Quantifying "Transitional" Soil Behaviour

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ABSTRACT

The last decade has seen an increasing amount of research on so called "transitional" soils that are characterised by incomplete convergence to unique normal compression lines and/or critical state lines in simple laboratory tests. This topic has often provoked reaction, perhaps because some have seen it as a challenge to critical state frameworks of soil behaviour. A particular issue is whether incomplete testing or other test defects might cause such an apparent behaviour. Confusion around the topic has not been helped by the wide range of degrees of convergence seen for different materials and differences seen between convergence in compression and shearing. This paper proposes a unifying means of plotting laboratory test data from such soils that will hopefully provide a rational framework for such discussions, since it makes explicit the degree of convergence towards unique volumetric states for different forms of loading. Data are examined for three "transitional" soils, which show that for these soils bringing about convergence would require strains that are beyond those that may easily be applied and that the lack of convergence cannot solely be an artefact of test defects. Plastic volumetric strain was found to cause much faster convergence than plastic shear strain.

Introduction

The central tenet of critical state soil mechanics is that continued shearing will eventually lead to a state of constant stress, volume and fabric to be reached that is independent of the starting condition. For many soils of natural origins that must be regarded as a target, which cannot always be easily reached because of a structure of the soil that is difficult to break down at the strains applied by standard laboratory tests. Some authors have therefore defined different critical state lines in the volumetric plane for natural and reconstituted samples of the same soil (e.g. Cotecchia & Chandler, 2000; Hosseini-Kamal et al., 2014) because the volumetric states at the ends of their tests are reasonably stable and can be regarded as pseudo critical states. For some soils this is simply a pragmatic choice and it could be envisaged that much larger strains or other types of loading might give convergence, but in others very robust forms of natural fabric can only be broken down by very severe mechanical remoulding (e.g. Fearon & Coop, 2000).

In reconstituted soils the fabric created by the preparation method often has an effect on subsequent behaviour (e.g. Santucci et al., 1998; Madhusudhan & Baudet, 2014). These effects may be related to, but should not be confused with, the so-called "transitional" behaviour in which the initial specific volume of a sample has a persisting effect on the subsequent behaviour, often regardless of preparation method. In these cases standard laboratory tests such as oedometers or triaxials may give locations of the normal compression and critical state lines that appear to depend on the initial density. Samples that have different specific volumes at similar stress states must have different fabrics, but Shipton & Coop (2015) found that for the simple sand / kaolin mixed soil, the sample preparation technique per se had no significant influence beyond the initial sample densities that they enabled to be created.

While the soils that have been used in the work described in this paper are simple laboratory created mixtures of fairly standard soil particles, this transitional behaviour has also been reported in reconstituted samples of natural soils, for example saprolites, alluvial and lagoon sediments, glacial till and natural sands (Ferreira & Bica, 2006; Nocilla et al., 2006; Ponzoni et al., 2014; Altuhafi et al., 2010; Ventouras & Coop, 2009) as well as a number of "man-made" materials such as mine tailings and rock fill (Coop, 2015; Xiao et al., 2016). In some of these materials standard tests seem to give normal compression or critical state lines that are, to all intents and purposes, parallel, while in others a slow convergence with increased stress level is clear, but often without the possibility that a unique normal compression or critical state line is defined before the zero void asymptote is approached. While all "transitional soils" must have an influence of initial density on the compression behaviour, some seem to give better convergence for critical states, indicating that volumetric and shear strains both affect the convergence but perhaps differently (Ponzoni et al., 2014).

The subject of transitional soils provokes controversy as it challenges some of our common assumptions and the idea that fabrics in reconstituted soils can have such persistent effects seems to be less readily accepted than for natural soils. The fact that it has proven impossible to predict which soils might be transitional and which not simply from grading has added to the apparent complexity, even if Ponzoni et al. (2017) have recently shown how mineralogy and grading may interact to cause it. The purpose of this paper is to navigate a way out of the complexity of these complex and often apparently contradictory data, hopefully offering a means to break through entrenched positions on how we should interpret the behaviour. Data from examples of "transitional soils" are first used to illustrate the problems of identifying unique behaviour and then a means of quantifying the rates of convergence to unique volume/stress states is given that allows comparison between data from different types of loading, highlighting when and how convergence might be brought about.

Materials and Testing

The materials used were not chosen to represent any particular natural soil and exactly what soils were tested is not of especial relevance because they were chosen mostly as examples that would clearly demonstrate the methods proposed in this paper. Even so, the gap graded soil was initially chosen by Martins et al. (2001) to have a similar grading to their natural saprolite.

This soil, of 75% quartz sand and 25% kaolin, was extensively tested by Shipton & Coop (2012, 2015) who used Thames valley sand for the quartz fraction (Takahashi & Jardine, 2007), highlighting normal compression lines and critical state lines in the e:lnp' (e void ratio, p' mean normal effective stress) plane that they interpreted as parallel and dependent on initial density. The raw data and testing details for this soil are not repeated here, as only a few new tests were needed to fill gaps in the existing data and details of procedure and data are given in Shipton & Coop (2012, 2015). This soil is referred to as sand with plastic fines (SPF).

The other two soils were based on the fractally graded sands that Altuhafi & Coop (2011) showed had transitional, or non-convergent, compression behaviour in oedometer tests, even at stresses over 100MPa. The gradings of the quartz and crushed limestone sands were not quite the same as those Altuhafi & Coop used, because the large amounts of soil needed here meant that the finer end of the grading could not be controlled to be fractal, but was simply created by adding commercial quartz or calcium carbonate silts. The quartz sand was Leighton Buzzard sand (LBS) and the crushed limestone (LMS) was supplied from China. The coarser part of the grading was controlled to be fractal using the mass method between 63 and 600μm with a fractal dimension of 2.57, as for Altuhafi & Coop. The gradings of all three soils are shown in Fig.1. Soils that are nominally fractal, like this, may seem to be purely artificial, but Coop et al. (2004) found that intense shearing would give a terminal grading of this type and they occur naturally in tills sheared under glaciers (Altuhafi et al., 2010).

The tests carried out were relatively straightforward, a key motivation behind the work being to see what compatibility there is between the degrees of convergence in different types of simple test. The oedometer tests, summarised in Table 1, were generally carried out in 50mm diameter fixed rings, but to reach the highest stresses smaller diameters of 30 or 20mm were used, but these had a floating ring design to minimise wall friction.

Three different triaxial apparatus were used, each of a stress path type, one with GDS pressure/ volume controllers with a sample size of 50mm diameter and 100mm height and two using Imperial College pneumatic control systems with sizes of 50mm diameter and 100mm height and 38mm diameter and 76mm height. The volumetric strains were measured either with the GDS controller or Imperial College volume gauges and the axial strains both by local LVDTs attached to the samples and an external LVDT mounted outside the cell chamber. Because of the large strains needed to examine whether unique critical states could be reached, only the external LVDT data are presented here, but the internal strain data showed good agreement up to the point where they went out of their measuring range. Also because of the large strains, a suction cap (Atkinson & Evans, 1985) was used to hold the sample firmly to the axial loading system, ensuring that it remained upright and concentric. Good saturation, with B values over 96%, was achieved by carbon dioxide circulation prior to water saturation, followed by the application of back pressures over 200kPa. For the LBS and LMS soils shearing was started at 0.05%/h at axial strains below 0.1%, to achieve good definition of the small strain stiffnesses, although these data are not discussed here. The rates were then gradually increased to a maximum of 0.4%/hour at large strains. Similar strain rates were used in drained and undrained tests, but the LMS and LBS soils were free draining and the slower rates at the start of the tests were only to ensure that the small strain data could be collected. The strain rates were only increased gradually to avoid large accelerations, while reaching a speed that was fast enough to finish each test in reasonable time, each shearing stage test typically lasting 3-4 days overall.

Even if the bulk of the triaxial tests in Tables 2 and 3 were of a fairly standard type, several tests using lubricated end platens were carried out in the LBS to verify the conclusion of Shipton and Coop (2015) that they did not make a significant difference. This is not to say that lubricated end platens are not an important means of improving test quality, just that within the large void ratio differences seen by Shipton & Coop, their effect was secondary. The high compressibility of the soils during both isotropic compression and subsequent shearing meant that in some cases the strains that could be reached in shearing were limited by the available stroke of the apparatus, and these are highlighted in the tables.

The LBS and LMS samples were all made by dry compaction, using under-compaction, since water pluviation led to segregation and layering while wet compaction gave rise to high suctions in the LMS. Some additional tests to fill gaps in the data for SPF utilised moist compaction or making samples by compression of a slurry, but as Shipton & Coop (2015) showed, the preparation method does not affect whether or how quickly the volume states converge in this soil. All of these samples were visually homogeneous. The compaction used for each sample varied because a wide range of different initial void ratios was required, and it was not required for all of them to have the same initial value. The initial void ratio of each sample is given in Tables 1-3.

A particular problem with transitional soils has been the identification of the fabric that gives rise to the slow convergence, particularly since the wide range of particle sizes means that it is difficult to know at what scale to examine the fabric with techniques such as SEM (e.g. Nocilla et al., 2006). However, Mercury Intrusion Porosimetry allows a wide range of scales to be investigated simultaneously, and Todisco et al. (2018) used this method to determine that for each soil discussed here there is a micro-fabric that is difficult to break down in conventional laboratory tests. Isotropy of the shear moduli measured with bender elements also indicated that the fabrics are isotropic and so they must be heterogeneous at the micro-scale.

Quantifying convergence is critically dependent on the accuracy of the void ratio measurements. The philosophy adopted here, as in previous similar work on transitional soils, was that it was not adequate just to evaluate that accuracy from the estimated accuracy of individual measurements made, such as weights or dimensions, which typically gives an optimistic assessment. Instead, a positive verification is made by means of multiple measurements of void ratio on the same sample, utilising dimensions and weights both at the start and end of each test that were as independent as possible, along with the measured volumetric strains during testing, as described by Rocchi & Coop (2014). At least three measurements were therefore made of the initial void ratio of each sample and the accuracy estimated from the difference between the highest and lowest value, discarding tests where the accuracy was worse than ±0.03. The specific gravities measured were respectively 2.61, 2.72 and 2.64 for the LBS, LMS and SPF.

Compression and Shearing Data

Isotropic and oedometric compression data are given in Fig.2 for the LBS and LMS soils. The data for the SPF soil were presented in detail by Shipton & Coop (2015) and are not repeated here. In both cases the compression curves steepen slowly and in neither case are there well defined yield points and unique normal compression line as might be expected in, for example, a uniformly graded sand at higher stresses (e.g. Coop & Lee, 1993). In isotropic loading there is little convergence of the compression paths, but in one-dimensional loading there is more, partly because of the higher stresses reached, but also because yield in oedometric compression would be expected at a lower stresses than for isotropic. However, there is little space remaining at very high stresses in the $e{:}\log\sigma'_v$ plane between the ends of the oedometer tests and the zero asymptote for any normal compression line to exist. The isotropic compression tests with and without lubricated end platens do not differ significantly within the context of this lack of convergence.

Example shearing stress-strain data for the LMS soil are given in Fig.3; space precludes giving all the data for the LMS soil and so tests with a range of effective cell pressures, different initial void ratios and both drained and undrained are given. The raw data for the LBS are also not shown, but these were very similar in nature. For brevity only the volumetric strains ε_v for the drained tests are given, since the undrained stress paths are given in Fig.4. The shear strain, ε_s, has been defined as:

$$\varepsilon_s = \varepsilon_a - \varepsilon_v/3 \qquad (1)$$

where ε_a is the axial strain. In each case an indication of the range of initial void ratio, e_i, values for the various tests is given so that effects of sample density may be identified more easily. For clarity the stress-strain data are separated into separate plots for looser and denser samples. There is of course some scatter in the data and some tests were less complete than others due to apparatus limitations. However, most of the tests do reach large shear strains and in each case the volumetric strains become reasonably constant.

While the key point of discussion of this paper is the lack of convergence to unique volumetric states, with some scatter the loose and dense samples do reach unique stress ratios. But perhaps the most noticeable feature of the data is that there is a surprisingly small range of behaviour for the different densities, most of the samples being mildly compressive with relative small volume changes and no peak strengths. None of the samples showed any clear visible strain localisation, as might be expected from their compressive, strain-hardening mode of behaviour. The lack of diversity of behaviour will be shown to be related to the slow convergence towards a unique critical state line. But this should not be thought typical of all well-graded soils and using similar simple apparatus and techniques the usual clear unique critical state line can generally be observed in the e:lnp' plane (e.g. Vilhar et al., 2013), with behaviour ranging from strain-hardening and compressive to dilative and strain-softening, depending on the initial state parameter.

The paths followed by the tests are given in the q':p' and e:lnp' planes in Figs.4 and 5. A unique critical state line is clearly defined in stress space, irrespective of initial density, the final value of q'/p' = M being reached at only 15-20% axial strain. The stress paths for the higher pressure tests are omitted on Fig.4 for brevity, but they defined the same M values. In contrast, the paths followed in the volumetric plane do not converge to a unique critical state line within the range of strains that could be applied, even allowing for a few highlighted tests that are less complete. The volumetric strains were reasonably constant at the end of almost all of the tests, so it is difficult to see what strains might be needed to achieve full convergence. The lubricated end platens used for some of the LBS tests do not make a significant difference to this pattern of behaviour. The degree of convergence does seem better at larger stresses but it is clear that fully convergent paths could not be expected from triaxial tests until stress levels in the 100s of MPa, at which point there would again be little available room in the e:lnp' plane to fit a useful critical state line before the zero asymptote is approached.

Observing similar behaviour for the SPF soil, Shipton & Coop (2015) made the pragmatic choice to define pseudo critical states at the end of test states, so that a different critical state line could be identified in the e:lnp' plane for groups of samples with similar initial void ratios. The approach adopted here is to avoid any such controversial choices, and simply to quantify the rate at which convergence is occurring so that it can be estimated when unique volumes might be achieved.

<u>Quantifying Convergence</u>

To cope with data that were similarly difficult to interpret, Ponzoni et al. (2014) proposed quantifying convergence using two methods. The first was simply to take the starting and ending void ratios of oedometer tests and quantify to what extent initial differences of void ratio were preserved at the highest stress reached. This was done by plotting the initial void ratios against those at the highest stress level reached, and calculating a gradient of that graph. Convergence to a unique normal compression line would mean that the final void ratios would be independent of the initial values so this gradient would be zero, while compression paths that had no convergence at all would give a gradient of 1, since the final void ratios would be directly dependent on the initial values.

The second method was to take the apparent critical states from the ends of triaxial tests where reasonably constant volumes had been reached and assume that tests on samples that had had different initial void ratios had reached different critical state lines at the end of shearing in the e:lnp' plane. The gradients of the critical state lines, λ, were all assumed to be the same, but each with a different intercept at 1kPa, Γ. The method then quantified how the Γ values changed with the initial void ratios of the samples.

There were several difficulties of these methods. Firstly it had been assumed, if only pragmatically, that e:lnp' critical state lines did exist, even if it was recognised that additional straining might cause more convergence. Secondly, these were assumed to be linear, which they might not be. Both methods were tied to quantifying convergence at specific states of the tests, i.e. the maximum stress reached in the oedometer or the maximum strain in the triaxial, rather than quantifying the progression of convergence as the tests proceeded. There was also no means of direct comparison between the two methods. The method proposed here overcomes these difficulties.

Quantifying Convergence in Compression

The rates of convergence are first discussed for the oedometric compression data. The method is identical for isotropic compression and the graphs for these stages are omitted for brevity as there is less convergence for them because of the lower stresses they reached. For each soil, in Fig.6 the current void ratio at any load level of a test is plotted against the initial void ratio, e_i, which is generally taken to be as close to a p' of 20kPa as possible for both the oedometer and triaxial tests for consistency. In some cases, notably the high pressure triaxial tests the initial stresses were a little higher that 20kPa, but the compression curves show that this makes only a small difference in the void ratio. The values of p' for the oedometer data in Fig.6 assumed a coefficient of earth pressure at rest, k_0=1-sinφ'. For each soil, examples are given of data for four different load levels; many more were considered but they are omitted from Fig.6 for clarity. For each stress level a best fit "convergence line" is determined by linear regression, the gradient of which, m, is calculated, the equations being given on the plots. This is similar to the method of Nocilla et al. (2006) and Ponzoni et al. (2014), the only difference being that the value of m is calculated at the various stress levels during the tests rather than solely at the end.

The values of the gradient m in Fig.6 decrease fairly consistently with increasing stress level indicating a slow but consistent convergence; m=0 would indicate full convergence to a unique normal compression line, but in no case is this reached. Within the slight data scatter there is no noticeable effect of the method of sample preparation for the SPF, as was concluded by Shipton & Coop (2015). In each case the data are well fitted by straight lines, but it is possible that this might not be the case for all soils and all possible values of e_i.

For the oedometer tests linear regression fits the data quite well on Fig.6, the data scatter is acceptable, and the decrease of gradient with increasing stress level is consistent. The same was true for the isotropic compression data. However, for the triaxial shearing data it is far more difficult to achieve such consistent data, as will be discussed below, and the gradients of the convergence lines were not so consistent, so that attempts were made to constrain the values. One possible means of constraint, shown for two example stress levels for each soil, is to force all of the chosen lines to pass through the origin. The equations for these are also shown on the figures. This constraint gives lines for the oedometer tests that fit reasonably well for the LMS and SPF soils, but poorly for the LBS. The consequence of such an assumption is that complete convergence of the compression paths for different initial void ratios to reach m=0 would only occur as the zero voids asymptote is approached, which may be correct for some soils but may not be for others.

The values of m from the isotropic and one-dimensional compression tests are plotted against log p' in Fig.7. Gradients that are for lines not constrained to pass through the origin have solid symbols and those for lines constrained to pass through the origin open symbols. The values of each generally reduce fairly consistently with p'. There is some scatter at higher stresses for isotropic compression because there were too few tests reaching these stresses, high pressure triaxial tests being rather more difficult to conduct than oedometers. As noted above, the effect of the constraint is significant for the oedometer tests on LBS, but less so for isotropic compression on the same soil. In each case a constraint to pass through the origin has the effect of increasing the gradient, which might be expected since the intercepts should not be negative because the current void ratio should always be lower than the initial value.

Quantifying Convergence in Shearing

For triaxial shearing similar gradients, m, were calculated at fixed values of shear strain, (0.1%, 1%, 5%, 10%, 15%, 20% and 30%) grouping data for tests that have similar current p' values at those strains. Figure 8 shows the data for ε_s^p=10%. This is the plastic component of shear strain, the elastic component, which was very small, having been deducted based on elastic shear moduli from bender element data. For the calculation of the plastic volumetric strains, ε_v^p, that are used later, a Poisson's ratio of 0.3 was assumed in the absence of any measurements of the elastic bulk modulus. All of the unconstrained lines with intercepts as well as the lines constrained to pass through the origin are shown, along with their equations, but examples are only given for a few stress levels in each figure for clarity, the calculation being repeated for other stress levels. In general the gradients of the lines, m, tend to reduce as stress level increases, although this is less clear for the LBS. The m values also decrease as the ε_s^p increases, but space preclude showing more examples. Using this method, data points may be plotted for any stress path, and the data for drained and undrained tests are not highlighted on the plot, since they were indistinguishable. The use of lubricated end platens for some of the LBS tests does not have any noticeable effect within the data scatter, as highlighted in Fig.8a.

The data in Fig.8 are quite scattered, especially for the unconstrained gradients, even if 20-30 triaxial tests were carried out for each soil. This gives some inconsistent trends in the change of m with p'. The m values are, however, lower than those for isotropic compression, as they should be since shearing can only give additional convergence beyond that achieved in the isotropic compression prior to shearing. The amount of test data needed to reduce the scatter significantly would therefore be prohibitive, so some form of data conditioning is needed. While it may be unfair to constrain the lines to pass through the origin, as discussed above, some form of constraint is required to avoid so much scatter in the shearing data.

Applying Constraints to the Convergence Lines for Shearing

It might be expected that the regression lines on Fig.8 that are not constrained to pass through the origin should move consistently as the shear strains increase. On Fig.9 the intercepts of these lines for all strain levels are given for the LMS, to give one example, showing data for all the stress levels considered, not just those that are on Fig.8b. At first sight the intercepts look quite scattered. This arises from small inaccuracies of void ratio measurement, even if this was carefully controlled and so was small. If the accuracy of void ratio cannot be improved, which would be difficult, then the only way to reduce this scatter of the intercepts is to carry out even more tests, which becomes prohibitive. Nevertheless, there are important features in the data that can be seen. At each strain level a mean intercept is shown for all the stress levels. It might be expected that the intercepts should vary systematically rather than jumping randomly and it is noticeable that generally the mean values do vary quite consistently. However, the data have been further conditioned by drawing trends through the mean values, and it is these trend lines that were then used in the analysis. If these intercepts are constrained to vary systematically then the gradients will also. These trend lines on Fig.9 been assumed to be straight for convenience and because the data scatter does not allow a better choice. The trend lines have a few constraints, for example that they may not increase indefinitely or decrease below zero. Having chosen these lines, intercepts for the convergence lines are calculated for each value of shear strain from the line, not the mean value data point at that strain. New gradients m are then calculated forcing the regression lines on graphs like those shown in Fig.8 through the imposed intercept. These new lines that are forced through the chosen intercepts are not shown on Fig.8 to avoid clutter, but they are quite similar to the unconstrained lines.

The impact of the constraint to the intercept on the gradients m for the shearing data are illustrated for the LMS soil in Fig.10. The completely unconstrained values, identified with open symbol type data points are very scattered. A hard, and perhaps unrealistic constraint, of making all the chosen lines pass through the origin does of course give gradients that are much more consistent, as shown by the cross-type symbol data points. However, this constraint also changes the overall trend, increasing the m values significantly. Instead, the proposed "soft" constraint of imposing intercepts calculated from graphs like Fig.9 helps to reduce scatter while not changing the overall trends and values significantly, as shown by the grey filled data points. In each case, however, while m does reduce with increasing shear strain, the m values are far from reaching the value of zero which would indicate a unique critical state line in the e:lnp' plane. Similar constraints to have consistently evolving intercepts on the regression lines used to calculate m could also be used for the oedometer and isotropic compression data, but it was not found necessary and the unconstrained values could be used.

Convergence Surfaces

To compare the m values for different types of tests, the assumption made is that the degree of convergence will depend simply on the plastic volumetric and shear strains that are applied to the soil, and not on the apparatus applying them. A similar dependence on plastic strains is made in the damage functions that define the rates of destructuration for many models for natural soils (e.g. Kavvadas & Amorosi, 2000), which seems appropriate since the slow convergence is known to result from the difficulty in breaking down the initial fabric. The strains used are cumulative from the start of each test at p'=20kPa since it is the overall strain that the soil has experienced that should determine the breakdown of structure.

To construct a convergence surface a three-dimensional graph is drawn relating the m values to the plastic shear and volumetric strains, ε_s^p and ε_v^p, using all three methods of deriving m, from isotropic compression, oedometric loading and triaxial shearing. This is shown for the LMS in Fig.11 using the m values that were constrained with the hard constraint so that the convergence lines for all types of loading all pass through the origin of the convergence graphs. This is shown in preference to the completely unconstrained data since the data for completely unconstrained m values are too scattered in shearing.

The resulting graph is difficult to understand and so an annotated version is shown in Fig11b. The graph is quantifying how quickly tests on samples of different initial void ratios approach convergence to unique void ratios, for example on a unique normal compression line (isotropic or one-dimensional) or unique critical state line, when m will be zero. Isotropic compression runs from the start of the surface at zero strains and m=1 (no convergence yet) almost following the zero shear strain axis. For isotropic compression tests estimates of the shear strains were calculated using the measured volumetric strains combined with the axial strains from the internal axial strain transducers, although those shear strains were of course very small.

For the oedometer tests the ratio of total volumetric to shear strains is fixed, and the path of the data points therefore lies diagonally across the graph at a ratio of strains of about 2/3 with m decreasing as the tests proceed. The ratio is not quite 2/3 because here we plot plastic, not total strains. For the isotropic and oedometric compression each point represents all of the tests conducted on that soil and so a strain must be assigned to each the m values calculated, for example, from Fig.6b. The value chosen is the mean strain reached by all the tests at that stress level, since the variation between tests was not large.

The triaxial shearing data define a series of points at the chosen shear strain values, running across the graph with m decreasing as shear strain increases, while the volumetric strain also increases but by much less. The starting point for triaxial shearing is after isotropic compression has been applied, so the paths will start on the isotropic compression path, tests at lower stresses starting closer to the start of the graph at m=1 and zero strains, while high pressure triaxial tests will have already had some significant reduction in m during isotropic loading.

A surface has been fitted through all of the data points and, given the data scatter, this has simply been assumed to be an inclined flat surface. To help visualisation in a two-dimensional image, firstly the coordinates of the corners of the surface are highlighted, and then the locations of the data points are clarified with a vertical line extending from each point to the surface, points lying below the surface having a solid line and those above a dashed one. In fitting the surface, least squares regression was used but weighting was applied to each data point for the number of tests used to derive it, so greater weight is given to the oedometer data points. The plane chosen has been constrained to pass through m=1 at zero strains, as it must do. There is no reason why the surface must be flat, and it would be expected, for example, that if it does approach m=0 then it would become asymptotic to that boundary since it cannot cross it. But to define the precise curved shape would again require a very large number of tests and for the present purpose there are data both above and below the chosen plane, so it seems to be a reasonable choice.

Some of the LMS data plot with small negative ε_v^p values because the small plastic volumetric strains during isotropic compression were less than the dilation during shear. This illustrates one defect of the current formulation, which is that plastic straining should destructure whether it is positive or negative and some means of combining them better needs to be devised, which be, for example, some form of work done, but this is beyond the current scope and will not frequently be a problem for transitional soils that are generally compressive in shearing. However, a Cam Clay style work equation would be unlikely to work because the surface clearly shows that convergence from the breakdown of the initial fabric is brought about much more rapidly by volumetric than shear strains and in this respect is similar to some destructuration models for natural clays (e.g. Kavvadas & Amorosi, 2000).

Each type of test has its own defects and inaccuracies, for example wall friction in oedometer tests or in a triaxial the effects of end restraint or strain localisation. It was shown above that within the large differences of void ratio caused by lack of convergence, the effects of end restraint are not significant, but Fig.11 provides further confirmation that it is not test defects that inhibit reaching unique states since the m values at given strains are broadly similar for different types of test. The key feature of the plot is that the degree of “incompleteness” of the tests is explicit and it is clear, for example, that continued shearing in a triaxial test will not bring about convergence.

As was highlighted above, using constrained values of m with zero intercepts imposed, may not be a desirable assumption, even if the lines derived fit the data reasonably well in Figs.6 and 8. Figure 12 therefore shows surfaces for all three soils that are plotted using the m values that are partially constrained or have the "soft" constraint for shearing. Here the completely unconstrained values are used for isotropic and oedometric compression, but for triaxial shearing the intercepts are constrained to change consistently, as in Fig.9. Relaxing the constraint in this way does make m more sensitive to shear strain for the LMS, but it is still the volumetric strain that dominates convergence. It can also be seen that at high shear strains the data points are tending to plot above the flat planes, indicating that they should perhaps curve to shallower gradients with respect to ε_s^p. The key features of the graphs remain, though, that simple tests of the type carried out here, even at high pressures, will never bring about convergence, and will never give unique normal compression or critical state lines for these types of soil. If convergence were to be found, it would require enormous stress and/or strain levels, well beyond most engineering relevance and possibly so close to the zero void boundary as to make it difficult to define any useful normal compression or critical state line.

Conclusion

Transitional behaviour is a controversial subject that can provoke entrenched positions, as have often been experienced by the authors. Not least, the name of this mode of behaviour is unhelpful as it risks confusion with other uses of "transitional" in soil mechanics, such as transitional fines content. The terminology is difficult to change and it may be that the two uses are related through the grading. The use of "transitional" here was originally supposed to indicate a transition between clay and sand modes of behaviour, although this has probably become obsolete as a justification as more examples are found. This paper has tried to present a means of quantifying rates of convergence towards unique volumetric states in compression or shearing, which should help focus the discussion. Using this method it is clear that for the three soils presented there is no possibility that simple laboratory element tests such as triaxial or oedometer tests can get anywhere near full convergence of the volumetric states such as to give useful unique normal compression or critical state lines. It is also clear that this is not the result of defective testing techniques but must be related to the persistence of fabric effects, as highlighted by Todisco et al. (2018). If we wished to bring about that convergence we would need other apparatus able to impose very much larger strains and/or stress levels which may then have little relevance to engineering practice. Finally, while the soils tested here may be somewhat unlikely artificial soil gradings, it should be recalled that transitional behaviour has been observed in many natural soils and so this unifying method will be a useful new tool to interpret data from such soils. Of course the technique could also be used for soils that are not transitional, and would be useful in assessing how quickly unique normal compression lines or critical state lines are reached and in understanding whether data are consistent between different forms of loading.

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NOMENCLATURE

e	void ratio
e_i	initial void ratio
k_0	coefficient of earth pressure at rest
LBS	Leighton Buzzard sand
LMS	crushed limestone
M	stress ratio at critical state
m	convergence parameter
p'	mean normal effective pressure

q deviatoric stress

SPF sand plastic fine (75% sand – 25% kaolin)

ε_s shear strain (ε_s^p plastic component)

ε_v volumetric strain (ε_v^p plastic component)

Γ intercept of critical state in e:lnp' plane at p'=1kPa

λ gradient of critical state or normal compression lines in e:lnp' plane

φ' angle of shearing resistance

σ'_v vertical effective stress

Table 1 Details of oedometer tests on LBS and LMS samples.

Type of soil	Initial void ratio, e_i	Final void ratio, e_{final}	Accuracy of the initial void ratio
LBS	0.597	0.322	±0.015
	0.389	0.292	±0.019
	0.534	0.303	±0.003
	0.453	0.298	±0.012
	0.411	0.291	±0.003
	0.378	0.297	±0.001
LMS	0.812	0.328	±0.017
	0.611	0.253	±0.023
	0.512	0.273	±0.002
	0.537	0.317	±0.021
	0.645	0.249	±0.005
	0.511	0.202	±0.002
	0.410	0.230	±0.007
	0.431	0.248	±0.009
	0.359	0.225	±0.005

Table 2 Details of triaxial tests on LBS samples

Test no.	Initial void ratio, e_i	Void ratio end of shearing	p'compression [kPa]	p'end of shearing [kPa]	Accuracy of e
LBD1	0.582	0.484	100	180	±0.014

LBD2*	0.583	0.472	500	930	±0.014
LBU3	0.555	0.500	500	460	±0.006
LBD4	0.551	0.466	500	940	±0.004
LBD5	0.524	0.445	500	950	±0.01
LBD6	0.544	0.461	1000	1830	±0.03
LBD7[+]	0.472	0.443	100	190	±0.04
LBD8[+]	0.435	0.422	100	200	±0.003
LBD9	0.420	0.411	100	190	±0.01
LBD10	0.435	0.369	300	550	±0.02
LBD11	0.497	0.420	500	990	±0.013
LBD12*[+]	0.457	0.386	500	1100	±0.007
LBD13	0.437	0.388	500	840	±0.007
LBU14	0.412	0.358	500	380	±0.01
LBD15	0.436	0.281	5300	9680	±0.003

* sheared to less than 10% shear strain, [+] lubricated ends.

Table 3 Details of triaxial tests on LMS samples

Test no.	Initial void ratio, e_i	Void ratio end of shearing	p’ compression [kPa]	p’end of shearing [kPa]	Accuracy of e
LMD1	0.605	0.555	50	80	±0.003
LMU2	0.607	0.520	720	475	±0.008
LMU3	0.548	0.511	95	40	±0.012
LMD4	0.549	0.453	300	650	±0.004
LMD5	0.506	0.432	300	610	±0.008
LMD6	0.538	0.410	570	1250	±0.022
LMD7	0.531	0.382	1020	2240	±0.010
LMD8	0.513	0.341	3080	6250	±0.007
LMD9	0.445	0.453	50	130	±0.01
LMD10	0.415	0.416	50	130	±0.007
LMU11	0.409	0.405	50	750	±0.007
LMD12	0.464	0.422	100	170	±0.026
LMD13	0.395	0.373	200	930	±0.002
LMD14	0.388	0.368	200	490	±0.002
LMD15*	0.492	0.389	500	520	±0.05
LMD16	0.471	0.356	470	1050	±0.012
LMD17	0.425	0.387	500	1140	±0.005
LMU18	0.390	0.356	500	1630	±0.007
LMD19	0.415	0.288	2400	5100	±0.01
LMD20	0.373	0.234	3880	7860	±0.009
LMD21	0.350	0.300	1000	2150	±0.008

*constant p' shearing.

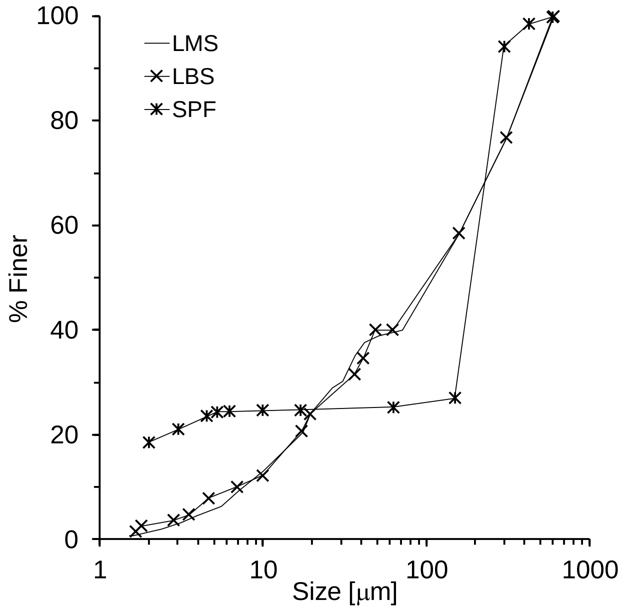
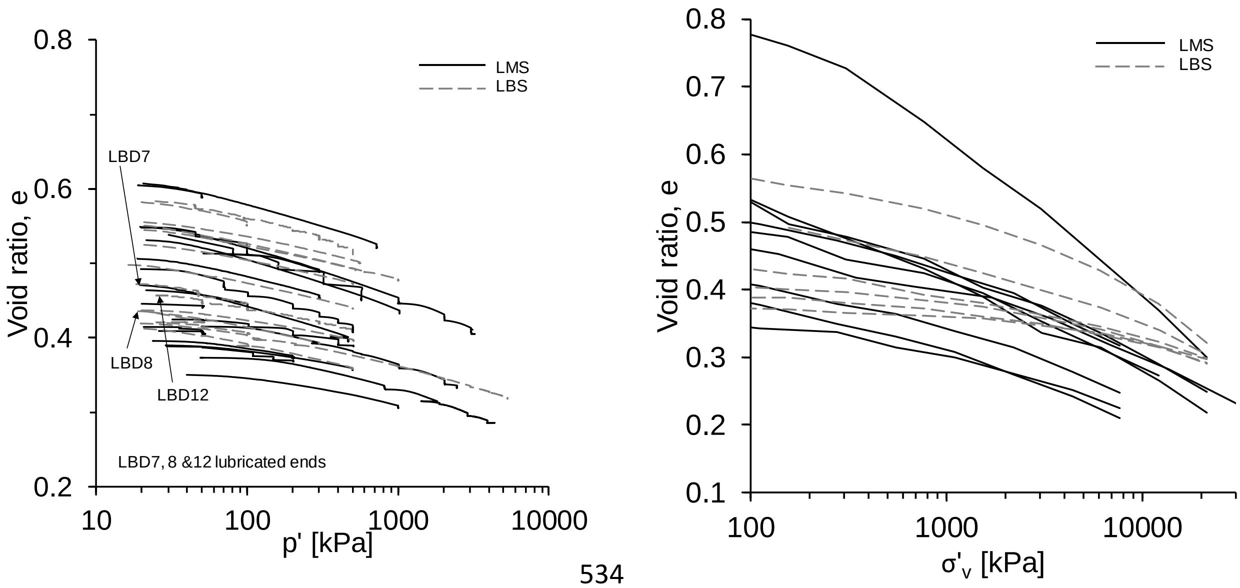
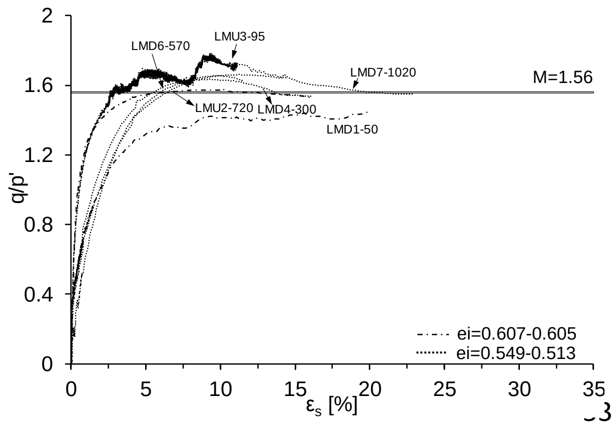


Fig.1 Gradings of the three soils.

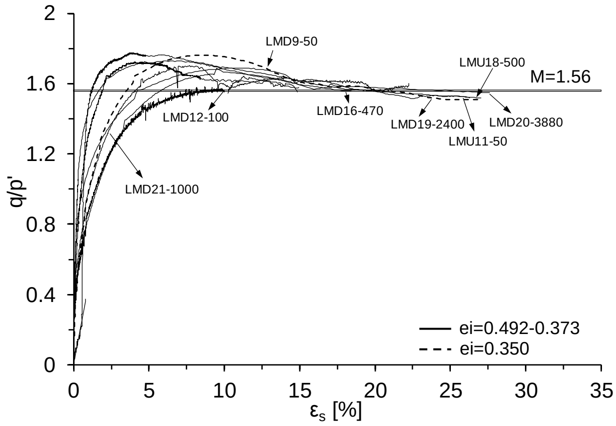


(a) (b)

Fig.2 Compression data for LMS and LBS soils, (a) isotropic, (b) oedometric



(a)



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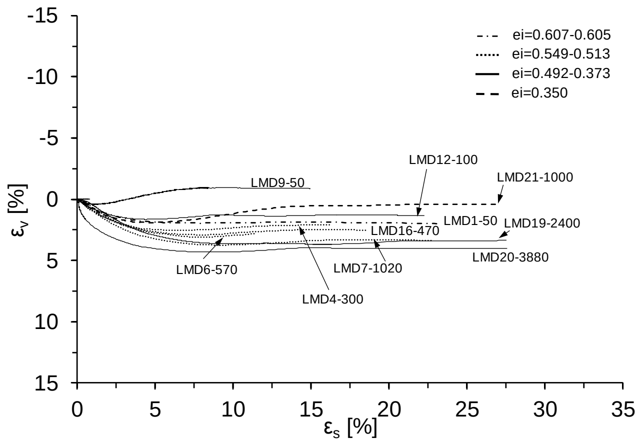
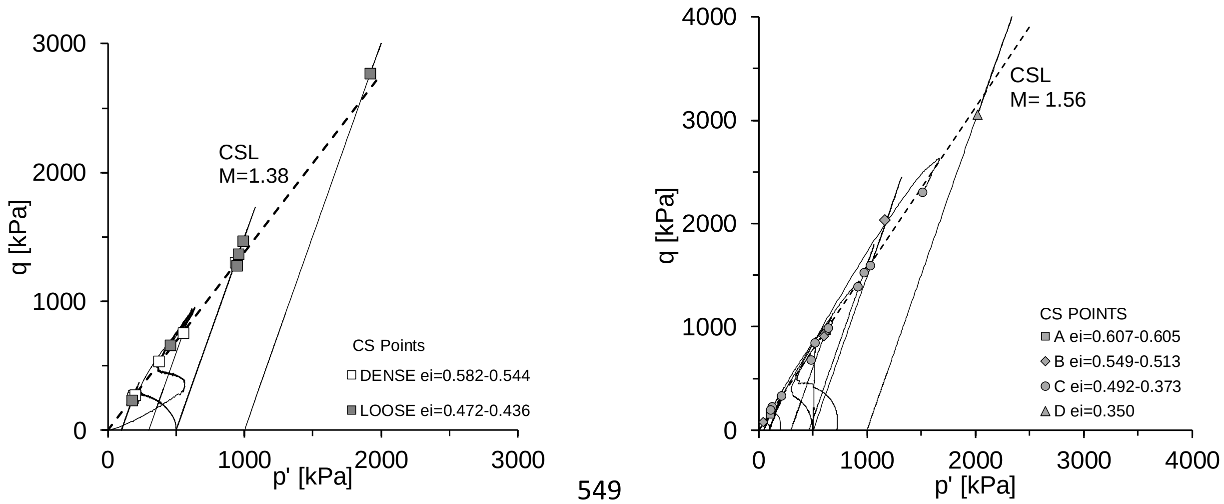


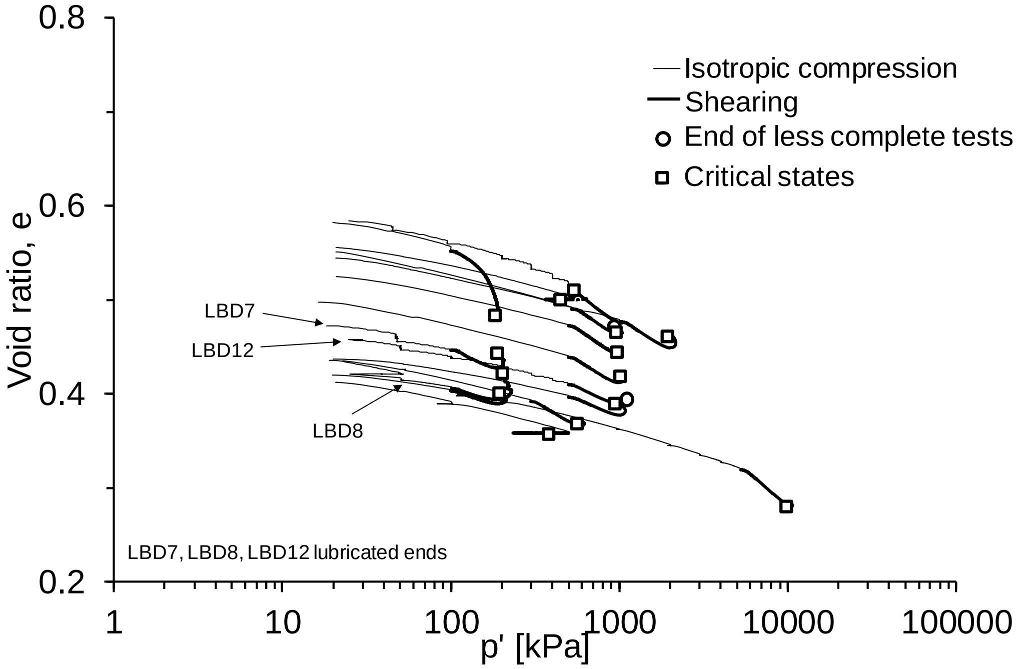
(c)

Fig.3 Example triaxial stress-strain data for the LMS soil, (a) stress ratio for looser samples, (b) stress ratio for denser samples, (c) volumetric strains for drained tests on samples of all densities.

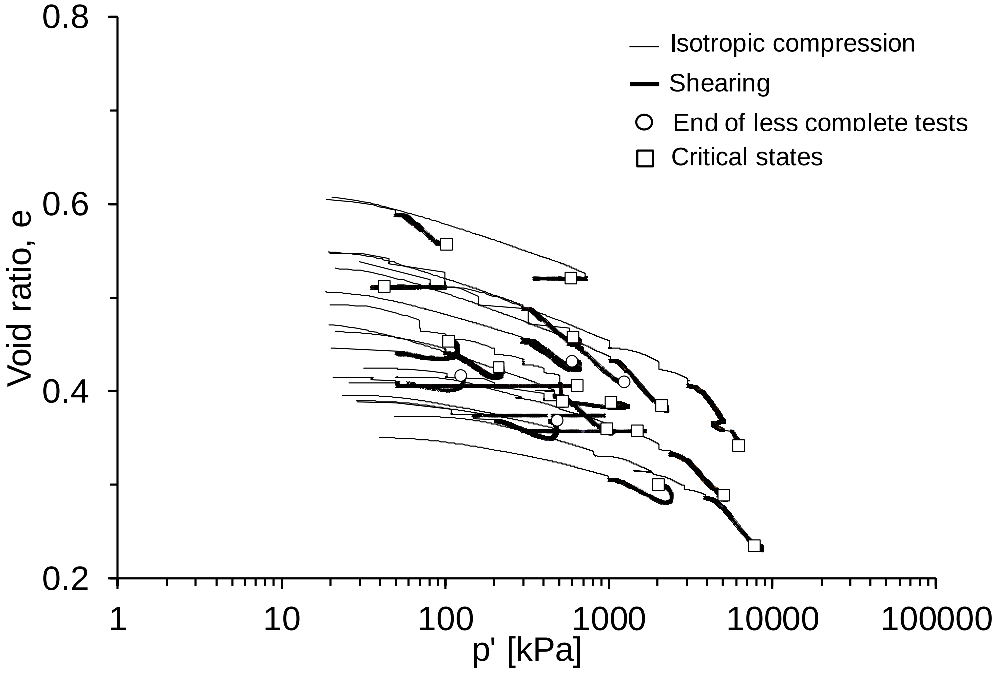


(a) (b)

Fig.4 Stress paths (a) LBS (b) LMS

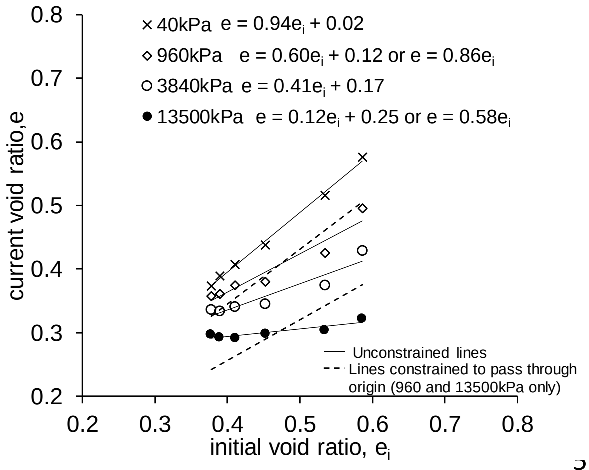


(a)

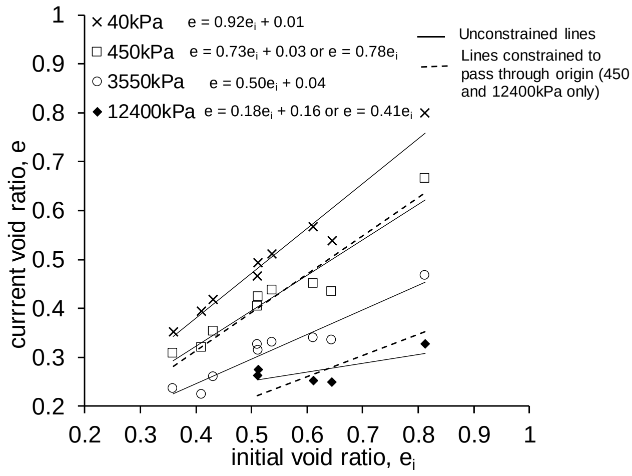


(b)

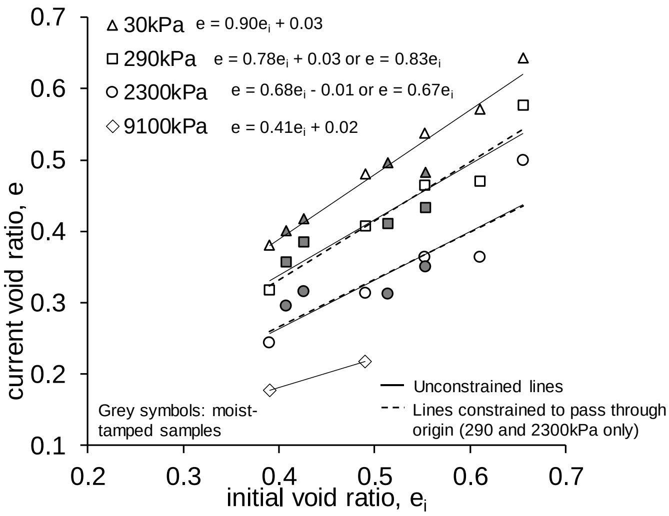
Fig.5 Paths followed in the volumetric plane (a) LBS (b) LMS



(a)

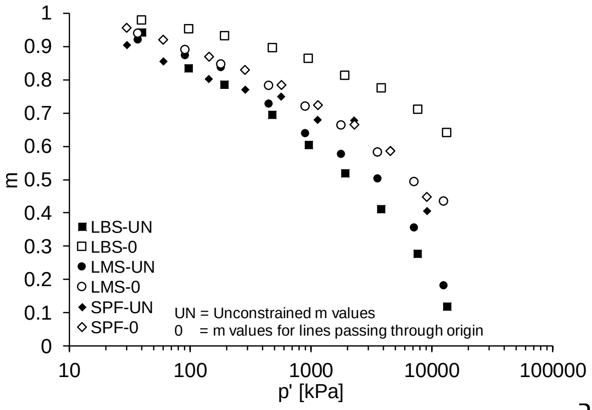


(b)

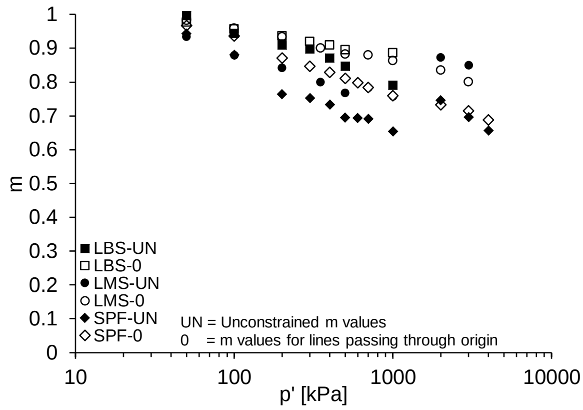


(c)

Fig.6 Convergence lines for oedometer tests on the three soils, (a) LBS, (b) LMS and (c) SPF

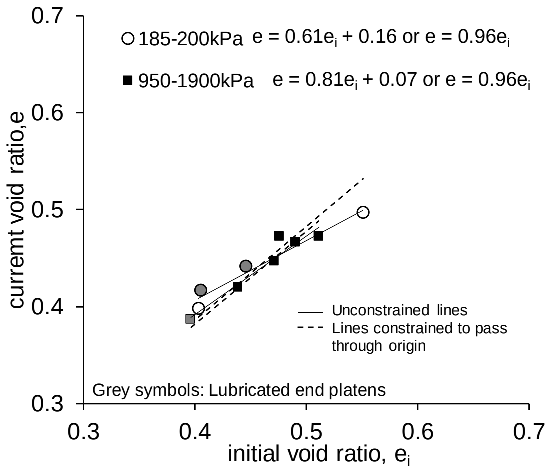


(a)

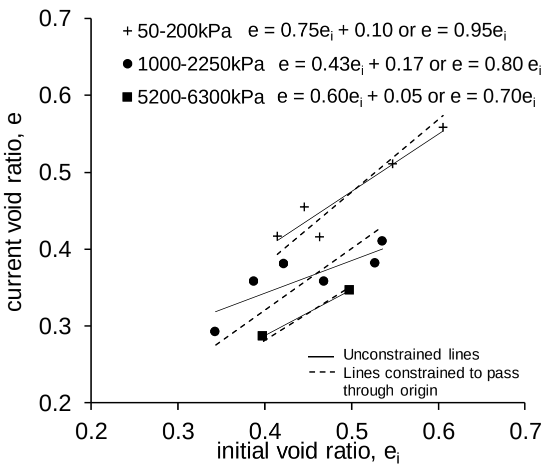


(b)

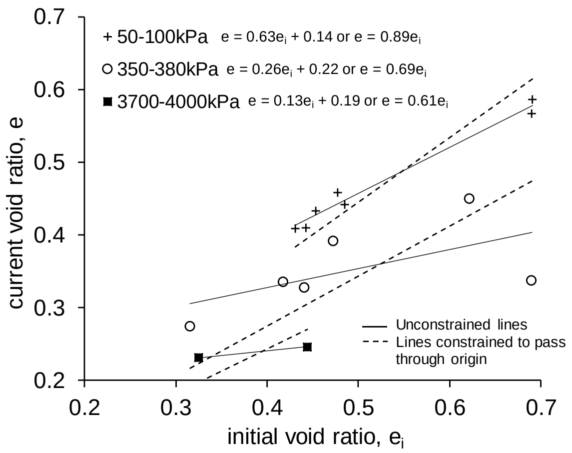
Fig.7 Summary of m values for (a) one-dimensional and (b) isotropic compression



(a)



(b)



(c)

Fig.8 Convergence lines at $\varepsilon_s^p = 10\%$ during triaxial shearing of the three soils, (a) LBS, (b) LMS and (c) SPF

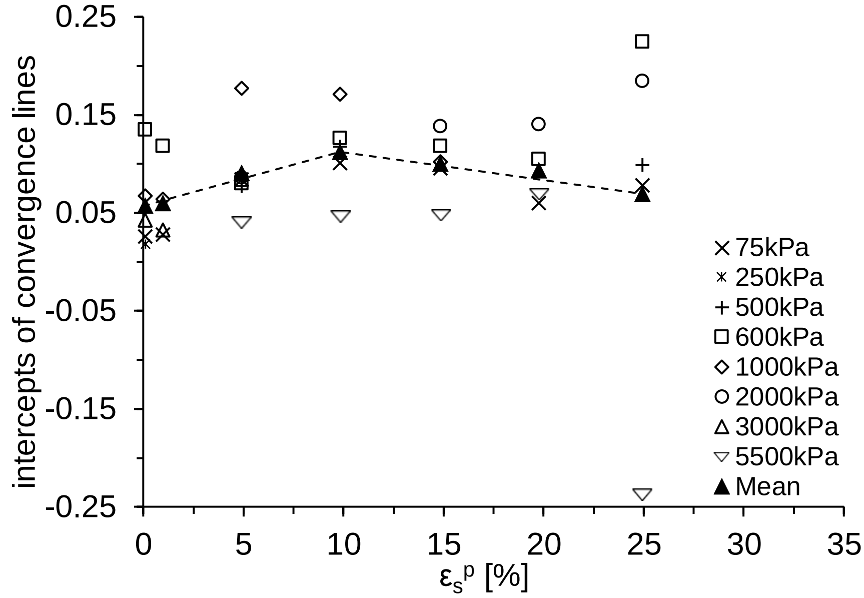


Fig.9 Evolution of intercepts of shearing convergence lines with shear strain for LMS.

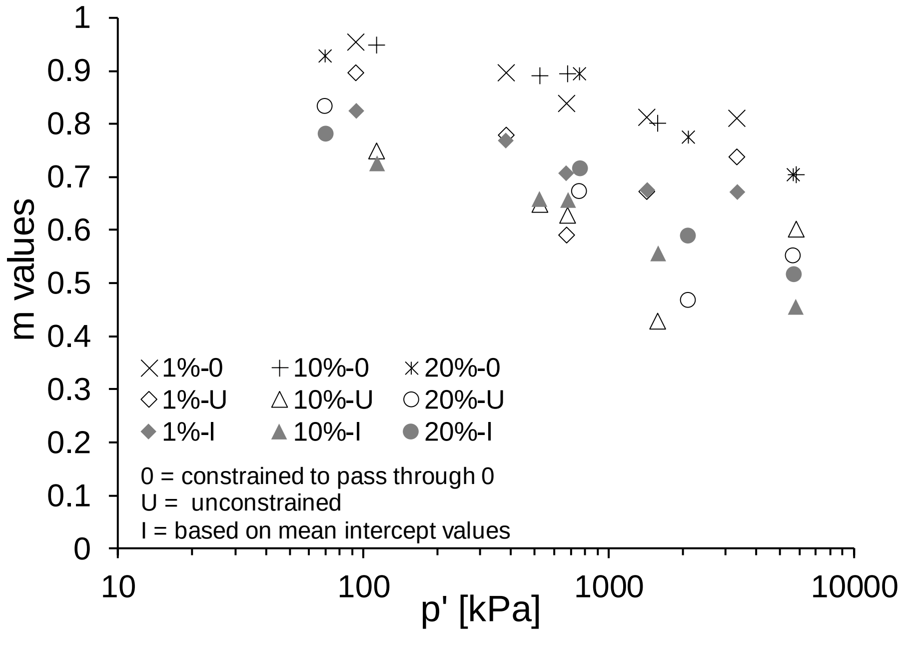
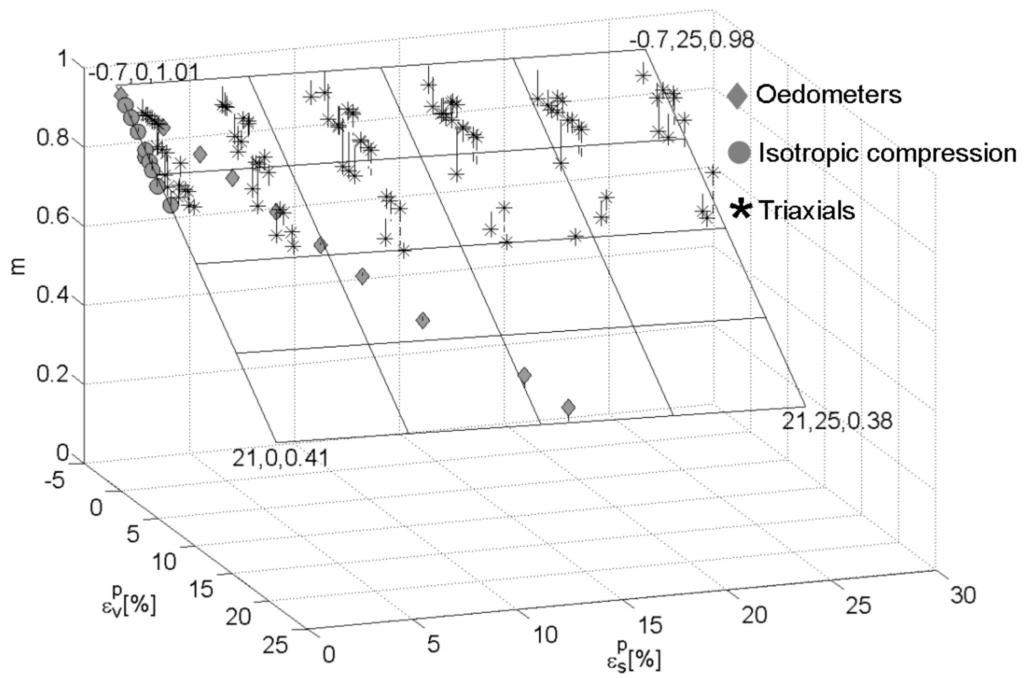
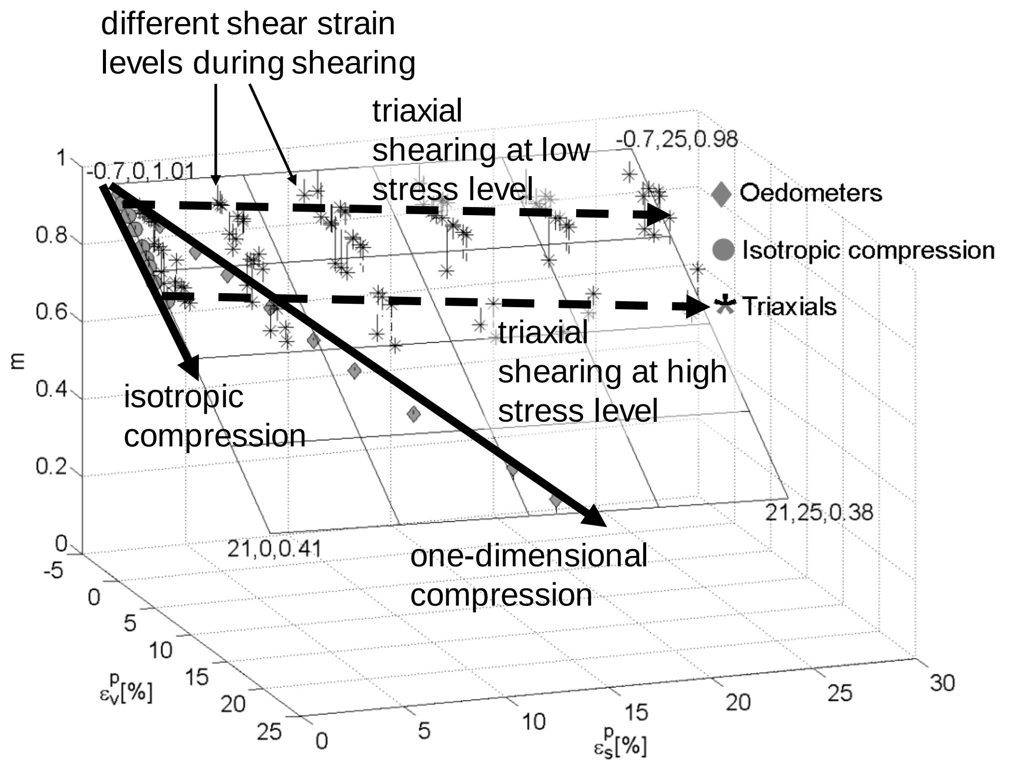


Fig.10 Evolution of shearing m values with stress level and shear strain for LMS.

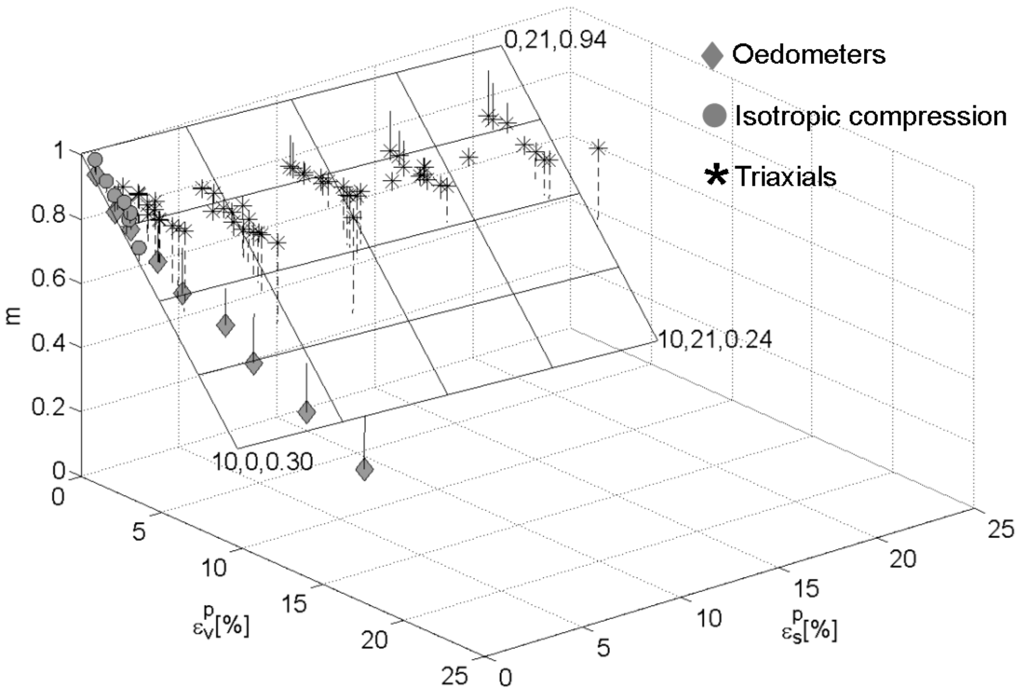


(a)

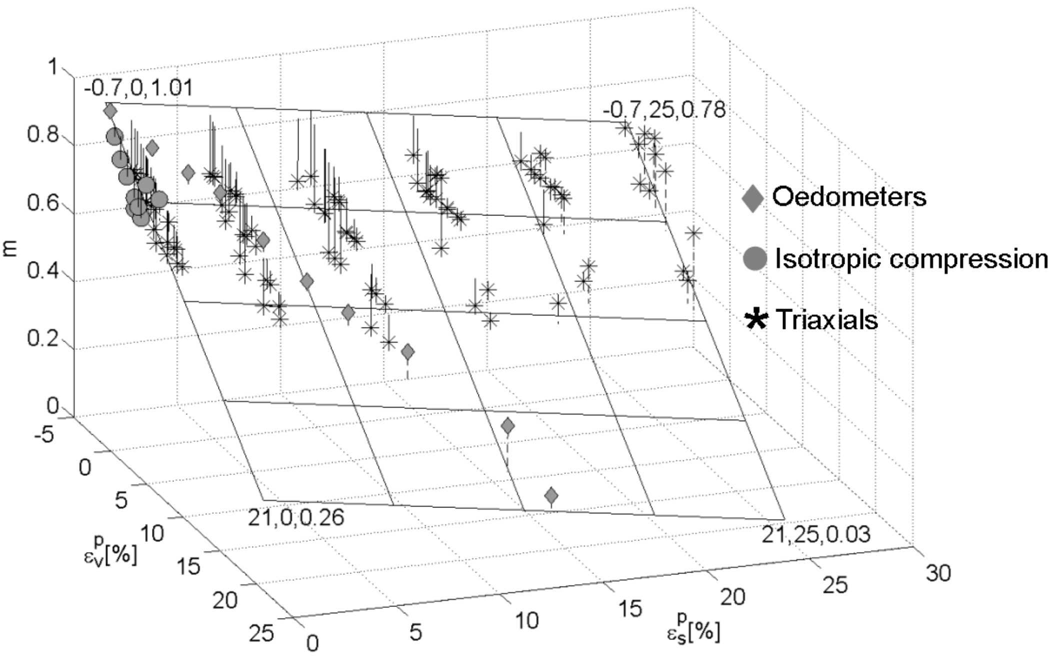


(b)

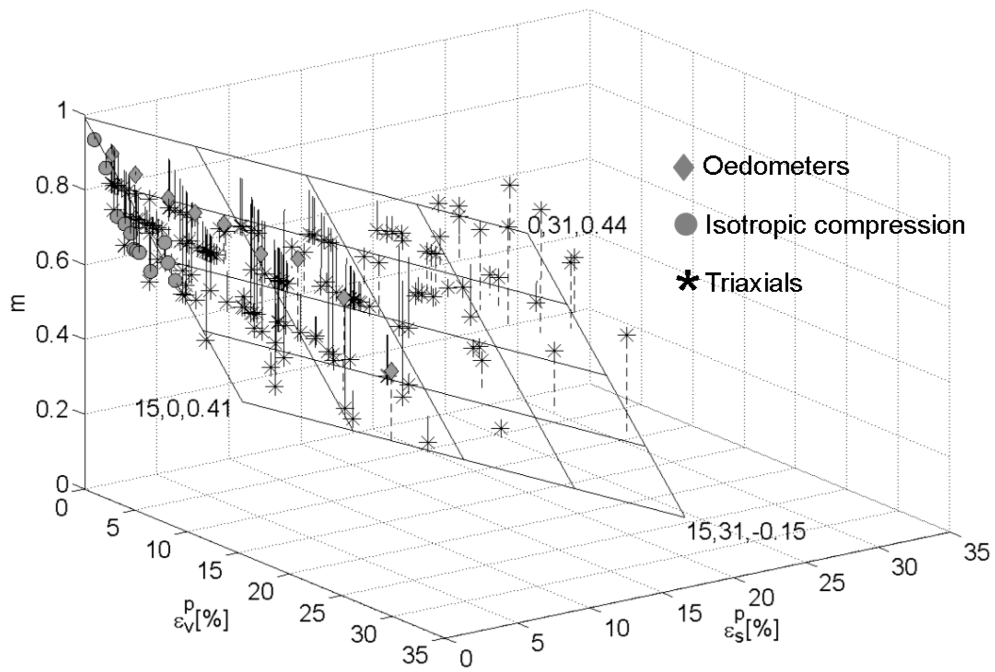
Fig. 11 (a) Convergence surface for constrained m values of LMS, (b) surface with explanatory annotation.


Oedometers
Isotropic compression
Triaxials
0,21,0.94
10,21,0.24
10,0,0.30
m
ε_v^p[%]
ε_s^p[%]

a)


Oedometers
Isotropic compression
Triaxials
-0.7,0,1.01
-0.7,25,0.78
21,0,0.26
21,25,0.03
m
ε_v^p[%]
ε_s^p[%]

b)



c)

Fig.12 Convergence surfaces with partial unconstraint of m values, a) LBS, b) LMS and c) SPF.