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ORIGINAL ARTICLE

Contribution of demography to economic growth

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Abstract From 1850 to 2000, in Western European countries life expectancy rose from 30–40 to 80 years and the average number of children per woman fell from 4 to 5 children to slightly more than one. To gauge the economic consequences of these demographic trends, we implement an overlapping generations model with heterogeneity by level of education in which individuals optimally decide their consumption of market- and home-produced goods as well as the time spent on paid and unpaid work. We find that around 17% of the observed increase in per-capita income growth from 1850 to 2000 was due to the demographic transition. Around 50% of the demo-

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graphic contribution is explained by the increase in the average productivity per worker (productivity component), which arises from the change in the population's age structure and the rise in households' saving rate. The remaining 50% is explained by the higher growth rate of workers relative to the total population (translation component).

Keywords Demographic dividend · Fertility, Mortality · Per-capita income growth · Overlapping generations

JEL Classification D58 · E27 · J11 · N30

1 Introduction

The importance of the demographic transition on per-capita income growth was neglected for a long time, mainly because of a myriad of inconsistent correlations between population and economic growth (Kelley 1988).¹ It was not until the 1990s, using empirical convergence models à la Barro (1991, 1997), that several scholars were able to better isolate the effect of demography on economic growth (Kelley and Schmidt 1995; Bloom and Williamson 1997, 1998). Their main finding was that demography has a strong and positive effect on economic growth when the working-age population grows faster than the dependent population, known as *first demographic dividend*. Later on, Kelley and Schmidt (2005) added an important contribution by considering, in their convergence model, that changes in the age distribution of the population (known as the *translation component*) were likely to affect the productivity of workers (the *productivity component*). By doing so, they estimated demography to account for 20% of the per-capita income growth worldwide between 1965 and 1990, which was validated for the EU by several scholars (Prskawetz et al. 2007).

Despite these recent findings there are still many unanswered questions (Williamson 2013). For instance, to what extent does demography influence economic growth over a longer time span? What is the historical impact of demographic changes on economic growth? The demographic dividend literature has extensively used cross-country panel data for the period 1950–2010, which historically coincides with the period of most rapid population growth. However, as far as we know, there has been no study on the impact of demographic change on economic growth starting in the nineteenth century, exactly at the onset of the demographic transition in Europe (Livi-Bacci 2000; Lee 2003).

The aim of this paper is to assess the impact of the demographic transition on per-capita income growth along the period 1850–2000. We focus our analysis on Spain, since it is of great interest to economists, demographers, and historians due to the availability of historical data and the similarities with the East Asian “tiger economies” in the second half of the 20th century (Prados de la Escosura and Rosés 2010a). Spain started the demographic transition later than northern European countries (Livi-Bacci 2000). In 1850 the Spanish population size was around 15 million inhabitants, the

¹ We use per-capita income growth and economic growth interchangeably in this article. The demographic transition refers to the transition from high birth and death rates to low birth and death rates.

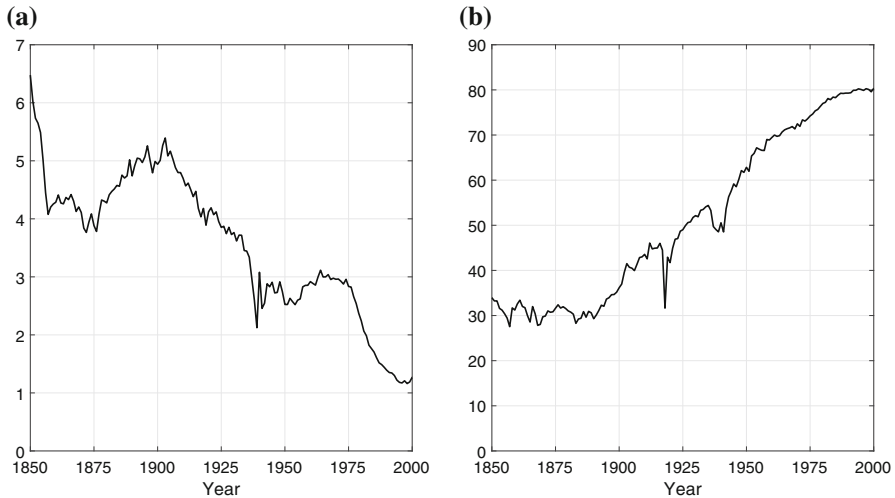


Fig. 1 Spanish total fertility rate (TFR) and life expectancy at birth: Period 1850–2000. **a** Total fertility rate (TFR), **b** life expectancy at birth. *Source* Authors’ estimates. *Notes* The TFR is the average number of children that would be born to a woman over her lifetime

average woman expected to have between four and five children, and life expectancy at birth was close to 30 years—due to an extremely high infant mortality—(Ramiro Fariñas and Sanz-Gimeno 2000). In 2000, the Spanish population was over 40 million people, the average woman expected to have 1.23 children, and life expectancy at birth was close to 80 years (see Fig. 1). The Spanish population also witnessed an economic revolution during this period. According to Prados de la Escosura and Rosés (2009) the average labor income per worker rose from the equivalent of 3.000–3.500 euros per year in 1850 to more than 33.500 euros in 2000.² Moreover, the average number of hours worked declined by 36% points and the entrance into the labor market was delayed due to the educational expansion. Indeed, 64% of the cohort born in 1850 was illiterate and 34% had only primary education (Nuñez 2005). In 2000, by contrast, the average number of years of schooling was 8.4 for adults (Barro and Lee 2013). Thus, the increase is even more remarkable if we focus on the wage rate per hour worked which rose from the equivalent of 1.2 euros in 1850 to 19.2 in 2000 (Prados de la Escosura and Rosés 2009).

The literature has frequently used convergence models to show the role of demographic change on economic growth. However, the results of these econometric models usually suffer from endogeneity problems (Feyrer 2007). More importantly, it is not possible to extend this kind of analysis further in the past because of the lack of data. A new approach to answer this old question is to estimate the demographic dividend using overlapping generation models (OLG) since the accumulation of capital and labor are modeled endogenously. For instance, Sánchez-Romero (2013) followed this strategy to analyze the evolution of the demographic dividend in Taiwan. He inves-

² All figures are measured at constant prices of 2010.

tigates the economic impact of different demographic scenarios in order to assess the demographic dividend, obtaining similar results to those obtained using growth regression models.

In this paper, we follow a similar strategy by implementing an OLG model. Nevertheless, our paper differs from [Sánchez-Romero \(2013\)](#) in two main aspects. First, the period of analysis increases from 40 to 150 years. Second, the model is extended by introducing household production and non-homothetic preferences. We follow the works of [Greenwood et al. \(2005\)](#) and [Ramey \(2009\)](#) in order to account for the impact of technological progress on the value of time. In addition, we follow the work of [Restuccia and Vandenbroucke \(2013\)](#), that assume non-homothetic preferences, in order to assess the impact of the increase in longevity and technological progress on the simultaneous reduction in hours worked and the increase in schooling time during the last hundred and fifty years in the US. Thus, the combination of home-production with non-homothetic preferences allows us to account for the historical reduction in paid hours. The costs of rearing children to households are introduced in the utility function through the family size ([Browning and Ejrnæs 2009](#)) and in the time constraint. Given that we have detailed demographic information for the period 1850–2000, our family size not only changes over time but also by age of the household head. The units of equivalent adult consumption of market and home-produced goods rely on the AGENTA database ([Vargha et al. 2015](#); [Rentería et al. 2016](#)). In addition, to control for the educational dividend caused by the educational transition ([Crespo-Cuaresma et al. 2014](#)), we introduce heterogeneous agents that differ in their educational attainment, using data from the Wittgenstein Data Explorer ([Wittgenstein Centre for Demography and Global Human Capital 2015](#)). Hence, the model takes as exogenously given inputs the evolution of vital rates (fertility and mortality) and human capital investments, ignoring their feedback effects.

Comparing our baseline scenario to a hypothetical economy whose population faces fertility and mortality rates which were prevailing in Spain in 1800, we find that the changes in the age structure of the population accounts for 16.8% of per-capita income growth for the period 1850–2000. This result lies within the possible range of values (i.e., 16–44%) found in the literature for the period 1950–2010 ([Kelley and Schmidt 2005](#)). Additional counterfactual experiments show that fertility explains 14.5% of the impact of demographic changes in per-capita income growth, while mortality explains 6.4% of the impact of demographic changes in per-capita income growth. Moreover, we have further decomposed the contribution of demography to per-capita income growth in the *translation component* (the difference between the growth rate of workers and the total population) and the *productivity component* (the growth rate of output per worker). Our results suggest that over this period of one hundred and fifty years, the translation component accounted for 50% of the total income growth, while the productivity component accounted for 50%. The growth rate of output per worker is explained by two main factors. First, the transition from a young age structure to an aging population, since this demographic process leads to an increase in the average age of asset holders—older households own more assets than younger households—and in the average age of workers—older households have a higher income. Second, through a rise in the propensity to save due to the longer life expectancy.

The paper is organized as follows. Section 2 details the theoretical model and its main theoretical implications. Section 3 presents the Spanish demographic transition, the economic data, and the model calibration. The contribution of the demographic transition on the per-capita income growth rate is presented in Sect. 4 and the impact of different model assumptions on our results are discussed in Sect. 5. Section 6 concludes.

2 The model

We implement a large-scale OLG model à la Auerbach and Kotlikoff (1987) in which heterogeneous households by level of education endogenously choose consumption and the times spent in the market and in home production. Demographics, the educational attainment, and technological progress are exogenous. Firms are assumed to operate in perfectly competitive markets and produce under constant returns to scale. To account for the full effect of demography on the economy, we assume Spain to be a closed economy. Hence, changes in the population structure might have an impact on input prices. Moreover, to better capture the accumulation of capital over time, we consider the historical evolution of public pension expenditures. Thus, our individuals contribute a fraction of their labor income to the pension system and receive pension benefits when retired.

2.1 Household preferences and home production

For expositional purposes, in this subsection we abstract from time subscripts. Households may belong to any of the three possible levels of education that we denote by $e \in \mathbf{E} = \{primary\ or\ less; secondary; tertiary\}$. Households derive utility from consumption of market-produced goods c^m , home-produced goods c^h , and leisure z . The period utility of a household, whose head has a level of education $e \in \mathbf{E}$, is given by

$$\begin{aligned}
 u_e^h(c^m, c^h, z) = & \phi_e^m \log \left(\frac{c^m}{1 + \eta(n)} - \bar{c}^m \right) + \phi_e^h \log \left(\frac{c^h}{1 + \eta(n)} - \bar{c}^h \right) \\
 & + \phi_e^z \frac{z^{1 - \frac{1}{\sigma_e}} - 1}{1 - \frac{1}{\sigma_e}}, \tag{1}
 \end{aligned}$$

where $\eta(n)$ is a function that transforms the number of children in the household by age (n) to the number of equivalent adult consumers, $\bar{c}^i > 0$ is the subsistence level of consumption of type $i \in \{m, h\}$, $\phi_e^i > 0$ is the relative weight of good $i \in \{m, h, z\}$ on the period utility, and $\sigma_e > 0$ is the elasticity of substitution on leisure. The set of parameters $\{\phi_e^m, \phi_e^h, \phi_e^z, \sigma_e\}$ depends on the level of education in order to better account for differences in the labor supply.

Home production requires intermediate goods and labor

$$f(c^x, h) = \left[\theta (c^x)^{\frac{\rho-1}{\rho}} + (1 - \theta) (h)^{\frac{\rho-1}{\rho}} \right]^{\frac{\rho}{\rho-1}}, \quad \text{with } \rho \geq 0, \tag{2}$$

where c^x stands for goods purchased in the market and used as intermediate goods for home production, h is the time spent on home production, θ is assumed to be positive and between zero and one, and ρ is the elasticity of substitution between the home input factors. According to Eq. (2) technological progress has an impact on home production through intermediate goods. Moreover, we implicitly assume that home-produced goods are always consumed in the household and not sold in the market; i.e., $c^h = f(c^x, h)$.

2.2 Household problem

In each year t households are heterogenous by their educational attainment (e), age (j), year of birth ($t - j$), assets accumulated (a), and the number of children raised at home (n). We denote by $\mathbf{x}_{e,j,t} = \{a_{e,j,t}, n_{j,t}\}$ the state variables of a household with a level of education e at age j in year t . Each household at age j in year t faces a probability to surviving to the next age of $\pi_{j+1,t+1}$, which is independent of other household characteristics.³ Individuals can live to a maximum of 100 years. Children are raised by the household between age 0 and 15 and do not work either in the market or at home.⁴ At the age of 16 (J_0) children start making decisions, leave their parents home, and establish their own households. After age 65 (J_R) all individuals retire. Adults are endowed with one unit of time that they distribute between market work (ℓ), household chores (h), child rearing ($h\nu(n)$), and leisure (z). Hours worked in the market are supplied in exchange of a (net of taxes) wage rate, whereas the time spent producing goods and rearing children is unpaid. Function $\nu(n)$ denotes the time spent rearing n children per unit of time devoted to home production. Hence, the functional form $h\nu(n)$ assumes, ceteris paribus, that individuals who either spend more hours producing goods at home or have more children also devote more time to childrearing and less time to labor. Annuity markets are absent and accidental bequests are distributed by the government to all households in the economy.

Households optimally choose their consumption of market goods, home production, intermediary goods, leisure time, and time spent on home production by maximizing their lifetime utility (V). Let the control variables of a household with a level of education e at age j in year t be $\mathbf{c}_{e,j,t} = \{c_{e,j,t}^m, c_{e,j,t}^h, c_{e,j,t}^x, h_{e,j,t}, z_{e,j,t}\} \in \mathbf{C}$. Thus, the household problem is equivalent to solving

$$V_{j,t}(\mathbf{x}_{e,j,t}) = \max_{\mathbf{c}_{e,j,t} \in \mathbf{C}} \left\{ u_e^h(c_{e,j,t}^m, c_{e,j,t}^h, z_{e,j,t}) + \pi_{j+1,t+1} V_{j+1,t+1}(\mathbf{x}_{e,j+1,t+1}) \right\} \quad (3)$$

³ Although the literature shows that there exists a positive correlation between educational attainment and longevity, we do not have information on death rates by educational attainment for the period analyzed (Lleras-Muney 2005).

⁴ This assumption is necessary for reducing the complexity of the model. Existing studies for families working at the textile sector in Catalonia show that children above the age of 5 or 6 were progressively substituting the market work of their mothers as the number of offspring in the household increased (Camps-Cura 1998). Indeed, this pattern was common to many other countries (Bengtsson 2004). Moreover, the literature suggests that children between ages 0 and 17 in the US supplied on average 4 h per week doing household chores, which represents one-sixth of the total average time devoted to such work by a prime-age adult (Ramey 2009).

subject to the budget constraint

$$a_{e,j+1,t+1} = \begin{cases} R_t a_{e,j,t} + tr_{j,t} + (1 - \tau_t)w_{e,j,t}\ell_{e,j,t} - c_{e,j,t}^x - \bar{c}_{e,j,t}^m & \text{for } J_0 \leq j \leq J_R, \\ R_t a_{e,j,t} + tr_{j,t} + b_t - c_{e,j,t}^x - c_{e,j,t}^m & \text{for } j > J_R, \end{cases} \quad (4)$$

the time constraint

$$\ell_{e,j,t} + h_{e,j,t}[1 + \nu(n_{j,t})] + z_{e,j,t} = 1, \quad (5)$$

and the standard boundary conditions

$$a_{e,J_0,\cdot} = 0 \quad \text{and} \quad a_{e,100,\cdot} \geq 0. \quad (6)$$

Parameter R is the capitalization factor, tr is the accidental bequests distributed by the government to the household, τ is the social contribution rate to the pension system, $w_{e,j,t} = r_t^H \epsilon_{e,j}$ is the wage rate per hour worked of an individual with education e at age j in year t , which depends on the wage rate per efficient unit of labor (r_t^H) and the age-specific productivity by educational attainment ($\epsilon_{e,j}$), and b_t is the pension benefit received in year t .

The optimal consumption path of market goods can be characterized by the Euler condition augmented by household size and subsistence level (see ‘‘Appendix A’’):

$$\tilde{C}_{e,j+1,t+1}^m / \tilde{C}_{e,j,t}^m = \pi_{j+1,t+1} R_{t+1} \text{ with } \tilde{C}_{e,j,t}^m = c_{e,j,t}^m - \bar{c}^m [1 + \eta(n_{j,t})]. \quad (7)$$

Equation (7) indicates that households smooth the consumption above the subsistence level \bar{c}^m for all household members. The introduction of \bar{c}^m and $\eta(n_{j,t})$ are key for explaining the historical decline in the number of hours worked. This is because when labor productivity is low and the number of children is high, households need to work more hours in the market in order to finance the minimum consumption level of goods. Afterwards, as productivity rises, households need less hours in the market to finance the minimum consumption expenditure (Restuccia and Vandenbroucke 2013). Moreover, given that the equivalent adult consumers multiply \bar{c}^m in (7), the same reasoning applies to $\eta(n_{j,t})$. Thus, increases in the number of equivalent consumers force individuals to supply more hours to the market. However, this effect might be offset by the subsequent increase in the demand for childrearing within the household as we will explain below.

The optimal hours worked in the market (or intensive labor supply) are given by the difference between the total available time and the sum of leisure and unpaid work,

$$\ell_{e,j,t} = 1 - z_{e,j,t} - h_{e,j,t}[1 + \nu(n_{j,t})], \quad (8)$$

where the optimal conditions of leisure and unpaid work are

$$z_{e,j,t} = \left(\frac{\phi_e^z \tilde{C}_{e,j,t}^m}{\phi_e^m c_{e,j,t}^m} \frac{c_{e,j,t}^m}{(1 - \tau_t)w_{e,j,t}} \right)^{\sigma_e}, \quad (9)$$

$$h_{e,j,t} = \frac{\phi_e^h c_{e,j,t}^h \tilde{C}_{e,j,t}^m}{\phi_e^m \tilde{C}_{e,j,t}^h c_{e,j,t}^m} \frac{c_{e,j,t}^m}{y_{e,j,t}^h + \left(\frac{\theta}{1-\theta} y_{e,j,t}^h\right)^\rho}, \quad (10)$$

respectively. The term $y_{e,j,t}^h = (1 - \tau_t)w_{e,j,t}[1 + v(n_{j,t})]$ is the opportunity cost of an hour devoted to home production and $\tilde{C}_{e,j,t}^h$ is equal to $c_{e,j,t}^h - \bar{c}^h[1 + \eta(n_{j,t})]$. The non-homotheticity between market consumption and leisure, see Eqs. (1) and (9), implies that a rise in productivity leads to an increase in leisure and a decline in paid hours. This is reflected in the second ratio inside the parenthesis in Eq. (9). The strength of this positive effect on leisure is nonetheless liable to diminish as households become wealthier, since $\tilde{C}_{e,j,t}^m$ will converge toward $c_{e,j,t}^m$.

Equation (10) makes explicit the importance of the minimum consumption of home-produced goods (\bar{c}^h) and the elasticity of substitution between home input factors (ρ) for the evolution of home labor. For example, if we assume that $\bar{c}^h = 0$ (i.e. $c_{e,j,t}^h = \tilde{C}_{e,j,t}^h$) and that $\rho = 1$, then an increase in productivity leads households to increase their time spent on unpaid work. However, for a sufficiently high value of $\bar{c}^h > 0$, an increase in productivity will make the marginal utility of home-produced goods to decline faster, which reduces the time spent on unpaid work. Similar to the effect of \bar{c}^m on paid hours, a rise in productivity leads to a drop in home labor, since individuals substitute home labor for intermediate goods. Indeed, in the interior solution, the ratio of intermediate goods to labor in home production is

$$\frac{c_{e,j,t}^x}{h_{e,j,t}} = \left(\frac{\theta}{1-\theta} y_{e,j,t}^h\right)^\rho. \quad (11)$$

Equation (11) shows that either a rise in wages, or the time spent rearing children per hour of home production, or a drop in the contribution rate, raises the ratio of intermediate goods to home labor for any $\rho > 0$.

The parameter ρ is also crucial for explaining the impact of $v(n)$ on both the total unpaid work (i.e. $h[1 + v(n)]$) and paid work. We can distinguish three cases. If $\rho < 1$, an increase in the time spent rearing children per unit of home labor will raise the number of total unpaid hours worked and reduce that of paid hours. This is because the increase in the marginal cost of home labor cannot be offset with the rise in intermediate goods, given that c^x and h are close complements. Moreover, given that leisure does not depend on $v(n)$, the increase in $v(n)$ has the opposite effect on paid hours. If $\rho = 1$, the increase in $v(n)$ will have no effect on the total time devoted to unpaid labor, since households offset the rise of $v(n)$ with a proportional increase in intermediate goods. And if $\rho > 1$, a rise in $v(n)$ leads to a drop in the total number of unpaid hours because households substitute home labor for intermediate goods.

Finally, households can, in addition to supplying labor to the market, specialize in home production when the marginal rate of substitution between leisure and consumption is greater than the effective wage rate per hour worked:

$$\frac{\partial u_e^h}{\partial z_{e,j,t}} \Big/ \frac{\partial u_e^h}{\partial c_{e,j,t}^m} > (1 - \tau_t)w_{e,j,t}. \quad (12)$$

As a consequence, households with a low (net) wage rate—that is, those with lower education—have a higher incentive to leave the labor market.

2.3 Firms

In order to obtain clear-cut results for the contribution of demography on economic growth, we use a simple production setting. Firms operate in a perfectly competitive environment and produce one homogenous good combining capital and labor, according to a standard Cobb–Douglas production function

$$Y_t = K_t^{\alpha_t} (A_t L_t)^{1-\alpha_t}, \tag{13}$$

where α_t denotes the share of capital in year t , K_t is the stock of physical capital, A_t is labor-augmenting technology, and L_t is the stock of human capital. Output can be either consumed, used as an intermediary good for home production, or used as an investment good. Labor-augmenting technology, A_t , is assumed to grow at the exogenous rate of g_t^A , i.e. $A_{t+1} = A_t(1 + g_t^A)$. Workers with different levels of education are assumed to be perfect substitutes. Hence, the stock of human capital is $L_t = \sum_{j=J_0}^{J_R} N_{j,t} \int_{\mathbf{E}} \epsilon_{e,j} \ell_{e,j,t} dE_{t-j}(e)$. The rental price of physical capital (r_t^K) and human capital (r_t^H) equals their marginal products, i.e., $r_t^K = \alpha_t \frac{Y_t}{K_t} - \delta_t$ and $r_t^H = (1 - \alpha_t) \frac{Y_t}{L_t}$, respectively, where δ_t is the depreciation rate of physical capital in year t .

2.4 Government

Our government has two objectives. First, the government distributes each period any positive accidental bequests in a lump-sum fashion to the generation containing the decedent’s children, if they are older than 16 years.⁵ For those decedents younger than age 44 (=28 + 16), their positive wealth is assumed to be distributed within the same birth cohort, since by definition individuals below age 16 cannot inherit wealth. Given that there are no other taxes, debts left at death are financed by social security taxes. Second, the government runs a pay-as-you-go pension system. In each period the government sets the social contribution rate τ_t that workers contribute to guarantee the pension benefits received by pensioners in year t plus the debt left by individuals dying in year t . The budget constraint of the pension system is then given by

$$\tau_t r_t^H L_t = \sum_{j>J_R} b_t N_{j,t} + D_t \quad \text{for all } t. \tag{14}$$

⁵ According to the estimated age-specific fertility rates, the average generational gap between 1850 and 2000 was 28 years. Similar modeling strategies have been followed by Wolff (1999).

The left-hand side of (14) is the total social contributions paid by workers in year t , while the right-hand side gives the total pension benefits claimed by pensioners plus the debt in year t .

The definition of the market equilibrium is standard and can be found in “Appendix B”.

3 Data and calibration

In this section we first describe the exogenous information used in the overlapping generations model: Demographics, capital depreciation rates, labor shares, the evolution of the educational distribution, age-specific productivities by educational attainment, pension replacement rates, and our own reconstruction of the labor-augmenting technological progress. We conclude the section by briefly explaining the calibration strategy and the model parameters.

3.1 Demographics

The economic model implemented in this paper requires demographic data by single years of age on death rates, fertility rates, and population size for a time-span larger than the period analyzed (1850–2000). Historical demographic information from Spain was frequently incomplete and often inconsistent among different sources. Fortunately, important efforts constructing homogeneous demographic time series have already been made by several scholars. For instance, [Maluquer de Motes \(2008\)](#) provides homogeneous total population series from 1850 to 2001, [Ramiro Fariñas and Sanz-Gimeno \(2000\)](#) estimate infant and childhood mortality time series from 1790 to 1960, and [Reher \(1991\)](#) calculates for the historic region of Castilla La Nueva (New Castile) vital series of births, marriages, and deaths by using parish registers from 1550 to 1900, among others.

To construct a demographic database consistent with our economic model (i.e., a unisex closed population), we combine two demographic methods widely used in population reconstruction: *Inverse Projection* (IP) and *Generalized Inverse Projection* (GIP) ([Lee 1985](#); [Oeppen 1993](#)). The IP method is used to calculate net migration rates, while the GIP method is used to reconstruct consistent data on population size by age $N_{j,t}$ and age-specific vital rates, i.e., age-specific fertility rates $f_{j,t}$ and age-specific conditional survival probabilities $\pi_{j,t}$. GIP is a non-linear optimization that produces constrained demographic projections with *a priori* information ([Oeppen 1993](#)). The GIP method is explained in more detail in “Appendix C”. Both models are implemented using census data for years 1857, 1860, 1877, 1887, 1900–1970 from [INE \(2015b\)](#), 1981 from [INE \(2015c\)](#), and the years 1991 and 2001 from [INE \(2015d\)](#). Age-specific fertility rates from 1922 to 2012 are taken from the [Human Fertility Collection \(2015\)](#). Total population size from 1787 to 2000 is obtained combining data from [INE \(2015b\)](#), the [Human Mortality Database \(2015\)](#), and [Maluquer de Motes \(2008\)](#). Data on total births and deaths come from the [INE \(2015b\)](#) and from the [Human Mortality Database \(2015\)](#).

Figures 1 and 2 show the evolution of three aggregate measures from 1850 to 2000 obtained with our population reconstruction: (i) total period fertility rate (TFR), (ii) period life expectancy at birth, and (iii) population distributions. Assuming that fertility and mortality patterns in year t prevail throughout life, the TFR is the average number of children that would be born to a surviving woman over her lifetime, while the life expectancy at birth indicates the number of years a newborn infant would live. From Fig. 1 we can observe that the levels of fertility and mortality in Spain were similar to those of western Europe before 1800 (Lee 2003). In this period, TFR ranged between four to five children per woman and life expectancy at birth was between 25 and 35 years (Livi-Bacci 2000). One main consequence of the late onset of the demographic transition in Spain was the slower population growth relative to other western European countries. Close to the end of the nineteenth and at the beginning of the twentieth century, the TFR started a secular decline from five to values slightly higher than one (Reher 2011), which was only interrupted during the Franco regime (1939–1975), and mortality rates decreased, especially for infants and children. The decline in fertility at the beginning of the twentieth century was triggered by improvements in child survival. The change in the fertility pattern transmitted across generations was materialized through duration-related fertility behavior, such as the delay in women’s age at first birth and birth spacing (Reher and Sanz-Gimeno 2007; Reher et al. 2008). The decline in fertility after 1975 was driven by a continuing delay in the mean-age at first birth caused by multiple factors such as youth job insecurity, difficulties accessing housing, low institutional support of childcare, the educational expansion, the use of modern contraceptive methods, among other reasons (Kohler et al. 2002; Esping-Andersen 2013; Baizan 2016). The decline in mortality translated into a remarkable increase in life expectancy at birth, which doubled from 40 to 80 years during the twentieth century. This trend was only transitorily stopped by drastic events: the Spanish flu of 1918, the Civil War (1936–1939), and the subsequent famine. Nowadays, Spain is the country with the second largest overall life expectancy at birth after Japan (OECD 2015b).

The demographic implications of the mortality and fertility processes summarized in Fig. 1 can be easily seen in Fig. 2. From 1850 to 2000, the Spanish population changed dramatically from a population clearly dominated by children and young adults until 1975 to a population that ages rapidly due to the fertility decline that started in 1975.

Combining the reconstructed fertility and mortality data with National Time Transfers Accounts (NTTA) estimates, we calculate $\eta(n_{j,t})$ and $\nu(n_{j,t})$ for a household at age j in year t as follows

$$\eta(n_{j,t}) = \sum_{x=0}^j \xi_{j-x}^m \sum_{k=0}^{N_C} k P_{x,t}(k), \tag{15}$$

$$\nu(n_{j,t}) = \sum_{x=0}^j \xi_{j-x}^h \sum_{k=0}^{N_C} k P_{x,t}(k), \tag{16}$$

where ξ_x^i are equivalent units of scale referring to an individual of age x for goods ($i = m$) and childrearing time ($i = h$), respectively, $P_{x,t}(k)$ is the probability of

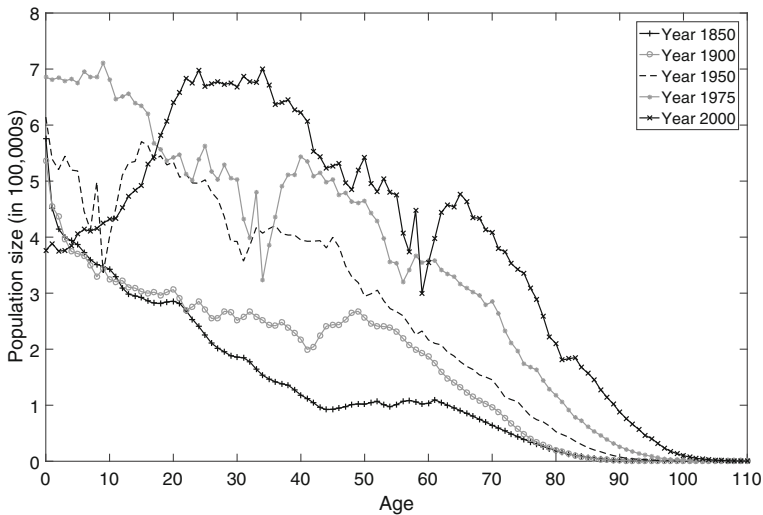


Fig. 2 Spanish population distribution (both sexes combined): Selected years 1850, 1900, 1950, 1975, and 2000. *Source* Authors' estimates

having k surviving children in year t when the household was x years old, and N_C is the maximum number of children per year. We use a standard profile for ξ_x^m that equals 0.4 from ages 0 to 4 and rises linearly with age until reaching 1 at the age of 18. The time spent rearing a child of age x per unit of home labor (ξ_x^h) is calculated pooling male and female time consumption profiles from age 0 to 18, taken from [Rentería et al. \(2016\)](#), and dividing it by the unpaid work profiles (without childrearing) of males and females combined between ages 28 to 46. This calculation suggests that a new born baby requires 3.65 times the time spent on home production; afterwards, ξ_x^h exponentially declines to 0 at the age of 16.

Figure 3 shows the evolution over the lifecycle of $\eta(n_{j,t})$ and $\nu(n_{j,t})$ for six selected cohorts (all other cohorts are omitted for presentation purposes). A value of 2 in Fig. 3a means that a household must finance the consumption of one additional adult.⁶ Over the one and a half centuries we observe two clear trends. First, mainly due to the postponement of childbearing, the age of the household at the maximum number of equivalent adult consumers progressively shifted to older ages from 34 (birth cohort born in 1820) to 42 (birth cohort born in 1970). Second, there was a progressive decline in the household size due to the drop in the total fertility rate (see left panel of Fig. 1).

Comparing Fig. 3a, b, we can notice that, for all cohorts, the maximum value of each profile is attained at older ages in 3a than in 3b. Remember that the highest value of ξ^m is attained at age 18, while the highest values of ξ^h are concentrated in the first years of life. As a result, we have the age at the maximum value of $\nu(n)$ moving from 31 (for the cohort born in 1820) to 35 (for the one born in 1970). The range of ages we have derived are in line with recent childcare profiles reported for

⁶ Since we assume a unisex model, one should multiply by two to approximate the values in Fig. 3a to a household run by two adults.

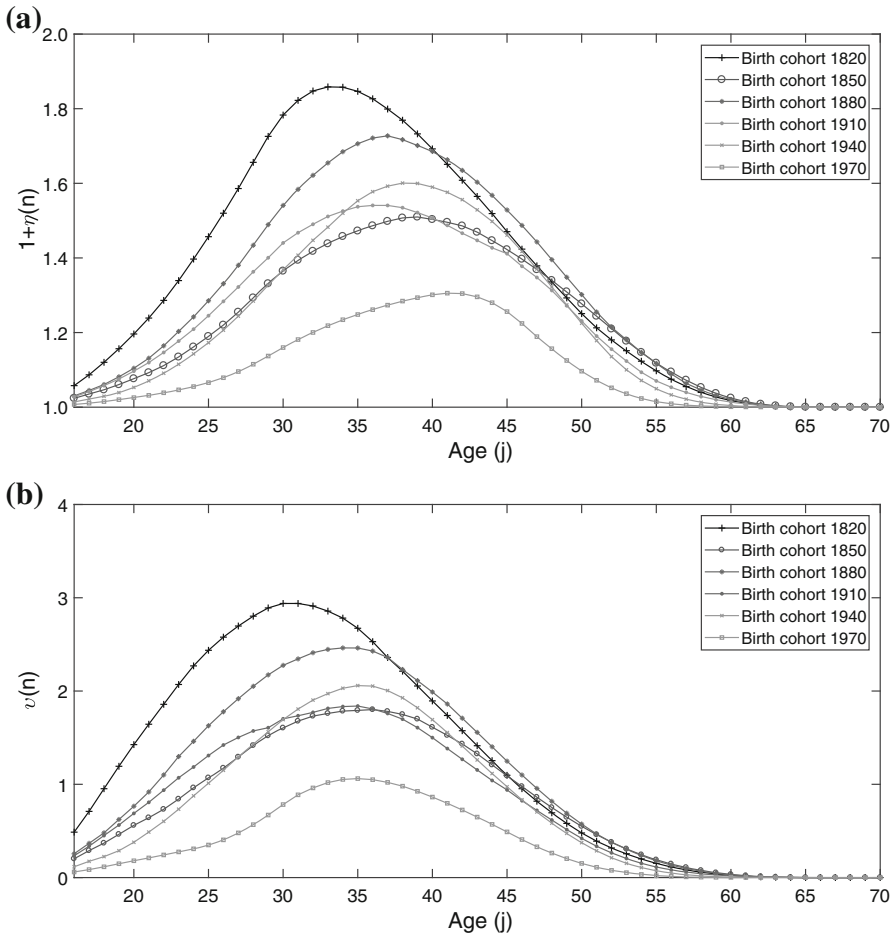


Fig. 3 Number of equivalent adults consumers and time spent rearing children per hour of household chores for selected birth cohort. **a** Equivalent adult consumers, $1 + \eta(n)$, **b** time spent rearing children per hour of household chores, $\nu(n)$. *Source* Authors’ estimates based on the population reconstruction and AGENTA data. Note: Profiles calculated under the assumption that children leave the parental house at age 16

14 European countries in [Vargha et al. \(2015\)](#). Under the assumption that ρ is lower than one ([Ramey 2009](#)), the difference between the peaks in $\eta(n)$ and $\nu(n)$ implies that there is a marginal tendency to allocate more time to home production early in life and, later on, to market production. Another important process for the labor supply is the drop in $\eta(n)$ and $\nu(n)$ across cohorts. Figure 3b shows that $\nu(n)$ declined from three (for the cohort born in 1820) to one (for the one born in 1970).⁷ A similar pattern is observed in Fig. 3a.

⁷ To test how sensitive Fig. 3b is to different ξ^h profiles, we computed the value of $\nu(n)$ for different ξ^h values. We found that a reduction in ξ_0^h of 60% (from 3.65 to 1.5) implies an overall reduction in $\nu(n)$ of 30%.

3.2 Stock of physical capital

The stock of physical capital (\tilde{K}_t) is derived applying the perpetual inventory approach to gross fixed capital formation to the following categories: construction, transport equipment, machinery and equipment, and intangible fixed assets.⁸ The information on gross fixed capital formation for each category during the period 1850–2000 is taken from [Prados de la Escosura \(2003\)](#). We apply the standard formula:

$$\tilde{K}_t = \sum_i \tilde{K}_{i,t} \quad \text{with} \quad \tilde{K}_{i,t+1} = (1 - \delta_i)\tilde{K}_{i,t} + I_{i,t},$$

where $\tilde{K}_{i,t}$ is the stock of capital in category i in year t , δ_i is the depreciation rate of category i , and $I_{i,t}$ is the gross capital formation in category i in year t . The depreciation rates applied to each category are 2.1, 18.2, 13.8, and 15%, respectively ([Hulten and Wykoff 1981](#)).⁹ The total depreciation rate of capital (δ_t) is calculated so as to satisfy $\tilde{K}_{t+1} = (1 - \delta_t)\tilde{K}_t + \sum_i I_{i,t}$. Since the standard formula for calculating the initial capital stock, $\tilde{K}_{i,1850} = I_{i,1850}/(\delta_i + g_i)$, where g_i is the growth rate of investment in the first 10 years in category i , gives a suspiciously low value, we opted for choosing an initial capital stock that minimized the difference between $\tilde{K}_{i,1850}$ and $\tilde{K}_{i,1890}$. A similar strategy is shown in [Prados de la Escosura and Rosés \(2010a\)](#).

3.3 Stock of human capital

We reconstruct the stock of human capital \tilde{L}_t taking into consideration the following components:

$$\tilde{L}_t = \text{HW}_t \sum_{j=16}^{65} N_{j,t} \int_{\mathbf{E}} \text{WP}_{e,j} \text{LF}_{e,j} dE_{t-j}(e), \quad (17)$$

where HW_t = average annual hours worked per worker in year t , $N_{j,t}$ = population size at age j in year t , $\text{WP}_{e,j}$ = productivity per hour worked at age j with education e (or efficient labor units), $\text{LF}_{e,j}$ = labor participation rate at age j for individuals with education e , and $E_{t-j}(e)$ = educational distribution for the cohort born in year $t - j$.

Time series on annual hours worked per worker are taken from [Prados de la Escosura and Rosés \(2009\)](#) and adjusted from 1977 to 2000 with [OECD \(2015a\)](#) data on hours work. The productivity per hour worked by educational attainment is calculated deterministically by fitting a quadratic function to [MTAS \(2010\)](#) data on the average wage rate per hour worked across age by educational attainment. We represent these functions in [Fig. 4b](#). Using data from [INE \(2015a\)](#), we calculate LF profiles by educational attainment as the average labor force participation rate between 1987 and

⁸ From now on, we use the symbol $\tilde{\cdot}$ to distinguish the reconstructed stock variables based on actual data from the stock variables obtained in our simulations.

⁹ A more sophisticated calculation of the stock of productive capital, which distinguishes between capital input and capital quality along the period 1850–2000, can be seen in [Prados de la Escosura and Rosés \(2009\)](#).

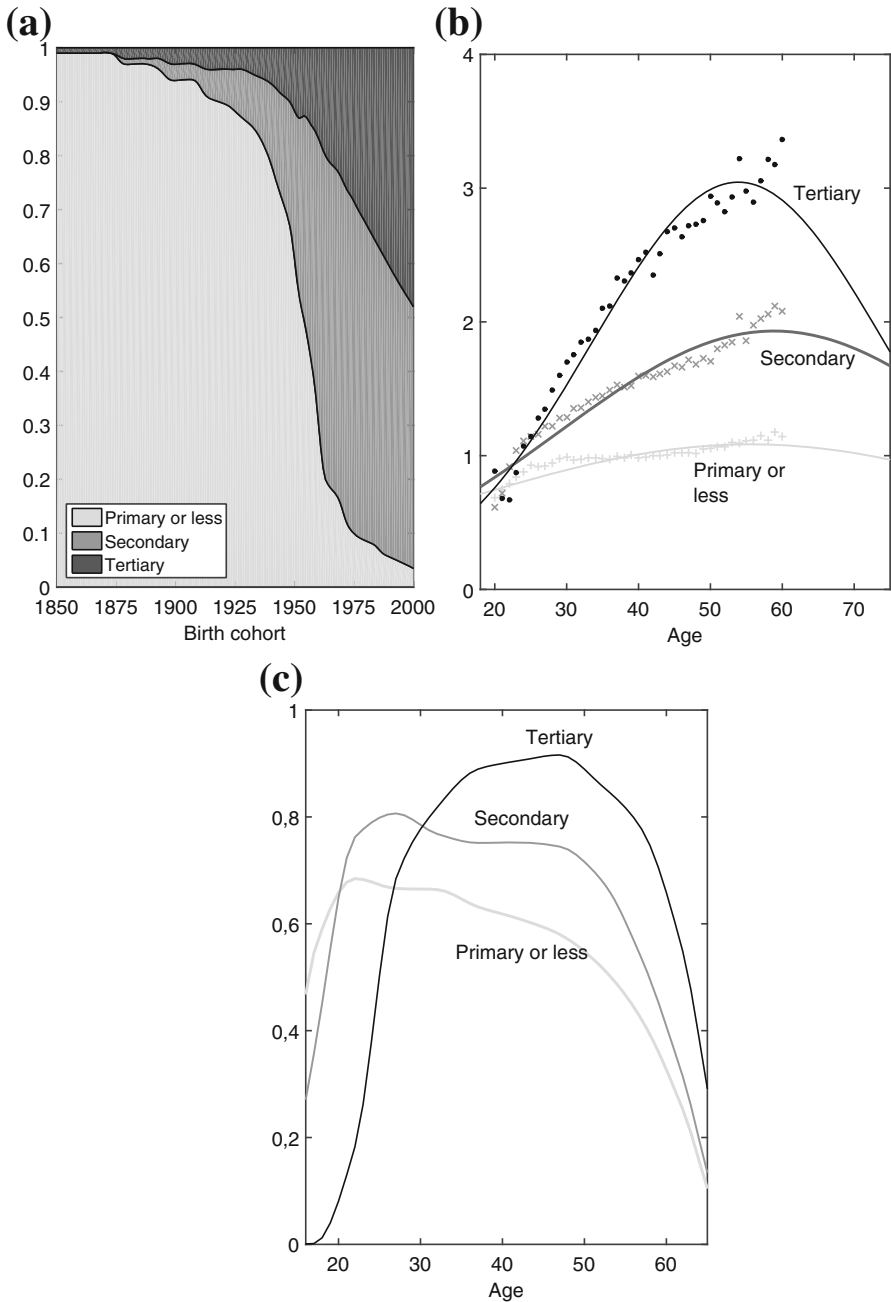


Fig. 4 Decomposition of the stock of human capital by educational attainment (both sexes combined). **a** The educational distribution, $E_c(e)$, **b** the endowment of efficient labor units, WP and **c** labor force participation rates, LF. *Sources* Educational distribution data is taken from the Wittgenstein Centre Data Explorer (Wittgenstein Centre for Demography and Global Human Capital 2015), the endowment of efficient labor units comes from MTAS (2010), and the average labor force participation rate between 1987 and 2013 is calculated using data from INE (2015a)

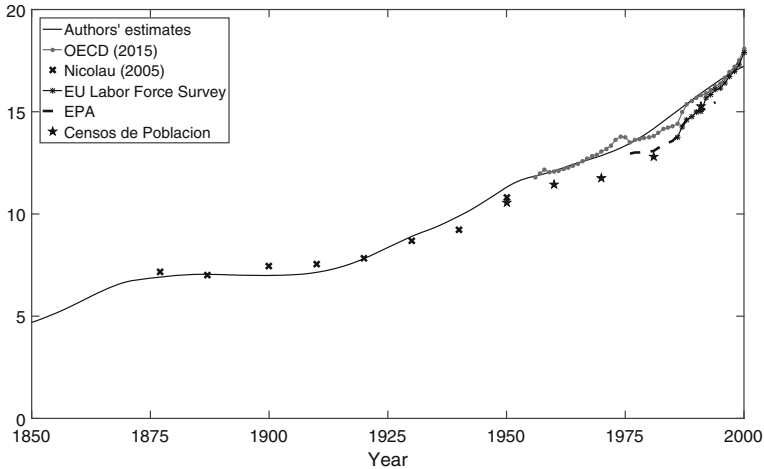


Fig. 5 Labor force, Spain 1850–2000 (both sexes combined, in millions)

2013 for each educational group. The LF profile by educational attainment, shown in Fig. 4c, is assumed to remain fixed in Eq. (17), while the supply of labor will be endogenously chosen by households in the OLG model. We choose the period 1987–2013 because it includes two periods in which the unemployment rate declined (1987–1991 and 1995–2007) and another two periods in which it increased (1991–1994 and 2008–2013). The educational distribution by birth cohort E_c in Spain is taken from the Wittgenstein Centre Data Explorer ([Wittgenstein Centre for Demography and Global Human Capital 2015](#)) for two reasons. First, this database offers information on historical reconstruction of educational attainment for the 20th century, and second, it provides harmonized projections until 2100 of the population by age and educational attainment ([Lutz et al. 2014](#)). We extract information for Spain on the shares of population by levels of education for the period 1970–2100, which allow us to calculate the educational distribution for each birth cohort born after year 1868.¹⁰ Since the available data is grouped by five-year age groups, we apply linear splines to interpolate the educational distribution for intermediate cohorts. Last, but not least, in order to guarantee an initial steady state, the educational attainment for cohorts born before 1868 are assumed to coincide with that of the cohort born in year 1868.

To check the validity of our reconstruction of the stock of human capital \hat{L}_t , we compare the labor force, which results from applying the formula $\sum_{j=16}^{65} N_{j,t} \int_{\mathbf{E}} \text{LF}_{e,j} dE_{t-j}(e)$, to existing estimates of the labor force from 1850 to 2000. Figure 5 shows that our labor force estimates before 1950 are very similar to those reported by [Nicolau \(2005\)](#) and it fits well to the most recent estimates from widely used databases which validates our strategy.

¹⁰ Given the positive correlation between educational attainment and life expectancy, individuals born close to 1868, who reached tertiary education are likely to be overrepresented. Nevertheless, due to the late onset of the educational expansion in Spain, our main results are not affected.

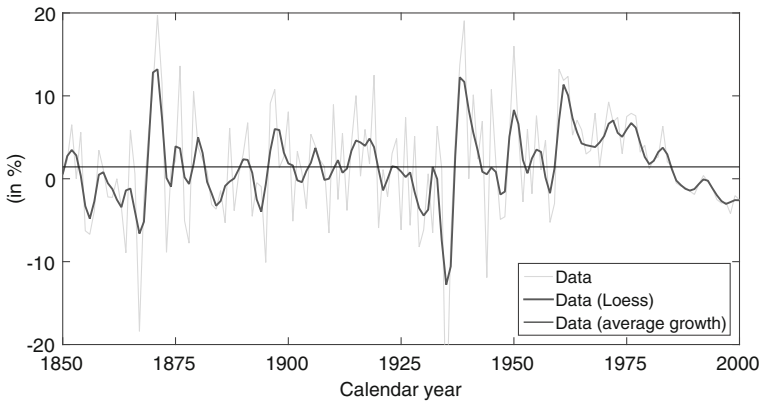


Fig. 6 Labor-augmenting technological progress, Spain 1850–2000. *Source* Authors’ calculations. *Note* The ‘Loess’ smoothing profile was computed using a bandwidth of 0.05 and rescaled to give an annual average growth rate of 1.40%

3.4 Labor-augmenting technology

The labor-augmenting technology is calculated using reconstructed input factors in Sects. 3.2 and 3.3. We use OECD (2015c) data on the total number of workers from 1956 in order to account for the rise in unemployment. Our estimation of the labor-augmenting technological progress is calculated applying the formula

$$\Delta \ln A_t = \Delta \ln(\tilde{Y}_t/N_t) - \Delta \ln(\tilde{L}_t/N_t) - \Delta \frac{\alpha_t}{1 - \alpha_t} \ln(\tilde{K}_t/\tilde{Y}_t), \tag{18}$$

where \tilde{Y}_t is the value added, \tilde{L}_t is the stock of human capital, and \tilde{K}_t is the stock of physical capital. From (18) we obtain that the annual average labor-augmenting technological progress from 1850 to 2000 was 1.40%. We assume no productivity growth before 1850 and we also assume that our estimated average productivity growth for the period 1850-2000 from 2012 onwards. Figure 6 shows the percentage change in the estimated labor-augmenting technological progress A_t .

3.5 Calibration

To perform our quantitative experiment we need to find the parameters such that the model is capable of reproducing some key historical facts of the Spanish economy. We proceed as follows:

In our baseline, we use the annual physical capital depreciation rate δ_t estimated in Sect. 3.2 and the labor share calculated by Prados de la Escosura and Rosés (2009). Hence, our modeled depreciation rate is, on average, close to 0.057 and the average labor share for the period 1850–2000 is 0.68. Since there is no information on the average pension benefit across cohorts, we calculate it indirectly. First, we decompose the ratio total public pension expenditures to compensation of employees using the identity

$$\frac{\text{Pension expenditures}_t}{\text{Comp. employees}_t} = \frac{\text{Avg. pension per indiv. } 65+_t}{\text{Avg. salary per indiv. } 16-64_t} \times \frac{\text{Pop. } 65+_t}{\text{Pop. } 16-64_t}. \quad (19)$$

The first term on the right-hand side of (19) is the average replacement rate of the pension system in year t , which hereinafter we denote by ψ_t . The second term on the right-hand side is the old-age dependency ratio. Data on total public pension expenditures from 1850 to 2000 is taken from [Comín and Díaz \(2005\)](#). Compensation of employees is calculated by multiplying the labor share from [Prados de la Escosura and Rosés \(2009\)](#) to the value added along the period analyzed. Second, the model uses ψ_t to calculate the pension benefits received for all individuals above age 65 as follows

$$b_t = \frac{\psi_t}{\sum_{j>J_R} N_{j,t}} \frac{r_t^H L_t}{\sum_{j=J_0}^{J_R} N_{j,t}} \quad \text{for all } t,$$

where the last term on the right-hand side is the average labor income of the working-age population. The average replacement rate before 1850 and after 2000 is assumed to stay constant at the levels observed in 1850 and 2000, respectively.

The model is comprised of sixteen parameters. Specifically, we have, for each educational group, the utility weights of each good on the household utility $\{\phi_e^m, \phi_e^h, \phi_e^z\}$ and the intertemporal elasticity of substitution on leisure σ_e . Then, common to all educational groups, we have the minimum consumption level for each good $\{\bar{c}^m, \bar{c}^h\}$ and the home-production technology $\{\theta, \rho\}$. To reduce the dimension of the parameter set, we impose without loss of generality that $\phi_e^m + \phi_e^h + \phi_e^z = 1$ and that work at home upon retirement accounts for 250 min per day ([Rentería et al. 2016](#)). Thus, using the first-order conditions, we can indirectly calculate ϕ_e^z as

$$\phi_e^z = \phi_e^h (1 - \theta) \frac{(1 - h)^{\sigma_e}}{h},$$

with $h = .26$ or, equivalently, 250 min per day out of 60 min \times (24–8) available hours. Moreover, we rely on the estimates of [Ramey \(2009\)](#) and set ρ at 0.95. All remaining behavioral parameters are structurally estimated using the model. Let us denote by λ the 9×1 vector of parameters left to be determined:

$$\lambda = [\lambda_e, \bar{c}^m, \bar{c}^h, \theta]' \quad \text{with } \lambda_e = \{\phi_e^h, \sigma_e\} \text{ for } e \in \mathbf{E}.$$

Further, we implement the following restrictions on the parameters, $\lambda \in \Lambda$. First, the intertemporal elasticities of substitution σ_e must be in the interval between zero and one. Second, the weight on the household utility of home-produced goods ϕ_e^h must be non-negative. Third, the minimum consumption level of market- and home-produced goods cannot be negative. And fourth, the share of intermediate goods on home production θ must be in the interval between zero and one.

For a given λ , we solve the model in order to obtain the optimal labor supply of individuals over their lifecycle, the average hours worked, the output per capita, and the consumption per capita. Then, we compute the function $F(\lambda)$ defined by

Table 1 Model parameters

Parameter	Symbol	Values		
<i>Firms technology</i>				
Capital share	α_t	Prados de la Escosura and Rosés (2010b)		
Capital depreciation rate	δ_t	Prados de la Escosura and Rosés (2009)		
Labor-augmenting technology	A_t	Authors' estimates		
<i>Home production</i>				
Elasticity of substitution on labor	ρ	0.952 Ramey (2009)		
Factor share	θ	0.430		
<i>Household preferences</i>				
Age at parental leave	J_0	16		
Retirement age	J_R	65		
Subsistence level market goods	\bar{c}^m	0.121		
Subsistence level home goods	\bar{c}^h	0.067		
		Level of education		
		Primary or less	Secondary	Tertiary
IES on leisure	σ_e	0.281	0.418	0.442
Weight of market goods	ϕ_e^m	0.056	0.049	0.053
Weight of home goods	ϕ_e^h	0.539	0.460	0.449
Weight of leisure	ϕ_e^z	0.405	0.491	0.498

$$F(\lambda) = \sum_{j=16}^{65} \sum_{e \in \mathbf{E}} \Phi_{e,j}^1(X; \lambda)^2 + \sum_{t=1851}^{2000} \sum_{i=\{l,y,c\}} \Phi_t^i(X; \lambda)^2, \tag{20}$$

where X denotes the exogenous information set of the economic model. The first term corresponds to the difference between the labor supply by educational attainment, shown in Fig. 4c, and the individual labor supply by educational attainment obtained with the model, i.e., $\Phi_{e,j}^1(X; \lambda) = \gamma LF_{e,j} - \frac{1}{27} \sum_{t=1987}^{2013} \ell_{e,j,t}$. To transform the participation rates to actual hours worked, we set γ to 0.32, which is equivalent to working 36 h per week out of a total of 112 h per week. The second term captures the difference between the observed average hours worked for the population between 16 and 65 years, income per capita, and consumption per capita from 1850 to 2000, and those obtained with the model. Thus, we search for the value of λ that minimizes the function $F(\lambda)$.

Table 1 reports the parameter values taken from the literature as well as those structurally estimated with the model. We can highlight in Table 1 three key parameters: the elasticity of substitution between input factors in home production (ρ), the subsistence level of market-produced goods (\bar{c}^m), and the subsistence level of home-produced goods (\bar{c}^h). The fact that the subsistence level of market- and home-produced goods are positive implies that the income effect dominated over the substitution effect when productivity was low. As a consequence, individuals had to work long hours in

order to finance the consumption of both goods in the nineteenth century. A similar result is obtained by Restuccia and Vandenbroucke (2013) analyzing the accumulation of human capital and the evolution of labor supply in the US for cohorts born between 1870 and 1970. It is also worth mentioning that the income effect varies over the lifecycle of the household, becoming stronger when the number of equivalent adult consumers in the household rises, and weaker when productivity increases. Another important remark is that \bar{c}^m and \bar{c}^h differ in magnitude. Hence, the rate of change of market-produced goods relative to home-produced goods will differ over time as productivity increases. This result has also been found recently by Moro et al. (2017). The other key parameter is ρ , which we took from Ramey (2009) given that we did not have enough data to estimate it structurally. As we have commented in Sect. 2.2 a value of ρ lower than one implies that home labor increases as productivity is on the rise. This effect is, however, offset by the existence of a positive minimum consumption of home-produced goods. The other effect of assuming a $\rho < 1$ is that total unpaid hours marginally fall and paid hours marginally rise with the decline in $v(n)$, see Fig. 3b.

Figure 7 shows the in-sample performance of the baseline model with respect to the targeted time series. In Fig. 7a, we can see how well the model replicates the average per-capita hours worked by level of education between 1987 and 2013. Figure 7b compares the observed average fraction of hours worked by the working-age population to that obtained in the baseline. The discrepancy between both figures from 1976 to 2000 is explained by the fact that we do not consider the risk of unemployment in the model, whereas the unemployment rate rose to values over 20% during this period. In Fig. 7c, d we show how well the model replicates the evolution of the income per capita (or economic growth) and the consumption per capita from 1850 to 2000.

The next section will apply the calibrated OLG model to the Spanish data to disentangle the contribution of demography to the observed economic growth.

4 Results

In this section we quantify the *Spanish demographic dividend* or, equivalently, the contribution of demography to Spain's economic growth from 1850 to 2000. In so doing, we first need to realize that assessing the demographic dividend over a long period of time by using the naïve demographic model

$$(Y/N)_{\text{gr}} = (Y/W)_{\text{gr}} + (W)_{\text{gr}} - (N)_{\text{gr}}, \quad (21)$$

where W stands for workers and 'gr' denotes the average growth rate, gives an incorrect measure, since the growth rate of the support ratio (i.e. $(W/N)_{\text{gr}}$) is zero in the long run. This is because in a stable population the growth rate of the population coincides with the growth rate of workers (Lotka 1939).

Table 2 shows the decomposition of the growth rate of per-capita output in Spain from 1850 to 2000. A naïve calculation using the first row of Table 2 suggests that only 5% (i.e., $= (W_{\text{gr}} - N_{\text{gr}}) / (Y/N)_{\text{gr}} = (.80 - .72) / 1.62$) of the Spanish economic growth from 1850 to 2000 is explained by demographic changes. However, long-run demo-

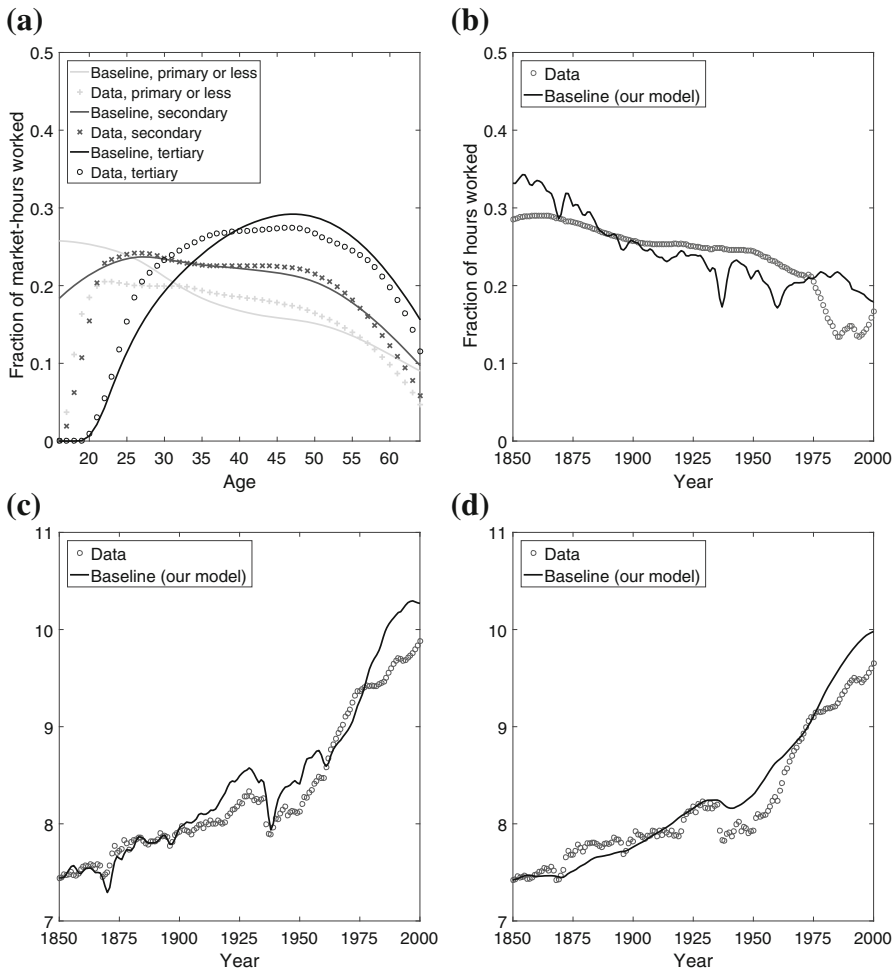


Fig. 7 In-sample performance of the model, Spain 1850–2000. **a** Fraction of per-capita hours worked by educational attainment, average of the 1987–2013 period, **b** average hours worked by working-age population, **c** income per capita (in logs) and **d** consumption per capita (in logs). *Source* See text on Fig. 4 for panels 7a, b. National accounts data are taken from [OECD \(2015c\)](#) and [Prados de la Escosura and Rosés \(2009\)](#)

graphic changes are translated into economic growth through productivity effects, known as the *productivity component* ([Kelley and Schmidt 2005](#)). For instance, some possible channels for demography to impact on productivity are: scale economies, population density, life-cycle savings, changes in the supply of labor, and changes in the human capital accumulation, among others.

In order to control for some of the above mentioned channels, in this article, we follow the same strategy as in [Sánchez-Romero \(2013\)](#) to assess the Spanish demographic dividend. First, we show that our model is capable of reproducing the evolution of per-capita income along the period 1850–2000. In this regard, Fig. 7c shows that our

Table 2 Per-capita output growth in Spain: 1850–2000 (annual average logarithmic rates in percent) *Source* Authors' estimations and [Prados de la Escosura and Rosés \(2009\)](#)

Period	Output per capita $(Y/N)_{gr}$	Output per worker $(Y/W)_{gr}$	Workers W_{gr}	Population N_{gr}
1850–2000	1.62	1.54	0.80	0.72
1850–1950	0.68	0.48	0.88	0.68
1951–1974	5.05	5.48	0.69	1.13
1975–2000	2.06	1.88	0.69	0.50

Bold values indicate the main results

model mimics well the small growth of per-capita income for the period 1850–1925, the period of stagnation from 1925–1950, and the *golden age* of rapid economic growth from 1950–1975. Second, based on the vital rates obtained for year 1800, we build a set of different demographic scenarios (from now on ‘experiments’) to disentangle the effect of demography for the period 1850 to 2000.¹¹ We propose the following three experiments:

- *Experiment 1* In this experiment we cancel the effect of fertility and mortality. This experiment gives the structure of the population in year 1800 under stable conditions. Life expectancy at birth is fixed at 31.5 years and the TFR is fixed at a value slightly above 5, which implies a young population structure (see the dotted line in Fig. 8a) and a constant annual population growth rate of 0.5% (see Fig. 8b). By comparing the economic outcomes of the baseline simulation to those of Experiment 1 we get the contribution of demography to economic growth.
- *Experiment 2* In this experiment we cancel the effect of the increase in longevity. This demographic scenario implies that the population would have increased until 1920 and it would have declined afterwards due to the fall in fertility below replacement level (see Fig. 8b). As a consequence, the age distribution of the population in year 2000 would have been older than under the baseline (see the triangle line in Fig. 8a).
- *Experiment 3* In this experiment we shut down the effect of the decline in fertility. The population growth rate would have continuously increased over the twentieth century until reaching a stable population growth rate of 3%. Thus, the age distribution of the population would have been even younger than under Experiment 1 (see circled line in Fig. 8a). Moreover, given the increasing population growth, during the period 1850–2000 the population growth rate would have been larger than the growth rate of the population between ages 16 and 65 (see Fig. 8b).

Notice that setting Experiments 2 and 3 allows us to separate, as completely independent factors, the effect of fertility and mortality on economic growth.

Given these three experiments, [Sánchez-Romero \(2013\)](#) shows that the impact of demography on per capita income growth can be easily estimated by calculating the relative contribution of each demographic factor to the observed economic growth,

¹¹ [Sánchez-Romero \(2013\)](#) shows that fixing birth and death rates at the levels prevailing at the beginning of the period analyzed underestimates the demographic impact. Hence, a more correct approach is to fix birth and death rates at least a generation before the period analyzed.

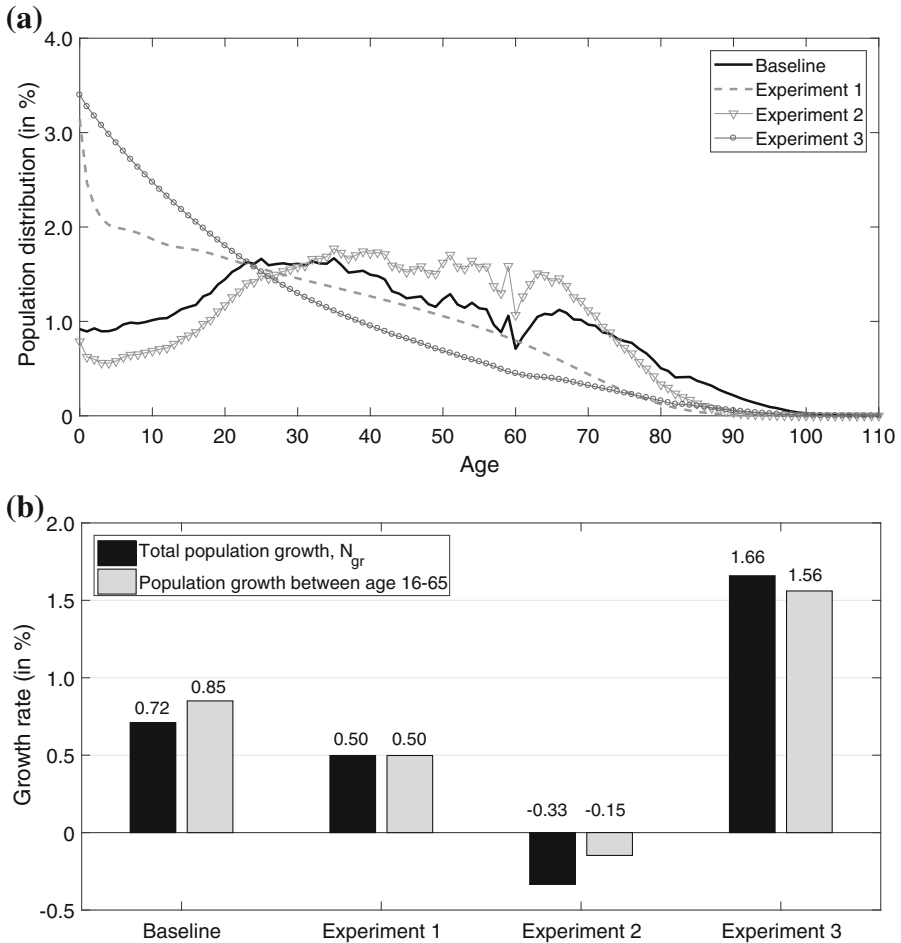


Fig. 8 Population characteristics under different experiments, Spain 1850–2000. **a** Population distribution in year 2000, $N_{j,2000}/N_{2000}$ and **b** population growth rates from 1850 to 2000. *Source* Authors’ calculation. *Notes* Experiment 1 assumes a fixed TFR around 5 and a life expectancy at birth of 31.5 years. Experiment 2 assumes a fixed life expectancy at birth of 31.5 years and a TFR as in the baseline. Experiment 3 assumes the TFR to be fixed around 5 and life expectancy evolving as in the baseline

which is replicated by our baseline model. To gain some intuition about our counterfactual experiments, let us assume per capita income growth from time t_0 to time t , $(Y/N)_{gr}$, is explained by an average exogenous increase in productivity (A_{gr}), by demographic changes (Dem), and by other exogenous factors (I). Thus, per capita income growth from time t_0 to t is given by

$$(Y/N)_{gr} = A_{gr} + Dem + I. (Baseline) \tag{22}$$

If we cancel—like in Experiment 1—the demographic changes (Dem), ceteris paribus other exogenous information (i.e. $A_{gr} + I$), the new per capita income growth during

the same period $(\widehat{Y/N})_{gr}$ will be given by

$$(\widehat{Y/N})_{gr} = A_{gr} + I. \text{ (Experiment)} \tag{23}$$

Thus, from (22) and (23) the contribution of demography to the observed per capita income growth can be calculated as follows

$$\frac{(Y/N)_{gr} - (\widehat{Y/N})_{gr}}{(Y/N)_{gr}} = \frac{Dem}{A_{gr} + Dem + I}. \tag{24}$$

The same intuitive calculation can be done either for each demographic factor (i.e. experiments 2 and 3) or for any other factor affecting per capita income growth. Next we explain how demography affects per capita income and its relative contribution. *Mortality and fertility effects.* Following Kelley and Schmidt (2005) our model captures the effect of demography on per capita income through two main channels:

- (i) the difference between the growth rate of workers and the growth rate of the population, or *translation component*, and
- (ii) through changes in the market labor supply and in household savings, caused by the rise in life expectancy and the fall in fertility, or *productivity component*. Notice the productivity component arises not only from changes in the behavior of individuals, but also from changes in the population’s age structure.

Given our economic setup, these two components are well-captured by the following decomposition of per capita income growth¹²

$$(Y/N)_{gr} = \underbrace{\left(\frac{\alpha}{1 - \alpha} K/Y \right)_{gr}}_{\text{Productivity component}} + (L/W)_{gr} + \underbrace{(W)_{gr} - (N)_{gr}}_{\text{Translation component}} + A_{gr}. \tag{25}$$

By comparing per-capita income growth in the baseline to that in the experiments, the first term on the right-hand side of Eq. (25) mainly captures the change in household savings, the second term L/W mainly reflects the change in the average hours worked, and the third term W/N reflects the change in the support ratio.¹³

¹² If we divide both sides of the Cobb–Douglas production function (13) by $Y_t^{\alpha_t}$, solving for Y_t , and dividing both sides by the total number of workers W_t , we get

$$\frac{Y_t}{W_t} = \left(\frac{K_t}{Y_t} \right)^{\frac{\alpha_t}{1-\alpha_t}} A_t \frac{L_t}{W_t}.$$

The growth in output per worker comes from the growth in the capital-output ratio, the growth in human capital per worker, and the labor-augmenting technological progress. Thus, substituting the output per worker in (21) gives (25).

¹³ Given that the model does not distinguish at each age between intensive labor supply (i.e. hours worked) and extensive labor supply (i.e. labor participation), W is the population between age 16 and 65 or working-age population.

Table 3 Source of per-capita output growth from 1850 to 2000 by demographic component (annual average logarithmic rates in percent) *Source* Authors’ calculations based on the OLG model

	Baseline		Experiment					
			1: Constant demography		2: Constant longevity		3: Constant fertility	
	I.a	<i>I.b</i> ^b	II.a	<i>II.b</i> ^b	III.a	<i>III.b</i> ^b	IV.a	<i>IV.b</i> ^b
$(Y/N)_{gr}$	1.63	<i>1.60</i>	1.36	<i>1.32</i>				
A: Input factors								
$(K/N)_{gr}^a$	0.61	<i>0.49</i>	0.48	<i>0.27</i>	0.55	<i>0.50</i>	0.52	<i>0.22</i>
$(L/N)_{gr}^a$	0.08	<i>0.18</i>	-0.06	<i>0.10</i>	0.04	<i>0.24</i>	-0.06	<i>-0.01</i>
TFP_{gr}^c	0.94	<i>0.94</i>	0.94	<i>0.94</i>	0.94	<i>0.94</i>	0.94	<i>0.94</i>
B.1: Productivity component								
$(K/Y)_{gr}^{rma}$	0.21	<i>0.04</i>	0.14	<i>-0.15</i>	0.17	<i>0.02</i>	0.18	<i>-0.15</i>
$(L/W)_{gr}$	-0.11	<i>0.03</i>	-0.19	<i>0.06</i>	-0.22	<i>0.08</i>	-0.09	<i>-0.01</i>
B.2: Translation component								
$(W/N)_{gr}$	0.14	<i>0.14</i>	0.00	<i>0.00</i>	0.18	<i>0.18</i>	-0.10	<i>-0.10</i>
A_{gr}	1.40	<i>1.40</i>	1.40	<i>1.40</i>	1.40	<i>1.40</i>	1.40	<i>1.40</i>

Experiment 1 shows the effect of fixing fertility and mortality at the level prevailing in 1800, Experiment 2 cancels the effect of the mortality decline, and Experiment 3 considers fertility rates to remain constant over time

Bold values indicate the main results and italic values indicate the complementary results

^aWeighted average growth rates using factor shares

^bValues in ‘.b’ columns correspond to the average annual growth rate under the assumption that the endogenous economic decisions observed in 1850 prevail until 2000. Therefore, the difference between the bold and italic numbers for each column gives the contribution of the behavioral change

^c TFP_{gr} stands for the growth rate of total factor productivity, which is equivalent to the weighted growth rate of the labor-augmenting technological progress $((1 - \alpha)A)_{gr}$

We summarize the simulation results in Table 3, which shows the source of per-capita income growth from 1850 to 2000. Column I reports the results obtained in the baseline, while columns II–IV show the results obtained in each experiment. We split Table 3 in two blocks of rows (A and B). The first block (A) shows the contribution of each input factor—physical and human capital—, both in per capita terms, to the growth rate of per-capita output. The second block (B) shows the decomposition of per-capita income growth in terms of the *productivity component* and the *translation component*; see Eq. (25). Recall that only by comparing the baseline to each experiment gives the contribution of each demographic factor to per-capita income growth. This is because the productivity component and the translation component are not only influenced by demographic factors but also by other exogenous factors such as the exogenous labor-augmenting technological progress (A), the educational expansion, the introduction of a public pension system, etc. Moreover, we include an additional (sub)column in Table 3, denoted with the extension .b, which shows for each simulation the decomposition of per-capita income growth that would result from holding the household’ saving rate and the household labor supply in year 1850 constant. Thereby, the difference between the values in columns .a and .b gives the contribution of the behavioral changes to the growth rate of each ratio $(\cdot)_{gr}$.

Table 3 shows that the average per-capita income growth rate from 1850 to 2000 in the baseline model is 1.63% per year, which is close to the actual 1.62% shown in Table 2. The contribution of the per-capita stock of physical capital (K/N) to the growth of per-capita income was 0.61% per year in the baseline (see column I.a), while the per-capita stock of human capital (L/N) contributed around 0.08% per year. We can also see in Table 3 that the growth rate of K/N is greater in column I.a (0.61%) than in column I.b (0.49%). This difference suggests that households reacted to the new economic and demographic environment by increasing their saving rate. In contrast, the smaller growth rate of L/N reported in column I.a (0.08%) compared to that in column I.b (0.18%) suggests that households reduced their labor supply during this period.

To disentangle whether the increase in savings and the decline in the labor supply are explained by demographic factors, or by other exogenous variables, we compare the experiments to the baseline. Thus, the difference in the growth rate of K/N between columns I.a and II.a suggests that demography accounts for an average increase in K/N of 0.13% per year ($= 0.61 - 0.48$) and for an increase in L/N of 0.14% per year ($= 0.08 - (-0.06)$). Looking at columns III.a and IV.a we can see that the contribution to the increase in K/N of the decline in fertility (0.09% $= 0.61 - 0.52$) is greater than the contribution of the increase in longevity (0.06% $= 0.61 - 0.55$). As shown in Fig. 8a (compare the solid black line to the circled gray line), the greater positive effect of fertility on K/N is explained by the aging of the population or, equivalently, by the change in the age structure of the population. Since older households have more wealth, the older population structure implies a greater per-capita wealth. In contrast, the lower impact of longevity on savings during the period 1850–2000 is the result of two opposite effects. First, a higher longevity negatively affects aggregate savings because the fall in mortality increases the population growth rate and, as a consequence, the average age of the population declines. Note that this effect is the opposite as the one explained for the decline in fertility. Second, the rise in longevity also has a positive effect on savings, since individuals increase their savings for retirement motive. Given that the net effect of longevity from 1850 to 2000 is positive, the latter effect dominates over the former effect during this period.

In order to analyze the impact of demography on the stock of human capital, we focus on the stock of human capital per worker (L/W) rather than on L/N . We can highlight two interesting results in Table 3. First, when the population is stable—i.e., fertility rates and mortality rates are constant—we can see in column II.a that L/W declines at a rate of 0.19% per year. This fall in L/W is explained by a reduction in the labor supply caused by the increase in the labor-augmenting technological progress. The intuition is simple. As productivity rises and households become richer, the non-homothetic preferences in Eq. (1) imply that households devote a higher fraction of their wealth to buy more leisure time and to reduce their working time. Second, comparing column I.a to II.a, we can see that demography accounts for an average increase in L/W of 0.08% per year ($= -0.11 - (-0.19)$). However, unlike the result obtained for the per-capita stock of physical capital, the positive effect of demography on L/W is explained by the increase in longevity rather than by the decline in fertility. From experiments 2 and 3 we get that the increase in longevity contributes to the rise in L/W by 0.11% per year ($= -0.11 - (-0.22)$), while the decline in fertility

Table 4 Demographic contribution to the growth rate of per-capita output $(Y/N)_{gr}$ from 1850 to 2000 by demographic component (in percent) *Source* Authors’ estimates based on Table 3

Experiment	Contribution $(Y/N)_{gr}$	Relative contribution		
		$(K/Y)_{gr}^a$	$(L/W)_{gr}$	$(W/N)_{gr}$
1: Constant demography	16.8	4.0	4.4	8.4
2: Constant longevity	6.4	2.3	6.7	− 2.7
3: Constant fertility	14.5	1.8	− 1.5	14.2

The contribution rate is calculated as $100 \times \frac{\text{Baseline} - \text{Experiment}}{\text{Baseline}}$. The relative contribution is calculated as $100 \times \frac{\text{Baseline} - \text{Experiment}}{\text{Baseline}(Y/N)_{gr}}$

Bold values indicate the main results and italic values indicate the complementary results

^aAverage growth rates calculated using the weights of the corresponding factor share $(\alpha/(1 - \alpha))$

reduces L/W by 0.02 (= $-0.11 - (-0.09)$) percent per year. The positive effect of longevity on L/W is due to the increase in the expected length of work, given that more households survive to retirement. Similar results have been derived using US data from 1850 to 1990 (Lee 2001). The negative effect of the decline in fertility on L/W is caused by a fall in hours worked. This is because households use the positive wealth effect from raising a lower number of children to purchase more leisure time.

Using the information displayed in Table 3 and applying Eq. (24), Table 4 reports the contribution of demography to the growth rate of per capita income during the period 1850–2000. The first row in Table 4 shows that demography contributed 16.8% points (= $(1.63 - 1.36)/1.63$) to the per-capita output growth in Spain from 1850 to 2000, instead of 5% estimated using the naïve demographic model; that is, the growth rate of the support ratio. The rise in longevity and the drop in fertility both had a positive and significant effect on per-capita income growth (see the last two rows in the first column of Table 4). In particular, the rise in longevity and the fall in fertility explain 6.4 (= $(1.63 - 1.53)/1.63$) and 14.5 (= $(1.63 - 1.40)/1.63$) percentage points, respectively, of the observed per-capita income growth between 1850 and 2000. Notice that since each demographic factor has a non-linear effect on the economic outcomes, the sum of both demographic components is not equal to the total effect. Moreover, the relative contributions of the translation component and the productivity component are reported in the last three columns of Table 4. The first row shows that the *translation component* accounts roughly for 50% (i.e., = $8.4/16.8$) of the demographic dividend along the period 1850–2000, while the *productivity component* explains the remaining 50%. If we study the relative contribution by demographic component, the second and third rows show that the rise in longevity mainly affected the *productivity component* (i.e., $(2.3 + 6.7)/6.4$), whereas the drop in fertility mainly affected the *translation component* (i.e., $14.2/14.5$).

5 Discussion of the results

Besides demography and the labor-augmenting technological progress, we have introduced in the OLG model four other exogenous sources of variation: (i) the increase in the number of people with secondary and higher education, or educational expansion;

Table 5 Contribution of other factors to per-capita income growth $(Y/N)_{gr}$ from 1850 to 2000 (in percent)
Source Authors' estimates

	Symbol $t = 1850$	Contribution $(Y/N)_{gr}$	Relative contribution		
			$\left(\frac{\alpha}{1-\alpha}K/Y\right)_{gr}$	$(L/W)_{gr}$	$(W/N)_{gr}$
Education	$E_t(\cdot)$	9.8	<i>-2.1</i>	<i>11.9</i>	<i>0</i>
Pen. replacement	ψ_t	-3.7	<i>-1.8</i>	<i>-1.9</i>	<i>0</i>
Capital share	α_t	7.1	<i>7.4</i>	<i>-0.3</i>	<i>0</i>
Capital depreciation	δ_t	-1.6	<i>-2.1</i>	<i>0.5</i>	<i>0</i>

The contribution rate is calculated as $100 \times \frac{\text{Baseline}-\text{Experiment}}{\text{Baseline}}$. The relative contribution is calculated as $100 \times \frac{\text{Baseline}-\text{Experiment}}{\text{Baseline}(Y/N)_{gr}}$

Bold values indicate the main results and italic values indicate the complementary results

(ii) the universalization of the pension system captured by a higher average replacement rate (ψ_t); (iii) the change in the capital share (α_t) over the period 1850–2000; and (iv) the progressive increase in the capital depreciation rate (δ_t). Under the assumption that all exogenous factors are independent, in this section we study the contribution of each factor to the growth rate of per-capita income. Hence, this exercise helps us to assess the relative importance of demography with respect to other economic factors.

To study the impact of the educational expansion on our results, we have compared our baseline simulation to a counterfactual experiment in which we fix the educational distribution of all cohorts to that observed for the cohort born in 1850. Table 5 shows that only 9.8% of the total increase in per-capita income growth from 1850 to 2000 is due to the educational expansion. This result does not imply that the contribution of education is generally small, but that the expansion of education in Spain occurred rather late compared to other European countries. Moreover, the value we obtain for the contribution of education on per-capita income growth is on the lower bound since we assumed education to have no impact on either the demographic transition (Murtin 2013; Crespo-Cuaresma et al. 2014) or the technological progress.

It is worth noting that the contribution of education to per capita income growth could also be interpreted as a contribution of demography. Indeed, there is growing evidence showing that changes in longevity may explain more than 30% of the increase in the number of years of education (Restuccia and Vandenbroucke 2013; Sanchez-Romero et al. 2016). Thus, demography may well account for 19.7% (i.e., = $16.8 + 0.3 \times 9.8$) of the observed per-capita income growth. Moreover, the model includes exogenously the quantity-quality trade-off (Becker et al. 1990). According to this theory, the decline in fertility can partly explain the increase in educational expenditure and the number of years of education (Lee and Mason 2009), which would yield values higher than 20%. We cannot rule out either the possibility that population density may also boost technological progress (Lee 1988; Kremer 1993; Galor and Weil 2000; Jones 2001), which will lead to even higher values. Nevertheless, since the causal relation between demography and education can go in both directions, in this article we opted for having a neutral position by taking the observed trends in those variables and ignoring their feedback effects.

In a lifecycle model the main driver for the accumulation of capital is the lack of income to finance the consumption during retirement, i.e., the *savings for retirement motive*. To avoid an unrealistic savings profile, we introduced the historical evolution of the social security system through our indirect estimation of the average replacement rate. Thus, by setting the average replacement rate to the value derived for the year 1850, we can have a rough assessment of the importance of the pension system on our results. Comparing the baseline to a counterfactual $\psi_t = \psi_{1850}$ for all t , we get that the expansion of the pension system has reduced per-capita income growth by 3.7%. Looking at Table 5 we note that almost 50% of the overall reduction in per-capita income is due to the fall in L/W and 50% to the reduction in K/Y . The decline in L/W is explained by the negative effect that a tax on labor income exerts on hours worked, while the reduction in savings is explained by the crowding out effect produced by the social security system.

From 1850 to 2000, one of the engines of growth was the progressive conversion of Spain from a rural economy to a system in which the industry and the tertiary sector play the most important roles. Although most of the consequences of the industrialization and tertiarization of the economy has already been captured by the labor-augmenting technological progress, this process is also partly reflected by the change in the labor share and the depreciation of capital. To capture the effect of the change in labor share and capital depreciation on our results, we run two additional experiments assuming a constant labor share of two-thirds, which corresponds to that reported by Prados de la Escosura and Rosés (2010b) for 1850, and another with a constant capital depreciation rate at the level of 1850. Prados de la Escosura and Rosés estimate that the labor share in Spain was below two-thirds from the beginning of the twentieth century until the 1930s, it was above two-thirds from 1930 until World War II, witnessed a sharp decline at the beginning of the 1950s, and recovered to pre-Civil War levels after the Stabilization Plan in 1959. The initial capital depreciation rate is estimated at 5.24%, while the capital depreciation rate at the beginning of the twenty-first century is slightly above 6%. We also include in the capital depreciation series the estimated 7% productive capital destruction during the Civil War (Prados de la Escosura and Rosés 2010a). Table 5 reports that the change in the labor share over the period 1850–2000 positively contributed by 7.1% points to per-capita income growth, whereas the increase of the capital depreciation affected per capita income growth negatively by 1.6%.

As a final remark, it is also interesting to study what will be the impact of demography if the model only considers the adult population, which is the standard practice in many overlapping generation models. Provided the same parameter values reported in Table 1, we obtain that the total contribution of demography on per capita income growth in the model without children (aged 0–15) is only 6.5%. Thereby, we conclude that the standard practice in OLG models underestimates the actual contribution of demography on per capita income growth by 10% points.

6 Conclusion

In this paper we studied the contribution of changes in the demographic structure on per-capita income growth. To shed light on this old topic, we used a new approach by

considering an OLG model populated by households that are heterogeneous by level of education, who optimally choose the consumption of market- and home-produced goods, and the time spent working in the market and in home production. Then, we validated our approach by showing that our model is capable of replicating the observed increase in income and consumption per capita, and the pronounced decline in hours of work from 1850 to 2000.

Our main finding suggests that the rise in longevity and the drop in fertility account for around 17% of the increase in per-capita income growth from 1850 to 2000. An analysis by demographic component gives that the drop in fertility explains 14.5% of the economic growth, while the rise in longevity explains 6.4%. We also studied the contribution of the demographic transition in terms of the *translation component* (population structure) and the *productivity component* (changes in labor supply and capital accumulation). Our results suggest that the translation component accounts for 50% of the total income growth from 1850 to 2000, and hence the productivity component accounts for 50%. By running additional counterfactual experiments we obtained two other key findings. First, we showed that, after productivity, demography is the most important factor explaining per-capita income growth for the country analyzed. Second, we showed that OLG models highly underestimate the impact of demography on per-capita income growth when dependent children are not included.

Our estimated contribution of demography to economic growth from 1850 to 2000 should, however, not be taken as a fixed number. We believe the value around 17% points is likely to be the minimum contribution of demography to per-capita income growth. Indeed, there is growing evidence showing that changes in longevity may explain to a large extent the educational transition and the change in labor supply (Cervellati and Sunde 2013; Restuccia and Vandenbroucke 2013; Sanchez-Romero et al. 2016). This figure could be even higher if we consider that population density may boost technological progress (Lee 1988; Kremer 1993; Galor and Weil 2000; Jones 2001; Croix et al. 2008). Thus, further research is needed to investigate other interactions such as the interplay among demographic change, the education transition, and productivity growth.

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Appendices

A Household problem

For notational simplicity and without loss of generality we get rid of subscripts that denote the level of education e and time t . The optimal allocation of time and con-

sumption of a household, whose head is of age j , is obtained by maximizing (3) with respect to c_j^m, c_j^h, c_j^x, h_j , and z_j , subject to (4), the time constraint, and the boundary condition.

The first-order conditions (FOC) are:

$$c_j^m : \frac{\partial u_e^h}{\partial c_j^m} = \pi_{j+1} \frac{\partial V_{j+1}(\mathbf{x}_{j+1})}{\partial a_{j+1}}, \tag{26}$$

$$c_j^h : \frac{\partial u_e^h}{\partial c_j^h} = \frac{\partial u_e^h}{\partial c_j^m} p_j^* + \lambda_1, \tag{27}$$

where p_j^* is the shadow price of home-produced goods and services in a household whose head is age j and λ_1 is the Lagrange multiplier associated to home production.

$$c_j^x : \frac{\partial u_e^h}{\partial c_j^h} \frac{\partial f^h}{\partial c_j^x} = \frac{\partial u_e^h}{\partial c_j^m}, \tag{28}$$

$$h_j, z_j : \frac{\partial u_e^h}{\partial c_j^h} \frac{\partial f^h}{\partial h_j} = \frac{\partial u_e^h}{\partial z_j} (1 + v(n_j)) \quad \text{for all } j. \tag{29}$$

If households supply their labor in the market, the time spent on home production satisfies

$$h_j : \frac{\partial u_e^h}{\partial c_j^h} \frac{\partial f^h}{\partial h_j} = \frac{\partial u_e^h}{\partial c_j^m} (1 - \tau_j) w_j (1 + v(n_j)). \tag{30}$$

The envelope condition (EC) is

$$a_j : \frac{\partial V_j(\mathbf{x}_j)}{\partial a_j} = \pi_{j+1} \frac{\partial V_{j+1}(\mathbf{x}_{j+1})}{\partial a_{j+1}} R_j. \tag{31}$$

Combining (26) and (31) gives (7). Equation (11) is obtained dividing (30) by (28).

B Market clearing conditions

Let $j \in \mathbf{J} = \{0, \dots, 100\}$, $t \in \mathbf{T} = \{1500, \dots, 2500\}$, and $e \in \mathbf{E}$. Given initial values $\{\bar{c}^m, \bar{c}^h, \phi_e^m, \phi_e^h, \phi_e^z, \sigma_e, \theta, \rho, \alpha_t, \delta_t, g_t, \psi_t, J_0, J_R\}_{e \in \mathbf{E}, j \in \mathbf{J}, t \in \mathbf{T}}$, demographics $\{N_{j,t}, n_{j,t}, \pi_{j,t}\}_{j \in \mathbf{J}, t \in \mathbf{T}}$, the educational distribution $E_t(e)$ for cohorts born at time $t \in \mathbf{T}$, and the age-specific productivity endowment by educational attainment $\{\epsilon_{j,e}\}_{e \in \mathbf{E}, t \in \mathbf{T}}$, a recursive competitive equilibrium is a sequence of a set of household policy functions $\mathbf{c}_{e,j,t} \in \mathbf{C}$, government policy functions $\{tr_{j,t}, \tau_t\}_{j \in \mathbf{J}, t \in \mathbf{T}}$, and factor prices $\{r_t^H, r_t^K\}_{t \in \mathbf{T}}$ such that

1. Factor prices equal their marginal productivities.

2. The government's budget constraint (14) is satisfied and all accidental bequests equal all transfers given

$$\sum_{j=16}^{100} N_{j,t} (1 - \pi_{j,t}) \int_{\mathbf{E}} a_{e,j,t} dE_{t-j}(e) = \sum_{j=16}^{100-28} N_{j,t} \pi_{j,t} tr_{j,t}$$

where

$$tr_{j,t} = \max\{0, \xi_{j,t}\},$$

$$D_t = \sum_{j=16}^{100} N_{j,t} \max\{-\xi_{j,t}, 0\}$$

with

$$\xi_{j,t} = \begin{cases} \frac{N_{j+28,t}(1-\pi_{j+28,t})}{N_{j,t}\pi_{j,t}} \int_{\mathbf{E}} a_{e,j+28,t} dE_{t-j-28}(e) + \frac{1-\pi_{j,t}}{\pi_{j,t}} \int_{\mathbf{E}} a_{e,j,t} dE_{t-j}(e) & \text{if } 16 \leq j \leq 44, \\ \frac{N_{j+28,t}(1-\pi_{j+28,t})}{N_{j,t}\pi_{j,t}} \int_{\mathbf{E}} a_{e,j+28,t} dE_{t-j-28}(e) & \text{if } j > 44. \end{cases}$$

3. Given the factor prices and government policy functions, household policy functions satisfy Eqs. (7)–(2), and the commodity of home production clears:

$$\sum_{j=16}^{100} N_{j,t} \int_{\mathbf{E}} f(c_{e,j,t}^x, h_{e,j,t}) dE_{t-j}(e) = \sum_{j=17}^{100} N_{j,t} \int_{\mathbf{E}} c_{e,j,t}^h dE_{t-j}(e)$$

4. The stock of physical capital and the labor input are given by:

$$K_t = \sum_{j=16}^{100} N_{j,t} \int_{\mathbf{E}} a_{e,j,t} dE_{t-j}(e)$$

$$L_t = \sum_{j=16}^{100} N_{j,t} \int_{\mathbf{E}} \epsilon_{e,j} \ell_{e,j,t} dE_{t-j}(e)$$

5. The commodity market clears:

$$Y_t = C_t + S_t$$

where the total consumption of market goods $C_t = \sum_{j=16}^{100} N_{j,t} \int_{\mathbf{E}} c_{e,j,t}^m + c_{e,j,t}^x dE_{t-j}(e)$ and S_t is gross savings in year t .

C Population reconstruction

Time is discrete. Individuals are assumed to live for a maximum of 100 years. Let the survival probability to age j in year t be

$$S_{j,t} = \prod_{x=0}^{j-1} \pi_{x,t-x} \quad \text{with } S_{0,\cdot} = 1, S_{100,\cdot} = 0, \tag{32}$$

where $\pi_{j,t}$ is the conditional probability (of being alive at age j in year t) of surviving to age $j + 1$ (with $\pi_{j,\cdot} = 0$, for all $j \geq 100$). Let $N_{j,t}$ be the size of the population at age j in year t . We assume a closed population. Thus, the population at time $t + 1$ is given by the population in year t plus the total number of births in year t , denoted B_t , less the total number of deaths during the year D_t . The dynamics of the population can be written in matrix notation using a Leslie matrix (Leslie 1945; Preston et al. 2002)

$$\mathbf{N}(t + 1) = \mathbf{\Gamma}(t)\mathbf{N}(t), \tag{33}$$

with

$$\begin{aligned} \Gamma_{j,1}(t) &= \frac{L_{0,t}}{2S_{0,t}} \left(f_{j,t} + f_{j+1,t} \frac{L_{j+1,t}}{L_{j,t}} \right) f_{fab}, \\ \Gamma_{j+1,j}(t) &= \frac{L_{j+1,t}}{L_{j,t}}, \text{ for } j \in \{1, \dots, 99\} \quad \text{at time } t, \end{aligned} \tag{34}$$

where $L_{j,t} = \frac{S_{j,t} + S_{j+1,t}}{2}$ is the person years lived by the cohort between ages j and $j + 1$ in period t , $f_{j,t}$ is the age-specific fertility rate at age j in year t , f_{fab} is the fraction of females at birth (we assume $f_{fab} = 0.4886$, which is the standard value in the demographic literature).

To reconstruct the population in Eqs. (33) and (34), we use a simplified version of a GIP model that matches the specific characteristics of our economic model: one gender without distinction between parity and region of birth, among others. The objective function used to solve the problem is:

$$\begin{aligned} \min_{\{\alpha_t^1, \alpha_t^2, \alpha_t^3, \mu_t, \beta_t\}} & \sum_{t \in \mathbb{D}} \left(1 - \hat{D}_t / D_t \right)^2 + \sum_{t \in \mathbb{B}} \left(1 - \hat{B}_t / B_t \right)^2 + \sum_{t \in \mathbb{N}} \left(1 - \hat{N}_t / N_t \right)^2 \\ & + \sum_{t \in \mathbb{E}} \left(1 - \hat{e}_{0,t} / e_{0,t} \right)^2 + \sum_{t \in \mathbb{T}} \left(1 - \hat{\text{TFR}}_t / \text{TFR}_t \right)^2 + \sum_{t \in \mathbb{C}} \sum_{a=0}^{\Omega-1} \left((N_{a,t} - \hat{N}_{a,t}) / N_t \right)^2 \\ & + \sum_{t=t_0}^T \sum_{i=0}^2 \left(\alpha_{t+1}^i - \alpha_t^i \right)^2 + \sum_{t=t_0}^T (\mu_{t+1} - \mu_t)^2 + \sum_{t=t_0}^T (\beta_{t+1} - \beta_t)^2, \end{aligned} \tag{35}$$

subject to Eqs. (33)–(34) and to

$$\sum_{k=1}^3 \alpha_t^k f_j^{(k)} = f_{j,t} \quad \text{with } \sum_{k=1}^3 \alpha_t^k = 1, \tag{36}$$

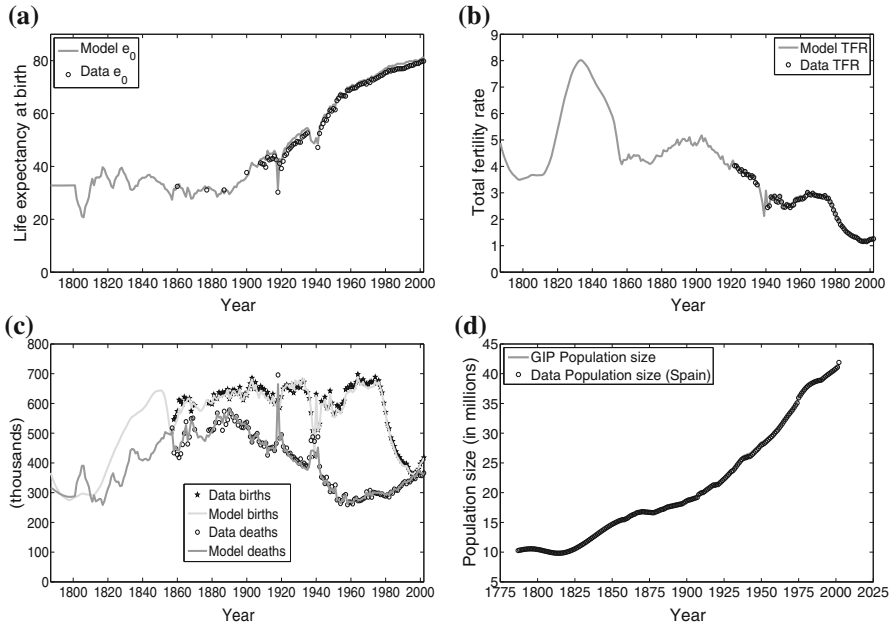


Fig. 9 In-sample performance of the GIP model to existing demographic data. Spain: Selected year between 1787 and 2000. **a** Life expectancy at birth, **b** total fertility rates, **c** total births and deaths and **d** total population

$$\prod_{x=0}^{j-1} \pi_{x,t} = \frac{e^{2(\mu_t + \beta_t Y^1(x) + (1-\beta_t)Y^2(x))}}{1 + e^{2(\mu_t + \beta_t Y^1(x) + (1-\beta_t)Y^2(x))}} \quad \text{with } \beta_t \in [0, 1], \quad (37)$$

where $\{\alpha_t^1, \alpha_t^2, \alpha_t^3, \mu_t, \beta_t\}$ are the corresponding parameters for fertility and mortality, respectively; $f_x^{(i)}$ and $\{Y^1(\cdot), Y^2(\cdot)\}$ are actual age-specific fertility rates and two Brass logit model standards—where $Y^1(\cdot), Y^2(\cdot)$ are associated to high mortality and low mortality rates, respectively—; and $\mathbb{I} \equiv \{\mathbb{D}, \mathbb{B}, \mathbb{N}, \mathbb{E}, \mathbb{T}, \mathbb{C}\}$ are the sets of deaths, births, total population, life expectancy, total fertility rates, and censuses used in the calculation. Crude migration rates are obtained using *inverse population projection* and are exogenous to the GIP model. Since GIP suffers from weak ergodicity, we use an initial population growth rate consistent with historical data prior to 1800 based on [Livi-Bacci and Reher \(1991\)](#) and [Reher \(1991\)](#).

Figures 9 and 10 show the in-sample performance of our population reconstruction with the existing demographic information. The demographic information used in Eq. (35) is depicted in these two figures. Specifically, in Fig. 9 we plot the life expectancy, total fertility rate, total number of births, total number of deaths, and the total population; whereas in Fig. 10 we compare the population distribution for some selected census years to the associated censuses.

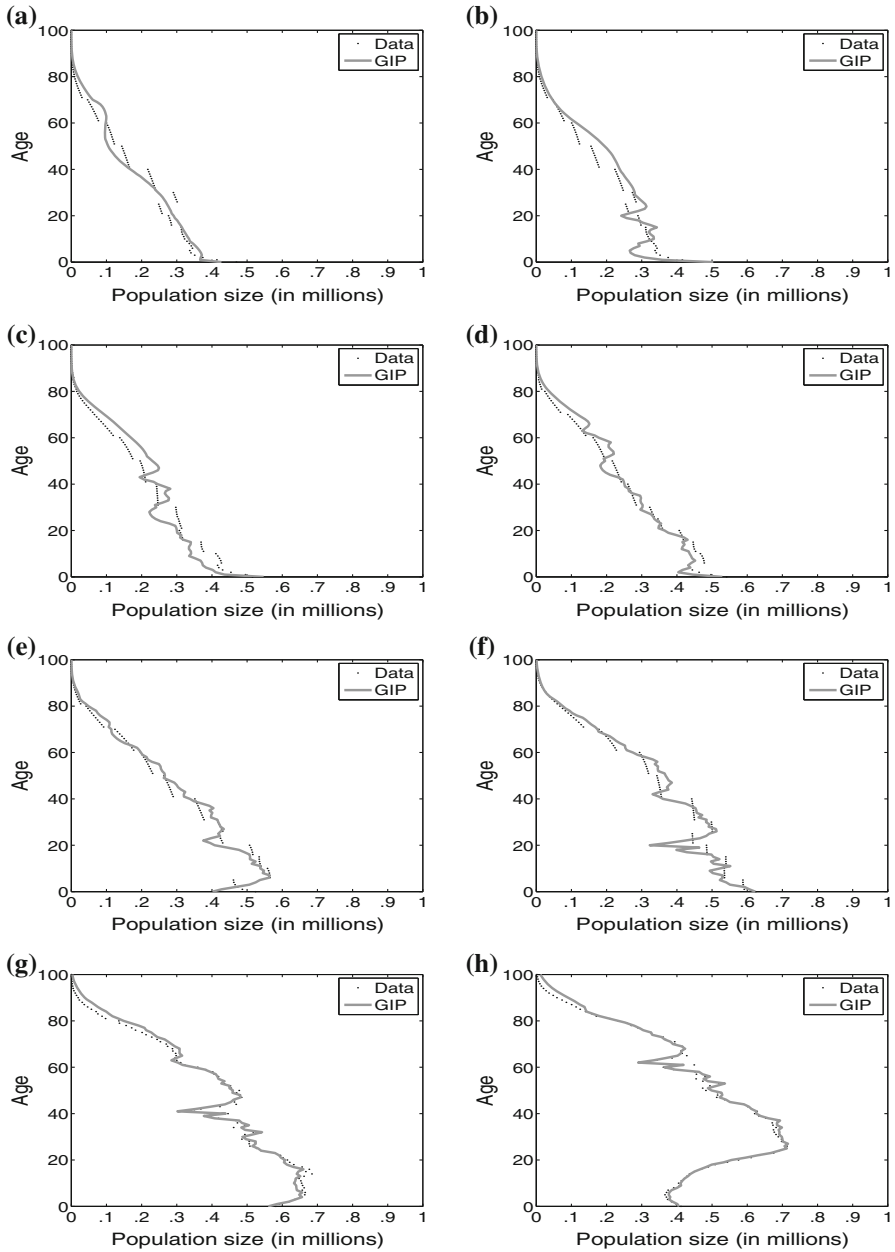


Fig. 10 In-sample performance of the GIP model to existing census data. Spain: Selected year between 1857 and 2002. **a** Census 1857, **b** census 1877, **c** census 1900, **d** census 1920, **e** census 1940, **f** census 1960, **g** census 1980 and **h** census 2002

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