Effective focal length through a microscope

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The optical power of a thick spherical lens and its Coddington shape factor are essential magnitudes that characterize its image quality. We propose an experimental procedure and apparatus that allow accurate determination of those magnitudes for any spherical lens from geometrical measurements. It overcomes the drawbacks of other devices that need of the refractive index or may damage the lens surfaces, like spherometers, and provides similar results to commercial lensmeters.

Keywords: Thick lens; Effective focal length; Shape factor

1. Introduction

The optical power and the shape factor are essential magnitudes that characterize the image quality of any lens, including ophthalmic, contact or intraocular lenses. In fact, optical aberrations are highly dependent on the shape and the index of refraction of the lens [1]. Despite the determination of the effective focal length (EFL) of a lens is an old topic in optical metrology, new works are still published [2]. Some methods provide very accurate measurements of EFL [3]; however, they need of sophisticated material with complex experimental setups. Spherical lenses are the most common ones since they are simpler and cheaper to manufacture. The optical power and the shape factor are usually computed from the measurement of the curvature radii of its surfaces through a spherometer [4]. However, this procedure cannot be translated to delicate lenses since the spherometer may produce scratches on the optical surface and damage the treatment layers. We present a simple method to obtain the EFL and shape of a lens just through measuring the location of some points on its surfaces.

2. Methods

A spherical surface centred at $C=\left(x_c, y_c, z_c\right)$ and curvature radii r can be represented by:

$$\left(x-x_c\right)^2+\left(y-y_c\right)^2+\left(z-z_c\right)^2=r^2 \tag{1}$$

Let us consider n points located on a spherical shell at the Cartesian coordinates $P_p=\left(x_p, y_p, z_p\right)$, with $p=0, \ldots, (n-1)$; and let us suppose that the origin of the spatial reference system is set on one of these points, P_0. The remaining $(n-1)$ points with respect to P_0 can be expressed as:

$$x_p=x_0+\Delta x_p;\quad y_p=y_0+\Delta y_p;\quad z_p=z_0+\Delta z_p \tag{2}$$

From expressions (1) and (2), we make the changes $x'_0=\left(x_0-x_c\right), y'_0=\left(y_0-y_c\right), z'_0=\left(z_0-z_c\right)$

and get a system of $(n-1)$ equations with three unknowns $\left(x'_0, y'_0, z'_0\right)$. A system of equations has solution if it is compatible and determinate, i.e. it should have, at least, 3 equations. So, the minimum number of points must be n =4. Therefore, we get, in matrix form:

$$\begin{pmatrix} \Delta x_1 & \Delta y_1 & \Delta z_1 \\ \Delta x_2 & \Delta y_2 & \Delta z_2 \\ \Delta x_3 & \Delta y_3 & \Delta z_3 \end{pmatrix} \begin{pmatrix} x'_0 \\ y'_0 \\ z'_0 \end{pmatrix} = -\frac{1}{2} \begin{pmatrix} (\Delta x_1)^2 + (\Delta y_1)^2 + (\Delta z_1)^2 \\ (\Delta x_2)^2 + (\Delta y_2)^2 + (\Delta z_2)^2 \\ (\Delta x_3)^2 + (\Delta y_3)^2 + (\Delta z_3)^2 \end{pmatrix} \rightarrow AX'_0 = B \tag{3}$$

We just must solve $X'_0 = A^{-1}B$ and finally obtain the curvature radius of the sphere as:

$$x'^2_0 + y'^2_0 + z'^2_0 = r^2 \tag{4}$$

Although the presented method is clear, we notice that equation system in (3) is very sensitive to small changes, even within the experimental error. This happens because the matrix A is often ill-conditioned, i.e. the system of equations is highly unstable. In order to avoid the ill-conditioning of A, it is necessary to increase the number of sampled points in the spherical shell and, accordingly, to obtain a new system of equations. Now, instead of expression (3), the system is an overdetermined linear system of m equations with 3 unknowns $A_{m\times3}X'_0 = B_{m\times1}$. If $A_{m\times3}$ is full rank, the approximate and unique solution will be the vector $X'_0 \in \mathbb{R}^m$ that minimizes the value $\left(\left\|A_{m\times3}X'_0 - B_{m\times1}\right\|_2\right)^2$, i. e. the Euclidean norm. This approach leads to the following system of normal equations: $X'_0 = \left[A_{m\times3}\right]^+ B_{m\times1}$ where $\left[A_{m\times3}\right]^+$ is the Moore-Penrose pseudo-inverse matrix of $A_{m\times3}$.

Therefore, if we manage to measure three points over a shell with respect to a fourth one, we can determine the curvature radius of the lens. We select points on one surface of a spherical lens, just by marking them with black ink. Then, we place the lens on a microscope plate, with the marks oriented towards the objective. We have used a microscope (Alphaphot-2 microscope from Nikon with a 10x objective and NA=0.25) with a plate that can be moved in X and Y directions and vertically (Z axis). Two gauges of 0.1 mm sensitivity provide the position in the XY plane, while a 0.0022 mm sensitivity micrometer measure vertical movements. To increase the sensitivity in the XY plane, an optical mouse is mounted attached to the vertical displacement. The mouse shows the horizontal movement on a computer screen (1280x1024 px). The cursor location is related to the horizontal movement through a factor of 0.021 mm/px, so the sensitivity is improved in 5 times.

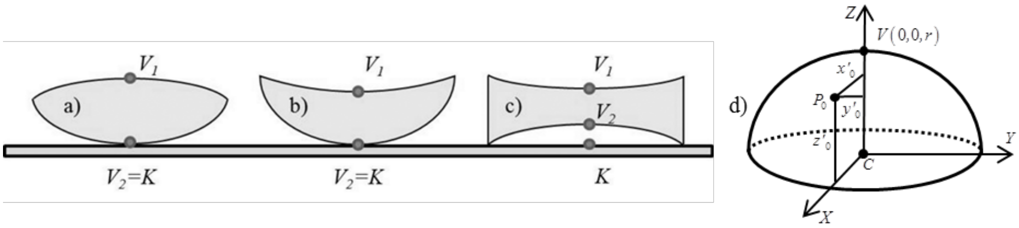

Fig. 1 (a) Biconvex lens. (b) Meniscus. (c) Biconcave lens (d) Location of P_0 on a spherical surface

The measuring process begins by setting the origin P_0. So, we first simply focus one of the marks on and, then, focus the 3 remaining points on and measure the displacements $\left(\Delta x_p, \Delta y_p, \Delta z_p\right)$ with respect to the origin point. Hence, we can obtain the curvature radius (4) for each surface. Curvature radii help us to obtain the central thickness of the lens. In the scheme in Fig. 1 (d), we start focusing

P_0 on and then we look for focusing $V(0,0,r)$ on, just by moving the plate of the microscope. It consists of a horizontal displacement, x'_0 and y'_0 from (3), and a vertical shift until the vertex. Once reached V_1, the determination of the central thickness is different depending on the shape of the lens. In cases Fig. 1(a) and Fig 1(b), central thickness is obtained just by looking for the point K, which is directly marked on the supporting plate. The lens is removed from the microscope and the plate is focused. The vertical movement of the microscope is directly the central thickness of the lens. The process is more complex in the case Fig. 1(c). If we look at Fig. 2, from V_1, we can horizontally displace the plate and look for the point *A*, just in the border of the lens. That displacement corresponds to *e*. Then, we go back to V_1 and remove the lens to focus on the plate. The distance from V_1 to the base is *g*. Then, central thickness is $d = g + \sqrt{r_2^2 - e^2} - r_2$. The last magnitude that we need is the refractive index. Following Fig. 2, we start focusing V_1 on. Without removing the lens, we focus the opposite vertex on, by displacing the plate $|s'_2|$. Since we are looking at V_2 through the lens, what we see is the image V'_2. This image is given by the upper surface, with radius r_1, that separates a medium of refractive index *n* from air.

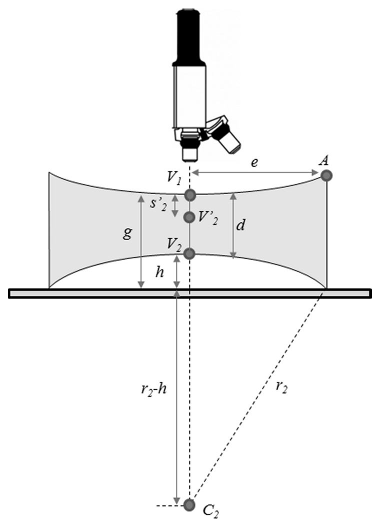

Fig. 2. Scheme that shows distances needed to obtain the central thickness and the refractive index.

From Geometrical Optics, and taking into account the sign convention for the curvature radii (biconvex: $r_1>0$, $r_2<0$; meniscus: $r_1>0$, $r_2>0$, biconcave: $r_1<0$, $r_2>0$) and for distances ($s'_2<0$ in all cases), we can deduce that the refractive index and the optical power of a thick lens.

$$n = \frac{d\left(r_1 - |s'_2|\right)}{s'_2\left(r_1 - d\right)}; \quad P = \frac{r_2 + |s'_2| - g - \sqrt{r_2^2 - e^2}}{r_2 |s'_2| \left(g + \sqrt{r_2^2 - e^2} - r_2 - r_1\right)} \left\{ r_2 + r_1 \frac{r_2 + r_1 - g - \sqrt{r_2^2 - e^2}}{|s'_2| - r_1} \right\} \tag{5}$$

Geometrical Optics determines the back vertex power of a thick lens as $P_b = P \Big/ \left(1 - d\frac{n-1}{nr_1}\right)$

3. Results

We apply the technique to measure the shape factor and power of an ophthalmic meniscus contact lens. First, the measured back vertex power of the lens through a lensmeter (Nidek LM-770) results $P_b = 4.00 \pm 0.12\, D$. Next, we have sampled points at the two surfaces of the lens and have computed the coordinates of the origins for both surfaces. In Fig. 3, we show the variation of the condition number with the number of sampled points for both surfaces. As can be seen, the system tends to stabilize after around 15 samples.

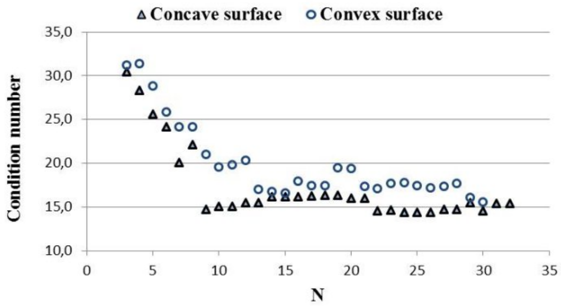

Fig. 3: Variation of the condition number with the number of sampled points for both surfaces

Obtained parameters in Table 1 lead us to conclude that the curvature radii are 68.09±0.08 mm and 151.9±0.1 mm, for convex and the concave sides of the lens, respectively. Finally, we measured $e = 22.49 \pm 0.03\,mm$, $g = 6.982 \pm 0.003\,mm$ and $s'_2 = -3.693 \pm 0.007\,mm$, so the optical power and the shape factor are $P = 3.92 \pm 0.06\,D$ and $q = 2.625 \pm 0.005$, respectively. Back vertex power results $P_b = 4.02 \pm 0.06\,D$, which implies a relative deviation below 0.5% compared to the value measured using the commercial lensmeter.

Table 1. Parameters obtained for each radius and its error

	$x'_0\left(mm\right)$	$y'_0\left(mm\right)$	$z'_0\left(mm\right)$
Convex surface	-14.001±0.004	-14.96±0.02	64.94±0.08
Concave surface	-10.08±0.01	-8.28±0.01	151.4±0.1

4. Conclusions

We have proposed a method for measuring the geometry and optical power of a spherical lens. It does not need of contact devices and it is able to provide the curvature radii and power of lenses of any size. Applying spherometers is not always possible if the lens is too small. We just have to accurately measure the location of different points on the surfaces. In principle, this can be achieved simply by changing the microscope. At last, but not least, the method does not need of the refractive index, whereas other instruments used to measuring optical power or curvature radii do.

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