

Multi-Pair Two-Way Massive MIMO Relaying with Zero Forcing: Energy Efficiency and Power Scaling Laws

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Abstract—In this paper, we study a multi-pair two-way half-duplex decode-and-forward (DF) massive multiple-input multiple-output (MIMO) relaying system, in which multiple single-antenna user pairs can exchange information through a massive MIMO relay. For low-complexity transmission, zero-forcing reception/zero-forcing transmission (ZFR/ZFT) is employed at the relay. First, we analytically study the large-scale approximations of the sum spectral efficiency (SE). Furthermore, we focus on three specific power scaling laws to study the trade-off between the transmit powers of each pilot symbol, each user and the relay, and also focus on how the transmit powers scale with the number of relay antennas, M , to maintain a finite SE performance. Additionally, we consider a practical power consumption model to investigate the energy efficiency (EE), and illustrate the impact of M and the interplay between the power scaling laws and the EE performance. Finally, we consider the system fairness via maximizing the minimum achievable SE among all user pairs.

Index Terms—Decode-and-forward relaying, half-duplex, massive MIMO, Max-min fairness, power scaling laws, spectral efficiency and energy efficiency, zero-forcing.

I. INTRODUCTION

MASSIVE multiple-input multiple-output (MIMO) has become a key technology for the next-generation wireless communications with the potential of achieving higher system capacity and data rate demands via simultaneously serving a significant number of users [1]–[3]. This is possible since large array gain and spatial multiplexing gain can be provided [4], [5], leading to a huge improvement in the spectral efficiency (SE) and energy efficiency (EE) [6]. Generally, precoding is a commonly used technique in massive MIMO to ensure downlink transmission and optimize link performance [7]–[9].

In a multi-pair relaying system, where users can exchange information via a shared relay, the application of massive MIMO techniques has attracted great attention due to the potential of improving the network capacity, cellular coverage, system throughput, and enhancing the service quality for cell

edge users [10], [11]. Moreover, by deploying a large number of antennas at the relay, the spatial diversity can be amplified while boosting the achievable performance [1]. Initially, one-way relaying systems were studied for multi-pair massive MIMO relaying. For amplify-and-forward (AF) protocol, the power control problem was studied in [12]; in addition, for decode-and-forward (DF) protocol, the comparison of the achievable SE with different linear processing methods has been studied in [11], while [13] has investigated the outage performance of one-way DF relaying. However, one-way relaying might incur SE loss [14], [15]. In order to reduce the SE loss, two-way relaying is considered to improve the SE and extend the communication range while enabling bidirectional communication [16]–[18]. Theoretically, in two-way relaying systems, user pairs can exchange information via a shared relay in only two time slots, and the required time is much shorter than that in one-way relaying system [19], [20]. To this end, multi-pair two-way relaying with massive arrays has been widely studied, where more than one pair of users can be served to exchange information [16], [21].

Normally, the AF protocol is investigated in most studies of multi-pair two-way massive MIMO relaying, while the DF protocol is typically overlooked. However, the AF relaying might suffer from noise amplification [22]. In this case, DF two-way relaying is proposed as it can achieve better performance than AF relaying at low signal-to-noise ratios (SNRs) without noise propagation at the relay [22], [23]. Also, we recall that DF two-way relaying has the ability to perform separate precoding and power allocation on each relaying communication direction, at the cost of higher complexity [24]. In some previous studies with two-way relaying, full-duplex has been adopted [18], [20]. However full-duplex operation may not be practically feasible due to the huge intensity difference in near/far field of the transmitted/received signals. In this case, half-duplex operation, in which the relay transmits and receives in orthogonal frequency or time resources, has practical relevance and is considered in this paper [23], [25].

In the light of above, we study a multi-pair two-way half-duplex DF relaying system with zero-forcing (ZF) processing and imperfect CSI [22], [26]. This paper extends the work of [22], where only maximum ratio processing (MRC/MRT) is considered. A detailed analysis of the sum SE is presented and we characterize a practical power consumption model to analyze the EE performance of the proposed relaying system. In addition, power scaling scenarios which can improve the

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EE while maintaining the desired SE for large number of relay antennas [4] are studied in detail. Specifically, the main contributions of this paper can be summarized as following

- With a general multi-pair massive MIMO two-way relaying system employing the DF protocol, we present a new large-scale approximation of the SE with ZF processing and imperfect CSI when the number of relay antennas approaches infinity. To the best of our knowledge, no other prior work with this specific relaying system has derived similar expressions due to the difficulty in manipulating matrix inverses, which inherently kick in ZF type of analysis.
- We characterize a practical power consumption model derived from the relevant models in [27], [28]. It is utilized to analyze the EE performance of the proposed multi-pair two-way relaying system.
- We investigate three power scaling laws inspired from [16], [22]. Our study illustrates that there exists a trade-off between the transmit powers of each user, each pilot symbol and the relay; namely, the same SE, even the same EE can be achieved with different configurations of the power-scaling parameters. This provides great flexibility in practical system design and forms a roadmap to select the optimal parameters to maximize the EE performance in particular scenarios.
- Motivated by the Max-Min fairness studies in [17], [29], we formulate an optimization problem to maximize the minimum achievable SE among all user pairs with imperfect CSI in order to improve the sum SE and achieve fairness across all user pairs. The complexity analysis of the proposed optimization problem has also been investigated.

The structure of the paper is organized as follows: Section II introduces the multi-pair two-way half-duplex DF relaying system model with ZF processing and imperfect CSI. Section III presents a large-scale approximation of the SE and characterizes the EE and its corresponding power consumption model. Section IV demonstrates the power-scaling laws with different parameter configurations in form of the asymptotic SE, while Section V illustrates the study of Max-Min fairness. Our numerical results are depicted in Section VI. Finally, Section VII concludes this paper.

Notation: We use \mathbf{H}^T , \mathbf{H}^H , \mathbf{H}^* and \mathbf{H}^{-1} to represent the transpose, conjugate-transpose, conjugate and the inverse of matrix \mathbf{H} , respectively. Moreover, \mathbf{I}_M stands for an $M \times M$ identity matrix. In addition, $|\cdot|$, $\|\cdot\|$ and $\|\cdot\|_F$ denotes the Absolute value, Euclidean norm and Frobenius norm, respectively. Then, $\mathcal{CN}(\mathbf{0}, \mathbf{\Sigma})$ represents circularly symmetric complex Gaussian distribution with zero mean and covariance $\mathbf{\Sigma}$. Finally, $\mathbb{E}\{\cdot\}$ is the expectation operator and $\text{diag}(\cdot)$ shows the diagonal elements of a matrix.

II. SYSTEM MODEL

As shown in Fig. 1, we investigate a multi-pair two-way half-duplex DF relaying system, in which K pairs of single-antenna users, defined as $T_{A,i}$ and $T_{B,i}$, $i = 1, \dots, K$, exchange information via a shared relay T_R with M antennas, generally,

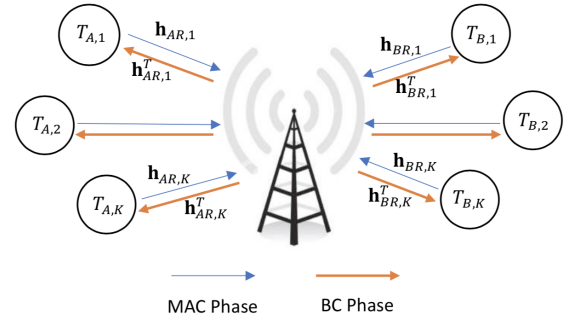


Fig. 1: Multi-pair two-way DF relaying system.

$M \gg K$. Moreover, we assume that there are no direct transmission links between user pairs. Normally, it is assumed that massive MIMO system operates in a time-division duplexing (TDD) mode [14], [30]. To this end, we assume that the proposed system is modeled as uncorrelated Rayleigh fading, works under TDD protocol and channel reciprocity holds [31], [32]. The uplink and downlink channels between $T_{X,i}$, $X = A, B$ and T_R are denoted as $\mathbf{h}_{XR,i} \sim \mathcal{CN}(\mathbf{0}, \beta_{XR,i} \mathbf{I}_M)$ and $\mathbf{h}_{XR,i}^T$, $i = 1, \dots, K$, respectively, while $\beta_{AR,i}$ and $\beta_{BR,i}$ represent the large-scale fading parameters which are considered to be constant in this paper for simplicity. Additionally, the channel matrix can be formed as $\mathbf{H}_{XR} = [\mathbf{h}_{XR,1}, \dots, \mathbf{h}_{XR,K}] \in \mathbb{C}^{M \times K}$, $X = A, B$.

For the proposed relaying system, the data transmission process can be divided into two phases with equal time slots. Generally, this two-phase protocol can be named as Multiple Access Broadcast (MABC) protocol [23]. In the first Multiple Access Channel (MAC) phase, all users transmit their signals to the relay simultaneously. Therefore, the received signal at the relay can be expressed as [28]

$$\mathbf{y}_r = \sum_{i=1}^K \left(\sqrt{p_{A,i}} \mathbf{h}_{AR,i} x_{AR,i} + \sqrt{p_{B,i}} \mathbf{h}_{BR,i} x_{BR,i} \right) + \mathbf{n}_r, \quad (1)$$

where $x_{XR,i}$ is the Gaussian signal transmitted by the i -th user $T_{X,i}$ with zero mean and unit power, $p_{X,i}$ is the average transmit power of $T_{X,i}$, $X = A, B$. \mathbf{n}_r is the vector of additive white Gaussian noise (AWGN) at the relay whose elements are independent and identically distributed (i.i.d) satisfying $\mathcal{CN}(0, 1)$. For low-complexity transmission, linear processing is applied at the relay. Thus, the transformed signal can be given by

$$\mathbf{z}_r = \mathbf{F}_{MAC} \mathbf{y}_r, \quad (2)$$

with $\mathbf{F}_{MAC} \in \mathbb{C}^{2K \times M}$, the linear receiver matrix in the MAC phase.

In the second Broadcast Channel (BC) phase, the relay first decodes the received information and then re-encodes and broadcasts it to users [22]. The linear precoding matrix $\mathbf{F}_{BC} \in \mathbb{C}^{M \times 2K}$ in the BC phase is applied to obtain the transmit signal of the relay as

$$\mathbf{y}_t = \rho_{DF} \mathbf{F}_{BC} \mathbf{x}, \quad (3)$$

where $\mathbf{x} = [\mathbf{x}_A^T, \mathbf{x}_B^T]^T$ represents the decoded signal and ρ_{DF} is the normalization coefficient determined by the average relay

power constraint $\mathbb{E}\{\|\mathbf{y}_t\|^2\} = p_r$. Therefore, the received signals at $T_{X,i}$, $X = A, B$ can be given by

$$z_{X,i} = \mathbf{h}_{XR,i}^T \mathbf{y}_t + n_{X,i}, \quad (4)$$

with the standard AWGN at $T_{X,i}$, $n_{X,i} \sim \mathcal{CN}(0, 1)$, $X = A, B$.

A. Linear Processing

Generally, the inter-pair interference and inter-user interference can be eliminated by linear processing in massive MIMO systems [4], [33]. In this paper, the basic linear processing scheme, ZF processing is applied at the relay to achieve low-complexity transmission. Thus, the linear processing matrices $\mathbf{F}_{MAC} \in \mathbb{C}^{2K \times M}$ and $\mathbf{F}_{BC} \in \mathbb{C}^{M \times 2K}$ for the proposed system defined above can be given by [10], [34]

$$\mathbf{F}_{MAC} = \left(\begin{bmatrix} \hat{\mathbf{H}}_{AR} & \hat{\mathbf{H}}_{BR} \end{bmatrix}^H \begin{bmatrix} \hat{\mathbf{H}}_{AR} & \hat{\mathbf{H}}_{BR} \end{bmatrix} \right)^{-1} \begin{bmatrix} \hat{\mathbf{H}}_{AR} & \hat{\mathbf{H}}_{BR} \end{bmatrix}^H, \quad (5)$$

$$\mathbf{F}_{BC} = \begin{bmatrix} \hat{\mathbf{H}}_{BR} & \hat{\mathbf{H}}_{AR} \end{bmatrix}^* \left(\begin{bmatrix} \hat{\mathbf{H}}_{BR} & \hat{\mathbf{H}}_{AR} \end{bmatrix}^T \begin{bmatrix} \hat{\mathbf{H}}_{BR} & \hat{\mathbf{H}}_{AR} \end{bmatrix}^* \right)^{-1}, \quad (6)$$

respectively. In (5)-(6) above, $\hat{\mathbf{H}}_{XR}$ are the estimated channels, $X = A, B$. To simplify the mathematical expressions in the following, we assume that $\mathbf{F}_{MAC}^{AR} \in \mathbb{C}^{K \times M}$, $\mathbf{F}_{MAC}^{BR} \in \mathbb{C}^{K \times M}$ represent the first K rows and the rest K rows of \mathbf{F}_{MAC} , respectively. Meanwhile, $\mathbf{F}_{BC}^{RB} \in \mathbb{C}^{M \times K}$, $\mathbf{F}_{BC}^{RA} \in \mathbb{C}^{M \times K}$ stand for the first K columns of and the rest K columns of \mathbf{F}_{BC} , respectively.

B. Channel Estimation

In massive MIMO systems, it is important to consider imperfect CSI for realistic scenarios [11]. In TDD systems, the standard way to estimate channels at the relay is to transmit pilots [27], [35]. In this case, among the coherence interval with length τ_c (in symbols), τ_p symbols are applied as pilot symbols for channel estimation [22]. Generally, we assume that all pilot sequences are mutually orthogonal and $\tau_p \geq 2K$ is required. Moreover, we assume that the minimum mean square error (MMSE) estimator is employed at the relay to estimate channels [11], [27], [36]. Therefore, we can have the channel estimates as

$$\mathbf{h}_{XR,i} = \hat{\mathbf{h}}_{XR,i} + \mathbf{e}_{XR,i}, \quad (7)$$

where $\hat{\mathbf{h}}_{XR,i}$ and $\mathbf{e}_{XR,i}$ are the i -th columns of the estimated matrix $\hat{\mathbf{H}}_{XR}$ and estimation error matrix \mathbf{E}_{XR} , respectively, while $\hat{\mathbf{H}}_{XR}$ and \mathbf{E}_{XR} are statistically independent, $X = A, B$. p_p represents the transmit power of each pilot symbol used for channel estimation, the elements in $\hat{\mathbf{h}}_{XR,i}$ and $\mathbf{e}_{XR,i}$ are Gaussian random variables with zero mean and variance $\sigma_{XR,i}^2 = \frac{\tau_p p_p \beta_{XR,i}}{1 + \tau_p p_p \beta_{XR,i}}$, $\tilde{\sigma}_{XR,i}^2 = \frac{\beta_{XR,i}}{1 + \tau_p p_p \beta_{XR,i}}$, $X = A, B$, respectively [11].

III. PERFORMANCE ANALYSIS

A. Spectral Efficiency

In this subsection, we focus on the SE performance of the proposed half-duplex DF two-way relaying system. Generally, the large-scale approximation of the SE can be derived when $M \rightarrow \infty$,

1) *Exact Expressions*: In the MAC phase, according to (1)-(2), the transformed signal at the relay determined by the i -th user pair can be expressed as

$$z_{r,i} = z_{r,i}^A + z_{r,i}^B, \quad (8)$$

where $z_{r,i}^X$ can be obtained by

$$\begin{aligned} z_{r,i}^X &= \underbrace{\sqrt{p_{X,i}} \left(\mathbf{F}_{MAC,i}^{AR} + \mathbf{F}_{MAC,i}^{BR} \right) \hat{\mathbf{h}}_{XR,i}^{XX,i}}_{\text{desired signal}} \\ &+ \underbrace{\sqrt{p_{X,i}} \left(\mathbf{F}_{MAC,i}^{AR} + \mathbf{F}_{MAC,i}^{BR} \right) \mathbf{e}_{XR,i}^{XX,i}}_{\text{inter-user interference}} \\ &+ \underbrace{\sum_{j \neq i} \sqrt{p_{X,j}} \left(\mathbf{F}_{MAC,i}^{AR} + \mathbf{F}_{MAC,i}^{BR} \right) \mathbf{h}_{XR,j}^{XX,j}}_{\text{inter-user interference}} + \underbrace{\mathbf{F}_{MAC,i}^{XR} \mathbf{n}_R}_{\text{noise}}, \end{aligned} \quad (9)$$

and $\mathbf{z}_r = \mathbf{z}_r^A + \mathbf{z}_r^B \in \mathbb{C}^{K \times 1}$, with $\mathbf{z}_r^X \in \mathbb{C}^{K \times 1}$, $X = A, B$. With the assistance of (8)-(9), when we take the i -th pair of users into consideration, the estimation error, inter-user interference and compound noise in $z_{r,i}$ can be given by

$$\begin{aligned} A_i &= p_{A,i} \left(\left| \mathbf{F}_{MAC,i}^{AR} \mathbf{e}_{AR,i} \right|^2 + \left| \mathbf{F}_{MAC,i}^{BR} \mathbf{e}_{AR,i} \right|^2 \right) \\ &+ p_{B,i} \left(\left| \mathbf{F}_{MAC,i}^{AR} \mathbf{e}_{BR,i} \right|^2 + \left| \mathbf{F}_{MAC,i}^{BR} \mathbf{e}_{BR,i} \right|^2 \right), \end{aligned} \quad (10)$$

$$\begin{aligned} B_i &= \sum_{j \neq i} p_{A,j} \left(\left| \mathbf{F}_{MAC,i}^{AR} \mathbf{h}_{AR,j} \right|^2 + \left| \mathbf{F}_{MAC,i}^{BR} \mathbf{h}_{AR,j} \right|^2 \right) \\ &+ \sum_{j \neq i} p_{B,j} \left(\left| \mathbf{F}_{MAC,i}^{AR} \mathbf{h}_{BR,j} \right|^2 + \left| \mathbf{F}_{MAC,i}^{BR} \mathbf{h}_{BR,j} \right|^2 \right), \end{aligned} \quad (11)$$

$$C_i = \left\| \mathbf{F}_{MAC,i}^{AR} \right\|^2 + \left\| \mathbf{F}_{MAC,i}^{BR} \right\|^2, \quad (12)$$

respectively. With the expressions of desired signals in (9) for $z_{r,i}^A$ and $z_{r,i}^B$, the SE of the specified user $T_{X,i}$ to the relay with $\text{SINR}_{XR,i}$, $X = A, B$, can be expressed as

$$\begin{aligned} R_{XR,i} &= \frac{\tau_c - \tau_p}{2\tau_c} \mathbb{E} \left\{ \log_2(1 + \text{SINR}_{XR,i}) \right\} \\ &= \frac{\tau_c - \tau_p}{2\tau_c} \mathbb{E} \left\{ \log_2 \left(1 + \frac{p_{X,i} \left(\left| \mathbf{F}_{MAC,i}^{AR} \hat{\mathbf{h}}_{XR,i} \right|^2 + \left| \mathbf{F}_{MAC,i}^{BR} \hat{\mathbf{h}}_{XR,i} \right|^2 \right)}{A_i + B_i + C_i} \right) \right\}. \end{aligned} \quad (13)$$

Additionally, the standard lower capacity bound associated with the worst-case uncorrelated additive noise is considered in this paper [22], [37]; therefore, the achievable SE of the i -th user pair in the MAC phase is given by Eqn. (14) on the top of next page.

In the BC phase, via applying \mathbf{F}_{BC} to generate the relay's transmit signal, the received signal at $T_{X,i}$ can be calculated by (4). Take $z_{A,i}$ as an example in Eqn. (15) on the top of next page, while $z_{B,i}$ can be obtained by replacing the subscripts "AR", "BR" in the channel vectors and corresponding vectors, the subscripts "RA", "RB" in linear precoding vectors, and "A", "B" in signal and noise terms with the subscripts "BR", "AR", the subscripts "RB", "RA", and "B", "A" in $z_{A,i}$, respectively. To this end, we can obtain the SE of the relay to the i -th user $T_{X,i}$, $X = A, B$ by Eqn. (16) on the top of next page.

$$R_{1,i} = \frac{\tau_c - \tau_p}{2\tau_c} \times \mathbb{E} \left\{ \log_2 \left(1 + \frac{p_{A,i} \left(|\mathbf{F}_{MAC,i}^{AR} \hat{\mathbf{h}}_{AR,i}|^2 + |\mathbf{F}_{MAC,i}^{BR} \hat{\mathbf{h}}_{AR,i}|^2 \right) + p_{B,i} \left(|\mathbf{F}_{MAC,i}^{AR} \hat{\mathbf{h}}_{BR,i}|^2 + |\mathbf{F}_{MAC,i}^{BR} \hat{\mathbf{h}}_{BR,i}|^2 \right)}{A_i + B_i + C_i} \right) \right\}. \quad (14)$$

$$z_{A,i} = \underbrace{\rho_{DF} \hat{\mathbf{h}}_{AR,i}^T \mathbf{F}_{BC,i}^{RA}}_{\text{desired signal}} + \underbrace{\rho_{DF} \mathbf{e}_{AR,i}^T \mathbf{F}_{BC,i}^{RA}}_{\text{estimation error}} + \underbrace{\rho_{DF} \sum_{j=1}^K \mathbf{h}_{AR,i}^T \mathbf{F}_{BC,i}^{RB} x_{A,j} + \rho_{DF} \sum_{j \neq i}^K \mathbf{h}_{AR,i}^T \mathbf{F}_{BC,j}^{RA} x_{B,j}}_{\text{inter-user interference}} + \underbrace{n_{A,i}}_{\text{noise}}, \quad (15)$$

$$R_{RX,i} = \frac{\tau_c - \tau_p}{2\tau_c} \mathbb{E} \left\{ \log_2(1 + \text{SINR}_{RX,i}) \right\} = \frac{\tau_c - \tau_p}{2\tau_c} \mathbb{E} \left\{ \log_2 \left(1 + \frac{|\hat{\mathbf{h}}_{XR,i}^T \mathbf{F}_{BC,i}^{RX}|^2}{|\mathbf{e}_{XR,i}^T \mathbf{F}_{BC,i}^{RX}|^2 + \sum_{j=1}^K \left(|\mathbf{h}_{XR,i}^T \mathbf{F}_{BC,j}^{RA}|^2 + |\mathbf{h}_{XR,i}^T \mathbf{F}_{BC,j}^{RB}|^2 \right) - |\mathbf{h}_{XR,i}^T \mathbf{F}_{BC,i}^{RX}|^2 + \frac{1}{\rho_{DF}}} \right) \right\}. \quad (16)$$

Meanwhile, the achievable SE of the i -th user pair in the BC phase is defined as the sum of the end-to-end SE from $T_{A,i}$ to $T_{B,i}$ and from $T_{B,i}$ to $T_{A,i}$ [22], [28],

$$R_{2,i} = \min(R_{AR,i}, R_{RB,i}) + \min(R_{BR,i}, R_{RA,i}). \quad (17)$$

Therefore, the sum SE of the multi-pair two-way DF relaying system can be expressed as

$$R = \sum_{i=1}^K R_i = \sum_{i=1}^K \min(R_{1,i}, R_{2,i}), \quad (18)$$

where R_i is the achievable SE of the i -th user pair for the proposed system determined by the minimum SE in MAC and BC phases [15], [24].

2) *Approximations*: Practically, the large-scale approximation of the SE for the i -th user pair studied in the following can be derived when the relay employs a great number of antennas, i.e., $M \rightarrow \infty$.

Lemma 1: When $M \rightarrow \infty$, the inner product of any two columns in the estimated channel matrix $\hat{\mathbf{H}}_{XR}$ can be defined as [10], [38]

$$\frac{1}{M} \cdot \hat{\mathbf{h}}_{XR,i}^H \hat{\mathbf{h}}_{XR,j} \rightarrow \begin{cases} \sigma_{XR,i}^2, & i = j \\ 0, & i \neq j \end{cases}. \quad (19)$$

Proof: Please see Appendix A.

With an increasing number of relay antennas, the channel vectors within $\hat{\mathbf{H}}_{XR}$ become asymptotically mutually orthogonal. As such, $\hat{\mathbf{H}}_{XR}^H \hat{\mathbf{H}}_{XR}$ can be assumed to approach a diagonal matrix [39]. Thus, according to *Lemma 1*, we can obtain

$$\frac{1}{M} \cdot \hat{\mathbf{H}}_{XR}^H \hat{\mathbf{H}}_{XR} \rightarrow \text{diag} \{ \sigma_{XR,1}^2, \sigma_{XR,2}^2, \dots, \sigma_{XR,K}^2 \}, \quad (20)$$

$$(\hat{\mathbf{H}}_{XR}^H \hat{\mathbf{H}}_{XR})^{-1} \rightarrow \text{diag} \left\{ \frac{1}{M \cdot \sigma_{XR,1}^2}, \frac{1}{M \cdot \sigma_{XR,2}^2}, \dots, \frac{1}{M \cdot \sigma_{XR,K}^2} \right\}, \quad (21)$$

$X = A, B$. Therefore, the linear processing matrices \mathbf{F}_{MAC} and \mathbf{F}_{BC} can be simplified when $M \rightarrow \infty$ as follows

$$\mathbf{F}_{MAC} \rightarrow \begin{bmatrix} (\hat{\mathbf{H}}_{AR}^H \hat{\mathbf{H}}_{AR})^{-1} \hat{\mathbf{H}}_{AR}^H \\ (\hat{\mathbf{H}}_{BR}^H \hat{\mathbf{H}}_{BR})^{-1} \hat{\mathbf{H}}_{BR}^H \end{bmatrix}, \quad (22)$$

$$\mathbf{F}_{BC} \rightarrow \left[\hat{\mathbf{H}}_{BR}^* (\hat{\mathbf{H}}_{BR}^T \hat{\mathbf{H}}_{BR}^*)^{-1}, \hat{\mathbf{H}}_{AR}^* (\hat{\mathbf{H}}_{AR}^T \hat{\mathbf{H}}_{AR}^*)^{-1} \right], \quad (23)$$

respectively, while the normalization coefficient ρ_{DF} defined in Section II can be given by

$$\rho_{DF} = \sqrt{E \left\{ \|\mathbf{F}_{BC}\|_F^2 \right\}} = \sqrt{\frac{M \cdot p_r}{\sum_{i=1}^K \left(\frac{1}{\sigma_{AR,i}^2} + \frac{1}{\sigma_{BR,i}^2} \right)}}. \quad (24)$$

Corollary 1: With the DF protocol and the properties of ZF processing [40], when $M \rightarrow \infty$, the large-scale approximations associated with \hat{R}_i ($R_i - \hat{R}_i \rightarrow 0$) can be given by

$$\hat{R} = \sum_{i=1}^K \hat{R}_i = \sum_{i=1}^K \min(\hat{R}_{1,i}, \hat{R}_{2,i}). \quad (25)$$

The approximations of the achievable SE in MAC and BC phases, and the SE from the user pair/relay to the relay/user pair can be expressed as

$$\hat{R}_{1,i} = \frac{\tau_c - \tau_p}{2\tau_c} \times \log_2 \left(1 + \frac{M(p_{A,i} + p_{B,i})}{\left(\frac{1}{\sigma_{AR,i}^2} + \frac{1}{\sigma_{BR,i}^2} \right) \left[\sum_{j=1}^K (p_{A,j} \tilde{\sigma}_{AR,j}^2 + p_{B,j} \tilde{\sigma}_{BR,j}^2) + 1 \right]} \right), \quad (26)$$

$$\hat{R}_{2,i} = \min(\hat{R}_{AR,i}, \hat{R}_{RB,i}) + \min(\hat{R}_{BR,i}, \hat{R}_{RA,i}), \quad (27)$$

$$\hat{R}_{AR,i} = \frac{\tau_c - \tau_p}{2\tau_c} \times \log_2 \left(1 + \frac{M p_{A,i}}{\left(\frac{1}{\sigma_{AR,i}^2} + \frac{1}{\sigma_{BR,i}^2} \right) \left[\sum_{j=1}^K (p_{A,j} \tilde{\sigma}_{AR,j}^2 + p_{B,j} \tilde{\sigma}_{BR,j}^2) + 1 \right]} \right), \quad (28)$$

$$\hat{R}_{RA,i} = \frac{\tau_c - \tau_p}{2\tau_c} \times \log_2 \left(1 + \frac{M \cdot p_r}{(p_r \tilde{\sigma}_{AR,i}^2 + 1) \sum_{j=1}^K \left(\frac{1}{\sigma_{AR,j}^2} + \frac{1}{\sigma_{BR,j}^2} \right)} \right). \quad (29)$$

$\hat{R}_{BR,i}$ and $\hat{R}_{RB,i}$ can be obtained by replacing the transmit powers $p_{A,i}$, $p_{B,i}$, and the subscripts "AR", "BR" with the

transmit powers $p_{B,i}$, $p_{A,i}$, and the subscripts "BR", "AR" in $\hat{R}_{AR,i}$ and $\hat{R}_{RA,i}$, respectively.

Proof: Please see Appendix B.

B. Energy Efficiency

Generally, the EE is defined as the ratio of the sum SE to the total power consumption of the proposed system and can be given by [4], [27], [41],

$$\epsilon = \frac{R}{P_{\text{total}}}, \quad (30)$$

where R denotes the sum SE defined in (18), P_{total} represents the total power consumption. In a practical system, the total power consumption consists of the transmitted signal power, the powers of operating static circuits and the RF components in each RF chain. Normally, each antenna is connected to one RF chain [27]. Therefore, the power consumption model for the users and the relay can be defined as [28],

$$\begin{aligned} P_{\text{tot},i} &= \frac{1}{2\tau_c} \left[\frac{(\tau_c - \tau_p)p_u + \tau_p p_p}{\zeta_i} + \tau_c \cdot P_{RF,i} \right] \\ &= \frac{1}{2\tau_c} \left[\frac{(\tau_c - \tau_p)p_u + \tau_p p_p}{\zeta_i} \right] + \frac{1}{2} P_{RF,i} \end{aligned} \quad (31)$$

$$P_{\text{tot},r} = \frac{1}{2\tau_c} \left[\frac{\tau_c p_r}{\zeta_r} + \tau_c \cdot P_{RF,r} \right] = \frac{1}{2} \left(\frac{p_r}{\zeta_r} + P_{RF,r} \right), \quad (32)$$

respectively. Note that $P_{\text{tot},i}$ represents the total power at i -th user and $P_{\text{tot},r}$ indicates the total power at the relay; ζ_r and ζ_i denote the power amplifier efficiency for the relay and i -th user, respectively. The power consumption of the RF components for single-antenna users and the relay with M antennas can be defined as

$$P_{RF,i} = P_{DAC,i} + P_{\text{mix},i} + P_{\text{filt},i} + P_{\text{syn},i}, \quad (33)$$

$$P_{RF,r} = M(P_{DAC,r} + P_{\text{mix},r} + P_{\text{filt},r}) + P_{\text{syn},r}, \quad (34)$$

respectively. P_{syn} is the power consumption of the frequency synthesizer, P_{DAC} , P_{mix} and P_{filt} are the power consumed by the digital-to-analog converters (DACs), signal mixers and filters in the RF chain respectively [27]. As a result, the total power consumption P_{total} for the system can be re-expressed as

$$P_{\text{total}} = 2K \cdot P_{\text{tot},i} + P_{\text{tot},r} + P_{\text{static}}, \quad (35)$$

where P_{static} is the power of all the static circuits [4]. To simplify the power consumption model in the simulation, we assume that $\zeta_i = \zeta_r = \zeta$, $P_{DAC,i} = P_{DAC,r} = P_{DAC}$, $P_{\text{mix},i} = P_{\text{mix},r} = P_{\text{mix}}$, $P_{\text{filt},i} = P_{\text{filt},r} = P_{\text{filt}}$ and $P_{\text{syn},i} = P_{\text{syn},r} = P_{\text{syn}}$ for $i = 1, 2, \dots, K$.

IV. POWER SCALING LAWS

In this section, we investigate how the power-scaling laws affect achievable SE; and, in particular, how power reductions with M maintain a desired SE. In the following, we consider three power-scaling cases: a) only the transmit power of each pilot symbol is scaled; b) the transmit powers of data transmission at each user and the relay are scaled; c) all transmit powers are scaled, to demonstrate the interplay among the transmit power of each pilot symbol p_p , the transmit power

of each user p_u and the relay p_r . For simplicity, we assume that $p_{A,i} = p_{B,i} = p_u$, $i = 1, \dots, K$. We define that $\bar{R}_{1,i}$, $\bar{R}_{2,i}$, \bar{R}_i , $\bar{R}_{XR,i}$ and $\bar{R}_{RX,i}$, $X = A, B$, are asymptotic expressions of the achievable SE; additionally, without loss of generality in the following, we define that

$$\bar{R} = \sum_{i=1}^K \bar{R}_i = \sum_{i=1}^K \min(\bar{R}_{1,i}, \bar{R}_{2,i}), \quad (36)$$

$$\bar{R}_{2,i} = \min(\bar{R}_{AR,i}, \bar{R}_{RB,i}) + \min(\bar{R}_{BR,i}, \bar{R}_{RA,i}). \quad (37)$$

1) *Case A:* Only the transmit power of the pilot symbol is scaled by M with $p_p = \frac{E_p}{M^\gamma}$, where E_p is a constant and $\gamma > 0$. This case is said to achieve power savings in the channel training stage.

Corollary 2: For $p_p = \frac{E_p}{M^\gamma}$, with fixed p_u , p_r , E_p and $\gamma > 0$, when $M \rightarrow \infty$, we can present the asymptotic results as

$$R_i - \min(\bar{R}_{1,i}, \bar{R}_{2,i}) \rightarrow 0. \quad (38)$$

with

$$\bar{R}_{1,i} = \frac{\tau_c - \tau_p}{2\tau_c} \log_2 \left(1 + \frac{2 \frac{\tau_p E_p}{M^{\gamma-1}}}{\left(\frac{1}{\beta_{AR,i}^2} + \frac{1}{\beta_{BR,i}^2} \right) \left(\sum_{j=1}^K (\beta_{AR,j} + \beta_{BR,j}) + \frac{1}{p_u} \right)} \right), \quad (39)$$

$$\bar{R}_{AR,i} = \frac{\tau_c - \tau_p}{2\tau_c} \log_2 \left(1 + \frac{\frac{\tau_p E_p}{M^{\gamma-1}}}{\left(\frac{1}{\beta_{AR,i}^2} + \frac{1}{\beta_{BR,i}^2} \right) \left(\sum_{j=1}^K (\beta_{AR,j} + \beta_{BR,j}) + \frac{1}{p_u} \right)} \right), \quad (40)$$

$$\bar{R}_{RA,i} = \frac{\tau_c - \tau_p}{2\tau_c} \log_2 \left(1 + \frac{\frac{\tau_p E_p}{M^{\gamma-1}} p_r}{\sum_{j=1}^K (p_r \beta_{AR,j} + 1) \left(\frac{1}{\beta_{AR,j}^2} + \frac{1}{\beta_{BR,j}^2} \right)} \right), \quad (41)$$

and $\bar{R}_{BR,i}$ and $\bar{R}_{RB,i}$ can be obtained by replacing the subscripts "AR", "BR" in $\bar{R}_{AR,i}$ and $\bar{R}_{RA,i}$ with the subscripts "BR", "AR", respectively.

We can observe that Case A depends on the choice of γ to scale the transmit power of each pilot symbol. From (39)-(41), we can know that when we reduce p_p aggressively with $\gamma > 1$, \bar{R}_i approaches zero. In contrast, when $0 < \gamma < 1$, \bar{R}_i grows unboundedly. Additionally, when $\gamma = 1$, \bar{R}_i converges to a non-zero limit.

2) *Case B:* The transmit power of each pilot symbol p_p is fixed, while other transmit powers are scaled with $p_u = \frac{E_u}{M^\alpha}$, $p_r = \frac{E_r}{M^\beta}$, where $\alpha \geq 0$ and $\beta \geq 0$, and E_u , E_r are constants. In this case, the potential power savings in data transmission are studied.

Corollary 3: For $p_u = \frac{E_u}{M^\alpha}$, $p_r = \frac{E_r}{M^\beta}$, with fixed p_p , E_u , E_r and $\alpha \geq 0$, $\beta \geq 0$, when $M \rightarrow \infty$, we can obtain

$$R_i - \min(\bar{R}_{1,i}, \bar{R}_{2,i}) \rightarrow 0. \quad (42)$$

with

$$\bar{R}_{1,i} = \frac{\tau_c - \tau_p}{2\tau_c} \log_2 \left(1 + \frac{2 \times \frac{E_u}{M^{\alpha-1}}}{\left(\frac{1}{\sigma_{AR,i}^2} + \frac{1}{\sigma_{BR,i}^2}\right)} \right), \quad (43)$$

$$\bar{R}_{AR,i} = \frac{\tau_c - \tau_p}{2\tau_c} \log_2 \left(1 + \frac{\frac{E_u}{M^{\alpha-1}}}{\left(\frac{1}{\sigma_{AR,i}^2} + \frac{1}{\sigma_{BR,i}^2}\right)} \right), \quad (44)$$

$$\bar{R}_{RA,i} = \frac{\tau_c - \tau_p}{2\tau_c} \log_2 \left(1 + \frac{\frac{E_r}{M^{\beta-1}}}{\sum_{j=1}^K \left(\frac{1}{\sigma_{AR,j}^2} + \frac{1}{\sigma_{BR,j}^2}\right)} \right). \quad (45)$$

Similarly, $\bar{R}_{BR,i}$ and $\bar{R}_{RB,i}$ can be obtained by replacing the subscripts "AR", "BR" in $\bar{R}_{AR,i}$ and $\bar{R}_{RA,i}$ with the subscripts "BR", "AR", respectively.

This case investigates that when both p_u and p_r are scaled with M when $M \rightarrow \infty$, the effects of estimation error and inter-user interference eliminate; thus, only the noise at users and the relay remains to cause imperfection. When we cut down p_u and p_r aggressively, namely, 1) $\alpha > 1$, and $\beta \geq 0$, 2) $\alpha \geq 0$, and $\beta > 1$, 3) $\alpha > 1$, and $\beta > 1$, \bar{R}_i reduces to zero. On the other hand, when we reduce both p_u and p_r moderately, which is, $0 \leq \alpha < 1$ and $0 \leq \beta < 1$, \bar{R}_i grows unboundedly.

Furthermore, for a specific scenario where both the transmit powers of the relay and of each user are scaled with the same speed $\alpha = \beta = 1$, \bar{R}_i converges to a non-zero limit,

$$\bar{R}_{1,i} = \frac{\tau_c - \tau_p}{2\tau_c} \log_2 \left(1 + \frac{2E_u}{\left(\frac{1}{\sigma_{AR,i}^2} + \frac{1}{\sigma_{BR,i}^2}\right)} \right), \quad (46)$$

$$\bar{R}_{AR,i} = \bar{R}_{BR,i} = \frac{\tau_c - \tau_p}{2\tau_c} \log_2 \left(1 + \frac{E_u}{\left(\frac{1}{\sigma_{AR,i}^2} + \frac{1}{\sigma_{BR,i}^2}\right)} \right), \quad (47)$$

$$\bar{R}_{RA,i} = \bar{R}_{RB,i} = \frac{\tau_c - \tau_p}{2\tau_c} \log_2 \left(1 + \frac{E_r}{\sum_{j=1}^K \left(\frac{1}{\sigma_{AR,j}^2} + \frac{1}{\sigma_{BR,j}^2}\right)} \right). \quad (48)$$

We can see that the non-zero limit increases with respect to E_u and E_r , while decreasing with respect to the number of user pairs K . Also, if we apply $0 \leq \beta < \alpha = 1$, the approximation of the sum SE is determined by the SE performance in the MAC phase, which means that $\bar{R}_{1,i}$ given by (46) determines \bar{R}_i when $M \rightarrow \infty$. On the other hand, when $0 \leq \alpha < \beta = 1$, the determination of SE appears in the BC phase; thus, \bar{R}_i is determined by $\bar{R}_{RA,i}$ and $\bar{R}_{RB,i}$ given by (48).

3) *Case C*: This is a general case where all transmit powers are scaled, $p_p = \frac{E_p}{M^\gamma}$, $p_u = \frac{E_u}{M^\alpha}$ and $p_r = \frac{E_r}{M^\beta}$, with $\gamma \geq 0$, $\alpha \geq 0$ and $\beta \geq 0$, E_p , E_u and E_r are constants.

Corollary 4: For $p_p = \frac{E_p}{M^\gamma}$, $p_u = \frac{E_u}{M^\alpha}$, $p_r = \frac{E_r}{M^\beta}$ with fixed E_p , E_u , E_r and $\gamma \geq 0$, $\alpha \geq 0$, $\beta \geq 0$, when $M \rightarrow \infty$, we can obtain

$$R_i - \min(\bar{R}_{1,i}, \bar{R}_{2,i}) \rightarrow 0. \quad (49)$$

with

$$\bar{R}_{1,i} = \frac{\tau_c - \tau_p}{2\tau_c} \log_2 \left(1 + \frac{2 \times \frac{\tau_p E_p E_u}{M^{\alpha+\gamma-1}}}{\left(\frac{1}{\beta_{AR,i}^2} + \frac{1}{\beta_{BR,i}^2}\right)} \right), \quad (50)$$

$$\bar{R}_{AR,i} = \frac{\tau_c - \tau_p}{2\tau_c} \log_2 \left(1 + \frac{\frac{\tau_p E_p E_u}{M^{\alpha+\gamma-1}}}{\left(\frac{1}{\beta_{AR,i}^2} + \frac{1}{\beta_{BR,i}^2}\right)} \right), \quad (51)$$

$$\bar{R}_{RA,i} = \frac{\tau_c - \tau_p}{2\tau_c} \log_2 \left(1 + \frac{\frac{\tau_p E_p E_r}{M^{\beta+\gamma-1}}}{\sum_{j=1}^K \left(\frac{1}{\beta_{AR,j}^2} + \frac{1}{\beta_{BR,j}^2}\right)} \right). \quad (52)$$

Similarly, $\bar{R}_{BR,i}$ and $\bar{R}_{RB,i}$ can be obtained by replacing the subscripts "AR", "BR" with the subscripts "BR", "AR" in $\bar{R}_{AR,i}$ and $\bar{R}_{RA,i}$, respectively.

As expected, the sum SE depends on the choice of α , β and γ . Additionally, $\alpha + \gamma$ determines the SE in the MAC phase, while $\beta + \gamma$ determines the SE in the BC phase. When $\alpha = \beta > 0$ and $\alpha + \gamma = 1$, the trade-off between the transmit powers of each user/the relay and of each pilot symbol is displayed. In this case, if we reduce the transmit power of each pilot symbol aggressively, the channel estimate is corrupted and in order to compensate this imperfection, the transmit power of each user/the relay should be increased. The limit of the asymptotic SE under this case can be given by

$$\bar{R}_{1,i} = \frac{\tau_c - \tau_p}{2\tau_c} \log_2 \left(1 + \frac{2\tau_p E_p E_u}{\left(\frac{1}{\beta_{AR,i}^2} + \frac{1}{\beta_{BR,i}^2}\right)} \right), \quad (53)$$

$$\bar{R}_{AR,i} = \bar{R}_{BR,i} = \frac{\tau_c - \tau_p}{2\tau_c} \log_2 \left(1 + \frac{\tau_p E_p E_u}{\left(\frac{1}{\beta_{AR,i}^2} + \frac{1}{\beta_{BR,i}^2}\right)} \right), \quad (54)$$

$$\bar{R}_{RA,i} = \bar{R}_{RB,i} = \frac{\tau_c - \tau_p}{2\tau_c} \log_2 \left(1 + \frac{\tau_p E_p E_r}{\sum_{j=1}^K \left(\frac{1}{\beta_{AR,j}^2} + \frac{1}{\beta_{BR,j}^2}\right)} \right). \quad (55)$$

Moreover, we can see that when $\alpha > \beta \geq 0$ and $\alpha + \gamma = 1$, the limit of \bar{R}_i is determined by $\bar{R}_{1,i}$ according to (53), which means that we can improve the sum SE by increasing E_p and E_u . In addition, the sum SE of the proposed system is an increasing function of K based on (36). Meanwhile, when $0 \leq \alpha < \beta$ and $\beta + \gamma = 1$, the limit of \bar{R}_i is determined by $\bar{R}_{RA,i}$ and $\bar{R}_{RB,i}$ according to (55) which displays the trade-off between the transmit powers of each pilot symbol and of the relay.

V. MAX-MIN FAIRNESS ANALYSIS

With the assistance of the above mentioned SE analysis, and as a further step forward from the power scaling laws, in this section, our objective is to harness SE fairness among the user pairs. We achieve this purpose by maximizing the minimum achievable SE among all the user pairs; therefore, providing max-min fairness [17], [29].

A. Spectral Efficiency Fairness

For analytical simplicity, the large-scale approximation in Corollary 1 is employed and we assume that the pilot power p_p is determined in advance. Moreover, we define that $\mathbf{p}_A = [p_{A,1}, \dots, p_{A,K}]^T$, and $\mathbf{p}_B = [p_{B,1}, \dots, p_{B,K}]^T$. In this case, the optimization problem can be formulated as

$$\max_{\mathbf{p}_A, \mathbf{p}_B, p_r} \min_{i \in 1, \dots, K} \hat{R}_i \quad (56a)$$

subject to

$$0 \leq p_r \leq P_r^{\max}, 0 \leq p_{A,i} \leq P_u^{\max}, 0 \leq p_{B,i} \leq P_u^{\max}, \forall i \quad (56b)$$

$$\frac{1}{2} \left(\sum_{i=1}^K \frac{(\tau_c - \tau_p)(p_{A,i} + p_{B,i})}{\tau_c \zeta_u} + \frac{p_r}{\zeta_c} \right) + P_o \leq P^{\max} \quad (56c)$$

Here, P^{\max} is the total power constraint, P_u^{\max} and P_r^{\max} are the maximum powers of each user and the relay, respectively and $P_o = \frac{1}{2} \left[\frac{2K\tau_p p_p}{\tau_c \zeta_u} + (2K+M)(P_{DAC} + P_{mix} + P_{filt}) + (2K+1)P_{syn} + P_{static} \right]$ is determined in Section III. According to Corollary 1, we can rewrite the optimization problem (56) by introducing the auxiliary variables t, t_1, t_2 as follows

$$\max_{\mathbf{p}_A, \mathbf{p}_B, p_r, t, t_1, t_2} t \quad (57a)$$

$$\text{s.t. (56b), (56c)} \quad (57b)$$

$$t_1 + t_2 \geq t \quad (57c)$$

$$\frac{\tau_c - \tau_p}{2\tau_c} \log_2 \left(1 + \frac{\frac{M\sigma_{AR,i}^2 \sigma_{BR,i}^2}{\sigma_{AR,i}^2 + \sigma_{BR,i}^2} (p_{A,i} + p_{B,i})}{\sum_{j=1}^K (p_{A,j} \tilde{\sigma}_{AR,j}^2 + p_{B,j} \tilde{\sigma}_{BR,j}^2) + 1} \right) \geq t, \forall i \quad (57d)$$

$$\frac{\tau_c - \tau_p}{2\tau_c} \log_2 \left(1 + \frac{\frac{M\sigma_{AR,i}^2 \sigma_{BR,i}^2}{\sigma_{AR,i}^2 + \sigma_{BR,i}^2} p_{A,i}}{\sum_{j=1}^K (p_{A,j} \tilde{\sigma}_{AR,j}^2 + p_{B,j} \tilde{\sigma}_{BR,j}^2) + 1} \right) \geq t_1, \forall i \quad (57e)$$

$$\frac{\tau_c - \tau_p}{2\tau_c} \log_2 \left(1 + \frac{p_r M}{(p_r \tilde{\sigma}_{BR,i}^2 + 1) \sum_{j=1}^K \left(\frac{1}{\sigma_{AR,j}^2} + \frac{1}{\sigma_{BR,j}^2} \right)} \right) \geq t_1, \forall i \quad (57f)$$

$$\frac{\tau_c - \tau_p}{2\tau_c} \log_2 \left(1 + \frac{\frac{M\sigma_{AR,i}^2 \sigma_{BR,i}^2}{\sigma_{AR,i}^2 + \sigma_{BR,i}^2} p_{B,i}}{\sum_{j=1}^K (p_{A,j} \tilde{\sigma}_{AR,j}^2 + p_{B,j} \tilde{\sigma}_{BR,j}^2) + 1} \right) \geq t_2, \forall i \quad (57g)$$

$$\frac{\tau_c - \tau_p}{2\tau_c} \log_2 \left(1 + \frac{p_r M}{(p_r \tilde{\sigma}_{AR,i}^2 + 1) \sum_{j=1}^K \left(\frac{1}{\sigma_{AR,j}^2} + \frac{1}{\sigma_{BR,j}^2} \right)} \right) \geq t_2, \forall i \quad (57h)$$

According to the property of logarithm function $\log(\frac{a}{b}) = \log(a) - \log(b)$, we can observe that the SE of the proposed system can be defined as a difference of two concave functions

[42], we can rewrite the constraints with $\hat{R}_i = f(p_{A,i}, p_{B,i}, p_r) - h(p_{A,i}, p_{B,i}, p_r)$, where the specific mathematical expressions for $T_{X,i}$, $i = 1, \dots, K$ and $X = A, B$ can be given by

$$f_1(p_{A,i}, p_{B,i}, p_r) = \frac{\tau_c - \tau_p}{2\tau_c} \times \log_2 \left(\frac{M\sigma_{AR,i}^2 \sigma_{BR,i}^2}{\sigma_{AR,i}^2 + \sigma_{BR,i}^2} (p_{A,i} + p_{B,i}) + \sum_{j=1}^K (p_{A,j} \tilde{\sigma}_{AR,j}^2 + p_{B,j} \tilde{\sigma}_{BR,j}^2) + 1 \right), \quad (58)$$

$$f_{XR}(p_{A,i}, p_{B,i}, p_r) = \frac{\tau_c - \tau_p}{2\tau_c} \times \log_2 \left(\frac{M\sigma_{AR,i}^2 \sigma_{BR,i}^2}{\sigma_{AR,i}^2 + \sigma_{BR,i}^2} p_{X,i} + \sum_{j=1}^K (p_{A,j} \tilde{\sigma}_{AR,j}^2 + p_{B,j} \tilde{\sigma}_{BR,j}^2) + 1 \right), \quad (59)$$

$$f_{RX}(p_{A,i}, p_{B,i}, p_r) = \frac{\tau_c - \tau_p}{2\tau_c} \times \log_2 \left(p_r M + (p_r \tilde{\sigma}_{XR,i}^2 + 1) \sum_{j=1}^K \left(\frac{1}{\sigma_{AR,j}^2} + \frac{1}{\sigma_{BR,j}^2} \right) \right), \quad (60)$$

$$h_{XR}(p_{A,i}, p_{B,i}, p_r) = \frac{\tau_c - \tau_p}{2\tau_c} \log_2 \left(\sum_{j=1}^K (p_{A,j} \tilde{\sigma}_{AR,j}^2 + p_{B,j} \tilde{\sigma}_{BR,j}^2) + 1 \right), \quad (61)$$

$$h_{RX}(p_{A,i}, p_{B,i}, p_r) = \frac{\tau_c - \tau_p}{2\tau_c} \log_2 \left((p_r \tilde{\sigma}_{XR,i}^2 + 1) \sum_{j=1}^K \left(\frac{1}{\sigma_{AR,j}^2} + \frac{1}{\sigma_{BR,j}^2} \right) \right). \quad (62)$$

Based on Corollary 1, it is clear that for $\hat{R}_{1,i}$, $h_1(p_{A,i}, p_{B,i}, p_r) = h_{XR}(p_{A,i}, p_{B,i}, p_r)$, $X = A, B$. The specific functions $f(p_{A,i}, p_{B,i}, p_r)$ and $h(p_{A,i}, p_{B,i}, p_r)$ defined above are jointly concave with respect to $p_{A,i}, p_{B,i}, p_r$, $i = 1, \dots, K$ [42], [43], and the relaxed problem can be reformulated as

$$\max_{\mathbf{p}_A, \mathbf{p}_B, p_r, t, t_1, t_2} t \quad (63a)$$

$$\text{s.t. (56b), (56c), (57c)} \quad (63b)$$

$$f_1(p_{A,i}, p_{B,i}, p_r) - h_1(p_{A,i}, p_{B,i}, p_r) \geq t, \forall i \quad (63c)$$

$$f_{AR}(p_{A,i}, p_{B,i}, p_r) - h_{AR}(p_{A,i}, p_{B,i}, p_r) \geq t_1, \forall i \quad (63d)$$

$$f_{RB}(p_{A,i}, p_{B,i}, p_r) - h_{RB}(p_{A,i}, p_{B,i}, p_r) \geq t_1, \forall i \quad (63e)$$

$$f_{BR}(p_{A,i}, p_{B,i}, p_r) - h_{BR}(p_{A,i}, p_{B,i}, p_r) \geq t_2, \forall i \quad (63f)$$

$$f_{RA}(p_{A,i}, p_{B,i}, p_r) - h_{RA}(p_{A,i}, p_{B,i}, p_r) \geq t_2, \forall i \quad (63g)$$

We can find that the difficulty in solving (63) lies in the component $h_{XR}(p_{A,i}, p_{B,i}, p_r)$ and $h_{RX}(p_{A,i}, p_{B,i}, p_r)$, $X = A, B$. Therefore, the value of $(p_{A,i}, p_{B,i}, p_r)$ at k -th iteration is supposed to be $(p_{A,i}^{(k)}, p_{B,i}^{(k)}, p_r^{(k)})$. Since $h_{XR}(p_{A,i}, p_{B,i}, p_r)$ and $h_{RX}(p_{A,i}, p_{B,i}, p_r)$, $X = A, B$ are concave and differentiable on the considered domain, we can easily find the affine function as a Taylor first order approximation near $(p_{A,i}^{(k)}, p_{B,i}^{(k)}, p_r^{(k)})$ shown in Eqn. (64)-(65) on the top of next page [42], [43]. To update the objective in the $(k+1)$ -th iteration we replace $h_{XR}(p_{A,i}, p_{B,i}, p_r)$ and $h_{RX}(p_{A,i}, p_{B,i}, p_r)$, $X = A, B$ by their affine functions on the top of next page, respectively. Therefore, the optimization problem (63) at iteration $(k+1)$ -th with

$$h_{XR}^{(k)}(p_{A,i}, p_{B,i}, p_r) = \frac{\tau_c - \tau_p}{2\tau_c} \times \left\{ \log_2 \left(\sum_{j=1}^K (p_{A,j}^{(k)} \tilde{\sigma}_{AR,j}^2 + p_{B,j}^{(k)} \tilde{\sigma}_{BR,j}^2) + 1 \right) + \frac{\sum_{j=1}^K (p_{A,j} - p_{A,j}^{(k)}) \tilde{\sigma}_{AR,j}^2 + (p_{B,j} - p_{B,j}^{(k)}) \tilde{\sigma}_{BR,j}^2}{\ln 2 \left(\sum_{j=1}^K (p_{A,j}^{(k)} \tilde{\sigma}_{AR,j}^2 + p_{B,j}^{(k)} \tilde{\sigma}_{BR,j}^2) + 1 \right)} \right\}, \quad (64)$$

$$h_{RX}^{(k)}(p_{A,i}, p_{B,i}, p_r) = \frac{\tau_c - \tau_p}{2\tau_c} \times \left\{ \log_2 \left((p_r^{(k)} \tilde{\sigma}_{XR,i}^2 + 1) \sum_{j=1}^K \left(\frac{1}{\sigma_{AR,j}^2} + \frac{1}{\sigma_{BR,j}^2} \right) \right) + \frac{(p_r - p_r^{(k)}) \tilde{\sigma}_{XR,i}^2 \sum_{j=1}^K \left(\frac{1}{\sigma_{AR,j}^2} + \frac{1}{\sigma_{BR,j}^2} \right)}{\ln 2 \left((p_r^{(k)} \tilde{\sigma}_{XR,i}^2 + 1) \sum_{j=1}^K \left(\frac{1}{\sigma_{AR,j}^2} + \frac{1}{\sigma_{BR,j}^2} \right) \right)} \right\}. \quad (65)$$

convex constraints can be reformulated as

$$\max_{\mathbf{p}_A, \mathbf{p}_B, p_r, t_1, t_2} t \quad (66a)$$

$$\text{s.t. (56b), (56c), (57c)} \quad (66b)$$

$$f_1(p_{A,i}, p_{B,i}, p_r) - h_1^{(k)}(p_{A,i}, p_{B,i}, p_r) \geq t, \forall i \quad (66c)$$

$$f_{AR}(p_{A,i}, p_{B,i}, p_r) - h_{AR}^{(k)}(p_{A,i}, p_{B,i}, p_r) \geq t_1, \forall i \quad (66d)$$

$$f_{RB}(p_{A,i}, p_{B,i}, p_r) - h_{RB}^{(k)}(p_{A,i}, p_{B,i}, p_r) \geq t_1, \forall i \quad (66e)$$

$$f_{BR}(p_{A,i}, p_{B,i}, p_r) - h_{BR}^{(k)}(p_{A,i}, p_{B,i}, p_r) \geq t_2, \forall i \quad (66f)$$

$$f_{RA}(p_{A,i}, p_{B,i}, p_r) - h_{RA}^{(k)}(p_{A,i}, p_{B,i}, p_r) \geq t_2, \forall i \quad (66g)$$

To this end, via solving the optimization problem (66), the lower bound of the SE for each user pair can be obtained. Overall, the optimal solutions can be obtained by existing optimization tool (CVX) and the iterative procedure of Max-Min fairness analysis can be summarized in Algorithm 1.

Algorithm 1 General iterative algorithm

Initialization: Set the iteration index $k = 0$, define a tolerance $\epsilon > 0$ and initial values for $p_{A,i}^{(0)}$, $p_{B,i}^{(0)}$ and $p_r^{(0)}$, $i = 1, \dots, K$.

Repeat:

- 1: Solve optimization problem (76) and obtain the solutions $p_{A,i}^{(k)}$, $p_{B,i}^{(k)}$ and $p_r^{(k)}$, $i = 1, \dots, K$.
- 2: Update $(p_{A,i}^{(0)}, p_{B,i}^{(0)}, p_r^{(0)}) = (p_{A,i}^{(k)}, p_{B,i}^{(k)}, p_r^{(k)})$, $i = 1, \dots, K$.
- 3: Set $k = k + 1$.

Until:

- 4: $|t^{(k)} - t^{(k-1)}| < \epsilon$ or $k \geq L$ (maximum iteration number).

Output: $p_{A,i}^*$, $p_{B,i}^*$ and p_r^* , $i = 1, \dots, K$ as the solutions.

B. Complexity Analysis

Recall that in the optimization problem (66), logarithmic functions are deployed in the constraints; therefore, the successive approximation method which constructs polynomial approximations for all logarithmic terms is employed in CVX when solving this optimization problem [44]. To the best of our knowledge, the exact complexity of the successive approximation method in CVX has not been determined in the literature. Accordingly, we consider studying the complexity of the optimization problem with polynomial-approximation constraints to obtain the complexity lower bound of Algorithm 1.

Here, we make use of the fact that the optimization problem can be considered as a quadratically constrained quadratic program (QCQP) in epigraph form [45] after constructing polynomial approximations in CVX. With the assistance of previous studies [46], [47], the quadratic constraints can be rewritten as linear matrix inequalities (LMIs). Note that linear constraints can also be considered as LMI constraints. Since Algorithm 1 is an iterative process, solving the optimization problem by the interior-point method via CVX, as per the methodology in [48], we can similarly determine the lower bound of the complexity of Algorithm 1 via the following two parts:

1) *Iteration Complexity:* In Algorithm 1, with the given tolerance $\epsilon > 0$, the number of required iterations to achieve the ϵ -optimal solution can be given by

$$C_{iter} = \sqrt{\sum_{j=1}^{L_{num}} k_j} \cdot \ln\left(\frac{1}{\epsilon}\right) = \sqrt{6K^2 + 4K + 3} \cdot \ln\left(\frac{1}{\epsilon}\right), \quad (67)$$

where L_{num} is the number of LMI constraints and k_j represents the size of the j -th constraint, $j = 1, \dots, L_{num}$.

2) *Per-Iteration Complexity:* For each iteration, a search direction is generated by solving a system of n linear equations in n unknowns with $n = O(K^2)$ [48]. To this end, the computation cost per iteration can be obtained by

$$\begin{aligned} C_{per} &= n \cdot \sum_{j=1}^{L_{num}} k_j^3 + n^2 \cdot \sum_{j=1}^{L_{num}} k_j^2 + n^3 \\ &= n \cdot (3 + 2K + 8K^3 + 24K^4) + n^2 \cdot (3 + 2K + 4K^2 + 12K^3) \\ &\quad + n^3. \end{aligned} \quad (68)$$

Hence, the lower bound of the total complexity C_{total} of Algorithm 1 can be calculated by combining these two parts,

$$\begin{aligned} C_{total} &= C_{iter} \times C_{per} \\ &= \sqrt{6K^2 + 3K + 4} \cdot \ln\left(\frac{1}{\epsilon}\right) \cdot n \cdot [(3 + 2K + 8K^3 + 24K^4) \\ &\quad + n \cdot (3 + 2K + 4K^2 + 12K^3) + n^2]. \end{aligned} \quad (69)$$

VI. NUMERICAL RESULTS

We now present simulation results to verify the above studies. Unless specifically noted, the following parameters are employed in the simulation. We consider an LTE frame with [27] and we assume a coherence time $\tau_c = 196$ (symbols) and the length of the pilot sequences is $\tau_p = 2K$, the minimum requirement. For simplicity, we assume that the large-scale

fading parameters are $\beta_{AR,i} = \beta_{BR,i} = 1$ and each user has the same transmit power $p_{A,i} = p_{B,i} = p_u$, $i = 1, \dots, K$. For the proposed power consumption model, we assume that $\zeta = 0.38$, $P_{DAC} = 7.8$ mW, $P_{mix} = 15.2$ mW, $P_{filt} = 10$ mW, $P_{syn} = 25$ mW and $P_{static} = 2$ W.

A. Validation of Analytical Expressions

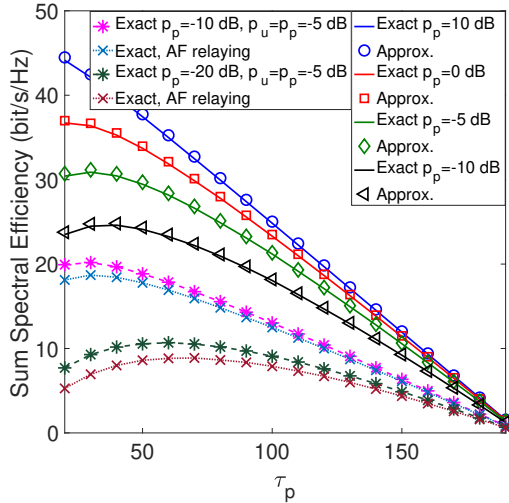


Fig. 2: Impact of τ_p for $M = 400$, $K = 10$, $p_u = 5$ dB and $p_r = 10$ dB.

Fig. 2 shows the sum SE vs the length of pilot sequence τ_p (in symbols). Note that the “Approx.” (Approximations) curves are obtained via applying Corollary 1, and the “Exact” (Exact results) curves are generated by (10)–(18). We can observe that the large-scale approximations closely match the exact results and the sum SE can be maximized with an optimal τ_p^* at low p_p . In contrast, the sum SE is a decreasing function of τ_p at moderate and high p_p . To this end, in order to achieve better sum SE performance, $\tau_p = 2K$, the minimum requirement, is deployed in the channel estimation phase. Moreover, AF relaying with ZF processing studied in [49]–[51] has been considered here as a benchmark to

further illustrate the performance of the proposed DF relaying system. It can be observed that when transmit powers are small resulting in lower SINRs, the performance of DF relaying can outperform that of AF relaying and an optimal τ_p^* can be obtained to maximize the sum SE with specific transmit power configurations. Since in our following simulations, we consider a more general power configuration, where SINRs are not small enough for DF relaying to outperform AF relaying; therefore, we only focus on the performance of the proposed DF relaying in the following numerical results.

B. Power Scaling Laws

1) *Case A*: It can be easily observed in Fig. 3 (a) that the power scaling law breaks down when $\gamma = 0$ and the sum SE grows unboundedly. For Case A, the curves named “Asy” (Asymptotic results) are presented according to Corollary 2. When $0 < \gamma < 1$, the sum SE is an increasing function of M . In contrast, when $\gamma = 1$, the sum SE progressively reaches a non-zero limit, and when $\gamma > 1$, the sum SE approaches zero gradually. Moreover, the sum SE is a decreasing function of γ ; since with larger γ , the system would experience a lower channel estimation accuracy, which results in worse system performance.

Fig. 3 (b) verifies the impact of M on the EE when different K and γ are applied. It is clearly shown that the sum SE saturates when M is large, while P_{total} increases linearly with M , and accordingly the EE peaks at a certain value of M . In this case, an optimal M^* can be selected to maximize the EE, especially when $0 \leq \gamma < 1$. Moreover, the EE decreases more significantly with a smaller K when M is large, and we can observe that larger K can introduce a larger optimal γ^* to obtain the maximum EE while achieving power savings.

2) *Case B*: Fig. 4 (a) investigates how the transmit powers of each user $p_u = E_u/M^\alpha$ and the relay $p_r = E_r/M^\beta$ affect the achievable SE. For Case B, the curves named “Asy” (Asymptotic results) are generated by Corollary 3. When $\alpha = 1$ and/or $\beta = 1$, the SE saturates to a non-zero limit. When $\alpha > 1$, $\beta > 1$, the sum SE gradually reduces to zero. On the other hand, when we cut down the transmit powers moderately, the

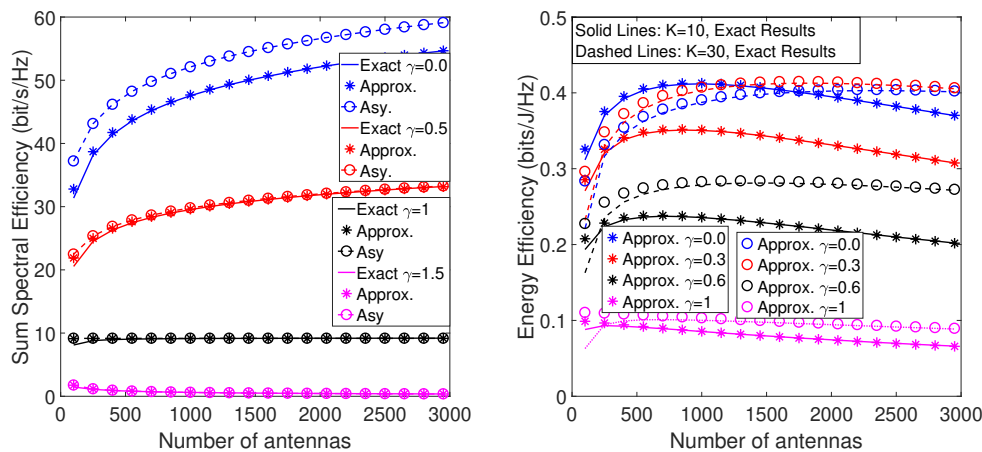


Fig. 3: (a) Sum SE and (b) EE v.s. number of relay antennas M for $K = 10$, $p_u = 5$ dB, $p_r = 10$ dB and $p_p = E_p/M^\gamma$ with $E_p = 5$ dB.

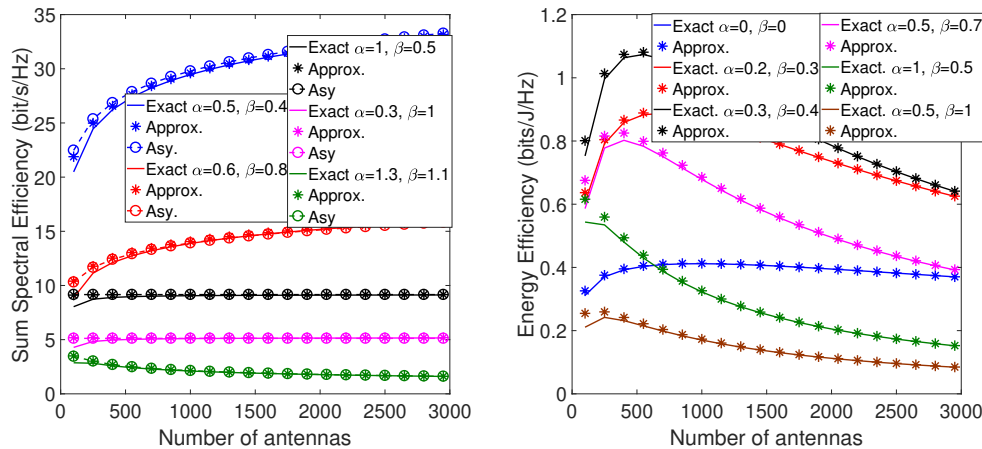


Fig. 4: (a) Sum SE and (b) EE v.s. number of relay antennas for $K = 10$, $p_p = 5$ dB, $p_u = E_u/M^\alpha$ with $E_u = 5$ dB and $p_r = E_r/M^\beta$ with $E_r = 10$ dB.

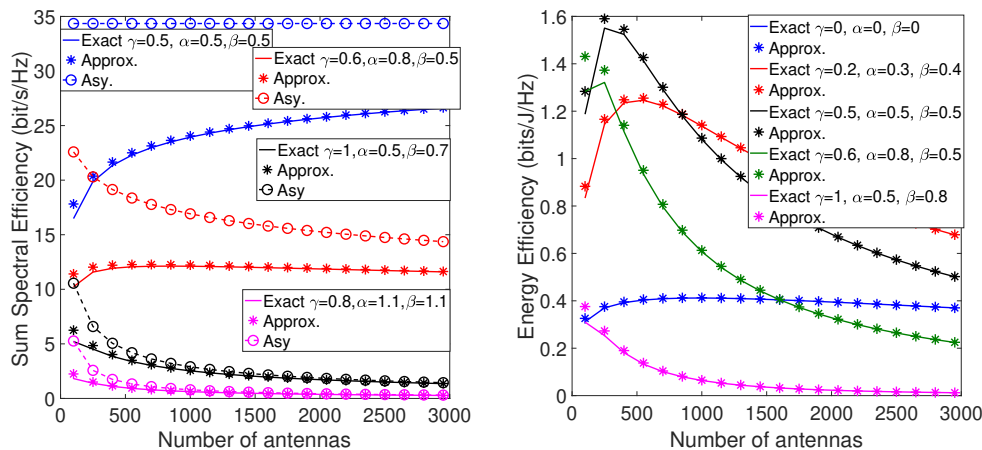


Fig. 5: (a) Sum SE and (b) EE v.s. number of relay antennas for $K = 10$, $p_p = E_p/M^\gamma$, $p_u = E_u/M^\alpha$ with $E_p = E_u = 5$ dB and $p_r = E_r/M^\beta$ with $E_r = 10$ dB.

sum SE grows unboundedly. The channel estimation accuracy keeps stable and the transmission phase plays an important role in the SE performance.

The impact of M on the EE with different α and β is investigated in Fig. 4 (b). We can see that when $0 < \alpha < 1$ and $0 < \beta < 1$, the EE performance is better than that without power scaling law and an optimal M^* can be obtained to maximize the EE. Therefore, the moderate power scaling in the transmission phase can help to optimize the EE performance. In contrast, when the transmit powers are reduced aggressively, the EE is a decreasing function with respect to M . With regards to this, by considering the trade-off between scaling parameters, appropriate values of α and β could be selected to optimize the EE performance.

3) *Case C*: Fig. 5 (a) verifies the trade-off between the transmit powers of each user, the relay and the pilot symbol. For Case C, the “Asy” (Asymptotic results) curves are obtained via Corollary 4. For the aggressive power-scaling scenario, the sum SE progressively converges to zero, as predicted. Moreover, with the moderate power-scaling parameters, $0 < \gamma < 1$, $0 < \alpha < 1$ and $0 < \beta < 1$, the sum SE increases with respect to M .

Fig. 5 (b) illustrates the impact of the number of relay antennas on the EE. It is clearly shown that the EE rises and then descends with respect to M while applying moderate power-scaling parameters; thus, we can obtain the optimal M^* to maximize the EE, e.g., with $\gamma = 0.2$, $\alpha = 0.3$ and $\beta = 0.4$, the maximum EE around 1.25 bits/J/Hz can be obtained when

TABLE I: Average Run Time (in seconds) for three scenarios of Algorithm 1

K		2	4	6	8	10
$M = 500$ $p^{max} = 23$ dB $p_p = 5$ dB	Algorithm 1	9.318	10.021	12.745	13.136	14.721
	Algorithm 1 with equal power allocation	6.485	9.548	13.233	15.936	16.110
	Uniform power allocation	4.100	5.529	7.504	8.997	11.298

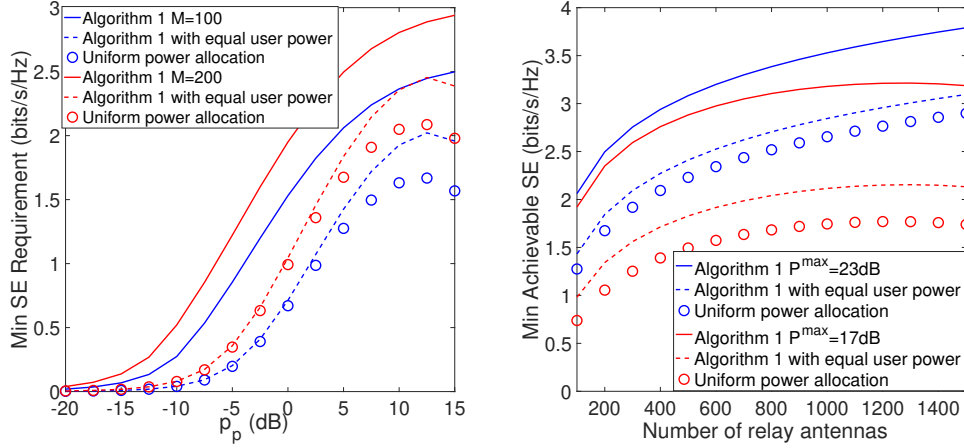


Fig. 6: Min achievable SE v.s. (a) p_p with $P_r^{max} = P_u^{max} = 23$ dB and (b) number of relay antennas with $p_p = 5$ dB, $P_r^{max} = P_u^{max}$ for $K = 10$ and $P_u^{max} = 10$ dB.

$M^* \approx 500$. On the other hand, when we cut down the transmit powers aggressively, the EE approaches to zero straightforwardly. Therefore, it is of crucial importance to determine the scaling parameters to optimize the EE performance in a specific power-limited scenario.

C. Max-Min Fairness

We consider three optimization scenarios with the minimum achievable SE among all user pairs: 1) Algorithm 1; 2) Algorithm 1 with equal user power, i.e. $p_{A,i} = p_{B,i} = p_u$, $i = 1, \dots, K$; 3) Uniform power allocation, i.e. $p_{A,i} = p_{B,i} = p_u$, $i = 1, \dots, K$, $2Kp_u = P_r$, $\frac{1}{2} \left(\sum_{i=1}^K \frac{(\tau_c - \tau_p)(p_{A,i} + p_{B,i})}{\tau_c \zeta_u} + \frac{P_r}{\zeta_c} \right) + P_o = P^{max}$. For a more practical comparison, all users' large-scale fading parameters are different and can be generated via $\beta_k = \sqrt{\frac{\mu}{D_k^\nu}}$, where μ is the large-scale fading coefficient, D_k is the distance between the k -th user and the relay, ν is the path-loss exponent [52]. Following our benchmark work in [22], we consider $\beta_{AR} = [0.3188, 0.4242, 0.5079, 0.0855, 0.2625, 0.8010, 0.0292, 0.9289, 0.7303, 0.4886]$, and $\beta_{BR} = [0.5785, 0.2373, 0.4588, 0.9631, 0.5468, 0.5211, 0.2316, 0.4889, 0.6241, 0.6791]$.

Fig. 6 (a) shows the minimum achievable SE versus p_p . It can be observed that the minimum achievable SE achieved via Algorithm 1 outperforms the other two scenarios, Algorithm 1 with equal user power and uniform power allocation, especially when p_p is large enough. Moreover, larger number of relay antennas can help to increase the minimum achievable SE with the same total power constraint P^{max} .

Fig. 6 (b) shows the minimum achievable SE with increasing number of relay antennas M . Similarly, a higher minimum achievable SE can be achieved by Algorithm 1 compared with the other two power allocation scenarios. The minimum achievable SE is an increasing function of M , especially with a larger total power constraint.

In Table I on the bottom of previous page, we display the average run time (in seconds) of three optimization scenarios defined above with a given tolerance $\epsilon = 10^{-5}$. We can observe that, the running times for all three scenarios are increasing

with respect to K . Then, uniform power allocation has the smallest number of constraints and, therefore, the running time for this scenario is the shortest.

VII. CONCLUSION

This paper has studied the sum SE and EE performance of a multi-pair two-way half-duplex DF relaying system with ZF processing and imperfect CSI. Note that this setup extends considerably a stream of recent papers on massive MIMO relaying by leveraging tools of Wishart matrix theory. In particular, a large-scale approximation of the achievable SE was deduced. Meanwhile, a practical power consumption model was characterized to study the EE performance. Furthermore, in view of approximations, three specific power scaling laws were investigated to present how the transmit powers of each pilot symbol, each user and the relay can be scaled to improve the system performance. These results have their own adding value as they translate mathematical formulations into system design guidelines for power savings. Finally, a formulated optimization problem was studied to optimize the minimum achievable SE among all user pairs. Our numerical results demonstrated emphatically that the proposed system with ZF processing is able to enhance the EE while preserving the SE performance with moderate system configurations. Moreover, the simulation results of the optimization problem demonstrated that our proposed max-min fairness scheme can achieve higher minimum achievable SE among user pairs compared with the benchmark schemes where equal user power and uniform power allocation are applied. In our future work, we will consider the application of multi-pair two-way DF relaying in TDD correlated massive MIMO systems as the subsequent subject.

APPENDIX A PROOF OF Lemma 1

In this appendix, we provide the calculations of Lemma 1. With the assumption that all estimated channels with $\hat{\mathbf{h}}_{XR,i} \sim \mathcal{CN}(\mathbf{0}, \bar{\sigma}_{XR,i}^2 \mathbf{I}_M)$ and $\hat{\mathbf{h}}_{XR,j} \sim \mathcal{CN}(\mathbf{0}, \bar{\sigma}_{XR,j}^2 \mathbf{I}_M)$ are mutually

independent when $i \neq j$, $i, j = 1, \dots, K$. When $M \rightarrow \infty$, we can have

If $i = j$,

$$\frac{1}{M} \hat{\mathbf{h}}_{XR,i}^H \hat{\mathbf{h}}_{XR,i} = \frac{1}{M} |\hat{\mathbf{h}}_{XR,i}|^2 = \frac{1}{M} \cdot M \tilde{\sigma}_{XR,i}^2 = \tilde{\sigma}_{XR,i}^2, \quad (70)$$

If $i \neq j$,

$$\frac{1}{M} \hat{\mathbf{h}}_{XR,i}^H \hat{\mathbf{h}}_{XR,j} = 0. \quad (71)$$

With the computation of (70)-(71), we can obtain *Lemma 1* in (19).

APPENDIX B

DERIVATION FOR APPROXIMATIONS OF THE SUM SEs

In this appendix, we present the detailed derivation for $\hat{R}_{1,i}$ and $\hat{R}_{RX,i}$, while $\hat{R}_{XR,i}$ can be obtained in a straightforward way. At first, some useful results widely used in the calculation are given in Lemma 2.

Lemma 2: Assume that $\mathbf{h}_i \sim \mathcal{CN}(\mathbf{0}, \sigma_i^2 \mathbf{I}_M)$ and $\mathbf{h}_j \sim \mathcal{CN}(\mathbf{0}, \sigma_j^2 \mathbf{I}_M)$ are mutually independent when $i \neq j$, $i, j = 1, \dots, K$. Therefore, we have

$$\frac{|\mathbf{h}_i^H \mathbf{h}_j|^2}{M^2} \rightarrow \begin{cases} \sigma_i^4, & i = j \\ \frac{1}{M} \sigma_i^2 \sigma_j^2, & i \neq j \end{cases}, \quad (72)$$

With the assistance of (20)-(23), Lemma 1 and Lemma 2, we derive the calculation of the corresponding approximations in the following. First, we focus on $\hat{R}_{1,i}$, $\hat{R}_{XR,i}$, $X = A, B$ in the MAC phase, consisting of four terms defined above. When $M \rightarrow \infty$, we can have

1) Desired signal power of $T_{X,i}$, $X = A, B$,

$$\begin{aligned} & p_{X,i} \left(|\mathbf{F}_{MAC,i}^{AR} \hat{\mathbf{h}}_{XR,i}|^2 + |\mathbf{F}_{MAC,i}^{BR} \hat{\mathbf{h}}_{XR,i}|^2 \right) \\ & \rightarrow \begin{cases} p_{A,i} |\mathbf{F}_{MAC,i}^{AR} \hat{\mathbf{h}}_{AR,i}|^2 \\ p_{B,i} |\mathbf{F}_{MAC,i}^{BR} \hat{\mathbf{h}}_{BR,i}|^2 \end{cases} \rightarrow \begin{cases} p_{A,i} \left| \frac{1}{M \sigma_{AR,i}^2} \hat{\mathbf{h}}_{AR,i}^H \hat{\mathbf{h}}_{AR,i} \right|^2 \\ p_{B,i} \left| \frac{1}{M \sigma_{BR,i}^2} \hat{\mathbf{h}}_{BR,i}^H \hat{\mathbf{h}}_{BR,i} \right|^2 \end{cases} \\ & \rightarrow \begin{cases} p_{A,i}, & X = A \\ p_{B,i}, & X = B \end{cases}, \end{aligned} \quad (73)$$

2) Estimation Error A_i ,

$$\begin{aligned} A_i & \rightarrow p_{A,i} \left(\left| \frac{1}{M \sigma_{AR,i}^2} \hat{\mathbf{h}}_{AR,i}^H \mathbf{e}_{AR,i} \right|^2 + \left| \frac{1}{M \sigma_{BR,i}^2} \hat{\mathbf{h}}_{BR,i}^H \mathbf{e}_{AR,i} \right|^2 \right) \\ & + p_{B,i} \left(\left| \frac{1}{M \sigma_{AR,i}^2} \hat{\mathbf{h}}_{AR,i}^H \mathbf{e}_{BR,i} \right|^2 + \left| \frac{1}{M \sigma_{BR,i}^2} \hat{\mathbf{h}}_{BR,i}^H \mathbf{e}_{BR,i} \right|^2 \right) \\ & \rightarrow \frac{(p_{A,i} \tilde{\sigma}_{AR,i}^2 + p_{B,i} \tilde{\sigma}_{BR,i}^2)}{M} \left(\frac{1}{\sigma_{AR,i}^2} + \frac{1}{\sigma_{BR,i}^2} \right), \end{aligned} \quad (74)$$

3) Inter-user Interference B_i ,

$$\begin{aligned} B_i & = \sum_{j \neq i} p_{A,j} \left(|\mathbf{F}_{MAC,i}^{AR} (\hat{\mathbf{h}}_{AR,j} + \mathbf{e}_{AR,j})|^2 + |\mathbf{F}_{MAC,i}^{BR} (\hat{\mathbf{h}}_{AR,j} + \mathbf{e}_{AR,j})|^2 \right) \\ & + \sum_{j \neq i} p_{B,j} \left(|\mathbf{F}_{MAC,i}^{AR} (\hat{\mathbf{h}}_{BR,j} + \mathbf{e}_{BR,j})|^2 + |\mathbf{F}_{MAC,i}^{BR} (\hat{\mathbf{h}}_{BR,j} + \mathbf{e}_{BR,j})|^2 \right) \\ & \rightarrow \sum_{j \neq i} p_{A,j} \left(|\mathbf{F}_{MAC,i}^{AR} \mathbf{e}_{AR,j}|^2 + |\mathbf{F}_{MAC,i}^{BR} \mathbf{e}_{AR,j}|^2 \right) \\ & + \sum_{j \neq i} p_{B,j} \left(|\mathbf{F}_{MAC,i}^{AR} \mathbf{e}_{BR,j}|^2 + |\mathbf{F}_{MAC,i}^{BR} \mathbf{e}_{BR,j}|^2 \right) \\ & \rightarrow \sum_{j \neq i} p_{A,j} \left(\left| \frac{1}{M \sigma_{AR,i}^2} \hat{\mathbf{h}}_{AR,i}^H \mathbf{e}_{AR,j} \right|^2 + \left| \frac{1}{M \sigma_{BR,i}^2} \hat{\mathbf{h}}_{BR,i}^H \mathbf{e}_{AR,j} \right|^2 \right) \\ & + \sum_{j \neq i} p_{B,j} \left(\left| \frac{1}{M \sigma_{AR,i}^2} \hat{\mathbf{h}}_{AR,i}^H \mathbf{e}_{BR,j} \right|^2 + \left| \frac{1}{M \sigma_{BR,i}^2} \hat{\mathbf{h}}_{BR,i}^H \mathbf{e}_{BR,j} \right|^2 \right) \\ & \rightarrow \frac{1}{M} \left(\frac{1}{\sigma_{AR,i}^2} + \frac{1}{\sigma_{BR,i}^2} \right) \sum_{j \neq i} (p_{A,j} \tilde{\sigma}_{AR,j} + p_{B,j} \tilde{\sigma}_{BR,j}), \end{aligned} \quad (75)$$

4) Noise C_i ,

$$C_i \rightarrow \left\| \frac{1}{M \sigma_{AR,i}^2} \hat{\mathbf{h}}_{AR,i}^H \right\|^2 + \left\| \frac{1}{M \sigma_{BR,i}^2} \hat{\mathbf{h}}_{BR,i}^H \right\|^2 \rightarrow \frac{1}{M} \left(\frac{1}{\sigma_{AR,i}^2} + \frac{1}{\sigma_{BR,i}^2} \right). \quad (76)$$

Substituting (73)-(76) into (13)-(14), we can obtain $\hat{R}_{1,i}$, $\hat{R}_{XR,i}$, $X = A, B$ in (26), (28).

Then, we focus on $\hat{R}_{RX,i}$ in the BC phase. Similarly, the corresponding terms in $\hat{R}_{RX,i}$, $X = A, B$ can be computed as following when $M \rightarrow \infty$,

1) Normalization coefficient,

$$\begin{aligned} \rho_{DF} & = \sqrt{\frac{p_r}{E \left\{ \|\mathbf{F}_{BC}\|^2 \right\}}} = \sqrt{\frac{p_r}{E \left\{ \sum_{i=1}^M \sum_{j=1}^{2K} |\mathbf{F}_{BC}(i, j)|^2 \right\}}} \\ & \rightarrow \sqrt{\frac{p_r}{\sum_{i=1}^K \left(\frac{1}{M \sigma_{AR,i}^2} + \frac{1}{M \sigma_{BR,i}^2} \right)}}. \end{aligned} \quad (77)$$

2) Desired signal,

$$|\hat{\mathbf{h}}_{XR,i}^T \mathbf{F}_{BC,i}^{RX}|^2 \rightarrow |\hat{\mathbf{h}}_{XR,i}^T \frac{1}{M \sigma_{XR,i}^2} \hat{\mathbf{h}}_{XR,i}^*|^2 \rightarrow 1, \quad (78)$$

3) Estimation error,

$$|\mathbf{e}_{XR,i}^T \mathbf{F}_{BC,i}^{RX}|^2 \rightarrow |\mathbf{e}_{XR,i}^T \frac{1}{M \sigma_{XR,i}^2} \hat{\mathbf{h}}_{XR,i}^*|^2 \rightarrow \frac{\tilde{\sigma}_{XR,i}^2}{M \sigma_{XR,i}^2}, \quad (79)$$

4) Inter-user interference,

$$\begin{aligned}
 & \sum_{j=1}^K \left(\left| \mathbf{h}_{XR,i}^T \mathbf{F}_{BC,j}^{RA} \right|^2 + \left| \mathbf{h}_{XR,i}^T \mathbf{F}_{BC,j}^{RB} \right|^2 \right) - \left| \mathbf{h}_{XR,i}^T \mathbf{F}_{BC,i}^{RX} \right|^2 \\
 &= \sum_{j=1}^K \left(\left| (\hat{\mathbf{h}}_{XR,i}^T + \mathbf{e}_{XR,i}^T) \mathbf{F}_{BC,j}^{RA} \right|^2 + \left| (\hat{\mathbf{h}}_{XR,i}^T + \mathbf{e}_{XR,i}^T) \mathbf{F}_{BC,j}^{RB} \right|^2 \right) \\
 &- \left| (\hat{\mathbf{h}}_{XR,i}^T + \mathbf{e}_{XR,i}^T) \mathbf{F}_{BC,i}^{RX} \right|^2 \\
 &\rightarrow \begin{cases} \sum_{j=1}^K \left| \mathbf{e}_{AR,i}^T \mathbf{F}_{BC,j}^{RB} \right|^2 + \sum_{j \neq i} \left| \mathbf{e}_{AR,i}^T \mathbf{F}_{BC,j}^{RA} \right|^2 \\ \sum_{j=1}^K \left| \mathbf{e}_{BR,i}^T \mathbf{F}_{BC,j}^{RA} \right|^2 + \sum_{j \neq i} \left| \mathbf{e}_{BR,i}^T \mathbf{F}_{BC,j}^{RB} \right|^2 \end{cases} \\
 &\rightarrow \begin{cases} \sum_{j=1}^K \left| \mathbf{e}_{AR,i}^T \frac{1}{M\sigma_{BR,j}^2} \hat{\mathbf{h}}_{BR,j}^* \right|^2 + \sum_{j \neq i} \left| \mathbf{e}_{AR,i}^T \frac{1}{M\sigma_{AR,j}^2} \hat{\mathbf{h}}_{AR,j}^* \right|^2 \\ \sum_{j=1}^K \left| \mathbf{e}_{BR,i}^T \frac{1}{M\sigma_{AR,j}^2} \hat{\mathbf{h}}_{AR,j}^* \right|^2 + \sum_{j \neq i} \left| \mathbf{e}_{BR,i}^T \frac{1}{M\sigma_{BR,j}^2} \hat{\mathbf{h}}_{BR,j}^* \right|^2 \end{cases} \\
 &\rightarrow \begin{cases} \sum_{j=1}^K \frac{\tilde{\sigma}_{AR,j}^2}{M\sigma_{BR,j}^2} + \sum_{j \neq i} \frac{\tilde{\sigma}_{AR,j}^2}{M\sigma_{AR,j}^2}, & X = A \\ \sum_{j=1}^K \frac{\tilde{\sigma}_{BR,j}^2}{M\sigma_{AR,j}^2} + \sum_{j \neq i} \frac{\tilde{\sigma}_{BR,j}^2}{M\sigma_{BR,j}^2}, & X = B \end{cases} \quad (80)
 \end{aligned}$$

By applying (77)-(80) to (16), we can obtain $\hat{R}_{RX,i}$, $X = A, B$, in (29). With other additional computations, we complete the proof of the SE approximations shown in Corollary 1.

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