Enhanced Vibration Damping by Means of Negative Capacitances

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ABSTRACT

The use of shunt of piezoelectric transducers to damp mechanical vibrations is an interesting approach thanks to its low cost and the light weight of the actuators used. Among the different ways to build the shunt impedance, the use of negative capacitances is very attractive because it allows for high damping performances with low power required by the control system. Negative capacitances do not exist in the actual world but they can be designed and built using circuits based on operational amplifiers.

The use of shunt circuits based on a negative capacitance coupled to a resistance allows to have a broadband control. This paper explains how to increase the bandwidth of the controller by adding to such a shunt circuit an inductance. The dynamics of the controlled system is solved analytically and the reason why the introduction of the inductance is able to give the mentioned improvement is made clear also using numerical simulations. Furthermore, this improvement also allows to increase the attenuation performance. The conditions necessary to assure the stability of the electro-mechanical system are found and explained.

KEYWORDS: Piezoelectric shunt; vibration control; negative capacitance; damping; dynamics

1 Introduction

This paper deals with vibration reduction by means of piezoelectric benders shunted with an electric impedance. This approach is very attractive when dealing with light structures. Indeed, the control method requires no, or few, power and it is based on lightweight actuators so that no load effects occur. Furthermore, no feedback signals and digital controllers are needed. Hence, such a control method is very cheap if compared to traditional active damping.

The piezoelectric actuator is linked to an electric impedance, as mentioned. This electric impedance can be realised with different layouts, depending on the kind of control required: mono-modal, multi-modal and broadband. This paper focuses on broadband control. The simplest way to build such a kind of controller is to connect the actuator to a resistance. Indeed, Hagood and von Flotow [1] proved that this control technique is effective and a proper choice of the resistance allows to focus the damping action on a given mode of the structure. Nonetheless, also the other modes of the structure are damped, even if the control action is less than that on the mode on which the resistance has been tuned. Moreover, the method is passive because the added element (i.e. the resistance) is passive and thus no power is fed to the system. The consequence is that this control is always stable, whatever resistance values is chosen.

A big issue related to this control technique is that the total amount of control action is very limited and the attenuation provided by this shunt impedance is often poor and unsatisfactory, even if the mode on which the resistance has been tuned is taken into account [2].

A method to enhance the attenuation provided by this kind of shunt is the addition of a second element into the shunt Impedance: a negative capacitance. This element does not exist in nature but it can be realised by employing an operational amplifier (OP-AMP). The increase of the vibration attenuation allowed by this further element was already evidenced in different works in the state of the art [3,4,5]. This addition poses some problems related to the stability of the whole electro-mechanical system (EMS) (i.e. vibrating structure + piezoelectric bender + shunt impedance) because the OP-AMP introduces some energy into the system and thus stability must be checked. If a value of the negative capacitance is chosen so that the stability of the EMS is assured, a great benefit in terms of vibration reduction can be observed.

This paper proposes a new shunt impedance constituted by the resistance, the negative capacitance and an inductance. The use of an inductance is already considered in literature but usually this element is used for mono-harmonic attenuation. Indeed, the circuit composed by the resistance, the inductance and the capacitance of the piezoelectric patch (here the piezoelectric patch is modelled as a capacitance in parallel to a charge source, see the Section 2), and eventually the added negative capacitance, is a resonant system, which is the electric equivalent of the tuned mass damper [6,7]. This paper will demonstrate that the addition of the inductance can be used even for broadband damping by choosing the values of the whole electric impedance with rules different from that used for mono-harmonic control.

The structure of the paper is the following. Section 2 described the model employed to describe the EMS, while Section 3 explains how to fix the values of the resistance and the inductance for broadband damping and explains the idea behind this paper. Furthermore, this section describes some numerical simulations to show the benefits provided by the addition of the inductance. Finally, Section 4 focuses on the stability of the EMS.

2 Analytical model of the electro-mechanical system

The model employed to describe the behaviour of the EMS is the one presented in the paper of Thomas et al. [2], which was used by its authors to find the optimal tuning of R and LR impedances shunted to a piezoelectric actuator and the associated vibration attenuation performances.

A generic elastic structure is considered, with one piezoelectric patch bonded on it (see Figure 1). U is the displacement of any point x of the structure at time t. A shunt impedance Z is connected to the piezoelectric actuator and V is the voltage between the electrodes of the piezoelectric patch, which is also the shunt terminal voltage. Q is the electric charge in one of the electrodes and, considering the convention of sign for V in Figure 1, Q is the charge in the upper electrode.

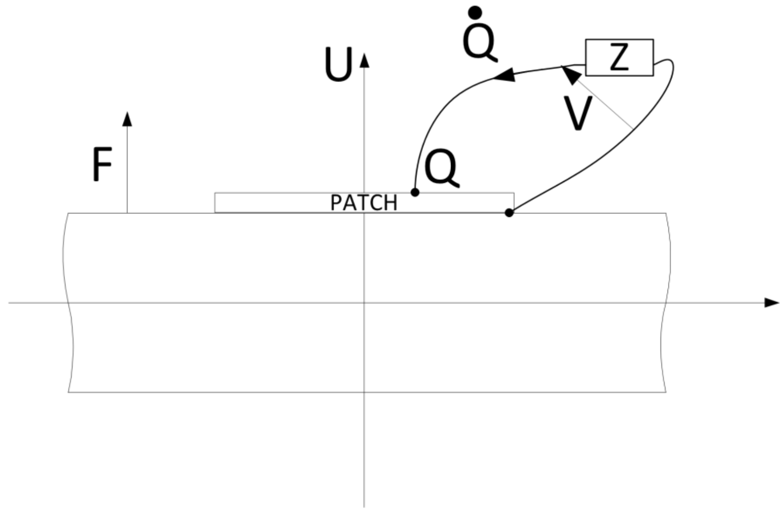

Figure 1: A piezoelectric patch bonded on a structure and shunted with an impedance *Z*. *F* is a force exciting the structure.

A reduced order model is obtained by expressing the displacement *U* in modal coordinates and considering *N* eigenmodes:

$$U(\boldsymbol{x},t) = \sum_{i=1}^{N} \phi_i(\boldsymbol{x}) q_i \qquad [1]$$

where q_i is the ith modal coordinate and ϕ_i is the ith eigenmode of the structure. The modal coordinates q_i are solutions of the problem [2]:

$$\ddot{q}_i + 2\xi_i\omega_i\dot{q}_i + \omega_i^2 q_i = F_i \qquad \forall i \in 1, \dots, N \qquad [2a]$$

$$C_p V - Q + \sum_{j=1}^{N} \chi_j q_j = 0 \qquad [2b]$$

Therefore, the motion of the EMS is described by *N* modal equations, corresponding to the balance law of mechanical forces. Since the model describes the whole electro-mechanical behaviour of the system, the dynamics of the structure is linked to the electric behaviour (described by Equation 2b) of the piezoelectric actuator and the shunt impedance by the term χ_j. Particularly, Equation 2b describes the balance of electric charges on the piezoelectric electrodes. ω_i is the ith eigenfrequency of the mechanical structure and ξ_i is the associated non-dimensional damping ratio. Here, ϕ_i, ξ_i and ω_i are related to the situation with the piezoelectric patch short circuited (i.e. $V = 0$). χ_j is a modal coupling coefficient, which is related to the energy transfer between the ith mode shape and the piezoelectric actuator. The χ_j coefficients can be computed by either a finite element model of the structure [8] or by an analytical approach [9]. Finally, C_p is the blocked (i.e. with $U(x,t) = 0 \quad \forall\, x \Rightarrow q_i = 0 \quad \forall i$) electric capacitance of the patch.

It is possible to demonstrate [5] that if one considers just one mode (under the hypothesis of low modal density), the Frequency Response Function (FRF) of the controlled system between a force and the response of the system around the ith mode considered can be expressed in the Laplace domain as:

$$H_i = \phi_i(x_f)\phi_i(x_m)\frac{ZC_{p,i}s + 1}{ZC_{p,i}s^3 + (1 + 2\xi_i\omega_i ZC_{p,i})s^2 + (2\xi_i\omega_i + \omega_i^2 ZC_{p,i} + \omega_i^2 C_{p,i}k_i^2 Z)s + \omega_i^2} \qquad [3]$$

Where $C_{p,i}$ is the capacitance of the piezoelectric patch between the ith and the (i+1)th modes, and Z is the shunt impedance. x_m is the point where the response of the structure is measured and x_f is the point where the disturbance forcing is applied. k_i is the modal electro-mechanical coupling factor, which is expressed as $k_i = \chi_i/(\omega_i\sqrt{C_{p,i}})$ and it is a very good approximation of the electro-mechanical coupling factor [2] $k_{eff,i} = \sqrt{(\omega_{OC,i}^2 - \omega_i^2)/\omega_i^2}$ (where $\omega_{OC,i}$ is the ith eigenfrequency of the EMS when the piezoelectric patch is open-circuited).

3 Electric impedance structure

The impedance Z used to shunt the piezoelectric patch is in this case the series of a resistance, an inductance and a negative capacitance. The additional element in respect to other papers in literature is the inductance (see Section 1). We start analysing the behaviour of an impedance made up by the series of just the resistance and the negative capacitance. An optimal value of the resistance $R_{i,opt}$ exists for each mode. This value is that able to maximise the attenuation for the ith mode. When this value is chosen, the other modes are damped but their attenuation is lower than that achievable with their own optimal values of the resistance. The value of $R_{i,opt}$ for each mode can be found minimising H_i^{max}, which is the maximum of the amplitude of H_i (see Equation 3).

The value of the resistance *R* must be fixed to the optimal value of the mode on which the damping action must be focused. Figure 2 shows the FRF between *V* and the charge Q_c (see Figure 3). This FRF is named H_c and can be seen as the FRF of the controller [7,10] able to damp the system and constituted physically by the shunt. Figure 2 clearly shows that the FRF of the controller is dependent on the value chosen for the resistance *R.*

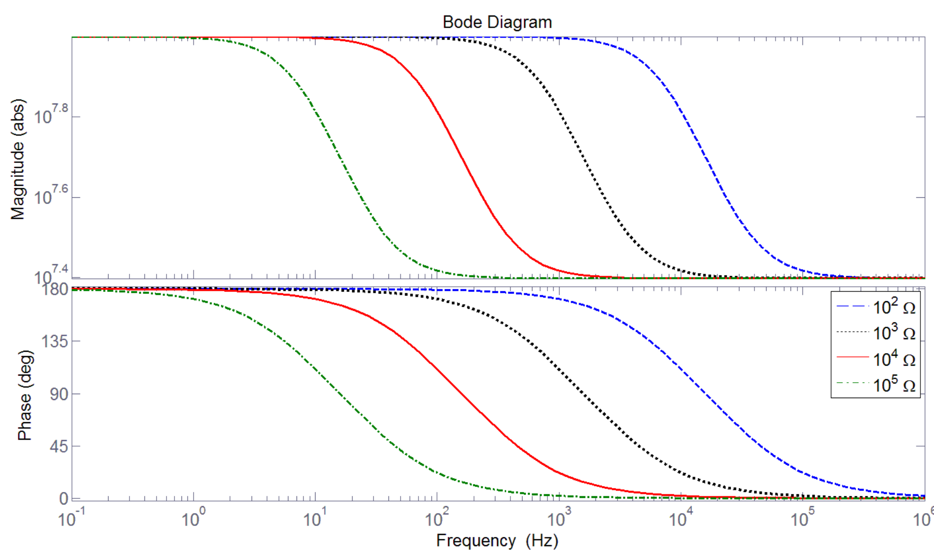

Figure 2: FRF V/Q_c (see Figure 3) for different values of R. Here $C_{p,i} = 40$ nF and $C_n = 50$ nF.

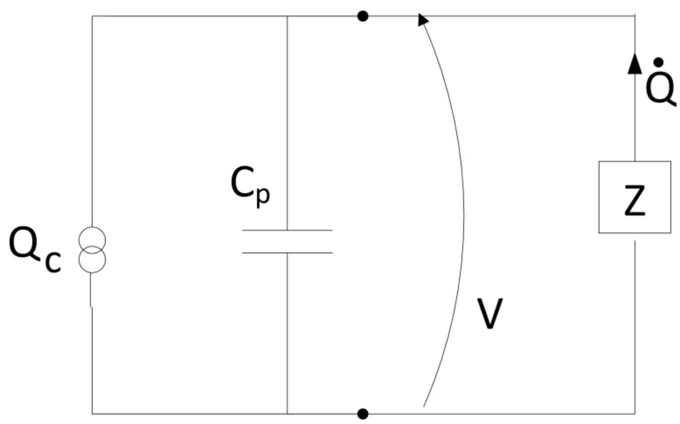

Figure 3: Electrical scheme of the shunt system; the piezoelectric patch is composed by the capacitance and the strain-induced charge source.

As for the value of the negative capacitance $-C_n$, the value assumed by C_n must be as close as possible to $C_{p,i}$ in order to maximise the effect of the negative capacitance and thus the attenuation provided by the shunt [5]. Section 4 will show that there is a threshold on the value of C_n which must be taken into account to assure EMS stability.

Now, it is important to explain why the addition of the inductance *L* allows to improve the attenuation performances. Let us suppose to be interested in damping the first three modes of a system. As soon as the value of C_n has been fixed, the value of *R* can be fixed as well. Particularly, *R* will be fixed to $R_{i,opt}$, where *i* is fixed to the mode on which the vibration attenuation must be focused. Then, the value of the inductance can be fixed. The circuit made up by *R*, *L*, $-C_n$ and C_p is a resonant system and thus has a single eigenfrequency (here named ω_e). Therefore, the FRF of the controller H_c shows an additional eigenfrequency at ω_e. Figure 4 shows H_c for ω_e tuned on the value of the first, second and third mode of a system chosen as an example. The eigenfrequencies on which ω_e must be tuned are not ω_i. Indeed, the negative capacitance with the current circuit layout (i.e. series layout) is able to shift the short-circuit eigenfrequencies towards the null frequency [3]. The new value of the short-circuit eigenfrequency ω_i^n is:

$$\omega_i^n = \sqrt{\omega_i^2 - \frac{\chi_i^2}{C_n - C_{p,i}}} \quad [4]$$

Hence, ω_e must be tuned on ω_i^n.

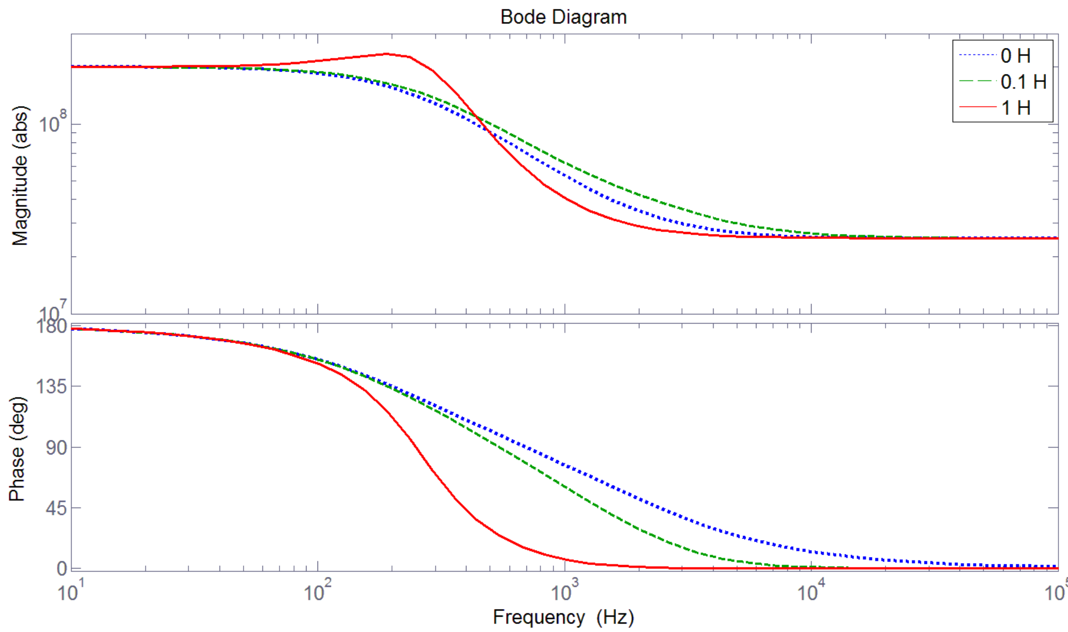

Figure 4: FRF V/Q_c (see Figure 3) for different values of L. Here $C_{p,i} = 40$ nF and $C_n = 45$ nF, R= 1.8 kΩ.

It is noticed that the effect of the controller tends to be null after ω_e. In our example we are interested in controlling the first three modes so that ω_e must be tuned on the eigenfrequency of the third mode. It is noticed that the value of *L* can be then calculated by using the expression linking ω_e to *L*:

$$\omega_e = \frac{1}{\sqrt{LC_{eq,i}}} \quad [5]$$

Where $C_{eq,i} = C_n C_{p,i}/(C_n - C_{p,i})$.

One could see the shunt as a mono-modal control. Actually, the value of *R* is much higher than the optimal value which should be used for a mono-modal resonant control [11] (indeed the value of *R* has been chosen with a criterion different from that used for classical *LR* impedances). This high value of *R* allows to have a broad peak in correspondence of ω_e for the FRF H_c (see Figure 4) and thus the control action is increased in the frequency range around ω_e. Therefore, the damping on all the modes at frequencies much lower than ω_e is not changed by the addition of *L*, while the modes ate frequencies not too far from ω_e becomes more damped by the addition of *L*. Figure 5 shows the attenuation improvements (in terms of A_r) with and without *L* for the modes of the system in Table 1 for a value of *R* chosen as $R_{4,opt}$ and ω_e fixed equal to the eigenfrequency of the fourth mode.

The attenuation improvement $A_{r,i}$ is defined as:

$$A_{r,i} = A_i(L = 0 \text{ and } R = R_{4,opt})\text{-}A_i\ (L \neq 0 \text{ and } R = R_{4,opt}) \quad [6]$$

Where A_i is the attenuation of the ith mode in decibel.

The figure indicates that the addition of the inductance is able to increase the attenuation on all the modes for about L <2 H. Obviously, the closer an eigenfrequency is to ω_e, the higher is the attenuation improvement provided by the addition of the inductance.

Mode	ω_i [rad/s]	ξ_i [%]	k_i
1	$2\pi 40$	0.2	0.20
2	$2\pi 100$	0.2	0.10
3	$2\pi 150$	0.1	0.08
4	$2\pi 200$	0.2	0.10

Table 1: Modal data of a four degree-of-freedom system.

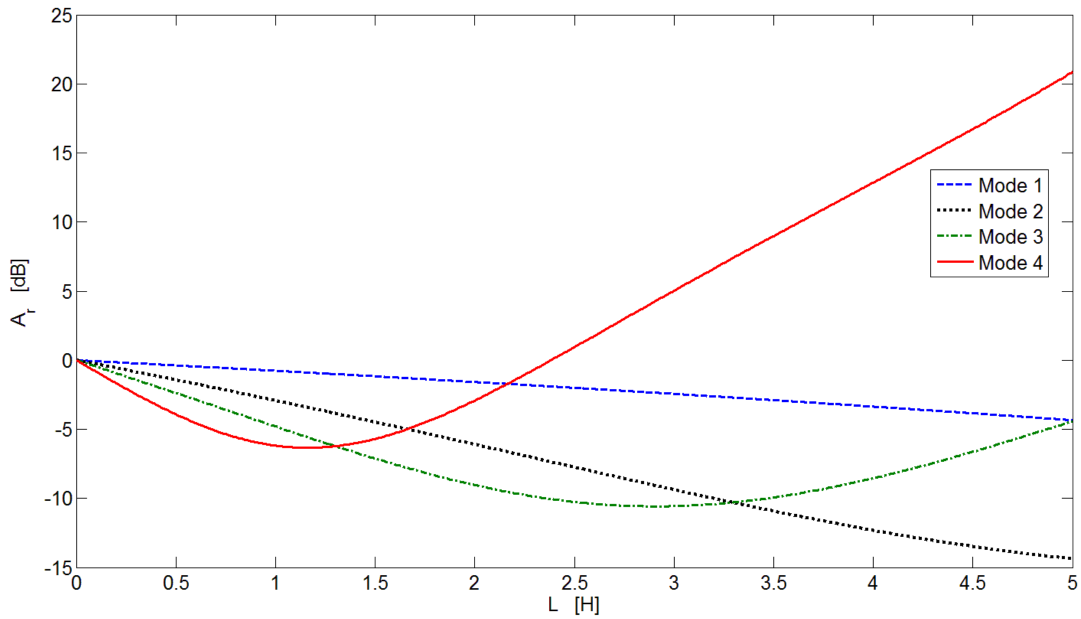

Figure 5: A_r as a function of L for the modes of the system in Table 1. Here $C_{p,4} = C_p = 40$ nF and $C_n = 45$ nF, R= 1.8 kΩ (very close to $R_{4,opt}$).

4 Stability of the electro-mechanical system

The use of a negative capacitance poses some issues related to EMS stability. We checked the stability conditions by applying the Routh-Hurwitz criterion on the denominator of H_i (see Equation 3). The result assuring stability is:

$$C_n > C_{p,i}(1 + k_i^2) \quad [7]$$

If this condition is fulfilled for each of the modes of the EMS, the strictest condition is given by the first mode [5] and the stability condition for the whole EMS becomes:

$$C_n > C_0 = C_{p,1}(1 + k_1^2) \quad [8]$$

Where C_0 is the capacitance of the piezoelectric patch at the null frequency. Therefore, the value of C_n cannot be made as close as desired to $C_{p,i}$ (as already mentioned in Section 3) but the threshold of Equation 8 must be always fulfilled.

5 Conclusion

This paper has dealt with vibration attenuation by means of piezoelectric patches shunted with electric impedances based on negative capacitances. A negative capacitance is able to increase a lot the attenuation performance of a pure resistive shunt. Thus paper explains how to increase its performance by adding an inductance into the shunt impedance layout. This added element is able to increase the broad-band attenuation. The condition for the stability of the electro-mechanical system is provided as well. The next step is the experimental validation of the strategy proposed.

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