Dear Author,

Here are the proofs of your article.

- You can submit your corrections **online**, via **e-mail** or by **fax**.
- For **online** submission please insert your corrections in the online correction form. Always indicate the line number to which the correction refers.
- You can also insert your corrections in the proof PDF and **email** the annotated PDF.
- For fax submission, please ensure that your corrections are clearly legible. Use a fine black pen and write the correction in the margin, not too close to the edge of the page.
- Remember to note the **journal title**, **article number**, and **your name** when sending your response via e-mail or fax.
- **Check** the metadata sheet to make sure that the header information, especially author names and the corresponding affiliations are correctly shown.
- **Check** the questions that may have arisen during copy editing and insert your answers/ corrections.
- **Check** that the text is complete and that all figures, tables and their legends are included. Also check the accuracy of special characters, equations, and electronic supplementary material if applicable. If necessary refer to the *Edited manuscript*.
- The publication of inaccurate data such as dosages and units can have serious consequences. Please take particular care that all such details are correct.
- Please **do not** make changes that involve only matters of style. We have generally introduced forms that follow the journal's style.
  Substantial changes in content, e.g., new results, corrected values, title and authorship are not allowed without the approval of the responsible editor. In such a case, please contact the Editorial Office and return his/her consent together with the proof.
- If we do not receive your corrections **within 48 hours**, we will send you a reminder.
- Your article will be published **Online First** approximately one week after receipt of your corrected proofs. This is the **official first publication** citable with the DOI. **Further changes are, therefore, not possible.**
- The **printed version** will follow in a forthcoming issue.

Please note

After online publication, subscribers (personal/institutional) to this journal will have access to the complete article via the DOI using the URL: http://dx.doi.org/[DOI].
If you would like to know when your article has been published online, take advantage of our free alert service. For registration and further information go to: http://www.link.springer.com.

Due to the electronic nature of the procedure, the manuscript and the original figures will only be returned to you on special request. When you return your corrections, please inform us if you would like to have these documents returned.

Metadata of the article that will be visualized in OnlineFirst

ArticleTitle	Longevity risk and economic growth in subpopulations: evidence from Italy	
Article Sub-Title		
Article CopyRight	Associazione per la Matematica Applicata alle Scienze Economiche e Sociali (AMASES) (This will be the copyright line in the final PDF)	
Journal Name	Decisions in Economics and Finance	
Corresponding Author	Family Name	**Levantesi**
	Particle	
	Given Name	**Susanna**
	Suffix	
	Division	
	Organization	Sapienza University of Rome
	Address	Viale Regina Elena 295-G, 00161, Roma, Italy
	Phone	
	Fax	
	Email	susanna.levantesi@uniroma1.it
	URL	
	ORCID	http://orcid.org/0000-0002-4644-4358
Author	Family Name	**Bozzo**
	Particle	
	Given Name	**Giuseppina**
	Suffix	
	Division	
	Organization	Sapienza University of Rome
	Address	Viale Regina Elena 295-G, 00161, Roma, Italy
	Phone	
	Fax	
	Email	
	URL	
	ORCID	
Author	Family Name	**Menzietti**
	Particle	
	Given Name	**Massimiliano**
	Suffix	
	Division	
	Organization	University of Calabria
	Address	Rende, Italy
	Phone	
	Fax	
	Email	
	URL	

ORCID

Schedule	Received	27 November 2019
	Revised	
	Accepted	11 February 2020

Abstract	Forecasting mortality is still a big challenge for Governments that are interested in reliable projections for defining their economic policy at local and national level. The accuracy of mortality forecasting is considered an important issue for longevity risk management. In the literature, many authors have analyzed the long-run relationship between mortality evolution and socioeconomic variables, such as economic growth, unemployment rate or educational level. This paper investigates the existence of a link between mortality and real gross domestic product per capita (GDPPC) over time in the Italian regions. Empirical evidence shows the presence of a relationship between mortality and the level of real GDPPC (and not its trend). Therefore, we propose a multi-population model including the level of real GDPPC and we compare it with the Boonen–Li model (Boonen and Li in Demography 54:1921–1946, 2017). The validity of the model is tested in the out-of-sample forecasting experiment.

Keywords (separated by '-')	Longevity risk - Mortality forecasting - Multi-population mortality models - Boonen–Li model
Footnote Information	

Decisions in Economics and Finance
https://doi.org/10.1007/s10203-020-00275-x

Longevity risk and economic growth in subpopulations: evidence from Italy

Giuseppina Bozzo[1] · Susanna Levantesi[1] · Massimiliano Menzietti[2]

Received: 27 November 2019 / Accepted: 11 February 2020

Abstract

Forecasting mortality is still a big challenge for Governments that are interested in reliable projections for defining their economic policy at local and national level. The accuracy of mortality forecasting is considered an important issue for longevity risk management. In the literature, many authors have analyzed the long-run relationship between mortality evolution and socioeconomic variables, such as economic growth, unemployment rate or educational level. This paper investigates the existence of a link between mortality and real gross domestic product per capita (GDPPC) over time in the Italian regions. Empirical evidence shows the presence of a relationship between mortality and the level of real GDPPC (and not its trend). Therefore, we propose a multi-population model including the level of real GDPPC and we compare it with the Boonen–Li model (Boonen and Li in Demography 54:1921–1946, 2017). The validity of the model is tested in the out-of-sample forecasting experiment.

Keywords Longevity risk · Mortality forecasting · Multi-population mortality models · Boonen–Li model

1 Introduction

In the twentieth century, life expectancy has considerably increased, raising the issue of longevity risk. An increasing attention is paid by Governments toward more reliable projections of survival probabilities so as to face uncertainty in future mortality and better estimate health and pension expenditure, and by insurance companies and pension schemes in order to face their obligations. There is an extensive literature on mortality forecasting especially in the category of extrapolation methods, including the Lee–Carter models family that is widely used in actuarial sciences. The extrap-

✉ Susanna Levantesi
susanna.levantesi@uniroma1.it

1 Sapienza University of Rome, Viale Regina Elena 295-G, 00161 Roma, Italy

2 University of Calabria, Rende, Italy

Journal: 10203 Article No.: 0275 TYPESET DISK LE CP Disp.:2020/2/25 Pages: 15 Layout: Small-Ex

olation models work on some latent factors, summarizing mortality trend along age, period and cohort. Therefore, future mortality rates will depend on unknown parameters. This problem can be overcome by explaining or substituting the latent factors with observable variables (e.g., economic growth, health expenditure, environmental conditions), which have a greater explanatory power compared to endogenous and unobservable factors. There is a growing attention in the analysis of the long-run correlations between mortality evolution and observable trends of socioeconomic variables. The related literature is quite extensive, especially on gross domestic product (used as a proxy for the economic growth). The relationship between mortality and gross domestic product has been investigated in several empirical studies, among them, the most recent are Hanewald (2011), Niu and Melenberg (2014), Boonen and Li (2017) and Seklecka et al. (2019). From the beginning of the twentieth century, many authors have observed that mortality rates tend to fluctuate with economic cycles and the literature was divided between those who argue that the relationship between mortality and economic cycles is pro-cyclical [e.g., (Tapia Granados 2008; Tapia Granados and Ionides 2011; Ruhm 2005)] and those who argues that mortality increases in times of economic instability [e.g., (Brenner 1983) and (Brenner 2005)]. In a pro-cyclical relationship, economic expansions imply increasing mortality rates, while recessions an opposite behavior. However, Brenner (2005) demonstrates that the economic growth occurred in the USA in the twentieth century led to a decrease in mortality rates. The impact of macroeconomic fluctuations on the mortality evolution has been typically discussed in a single-population framework, while an extension to a multi-population framework has been recently proposed by Boonen and Li (2017). They study the existence of a long-term relationship between economic growth and mortality for groups of closely related populations, and forecast mortality for each population by considering this relationship. They assume that the real gross domestic product per capita (GDPPC) of the countries within a group with similar socioeconomic conditions should not diverge and extend the Li–Lee multi-population model (Li and Lee 2005) by incorporating the GDPPC common trend for the whole group, instead of the population-specific GDPPC. The literature about multi-population models based on regional analyses is quite extensive. By way of example, Debón and Montes (2011) proposed a multi-population mortality model for the Spanish regions, while Danesi et al. (2015) compared ten different extensions of the Lee–Carter model (1992) for the Italian regions. The use of multi-population models in forecasting mortality is motivated by the need of coherent mortality forecasts for a group of populations, when the populations are similar for socioeconomic conditions and/or belong to a single population that has been classified according to gender, country area, income level and other meaningful characteristics. In this paper we investigate the relationship between mortality evolution and macroeconomic fluctuations over time for the Italian regions. It is well understandable that the regional populations share some common features and their mortality can be jointly modeled by a common time trend. In this perspective, we should consider a multi-population mortality model that is able to integrate this relationship by simultaneously modeling the regional populations. However, mortality improvements can be due to advances in economic growth, public health, lifestyle and government regulation that may differ region by region. There are many examples of regions that are inhomogeneous along multiple dimensions and this is reflected

Journal: 10203 Article No.: 0275 TYPESET DISK LE CP Disp.:2020/2/25 Pages: 15 Layout: Small-Ex

on their mortality experience. Our analysis is focused on Italy that we believe could provide a meaningful case study. Italian regions, in fact, are characterized by different types of socioeconomic development, living conditions and historical differences (which implies a geoeconomic division of the country) as well as different levels of mortality improvement. Indeed, during the last decades Italian regions have experienced significant improvements in mortality, but to different extents. Differently from the findings of the paper of Boonen and Li, we observe that Italian regional mortality is correlated to the level of real GDPPC and not to its trend. Therefore, we propose a multi-population mortality model including a population-specific term depending from this level, multiplying a common mortality trend. In order to check the predictive capacity of our model, we perform an out-of-sample test. The regional differences captured by our model may have important implications on longevity risk management in annuity business.

The paper is organized as follows. In Sect. 2 we illustrate two multi-population mortality models: the Li and Lee and the Boonen and Li models. In Sect. 3 we study the relationship between mortality and real GDPPC in the Italian regions through a cointegration analysis. Section 4 relies on the proposed multiplicative common factor model that includes real GDPPC level. The results of the out-of-sample test are also provided as well as a discussion on the GDPPC predictive power on regional mortality improvements. Section 5 concludes the paper.

2 Mortality modeling and real GDPPC

In the following we present the multi-population Li and Lee model (2005) and the Boonen and Li model (2017) that is a multi-population model including GDPPC. We assume that the number of deaths are modeled by a Poisson distribution (Brouhns et al. 2002), $D(x,t,i) \sim \text{Pois}(m(x,t,i) \cdot E(x,t,i))$, and the models' parameters are estimated by maximizing the corresponding log-likelihood function.

2.1 The augmented common factor model

Li and Lee (2005) proposed a multi-population generalization of the Lee–Carter model, known as the Augmented Common Factor (ACF), aiming to model mortality for a group of populations "in a coherent way, taking advantage of commonalities in their historical experience and age patterns, while acknowledging their individual differences in levels, age patterns, and trends." Hence, the ACF ensures that the Lee–Carter forecast of the central death rates of two or more populations within a group will not diverge in the long-run. This idea is based on the consideration that the past differences among similar populations belonging to a single group should not lead to a divergence in the long-run. The ACF model has the following parameterization:

$$\log m(x,t,i) = \alpha(x,i) + B(x)K(t) + \beta(x,i)k(t,i) + \varepsilon(x,t,i) \qquad (2.1)$$

Journal: 10203 Article No.: 0275 TYPESET DISK LE CP Disp.:2020/2/25 Pages: 15 Layout: Small-Ex

where $\alpha(x,i)$, $\beta(x,i)$ and $k(t,i)$ are population-specific parameters and $K(t)$ is an index of the general level of mortality over time. $B(x)$ is a age-dependent parameter indicating the sensitivity of $\log m(x,t,i)$ to $K(t)$. Both the parameters $B(x)$ and $K(t)$ are common to all the subpopulations. For the parameters identifiability, the following constraints are necessary:

$$\sum_x B(x) = 1; \quad \sum_t K(t) = 0; \quad \sum_x \beta(x,i) = 1; \quad \sum_t k(t,i) = 0 \tag{2.2}$$

2.2 The Boonen–Li model

Boonen and Li (2017) studied the long-term relationship between economic growth and mortality evolution, considering groups of populations related to each other. They analyzed four groups: countries with low mortality, Eastern European countries, former Soviet Union countries and Sweden. They explain the mortality of a group of populations through the common GDPPC trend using a principal component analysis, instead of looking at population-specific GDPPC. In practice, their model extends the ACF by including an additional component given by the common real GDPPC in log scale, $g(t)$. The Boonen–Li model (BL) has the following parameterization:

$$\log m(x,t,i) = \alpha(x,i) + B(x)K(t) + \beta(x,i)k(t,i) + \gamma(x)g(t) + \varepsilon(x,t,i) \tag{2.3}$$

where $\alpha(x,i)$, $\beta(x,i)$, $k(t,i)$, $B(x)$ and $K(t)$ have the same meaning as in the ACF model and $\gamma(x)$ is an index describing the age pattern of $g(t)$. The authors focused the analysis on the real GDP trend and transform the logarithm of the real GDPPC in each population to have a mean equal to 0. In order to identify the parameters, the model requires the same constraints of the ACF (Eq. 2.2) and an orthogonality constraint between $K(t)$ and $g(t)$: $\sum_t K(t)g(t) = 0$.

3 Analysis of the relationship between mortality and GDPPC

3.1 Data description and notation

We consider a dataset, provided by Istat (www.istat.it), collecting mortality rates and GDPPC from the Italian regions. We joined the small size regions (having population less than 500,000 units in 2017) to one of the neighbor regions,[1] obtaining 18 regions instead of the official 20. The dataset concerns the Italian regional population for ages 40–89 and years 1980–2016. The real GDPPC dataset covers the same time period. In order to exclude the accidental mortality (typically affecting the younger ages) from our analysis, we focus on adult ages. We define the index $i = 1, 2, \ldots, I$ as the i-th subpopulation among the I populations in the study, where $I \geq 2$ and consider the following data referred to the i-th population at time t: deaths among individuals

[1] Valle d'Aosta is joined to Piemonte (Piemonte-Valle d'Aosta) and Molise to Abruzzo (Abruzzo-Molise).

Journal: 10203 Article No.: 0275 TYPESET DISK LE CP Disp.:2020/2/25 Pages: 15 Layout: Small-Ex

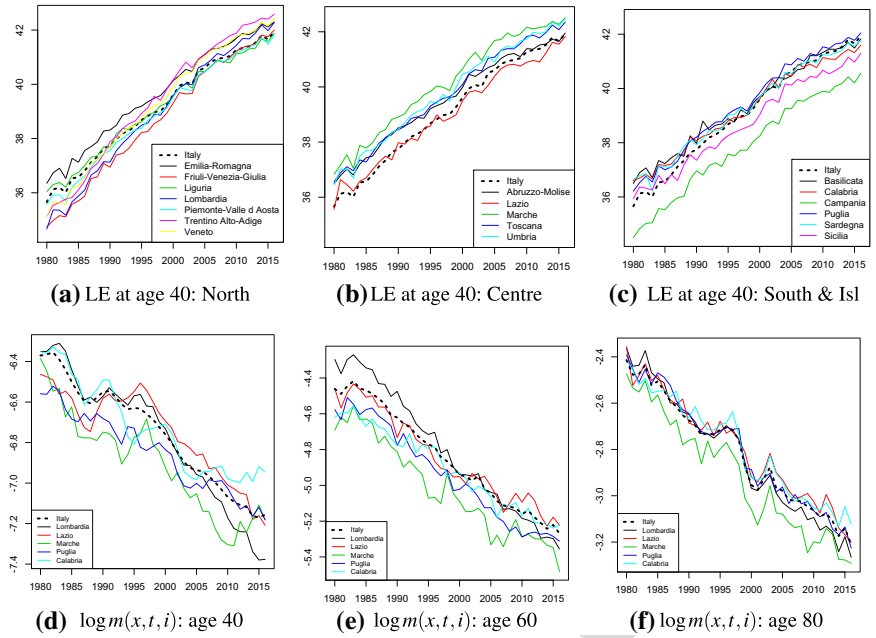
aged x, $d(x,t,i)$, exposure-to-risk aged x, $E(x,t,i)$, the corresponding central death rate, $m(x,t,i)$ and the logarithm of real GDPPC, $g(t,i)$. While the logarithm of real GDPPC for the group of populations is denoted by $G(t)$.

In Fig. 1 panel a, we plot the curtate life expectancy at age 40 for the Italian regions, divided by geographical location (North, Center and South and Islands).[2] In some regions the life expectancy converges to the national data, e.g., in Abruzzo-M. where the value was higher than the national level in 1980 but almost equal in 2016. In other cases, it remains far from the national value, e.g., in Campania. More generally, we cannot assert that regional life expectancies converge to the national one. Figure 1b shows $\log m(x,t,i)$ for three fixed ages (40, 60, 80) and for five regions that can be considered representative of the geographic location of the country. During the four past decades Italy experienced a significant improvement in mortality, and these plots seem to suggest that the level of this improvement over the period 1980–2016 follows different patterns among ages and regions, as already observed by Danesi et al. (2015). From the plots in panels a, b, c, it is not possible to state that there is an evident convergence of central death rates among regions. Their specific socioeconomic characteristics have probably led these subpopulations to experience a different mortality evolution. Therefore, a mortality model common to all the regions seems to be inadequate. In particular, a single latent factor for all regions could overestimate the evolution of mortality in some regions and underestimate it in others. On the other hand, independently modeling the mortality of each region would neglect the possible dependencies that regions of the same country should present as part of the same system. An adequate mortality model should include one or more common factors to take into account possible dependencies among regions, and one or more independent factors to take into account the regional divergences. The Italian regions show strong differences, not only from the point of view of the survival evolution, but also under social and economic aspects.

Among the Italian regions, a considerable gap in the real GDPPC level exists: for example the GDPPC is €34,233 in Lombardia against €15,738 in Calabria in 2017 (source: Istat). Therefore, the possible relationship between mortality trend and real GDPPC has to be deeply investigated in order to verify if the real GDPPC could be a possible candidate to explain the different evolution of mortality among the Italian regions. Both Hanewald (2011) and Niu and Melenberg (2014) analyzed this relationship through the comparison between the Lee–Carter mortality time index and the real GDPPC. Since our analysis refers to a group of subpopulations, we study the behavior of the ACF population-specific time index of mortality, $k(t,i)$ instead of the Lee–Carter time index. The values of $k(t,i)$ show considerable differences in the regional mortality trend (Fig. 2a, c–e). In particular, the latent factor shows a clear increasing trend for Abruzzo-M., Basilicata, Calabria, Campania, Sardegna and Sicilia, a clear decreasing trend for Friuli-V.G., Lombardia, Trentino A.A. and Veneto, and a stable trend for the remaining regions. The evolution of real GDPPC in log scale by region, $g(t,i)$, evidences appreciable differences between southern and northern regions (Fig. 2b, d–f: solid lines refer to regional parameter $g(t,i)$, while dashed line

[2] In the following, we abbreviate Abruzzo-Molise as "Abruzzo-M.," Emilia-Romagna as "Emilia-R.," Friuli-Venezia Giulia as "Friuli-V.G.," Piemonte-Valle D' Aosta as "Piemonte-V.D.," Trentino Alto-Adige as "Trentino A.A."

Journal: 10203 Article No.: 0275 TYPESET DISK LE CP Disp.:2020/2/25 Pages: 15 Layout: Small-Ex



(a) LE at age 40: North **(b)** LE at age 40: Centre **(c)** LE at age 40: South & Isl

(d) $\log m(x,t,i)$: age 40 **(e)** $\log m(x,t,i)$: age 60 **(f)** $\log m(x,t,i)$: age 80

Fig. 1 **a–c** Life expectancy (LE) at age 40; **d–f** logarithm of $m(x,t,i)$. Years 1980–2016. Solid lines refer to Italian regions, dashed line to Italy

to parameter $G(t)$ for Italy). The North of Italy is richer than the South and this gap has been widened by the last economic crisis. The comparison between left and right panels of Fig. 2 shows at a glance that mortality has improved over time at higher level in the regions with higher level of real GDPPC, suggesting that the driver of mortality evolution could be the level of GDP rather than its trend.

3.2 Cointegration analysis

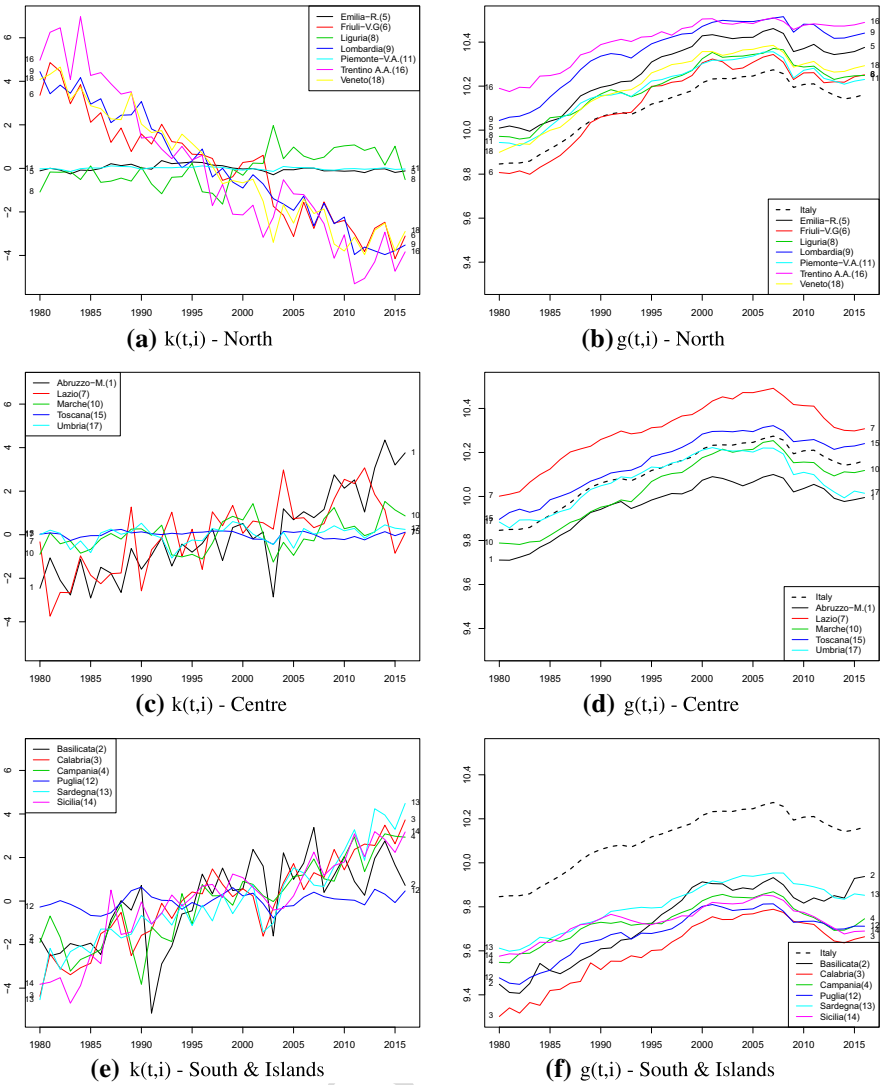
To further investigate the long-run relationship between GDP and mortality, we study the cointegration between $g(t,i)$ and the ACF population-specific time index, $k(t,i)$. The $g(t,i)$ and $k(t,i)$ time series are cointegrated if the following conditions hold:

- They are non-stationary (i.e., they have a unit root);
- Their linear combination is stationary.

This second condition is equivalent to state that there exists a constant b_1 such that the series $k(t,i) - b_1 \cdot g(t,i)$ is stationary.

We study the cointegration relationship using the Engle and Granger approach (Engle and Granger 1991). Following this approach, we preliminarily have to test the condition of non-stationarity of $g(t,i)$ and $k(t,i)$, using the Phillips–Perron (PP) test (Phillips and Perron 1988) that is a unit root test checking if the time series is integrated of order 1, $I(1)$. Secondly, if the condition of non-stationarity is confirmed by the test, the constant b_1 can be estimated using linear regression: $k(t,i) = b_0 + b_1 \cdot g(t,i) + u(t)$,

Journal: 10203 Article No.: 0275 TYPESET DISK LE CP Disp.:2020/2/25 Pages: 15 Layout: Small-Ex



(a) k(t,i) - North **(b)** g(t,i) - North

(c) k(t,i) - Centre **(d)** g(t,i) - Centre

(e) k(t,i) - South & Islands **(f)** g(t,i) - South & Islands

Fig. 2 $k(t,i)$ of ACF model (left panels) and $g(t,i)$ (right panels). Years 1980–2016

where b_0 is the intercept and $u(t)$ are the residuals. The $k(t,i)$ and $g(t,i)$ series are cointegrated if the residuals of the linear models $u(t)$ are stationary. This last condition can be validated by, e.g., the Augmented Dickey–Fuller (ADF) test on the residuals $u(t)$. The PP test can be applied by adding a constant, a constant and a linear trend, or neither. As regard to our data, we observe that $k(t,i)$ in the years 1980–2016 are characterized by a downward/upward trend depending from the region (Fig. 2 left panels), while $g(t,i)$ over the same period show similar patterns in all the regions (Fig. 2 right panels).

Journal: 10203 Article No.: 0275 TYPESET DISK LE CP Disp.:2020/2/25 Pages: 15 Layout: Small-Ex

Therefore, we perform the PP test on $k(t,i)$ including both a constant and a linear trend, while not-including a linear trend for $g(t,i)$. The tests' results carried out on the levels of the time series are shown in Table 1 (columns 3 and 4). The null hypothesis of non-stationarity can be rejected for $k(t,i)$ and accepted for $g(t,i)$.The PP test is also applied to the first differences of $k(t,i)$ and $g(t,i)$, even if such a transformation might miss the long-term properties of the data (see Hanewald (2011) for further details on the discussion of the opportunity of differencing a time series). Results given in Table 1 (columns 5 and 6) show that all the series become stationary after differencing. We can conclude that only the level of $g(t,i)$ satisfies the condition of non-stationarity required by the Engle–Granger procedure for cointegration.

Although the series are stationary, we still analyze the cointegration to get a definitive picture of the relationship between $g(t,i)$ and $k(t,i)$. This analysis is developed according to the ADF test, where the null hypothesis is the absence of cointegration (Table 1, columns 7 and 8). Critical values for the null of no cointegration are provided at the end of the table. As we expected, the results of the ADF test do not support the existence of a cointegration relationship between the two series. $g(t,i)$ and $k(t,i)$ are not cointegrated in 12 regions (i.e., the null hypothesis of no cointegration is accepted), except for Basilicata (cointegration verified at 1% level), Puglia (at 5% level) and Lazio, Piemonte V.D. and Toscana (at 10% level). The Engle–Granger procedure has been also used by Hanewald (2011) to test the cointegration between the Lee–Carter mortality index $k(t)$, and real GDP growth rate in six OECD countries and different age groups. Similarly to our case, Hanewald's results show a cointegration relationship in about one-quarter of the cases. Also, Niu and Melenberg (2014) applied Engle–Granger in order to directly study the long-run relationship between $\log m(x,t)$ and real GDPPC on log scale and found that these series are cointegrated. However, our analysis does not validate the hypothesis of cointegration between the evolution of mortality and the logarithm of GDPPC.

4 A multiplicative common factor model

As previously observed, mortality declined more in those regions characterized by a highest GDPPC level. Therefore, we focus on the level of $g(t,i)$, rather than on its trend, as a possible explanatory factor for describing Italian regions mortality evolution. To this aim, we define the following population-specific index, $m(i)$, as the mean of the ratio between the GDPPC at regional and national level over time:

$$m(i) = \frac{1}{T}\sum_{t}\left[\frac{g(t,i)}{G(t)}\right] \tag{4.1}$$

where T is the length of the time series. This index will be greater/lower than 1 in the wealthier/poorer regions (using GDPPC as proxy) with respect to the national value. In order to investigate the link between mortality and real GDPPC level, we compare $m(i)$ with the ratio, $r(i)$, between the slope of regional life expectancy (at age 40 over the period 1980–2016) with respect to the corresponding slope for the entire country. Hence, $r(i)$ is greater than 1 for those regions that have experienced a higher increase

Journal: 10203 Article No.: 0275 TYPESET DISK LE CP Disp.:2020/2/25 Pages: 15 Layout: Small-Ex

Table 1 Results of the PP test on the ACF population-specific index, $k(t,i)$, and real GDPPC on log scale, $g(t,i)$, in Italian regions. Levels of time series (columns 3–4) and first differences (columns 5–6)

Region		PP test: levels		PP test: first diff.		ADF test
i	Name	$k(t,i)$	$g(t,i)$	$k(t,i)$	$g(t,i)$	Test statistics
1	Abruzzo-M.	− 38.940***	− 0.566	− 42.921***	− 20.500**	− 0.887
2	Basilicata	− 31.054***	− 3.673	− 42.707***	− 27.766***	− 4.259***
3	Calabria	− 26.697***	− 0.299	− 43.094***	− 52.168***	− 0.934
4	Campania	− 24.864***	− 2.805	− 38.991***	− 23.154**	− 1.288
5	Emilia-R.	− 13.195	− 0.955	− 42.048***	− 21.461**	− 1.958
6	Friuli-V.G.	− 34.819***	− 0.917	− 50.146***	− 22.328**	− 1.225
7	Lazio	− 36.120***	0.238	− 48.929***	− 23.678**	− 3.040*
8	Liguria	− 27.088***	− 0.606	− 52.285***	− 19.602**	− 2.156
9	Lombardia	− 36.344***	− 0.832	− 52.112***	− 29.009***	− 0.574
10	Marche	− 20.939**	− 0.062	− 39.633***	− 19.948**	− 2.892
11	Piemonte-V.D.	− 21.895**	− 0.645	− 40.356***	− 26.629***	− 3.153*
12	Puglia	− 19.123**	− 1.163	− 33.767***	− 22.271**	− 3.494**
13	Sardegna	− 32.431***	0.309	− 43.368***	− 29.558***	− 0.316
14	Sicilia	− 22.309**	− 1.303	− 47.296***	− 21.174**	− 0.358
15	Toscana	− 21.196**	− 0.285	− 42.290***	− 22.791**	− 3.085*
16	Trentino-A.A.	− 25.693***	− 1.445	− 51.140***	− 35.945***	− 1.747
17	Umbria	− 25.739***	0.159	− 48.441***	− 31.669***	− 2.911
18	Veneto	− 38.940***	− 0.627	− 46.206***	− 24.436***	− 1.349

Engle–Granger cointegration approach: results of the ADF for the residuals of regression of $k(t,i)$ and $g(t,i)$ (column 7)
PP test $^{*}p < 0.10$, $^{**}p < 0.05$, $^{***}p < 0.01$. *ADF test* $^{+}-2.91$ for $p < 0.10$, $^{*}-3.17$ for $p < 0.05$, and $^{**}-3.73$ for $p < 0.01$. Lags = 1

in life expectancy than Italy. In order to be a good candidate to explain mortality, $m(i)$ should not be too sensitive to the estimation horizon. Therefore, we test its robustness using data from three periods of different extent: 1980–2006, 1980–2011, 1980–2016 (Table 2). We observe a correspondence between $m(i)$ (period 1980–2016) and $r(i)$ in 14 regions over 18. The values of $m(i)$ show very small variations over the time intervals included in our test. Overall, considering all the periods, $m(i)$ has values between 0.950 (Calabria 1980–2016) and 1.030 (Trentino-A.A. 1980–2016), while the percentage variation obtained by changing the estimation period is very small (under 1%). Therefore, $m(i)$ can be considered fairly stable over time.

Our analysis shows that regions characterized by a higher level of income show a more increasing trend in life expectancy compared to the national level, and the opposite is true for lower-income regions. Moving from this consideration, we use the $m(i)$ index (measuring the regions' level of wealth) to differentiate the evolution of mortality among the Italian regions and propose the following multi-population mortality model:

$$\log m(x,t,i) = \alpha(x,i) + m(i)B(x)K(t) + \varepsilon(x,t,i) \tag{4.2}$$

Journal: 10203 Article No.: 0275 TYPESET DISK LE CP Disp.:2020/2/25 Pages: 15 Layout: Small-Ex

Table 2 Values of $m(i)$ by different fitting periods and values of $r(i)$

Region		$m(i)$			$r(i)$
i	Name	1980–2006	1980–2011	1980–2016	1980–2016
1	Abruzzo-M.	0.992	0.987	0.986	0.861
2	Basilicata	0.965	0.961	0.963	0.856
3	Calabria	0.952	0.951	0.950	0.837
4	Campania	0.967	0.964	0.963	0.942
5	Emilia-R.	1.017	1.017	1.017	0.958
6	Friuli-V.G.	1.001	1.002	1.003	1.181
7	Lazio	1.017	1.021	1.020	0.918
8	Liguria	1.006	1.010	1.010	0.936
9	Lombardia	1.018	1.026	1.025	1.205
10	Marche	0.999	0.994	0.994	0.911
11	Piemonte-V.D.	1.014	1.009	1.009	1.020
12	Puglia	0.962	0.958	0.958	0.942
13	Sardegna	0.973	0.971	0.971	0.875
14	Sicilia	0.967	0.964	0.963	0.862
15	Toscana	1.005	1.006	1.006	0.927
16	Trentino-A.A.	1.028	1.029	1.030	1.223
17	Umbria	0.995	0.999	0.997	0.938
18	Veneto	1.004	1.010	1.011	1.168

Italian regions

Parameters are identified by applying the constraints, $\sum_x B(x) = 1$ and $\sum_t K(t) = 0$. The $m(i)$ index is an exogenous variable linking the mortality evolution over time, common to a group of subpopulations [and described by $K(t)$], to the GDPPC level of each subpopulation. Note that our model does not imply coherence of the forecasts as defined by Li and Lee (2005), consistently with the lack of coherence among Italian regions in the observed mortality evolution. The main advantages of our model are: the parsimony (number of parameters less than other models including GDP), the demographical significance due to an observable economic variable (the GDPPC level) and the ease of implementation.

In the forecasting, the time-dependent parameter $K(t)$ is modeled by an ARIMA (0, 1, 0):

$$K(t) = K(t-1) + \delta + \epsilon(t), \quad \varepsilon(t) \sim N(0, \sigma_K^2) \tag{4.3}$$

where δ is the drift parameter and $\varepsilon(t)$ are the error terms, normally distributed with null mean and variance σ_K^2. The model's goodness of fit is evaluated according to the Bayesian Information Criterion (BIC) providing a trade-off between the quality of the fit and the parsimony of the model. The BIC formula is: $BIC = -2 \log L + N \log n$, where n is the number of observations, N the number of free parameters to be estimated and L the maximized value of the likelihood function for the estimated model. Our model provides the best BIC result ($-151, 293$), compared to the BL model ($-151,$

Journal: 10203 Article No.: 0275 TYPESET DISK LE CP Disp.:2020/2/25 Pages: 15 Layout: Small-Ex

716), as a direct consequence of the parsimony: 985 parameters versus 2564 for the BL model.

4.1 Out-of-sample test

We check the model's predictive capacity through the out-of-sample test that is a traditional statistical test of a model's forecast performance. Empirical evidence from out-of-sample forecast performances is generally considered more reliable with respect to in-sample performances, usually more sensitive to outliers and data mining. In the test, the data set is split into an in-sample period (1980–2006), used to estimate the model's parameter and an out-of-sample period (2007–2016), used to evaluate the forecasting performance. The goodness of the out-of-sample test is measured through the Mean Absolute Percentage Error (MAPE) and Root Mean Square Error (RMSE). MAPE is defined by:

$$\text{MAPE} = 100 \cdot \frac{1}{\tau \cdot N} \cdot \sum_{x,t} \left| \frac{m_{x,t,i} - \hat{m}_{x,t,i}}{m_{x,t,i}} \right| \tag{4.4}$$

where N is the number of free parameters to be estimated, τ is the number of out-of-sample years and $\hat{m}_{x,t,i}$ are the central death rates forecasted by the model for the subpopulation i. While, RSME is defined by:

$$\text{RMSE} = \sqrt{\frac{\sum_{x,t} \left(m_{x,t,i} - \hat{m}_{x,t,i}\right)^2}{N}} \tag{4.5}$$

The forecast's accuracy of our model is compared to the BL model. In the latter, following the authors, the population-specific parameters $k(t, i)$ are forecasted by an AR (1) for each $i \in I$ in order to avoid a long-term divergence between mortality rates, while $G(t)$ is extrapolated by a RWD. In our model, $m(i)$ is calculated from the observed values of the GDPPC level over the in-sample period 1980–2006 and supposed to remain constant over the out-of-sample period (we checked its robustness in the previous section). Table 3 shows the results of the MAPE and RMSE for the out-of-sample forecast (period 2007–2016), where $G(t)$ is extrapolated by a RWD. Our model obtains the best performance in 8 regions (representing 44% of the country) according to MAPE and in 11 regions (61% of the country) according to RMSE. In light of these results, it is not possible to clearly determine which model is preferable between BL and our model.

4.2 Discussion on the GDPPC predictive power on regional mortality improvements

The BL model is based on the idea that the evolution of mortality is influenced by the GDPPC dynamics. Therefore, in the out-of-sample window, the model requires an assumption on the future evolution of GDPPC, and mortality projections depend on such an assumption. The $G(t)$ values, both historical (black line) and forecasted according to the BL model (blue line), are illustrated in Fig. 3 (panel a) with 95%

Journal: 10203 Article No.: 0275 TYPESET DISK LE CP Disp.:2020/2/25 Pages: 15 Layout: Small-Ex

Table 3 MAPE and RMSE

i	Name	MAPE		RMSE	
		Boonen–Li	Our model	Boonen–Li	Our model
1	Abruzzo-M.	11.81% (2)	11.78% (1)	0.003236 (1)	0.003373 (2)
2	Basilicata	10.37% (2)	9.70% (1)	0.004748 (2)	0.003326 (1)
3	Calabria	12.24% (2)	10.47% (1)	0.003664 (2)	0.002542 (1)
4	Campania	10.54% (2)	9.97% (1)	0.003899 (2)	0.003584 (1)
5	Emilia-R.	6.09% (1)	7.20% (2)	0.002987 (1)	0.003914 (2)
6	Friuli-V.G.	8.70% (2)	8.63% (1)	0.003305 (2)	0.003030 (1)
7	Lazio	9.11% (1)	10.12% (2)	0.003116 (2)	0.002969 (1)
8	Liguria	8.83% (1)	9.49% (2)	0.003122 (1)	0.003724 (2)
9	Lombardia	4.07% (1)	5.15% (2)	0.002676 (2)	0.002335 (1)
10	Marche	9.19% (1)	9.40% (2)	0.003006 (1)	0.003096 (2)
11	Piemonte-V.D.	5.48% (1)	6.39% (2)	0.002899 (1)	0.003416 (2)
12	Puglia	8.78% (2)	7.63% (1)	0.003143 (2)	0.002450 (1)
13	Sardegna	12.02% (2)	11.24% (1)	0.002509 (2)	0.001790 (1)
14	Sicilia	11.07% (2)	10.42% (1)	0.003704 (2)	0.002696 (1)
15	Toscana	7.12% (1)	7.74% (2)	0.003061 (1)	0.003548 (2)
16	Trentino-A.A.	7.49% (1)	8.64% (2)	0.003388 (2)	0.002679 (1)
17	Umbria	8.56% (1)	8.68% (2)	0.003112 (2)	0.002840 (1)
18	Veneto	4.77% (1)	6.69% (2)	0.003137 (1)	0.003215 (2)

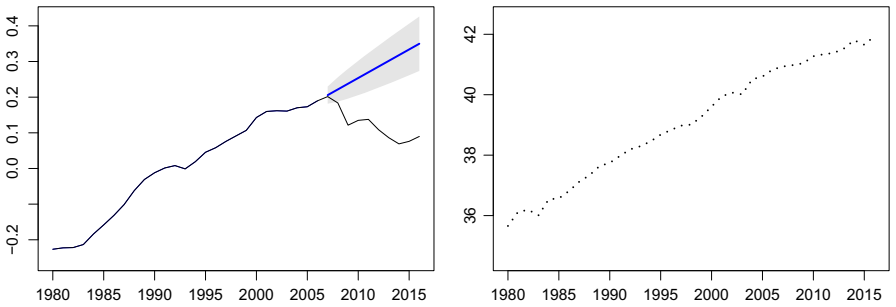
Italian regions. Years 2007–2016

confidence intervals. It is clearly evident that the historical $G(t)$ suffered a setback in 2008 due to economic crisis, bringing the country into a recession from which it has not yet come out. Since the GDPPC has grown in the estimation window (1980–2006), the BL model assumes that this growth will continue in the next years (2007–2016). As a consequence, the model assumes a positive impact of GDPPC on mortality trend in the period 2007–2016: as $\gamma(x)$ has a negative sign, an increase in GDPPC causes a reduction in mortality rates (see Eq. 2.3). According to the BL model, a decreasing GDPPC trend should produce a mortality increase or at least a slower reduction. Nevertheless, Italy has experienced a worsening of the economic conditions that, apparently, had no immediate consequences on mortality rates as shown by the evolution of life expectancy at age 40 (Fig. 3b). In light of these considerations, based on the Italian regions, the explanatory capacity of the BL model during a period of economic crisis could be questionable as the mortality rates obtained for 2007–2016 depend on the increased GDPPC assumption, not verified in Italy.

4.3 Application to annuity portfolios

In this section we measure the impact of applying our model to evaluate a life annuity portfolio. We consider three portfolio compositions: in the first (Italian portfolio) the insured population portrays the regional composition of the Italian population aged

Journal: 10203 Article No.: 0275 TYPESET DISK LE CP Disp.:2020/2/25 Pages: 15 Layout: Small-Ex



(a) Parameter $G(t)$, Italy. **(b)** LE at age 40, Italy.

Fig. 3 **a** Values of $G(t)$, historical (black line) and forecasted with the BL model (blue line) with 95% confidence intervals. **b** Life expectancy at age 40. Years 1980–2016. Italy

Table 4 Annuity values with different portfolio regional compositions

	Italy	North–East	South–West
$a(65, 2017)$	22.207	22.744	21.323
Relative change (%)	–	2.42%	−3.98%

65 in 2016, in the second (North–East portfolio) the composition of Emilia-Romagna, Veneto and Trentino Alto-Adige population, in the third (South–West portfolio) the composition of Campania, Calabria and Sicilia population. North–East regions are characterized by high GDPPC level while the South–West ones by low GDPPC level. Consequently, according to our model, the North–East portfolio will experience higher mortality improvements than the Italian population and the opposite for the South–West portfolio.

We define the expected present value of an immediate annuity of 1 m.u. per year, paid in arrears, if the policyholder is alive as:

$$a(x,t) = \sum_{h=1}^{\omega-x} {}_hp_{x,t} \cdot d(t, t+h) \tag{4.6}$$

where ω is the maximum attainable age, ${}_hp_x$ the probability of an individual aged x at time t to be alive at age $x+h$ and $d(t, t+h)$ the discount factor from time t to $t+h$.

We consider a portfolio of immediate annuities written on a cohort of 1000 individuals, all aged 65 in 2017. The interest rate is assumed to be deterministic and equal to zero, therefore $a(x,t)$ corresponds to the life expectancy at age x and time t.[3] The $a(x,t)$ values for the 18 regions are depicted in Fig. 4 and vary from 21.06 (Campania) to 22.94 (Trentino Alto-Adige), with a difference in life expectancy of almost 2 years.

The expected present value of the three annuity portfolios are reported in Table 4.

Our results show that the evaluation of an annuity portfolio with national life tables (as usual in actuarial practice) could lead to an overestimation/underestimation in pric-

[3] Life tables are extended until age 119 ($\omega = 120$) using a logistic extrapolation.

Journal: 10203 Article No.: 0275 TYPESET DISK LE CP Disp.:2020/2/25 Pages: 15 Layout: Small-Ex

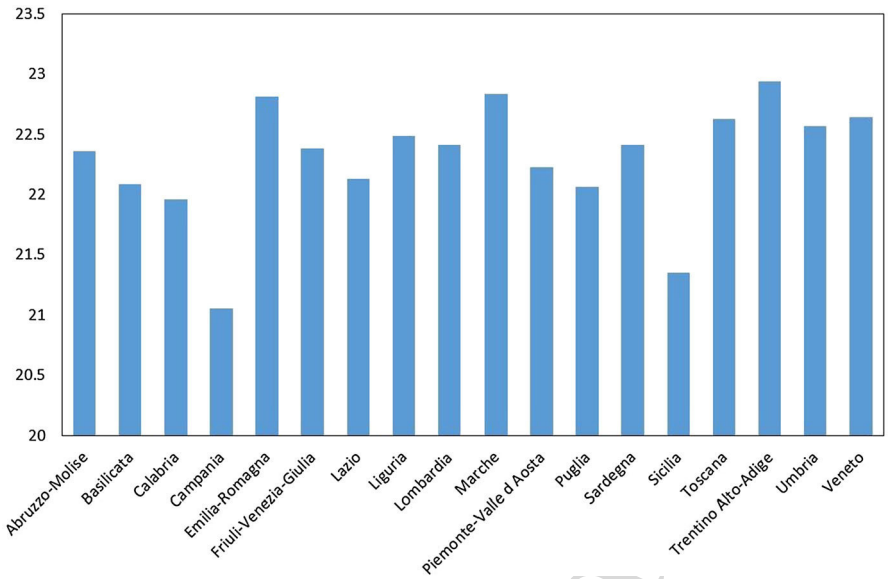


Fig. 4 Annuity values for a cohort aged 65 in 2017. Italian regions

ing and reserving. In this regard, our model, accounting for the regional heterogeneity, allows to reduce misestimations in actuarial evaluations.

5 Conclusions

During the period 1980–2016, Italy experienced a significant improvement in mortality with different regional patterns, probably due to specific socioeconomic characteristics. To deepen the understanding of this phenomenon, we study the long-term relationship between economic growth (using real GDPPC as a proxy) and mortality evolution at both regional and national level. We analyze the cointegration between the ACF population-specific time index of mortality and the logarithm of the regional real GDPPC, concluding that the evolution of mortality in Italy is not driven by the GDPPC trend. We observe that mortality declined more in the wealthiest regions, therefore we test a model that describes the mortality evolution by the GDPPC level and not by its trend. Our model is compared to the multi-population model of Boonen and Li, which includes a factor representing the GDPPC common trend, providing the best BIC results. The out-of-sample test confirms the validity of our model. It provides the best performance in term of RMSE for 61% of the country respect to the BL model. However, the level of the best performance decreases when considering MAPE (44% of the country). Our analysis shows that both the BL and our model are adequate to represent the Italian regional mortality. However, the BL model could overestimate the effects of an economic crisis on mortality rates. As soon as mortality data for a longer period is available, further analyses may be conducted by including the economic crisis in the estimation window in order to evaluating the performance

Journal: 10203 Article No.: 0275 TYPESET DISK LE CP Disp.:2020/2/25 Pages: 15 Layout: Small-Ex

of the two models. The application of our model to an annuity portfolio highlights the importance of including regional heterogeneity in actuarial evaluations. Therefore, it might be considered an useful tool for the longevity risk management of the annuity business in Italy.

References

Brenner, M.H.: Mortality and economic instability: detailed analyses for Britain and comparative analyses for selected industrialized countries. Int. J. Health Serv. **13**, 563–620 (1983)

Brenner, M.H.: Commentary: economic growth is the basis of mortality rate decline in the 20th century? Experience of US 1901–2000. Int. J. Epidemiol. **34**, 1214–1221 (2005)

Boonen, T.J., Li, H.: Modeling and forecasting mortality with economic growth: a multipopulation approach. Demography **54**, 1921–1946 (2017)

Brouhns, N., Denuit, M., Vermunt, J.K.: A poisson logbilinear approach to the construction of projected lifetables. Insur. Math. Econ. **31**(3), 373–393 (2002)

Danesi, I.L., Haberman, S., Millosovich, P.: Forecasting mortality in sub-populations using Lee-Carter type models: a comparison. Insur. Math. Econ. **62**, 151–161 (2015)

Debón, A., Montes, F., Martínez-Ruiz, F.: Statistical methods to compare mortality for a group with non-divergent populations: an application to Spanish regions. Eur. Actuar. J. **1**(2), 291–308 (2011)

Engle, R.F., Granger, C.W.J.: Long Run Economic Relationships: Readings in Cointegration. Oxford University Press, Oxford (1991)

Hanewald, K.: Explaining mortality dynamics: the role of macroeconomic fluctuations and cause of death trends. N. Am. Actuar. J. **15**, 290–314 (2011)

Lee, D., Carter, L.R.: Modeling and forecasting U.S. mortality. J. Am. Stat. Assoc. **87**, 659–671 (1992)

Li, N., Lee, R.D.: Coherent mortality forecasts for a group of populations: an extension of the Lee-Carter method. Demography **42**, 575–594 (2005)

Niu, G., Melenberg, B.: Trends in mortality decrease and economic growth. Demography **51**, 1755–1773 (2014)

Phillips, P.C.B., Perron, P.: Testing for unit roots in time series regression. Biometrika **75**, 335–346 (1988)

Ruhm, C.J.: Commentary: mortality increases during economic upturns. Int. J. Epidemiol. **34**, 1206–1211 (2005)

Seklecka, M., Lazam, N.M., Pantelous, A.A., O'Hare, C.: Mortality effects of economic fluctuations in selected eurozone countries. J. Forecast. **38**, 39–62 (2019)

Tapia Granados, J.A.: Macroeconomic fluctuations and mortality in postwar Japan. Demography **45**(2), 323–343 (2008)

Tapia Granados, J.A., Ionides, E.L.: Mortality and macroeconomic fluctuations in contemporary Sweden. Eur. J. Popul. **27**, 157–184 (2011)

Journal: **10203** Article No.: **0275** TYPESET DISK LE CP Disp.:**2020/2/25** Pages: **15** Layout: **Small-Ex**

Journal: 10203
Article: 275

Author Query Form

Please ensure you fill out your response to the queries raised below and return this form along with your corrections

Dear Author

During the process of typesetting your article, the following queries have arisen. Please check your typeset proof carefully against the queries listed below and mark the necessary changes either directly on the proof/online grid or in the 'Author's response' area provided below

Query	Details required	Author's response
1.	Kindly check and confirm that the corresponding author is correctly identified.	Yes. The corresponding author is correctly identified
2.	Journal instruction requires a city and country for affiliation 2; however, these are missing in affiliation. Please verify if the provided city and country are correct and amend if necessary.	The affiliation is correct.
3.	Please check the edit made in the article title.	I disagree the change in the article title. Please write sub-populations and not subpopulations
4.	As per the information provided by the publisher, Fig. 3 will be black and white in print; hence, please confirm whether we can add colour figure online to the caption.	Yes, I confirm it.