numerical simulation $\hat{\phi}_{h,i}$ can be expressed as

$$\phi_{h,i}\left(\mathbf{x},\boldsymbol{\theta}_{\text{el}},\boldsymbol{\Sigma}_{\phi}\right)=\hat{\phi}_{h,i}\left(\mathbf{x},\boldsymbol{\theta}_{\text{el}}\right)+\sigma_{\phi_h}\varepsilon_{\phi_{h,i}}, \tag{4}$$

where $\boldsymbol{\Sigma}_{\phi}$ is the covariance matrix and $\sigma_{\phi_h}\varepsilon_{\phi_{h,i}}$ is the error term composed by a normally distributed random variable $\varepsilon_{\phi_{h,i}}$ and its standard deviation σ_{ϕ_h}.

Despite the use of the gPCE based meta models, the computational burden could be still very high due to the large number of random variables. Indeed, both the elastic parameters $\boldsymbol{\theta}_{\text{el}}$ and the terms of the covariance matrices, $\boldsymbol{\Sigma}_f$ and $\boldsymbol{\Sigma}_{\phi}$, are updated. However, by assuming that only the components of a same mode are correlated, the number of variables considerably decreases, that is

$$\begin{gathered}\boldsymbol{\Sigma}_f=\begin{bmatrix}\sigma_{f_1}^2 & 0 & \cdots & 0\\ 0 & \sigma_{f_2}^2 & \cdots & 0\\ \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & \cdots & \sigma_{f_q}^2\end{bmatrix}\\ \boldsymbol{\Sigma}_{\phi}=\begin{bmatrix}\left[B_1\right] & & & \\ & \left[B_2\right] & & \\ & & \ddots & \\ & & & \left[B_q\right]\end{bmatrix}.\end{gathered} \tag{5}$$

In this way, $\boldsymbol{\Sigma}_{\phi}$ assumes a block form, as indicated in Equation 5, where each block indicates the covariance matrix of each mode, otherwise the terms are zero. The computational burden could be further reduced by assuming a covariance function for each block $\left[B_k\right]$. In this application, two components ϕ_r and ϕ_s of the same mode k are correlated through an exponential covariance function of the Euclidean distance d_{ϕ_r,ϕ_s}

$$\text{COV}\left(\phi_r,\phi_s\right)=\frac{1}{\lambda_k}\exp\left(-w_k d_{\phi_r,\phi_s}\right). \tag{6}$$

In this way, the terms of each block $\left[B_k\right]$ are described by two coefficients λ_k and w_k, which are respectively collected in the vectors $\boldsymbol{\lambda}$ and $\mathbf{w}$.

The procedure proposed by Sevieri et al. [13] is based on a hierarchical Bayesian model [24] with two levels. This particular architecture of the Bayesian process allows updating both mean values and standard deviations of the random variables and inserting information on more than one level. These two features are particularly advantageous in the prognosis phase of a dynamic SHM. The first level is represented by the hyper-parameters $\boldsymbol{\Xi}_{\text{el}}$, i.e. mean values and standard deviations, of the elastic random variables $\boldsymbol{\theta}_{\text{el}}$. Whereas, the second level is composed by the elastic parameters $\boldsymbol{\theta}_{\text{el}}$ themselves and the terms of the covariance matrix $\boldsymbol{\Sigma}_f$, in the case of frequencies, or the coefficients $\boldsymbol{\lambda}$ and $\mathbf{w}$, in the case of mode shapes. For the sake of simplicity, let's collect the parameters of the second level in the vector $\boldsymbol{\Theta}$. Once a set of new observations $\mathbf{y}$ is available, the prior distribution $p(\boldsymbol{\Theta},\Xi_{\text{el}}) = p(\boldsymbol{\Theta} \mid \Xi_{\text{el}})\, p(\Xi_{\text{el}})$ can be updated through the likelihood function $L(\mathbf{x},\boldsymbol{\Theta},\Xi_{\text{el}} \mid \mathbf{y})$, thus obtaining the posterior distribution $p(\boldsymbol{\Theta},\Xi_{\text{el}} \mid \mathbf{y})$

$$p(\boldsymbol{\Theta},\Xi_{\text{el}} \mid \mathbf{y}) = \kappa L(\mathbf{x},\boldsymbol{\Theta},\Xi_{\text{el}} \mid \mathbf{y})\, p(\boldsymbol{\Theta} \mid \Xi_{\text{el}})\, p(\Xi_{\text{el}}). \tag{7}$$

In the previous Equation, κ is the normalizing factor. By exploiting the large amount of data, which will be available due to the integration of the present procedure within a SHM system, and the Central Limit Theorem, the likelihood function can be written as

$$L(\mathbf{x},\boldsymbol{\theta}_{\text{el}},\Xi_{\text{el}},\boldsymbol{\Sigma}_f) \propto \prod_{i=1}^{l} \frac{\exp\left[-\frac{1}{2}\mathbf{r}_i^{f^T}(\mathbf{x},\boldsymbol{\theta}_{\text{el}},\Xi_{\text{el}})\boldsymbol{\Sigma}_f^{-1}\mathbf{r}_i^f(\mathbf{x},\boldsymbol{\theta}_{\text{el}},\Xi_{\text{el}})\right]}{\sqrt{|2\pi\boldsymbol{\Sigma}_f|}}, \tag{8}$$

in the case of frequencies, and as

$$L(\mathbf{x},\boldsymbol{\theta}_{\text{el}},\Xi_{\text{el}},\boldsymbol{\Sigma}_\phi) \propto \prod_{i=1}^{l} \frac{\exp\left[-\frac{1}{2}\mathbf{r}_i^{\phi^T}(\mathbf{x},\boldsymbol{\theta}_{\text{el}},\Xi_{\text{el}})\boldsymbol{\Sigma}_\phi^{-1}\mathbf{r}_i^\phi(\mathbf{x},\boldsymbol{\theta}_{\text{el}},\Xi_{\text{el}})\right]}{\sqrt{|2\pi\boldsymbol{\Sigma}_\phi|}}, \tag{9}$$

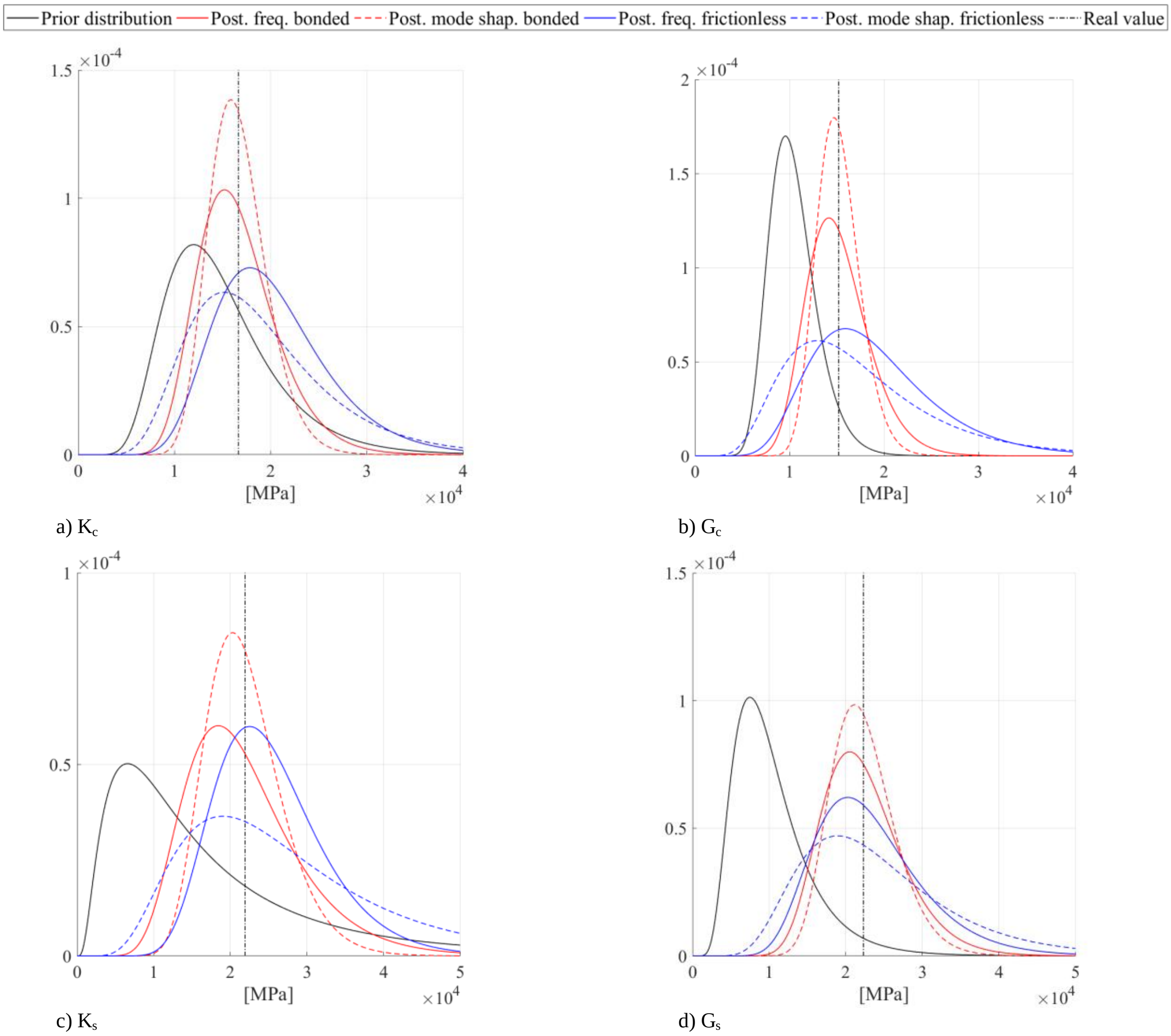
in the case of mode shapes. All entries in Equation 8 and 9 have been already defined except for $\mathbf{r}_i^f$ and $\mathbf{r}_i^\phi$ which are respectively the residuals of the frequencies and of the mode shapes. The residuals are the difference between the *i*-th observation and the corresponding prediction.

In the context of inverse problem, the coherence problem, previously mentioned (Section 3), is faced by modifying the numerical algorithm Markov Chain Monte Carlo (MCMC) [25] by introducing a reordering step based on the MAC matrix. More specifically, once the deterministic model, or its approximation, is solved (*i*-th step), the resulting numerical mode shapes are used with the experimental observations to calculate the *i*-th MAC matrix. Therefore, the numerical results are reordered coherently with the experimental ones, thus moving the highest MAC coefficients on the diagonal. The reordered numerical results are used to compute the residuals and to solve the inverse problem. In civil engineering field, the coherence between experimental and numerical modes is usually guaranteed by defining suitable objective functions or by using the concept of *system mode shapes* [26]. However, in the former case the predictive model of the modal characteristics can not be explicitly defined, while in the latter case numerical modes related to the soil are not discarded. Therefore, the modifications of MCMC, proposed by Sevieri et al. [13] seems to be more efficient in dam engineering field. The hyper-prior distributions (Table 3) are defined by using the material test results (Table 1). The distributions of the mean values are directly derived from Table 1, while for those of the standard deviations a C.o.V. equal to 10% is assumed. Non-informative prior distributions [22] are used for the terms of $\boldsymbol{\Sigma}_f$ and the coefficients collected in $\boldsymbol{\lambda}$ and $\mathbf{w}$, since no information about them are available. In this case of study, since no records of ambient vibrations are available, a high-fidelity model is used to simulate the experimental behavior of the dam. In this application, the high-fidelity model is a more refined version of the “bonded” one presented in Section 3. It is composed by 81276 quadratic tetrahedral mechanical elements C3D10 for the soil, 28794 quadratic tetrahedral mechanical elements C3D10 for the dam body, 57414 linear tetrahedral acoustic elements AC3D4 for the basin and 3100 linear hexahedral one-way infinite elements as boundary conditions for the soil domain. The elastic parameters of the high-fidelity model are reported in Table 3.

	K_C [MPa]	G_C [MPa]	K_S [MPa]	G_S [MPa]
s. d.	16667.0	15217.0	21930.0	22321.0

Table 3: Elastic parameters of the high-fidelity model.

The Bayesian procedure is applied separately for frequencies (Equation 3) and mode shapes (Equation 4), both in the case of "bonded" and "frictionless" model. The results in terms of comparisons between prior and posterior distributions of the parameters $\boldsymbol{\theta}_{el}$ are shown in Figure 6. In the case of "bonded" model the updating leads to the same values assumed for the high-fidelity model, either using frequencies or mode shapes predictive models. In the "frictionless" case the use of different predictive models leads to different results which are in contrast with the correct one, because of the lack of stiffens due to the absence of interaction between adjacent monoliths. These results highlight the need to perform predictive models which consider the state of the vertical contraction joints, i.e. close or open. An idea is to model the state of each vertical joint as random variable, and to correlate these states with the environmental conditions, i.e. reservoir level and temperatures.



a) K_c b) G_c c) K_s d) G_s

Figure 6: Comparison between prior and posterior distributions.

5 CONCLUDING REMARKS

Dynamic SHM systems, based on ambient vibrations, are powerful tools to control the health state of the structures and to reduce the uncertainties in the predictive models. However, due to the amount of uncertainties and the numerical complications which affect the FE models of concrete dams, dynamic SHM system are very rare in dam engineering field. Indeed, the dam-soil-reservoir interaction as well as the epistemic uncertainties lead to a large number of numerical modes with no experimental correlations.

In this paper, an Italian concrete gravity dams is used first to investigate the effect of the epistemic uncertainties on the modal behavior of the structure and then to apply a hierarchical Bayesian procedure to reduce them. Particular attention has been placed on the contribution of the vertical contraction joints behavior which must be considered in order to perform a reliable predictive model of the structure. Indeed, the opening-closing movement of the vertical contraction joints during the year leads to a strong variation of the modal behavior of the dam, which cannot be neglected. This paper shows how a hierarchical Bayesian framework, if integrated within a dynamic SHM of concrete dams, can successfully improve the performance of the SHM itself.

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