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Interdependencies between variables in fatigue analysis of a weight-optimised naval ship

Teresa Magoga[a,b]*, Roberto Ojeda[b], Stuart Cannon[a,b], and Giles Thomas[c]

[a]*Defence Science and Technology Group, 506 Lorimer Street, Fishermans Bend 3207, Australia*
[b]*Australian Maritime College – University of Tasmania, Launceston 7250, Australia*
[c]*University College London, United Kingdom*

Abstract

Modern warships are often constructed from aluminium alloys or high tensile steel, and their increasing range of operational roles indicates exposure to harsh seaway loads including slamming. These factors can lead to fatigue cracking, which can reduce operational availability. The objective of the present study is to improve understanding of the influence of variables in the fatigue analysis of a weight-optimised warship. The objective is met by analysing hull monitoring data acquired from a 56 m naval aluminium patrol boat, to determine the long-term importance of slamming and the correlation between the hourly number of slams, ship speed, and fatigue damage at two structural details. It was found that the effect of the ship's speed on the fatigue damage is not statistically significant. In addition, a sizable proportion of the fatigue damage accumulated at low to moderate ship speed, when the patrol boat experienced slamming, rather than at higher speeds. This may be due to voluntary and/or involuntary speed reduction, which is not typically taken into account in numerical fatigue analysis. That is, the use of long-term distributions of the wave environment and ship speed may mask the effects of voluntary and/or involuntary speed reduction on slamming occurrence and the fatigue damage. This finding can lead to improved requirements setting and through-life structural management of weight-optimised warships.

* Corresponding author. Tel.: +613-9626-8158.
E-mail address: Teresa.Magoga@dst.defence.gov.au

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1. Introduction

The analysis of a complex physical structure involves uncertainty and imprecision in parameters and models of different types. To predict the behaviour of a structure, the associated uncertainty and imprecision must be appropriately represented (Beer et al. 2013).

In ship design, factors including the construction quality, lifetime loads, and material properties are assumed. For naval ships, operations in often harsh environments and evolving mission requirements are additional sources of uncertainty (Magoga et al. 2019). These factors affect the structural fatigue life of a naval ship.

Fatigue damage of a structure occurs under the exposure to numerous cycles of stress peaks and troughs. The fatigue damage accumulates until the load-bearing capacity of the structural item falls below the applied load. In general, the primary sources of cyclic loads applied to the hull are wave action and impact loads such as slamming. A slam event occurs when a vessel experiences sufficiently large heave and pitch motions such that the bow emerges from the water and re-enters with a heavy impact. Slamming can have a considerable influence on the fatigue life of ships when compared to accounting for the global wave induced stresses alone (Thomas et al. 2006; Sheinberg et al. 2011; Magoga et al. 2017).

Fatigue analysis is an important part of the structural design of naval ships because cracking can lead to costly repairs and reduction of availability. Therefore, there is a need to articulate the uncertainties and interdependencies between the variables in structural fatigue life assessment (Magoga and Dwyer 2018).

For accurate fatigue analysis, information on the ship's materiel state and usage is required. One approach to obtain in-service information is through the implementation of a hull monitoring system (HMS). A HMS is a ship-board system that can monitor the hull response, sea state, and operational parameters of a marine structure. However, in general, hull monitoring is resource intensive. In fiscally constrained environments, there is a trade-off between the required accuracy and the cost of through-life fatigue management of complex structures (Molent and Aktepe 2000; Sabatino and Frangopol 2017). Therefore, consideration of this trade-off with respect to hull monitoring is required to ensure informed decision-making.

In response to the need to manage the operational availability of naval ships cost-effectively in the context of changing mission requirements and uncertainties associated with structural service life prediction, Magoga (submitted for publication) conducted a sensitivity study for a naval aluminium patrol boat. The tool used was Spectral Fatigue Analysis (SFA), which is a direct calculation method that explicitly includes the ship's operational profile and encountered wave environment. It was determined that the fatigue damage of welded details on the patrol boat is most sensitive to increasing significant wave height. A limitation of this study was that slamming loads were not included in the analysis.

To extend the above work, the aims of this paper are to establish the long-term importance of slamming in the fatigue assessment of a naval patrol boat, and to determine the rank correlation between the hourly number of slams, ship speed, significant stress, and fatigue damage. The impact of this research includes improved understanding of the uncertainties and interdependencies between the fatigue life and capability aspects of naval ships.

Nomenclature

ω	Wave frequency [rad/s]
τ	Rank correlation coefficient
ζ_α	Wave amplitude [m]
$\Delta\sigma$, S	Stress cycle [MPa]
$\Delta\sigma_{1/3}$	Significant stress cycle [MPa]
c_1, c_2	Coefficients of linear function
CDT	Cumulative Damage Theory
D	Fatigue damage
D_{total}	Fatigue damage based on total stress
D_{wave}	Fatigue damage based on 'wave-only' stress
GPS	Global Positioning System
$H_{1/3}$	Significant wave height [m]

HMS	Hull monitoring system
L_{WL}	Waterline length [m]
m	Inverse slope of S-N curve
n	Number of stress cycles
N	Number of stress cycles to failure
n_{obs}	Number of observations
N_{slam}	Number of slam events
R^2	Coefficient of determination
S_ζ	Wave energy spectrum
SFA	Spectral fatigue analysis
S-N	Fatigue resistance expressed as stress cycle (S) and number of cycles to failure (N)
T_z	Wave period [s]
v	Ship speed [kn]
v_{ave}	Average ship speed [kn]

2. Materials and method

The ship analysed is a 56 m patrol boat constructed from marine-grade aluminium alloys.

2.1. Hull monitoring system data

HMS data from the patrol boat is available. It includes measurements of strain, acceleration, and ship speed (from a Global Positioning System - GPS). However, a limitation of the HMS was that environmental parameters could not be recorded.

Measurements at two strain gauge locations, shown in Fig. 1, are analysed in the present paper. Data processing routines were developed in MATLAB (MathWorks 2015) to convert and filter the raw strain data to stress, and remove spikes due to electrical transients in sensor measurements (Magoga et al. 2017). The stress records were reduced into spectra of cycles using the rainflow counting method (Rychlik 1987). Table 1 shows the sample rates and the number of hours of usable data for the strain gauges and GPS.

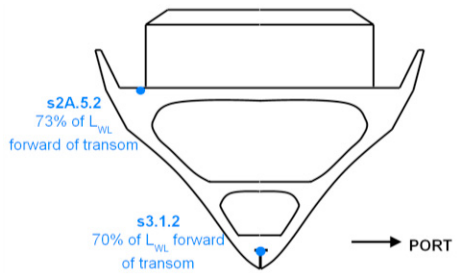

Fig. 1. Schematic of patrol boat cross-section showing analysed strain gauge locations (L_{WL} is waterline length)

Table 1. Sample rate and usable data for strain gauges and Global Positioning System

Sensor	Sample rate	Useable data
s3.1.2 and s2A.5.2	100 Hz	3304 hours
GPS	Approximately every 16 s	~9200 hours

2.2. Correlation of variables

Kendall's rank correlation coefficient (Kendall 1979) is a measure of the statistical association between the rankings of two variables. The rank correlation coefficient τ is calculated using Equation 1 (n_{obs} is the number of observations):

$$\tau = \frac{(number\ of\ concordant\ pairs) - (number\ of\ discordant\ pairs)}{n_{obs}(n_{obs}-1)/2} \tag{1}$$

Concordant pairs are those ordered in the same way, and discordant pairs are ordered differently. The value of τ ranges from −1 to +1, where ±1 indicates perfect association between two variables. The smaller the magnitude of τ, the weaker the relationship between the two variables.

2.3. Fatigue damage analysis

Cumulative Damage Theory (CDT) coupled with the S-N curve method is used to estimate the fatigue damage of the welded joints. CDT, also known as the Palmgren–Miner rule (Miner 1945), calculates the fatigue damage from each interval of the applied stress range as the ratio of the number of cycles (n) to the number of cycles to failure (N) that is determined from an S-N curve. The damage D caused by all cycles is calculated using Equation 2, where k is the number of stress ranges, a and m are S-N curve parameters, and n_i and N_i are the number of actual cycles experienced and cycles to failure for the i^{th} stress range increment:

$$D = \sum_{i=1}^{k} \frac{n_i}{N_i} = \frac{1}{a} \sum_{i=1}^{k} n_i (\Delta\sigma_i)^m \tag{2}$$

2.4. Fatigue resistance data

S-N curves from the aluminium structural design code Eurocode 9 (Technical Committee CEN/TC 250 1999) are used in the analysis. The S-N curves for most of the welded joints have three slopes.

2.5. Spectral fatigue analysis

The Finite Element Analysis package MAESTRO 11.2.2 is used in the present study. MAESTRO has a hydrodynamic analysis module (MAESTRO-Wave) that includes an implementation of strip theory that computes both panel pressures and sectional loads. The equations of motion are formulated based on the structural mesh, allowing equilibrium of the applied pressure and inertial force. The loads are based on the panel pressure integration (Ma et al. 2014). For cases when the Froude number is greater than 0.4, the option 'strip theory, 2.5D, high speed' is selected. This option uses a Rankine Source method with a forward speed correction term in the free surface computation (Zhao and Ma 2016).

MAESTRO's SFA module uses a database of stress Response Amplitude Operators for a ship to transform an input wave spectrum into a response spectrum. The wave energy spectrum S_ζ is defined by Equation 3, where ω is the wave frequency and ζ_α is the wave amplitude:

$$S_\zeta(\omega)\cdot d\omega = \frac{1}{2}\zeta_\alpha^2(\omega) \tag{3}$$

Similarly, the energy spectrum of the stress response σ(ω, t) can be defined by:

$$S_\sigma(\omega)\cdot d\omega = \left|\frac{\sigma_\alpha}{\zeta_\alpha}(\omega)\right|^2 S_\zeta(\omega)\cdot d\omega \tag{4}$$

The stress response spectrum is then calculated by:

$$S_\sigma(\omega) = \left| \frac{\sigma_\alpha}{\zeta_\alpha}(\omega) \right|^2 S_\zeta(\omega) \quad (5)$$

Using SFA, a vessel's lifetime exposure at sea is divided into 'cells' that represent combinations of sea state, ship heading with respect to the waves, and ship speed. Thus, the stochastic response in the cell becomes statistically stationary. After the computation of stresses, the expected values of short-term stress ranges are determined from an assumed Rayleigh distribution (Mansour and Liu 2008). The total fatigue damage is found by summing the fatigue damage from each short-term stress prediction, weighted by the corresponding occurrence probability of the operational and environmental conditions.

In MAESTRO, when a three-slope S-N curve is selected the stress spectrum is divided into a number of stress bins and CDT is used to estimate the fatigue damage.

3. Analysis and results

The uncertainty associated with the occurrence of slamming and its impact on fatigue damage is investigated via analysis of the strain data acquired from the patrol boat's HMS.

The results from SFA are used to establish the significance of the ship speed, heading relative to the dominant wave direction, significant wave height, and wave period to the fatigue damage.

3.1. Longer-term importance of slamming

Using the methods of Magoga et al. (2017), it was found that the patrol boat sustained slamming during 50 of the 3304 hours (refer to Table 1). Table 2 presents the Eurocode 9 detail types and the total fatigue damage D_{total} at the considered strain gauge locations. At s3.1.2 and s2A.5.2, 45% and 38% of the fatigue damage occurred during the 50 hours in which the patrol boat experienced slamming, respectively. The smaller percentage at s2A.5.2 is attributed to the dissipation of energy, associated with the slam event, through the structure.

Table 2. Eurocode 9 detail type, D_{total} accrued over 3304 hours, and D_{total} accrued over 50 hours in which slams were detected at strain gauge locations s3.1.2 and s2A.5.2

Strain gauge	Location	Eurocode 9 detail type	D_{total} over 3304 hours	D_{total} over 50 hours slams detected
s3.1.2	Flange of keel	11.3 (double sided butt-weld)	0.016	0.0072
s2A.5.2	Underside of main deck	5.6 (longitudinal weld)	0.019	0.0072

3.2. Correlation between hourly number of slams, ship speed, significant stress, and fatigue damage

A limited subset of the HMS data is used to correlate the number of slams per hour (N_{slam}), the average ship speed per hour (v_{ave}), the significant stress range ($\Delta\sigma_{1/3}$), the fatigue damage based on the low-frequency or 'wave-only' component of stress (D_{wave}), and the fatigue damage based on the total stress (D_{total}). This subset is comprised of 522 hours, and was created such that each observation of v_{ave} has a standard deviation of less than 1 kn to ensure minimal speed fluctuation during each hour. Heading changes are also relatively small. The normalised speed distribution based on the subset is given in Fig. 2. For comparison the long-term speed profile, over 9200 hours (refer to Table 1), is shown in Fig. 2. Additionally, scatterplots of N_{slam} and v_{ave} versus D_{total} at s3.1.2 are presented in Fig. 3.

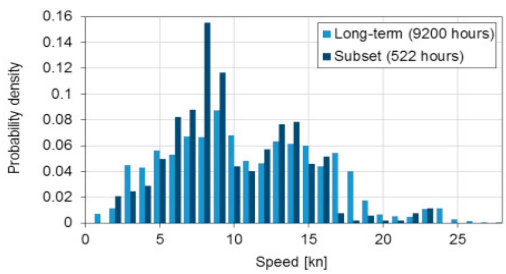

Fig. 2. Comparison between normalised speed histograms from 522 hour subset and 9200 hours (speed greater than 1 kn)

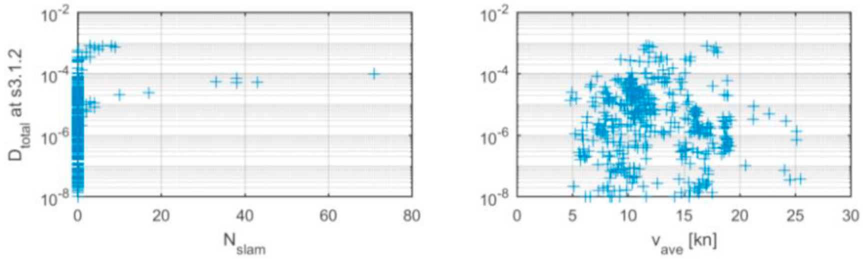

Fig. 3. Hourly D_{total} at s3.1.2 versus N_{slam}, v_{ave}, over 522 hour subset

Based on the 522 hour subset, the rank correlation coefficients (τ) between N_{slam}, v_{ave}, $\Delta\sigma_{1/3}$, D_{wave} and D_{total} per hour at s2A.5.2 and s3.1.2 are given in Table 3. The corresponding p-values are shown in Table 4. A p-value is a measure of the strength of the evidence against the null hypothesis that no correlation exists between two variables. Most of the p-values in Table 4 are less than 0.05, indicating that the results are statistically significant with a 95% confidence level. However, with the exception of $\Delta\sigma_{1/3}$ at s3.1.2, the τ values associated with v_{ave} are not statistically significant.

Table 3. τ correlation coefficients between N_{slam}, v_{ave}, $\Delta\sigma_{1/3}$, D_{wave}, and D_{total} at s2A5.2 and s3.1.2 based on 522 hour subset

	N_{slam}	v_{ave} [kn]	$\Delta\sigma_{1/3}$ [MPa] @ s2A.5.2	D_{wave} @ s2A.5.2	D_{total} @ s2A.5.2	$\Delta\sigma_{1/3}$ [MPa] s3.1.2	D_{wave} @ s3.1.2	**D_{total} @ s3.1.2**
N_{slam}	1							
v_{ave} [kn]	0.076	1						
$\Delta\sigma_{1/3}$ [MPa] @ s2A.5.2	0.232	-0.034	1					
D_{wave} @ s2A.5.2	0.251	-0.057	0.880	1				
D_{total} @ s2A.5.2	0.252	-0.042	0.888	0.933	1			
$\Delta\sigma_{1/3}$ [MPa] s3.1.2	0.220	-0.092	0.821	0.835	0.828	1		
D_{wave} @ s3.1.2	0.262	-0.077	0.750	0.821	0.795	0.813	1	
D_{total} @ s3.1.2	**0.262**	**-0.087**	**0.759**	**0.840**	**0.822**	**0.830**	**0.939**	**1**

Table 4. p-values for results in Table 3

	N_{slam}	v_{ave} [kn]	$\Delta\sigma_{1/3}$ [MPa] @ s2A.5.2	D_{wave} @ s2A.5.2	D_{total} @ s2A.5.2	$\Delta\sigma_{1/3}$ [MPa] s3.1.2	D_{wave} @ s3.1.2	**D_{total} @ s3.1.2**
N_{slam}	1							
v_{ave} [kn]	0.519	1						
$\Delta\sigma_{1/3}$ [MPa] @ s2A.5.2	$<10^{-3}$	0.089	1					
D_{wave} @ s2A.5.2	0.001	0.494	$<10^{-3}$	1				
D_{total} @ s2A.5.2	$<10^{-3}$	0.432	$<10^{-3}$	$<10^{-3}$	1			
$\Delta\sigma_{1/3}$ [MPa] s3.1.2	$<10^{-3}$	0.006	$<10^{-3}$	$<10^{-3}$	$<10^{-3}$	1		
D_{wave} @ s3.1.2	0.007	0.351	$<10^{-3}$	$<10^{-3}$	$<10^{-3}$	$<10^{-3}$	1	
D_{total} @ s3.1.2	**0.006**	**0.331**	**$<10^{-3}$**	**$<10^{-3}$**	**$<10^{-3}$**	**$<10^{-3}$**	**$<10^{-3}$**	**1**

The results shown in Table 3 indicate that, unsurprisingly, the values of τ between D_{wave} and D_{total} at the two strain gauge locations are the largest (both equal to or greater than 0.933). D_{total} as a function of D_{wave} can be modelled by Equation 6:

$$D_{total} = c_1 \times D_{wave} + c_2 \tag{6}$$

Using the non-linear least squares approach in MATLAB (MathWorks 2015) the coefficients c_1 and c_2 with confidence intervals, as well as the coefficient of determination R^2, are found as per Table 5.

Table 5. Coefficients with confidence intervals for Equation 6

Strain Gauge	c_1	c_2	R^2
s2A.5.2	2.05 (2.04, 2.06)	1.81×10^{-7} (-1.54×10^{-7}, 5.16×10^{-7})	0.997
s3.1.2	2.09 (2.08, 2.10)	-4.55×10^{-7} (-8.26×10^{-7}, -8.39×10^{-8})	0.998

The correlations between D_{total} at s3.1.2 and s2A.5.2, and $\Delta\sigma_{1/3}$ at s3.1.2 and s2A.5.2, are also strong (τ values of 0.822 and 0.821, respectively). The relationship between the latter two elements can be modelled by Equation 7:

$$\Delta\sigma_{1/3} \ at \ s2A.5.2 = 1.7 \times \left(\Delta\sigma_{1/3} \ at \ s3.1.2\right) + 1.6 \tag{7}$$

3.3. Correlation between operational and environmental parameters and fatigue damage based on SFA

Similarly, the Kendall rank correlation coefficient is used to establish the correlation between the ship speed (v), significant wave height ($H_{1/3}$), wave period (T_z), ship heading relative to the dominant wave direction (χ), and D at strain gauge s3.1.2 calculated using SFA. The SFA was reported by Magoga (submitted for publication). The speed profile used is based on the long-term data (refer to Fig 3). The wave scatter diagram (combined probability of $H_{1/3}$ and T_z) used is for Northern Australian Waters. The heading is uniformly distributed, which is typically assumed in fatigue strength assessment (DNV GL 2015).

The τ correlation coefficients are given in Table 6. Based on the SFA, the parameter of greatest significance to D is $H_{1/3}$, followed by T_z, v, and χ.

Table 6. τ correlation coefficients between v, χ, $H_{1/3}$, T_z, $\Delta\sigma_L$, and D at s3.1.2 calculated using SFA – input variables have unique probability distributions. All p-values less than 0.05.

	v [kn]	χ [deg]	$H_{1/3}$ [m]	T_z [s]	**D**
v [kn]	1				
χ [deg]	0	1			
$H_{1/3}$ [m]	0	0	1		
T_z [s]	0	0	0.217	1	
D	**0.109**	**-0.045**	**0.390**	**-0.182**	**1**

4. Discussion

The results presented in Table 2 show that 38% to 45% of the fatigue damage at the analysed locations accrued during a relatively small fraction of the patrol boat's time at sea. This result is significant, as it emphasises the importance of including the contribution of slamming in fatigue damage assessment of naval ships. This finding builds upon previous work that estimated the contribution of slamming to the fatigue damage at various welded details of the patrol boat though for a limited number of hourly records (Magoga et al. 2017).

As indicated in Tables 3 and 4, the number of slams per hour and the fatigue damage at the two strain gauge locations is of moderate statistical association. However, the correlation between the ship speed and fatigue damage is not statistically significant. Further, the correlation between the significant stress range at s3.1.2 and ship speed is very small and negative. Prima facie, these observations are counter-intuitive as it would be expected that the stresses and in turn the fatigue damage increase with ship speed. As shown in Table 6, based on SFA using linear hydrodynamic analysis, the correlation between the ship speed and fatigue damage at s3.1.2 is positive and statistically significant.

These results lead to the proposal that voluntary speed reduction and/or involuntary speed reduction influences the fatigue damage incurred in the in-service ship. Voluntary speed reduction occurs when the operator reduces the speed of the ship due to severe slamming or large accelerations. Involuntary speed reduction is due to the added resistance of the ship, and changes to the propeller efficiency, due to waves and wind. Both voluntary and involuntary speed reduction can depend on the significant wave height and the relative heading between the ship and waves (Faltinsen 1990; Petricic and Mansour 2011). These factors are inherently captured in the measured data but not in the SFA. As such, it is suggested that use of long-term distributions of the significant wave height, wave period and ship speed, which are assumed in numerical fatigue analysis, may mask the interdependencies between the variables that affect the probability of the vessel experiencing slamming and the fatigue damage.

A further implication of the present study is that, for the two locations considered (the keel and underside of the main deck), the stresses at one location can be transferred to the other via Equation 7. Thus, it may be sufficient only to instrument the keel.

Future work may include exploring the possibility to combine the slamming contribution to the fatigue damage with SFA by applying Equation 6. This would permit the occurrence of slamming to be treated as a variable in an expanded sensitivity analysis. In addition, a limitation of the HMS data used in this study is that it does not include environmental parameters, and ship operator inputs to capture voluntary speed reductions. Thus, there is merit in investigating hindcast wind-wave models to enable the wave environment to be coupled to the ship heading and speed. This information would considerably improve the fatigue life assessment and management of naval ships.

5. Conclusion

Hull monitoring data acquired from a 56 m naval aluminium patrol boat has been used to investigate the uncertainties and interdependencies between the variables in fatigue life analysis. Analysis of the results suggests that the use of long-term distributions of the significant wave height and wave period as well as ship speed may mask voluntary and/or involuntary speed reduction, which affects the probability of the ship experiencing slamming and in turn the fatigue damage. This work supports informed decision-making regarding the fatigue life and operational availability of naval ships.

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