Official URL:

https://doi.org/10.1007/978-3-319-60045-1_17

Graphical Representations of Multiple Agent Preferences

Nahla Ben Amor[1], Didier Dubois[2], Héla Gouider[1(✉)], and Henri Prade[2,3]

[1] LARODEC, University of Tunis, Tunis, Tunisie
nahla.benamor@gmx.com, hela.gouider@gmail.com
[2] IRIT, University of Toulouse, Toulouse, France
{dubois,prade}@irit.fr
[3] QCIS, University of Technology, Sydney, Australia

Abstract. A multiple-agent logic, which associates subsets of agents to logical formulas, has been recently proposed. The paper presents a graphical counterpart of this logic, based on a multiple agent version of possibilistic conditioning, and applies it to preference modeling. First, preferences of agents are supposed to be all or nothing. We discuss how one can move from the network to the logic representation and vice-versa. The new representation enables us to focus on networks associated to subsets of agents, and to identify inconsistent agents, or conflicting subsets of agents. The question of optimization and dominance queries is discussed. Finally, the paper outlines an extension where gradual preferences are handled.

Keywords: Possibilistic network · Multiple agent logic · Preferences

1 Introduction

Modeling preferences has been an active research topic in Artificial intelligence for about twenty years. Graphical and logical formalisms have been proposed for describing user's preferences compactly. Graphical representations are appealing for elicitation purposes, and offer a basis for local computation; see, [1] for an overview. Note that only a few graphical models have been proposed for modeling *multiple agent* preferences, based on different extensions of Conditional Preference networks (CP-nets) [5,8], or Generalized Additive Independence networks (GAI-nets) [7]. Besides, a multiple agent logic [2], where formulas are pairs of the form (p, A) made of a proposition p and a subset of agents A, has been advocated for handling beliefs: then (p, A) means '(at least) all agents in A believe that p is true'. But (p, A) may also have a preference reading ('(at least) all agents in A want p to be true').

The strong similarity of multiple agent logic with possibilistic logic and the existence of transformations between possibilistic logic and possibilistic networks [3] suggest to develop a graphical counterpart to multiple agent logic. When modeling preferences, multiple agent networks can be seen as a generalization

DOI: 10.1007/978-3-319-60045-1 17

of individual π-Pref nets (when possibility degrees are binary valued). In the following we investigate the interest of multiple agent networks (and of their graded extension) for handling preferences.

This paper is organized as follows. Section 2 defines conditioning in case of Boolean possibilities. Section 3 introduces multiple agent logic, and its graphical counterpart in a preference perspective. Section 4 presents the main steps for transforming one model into another. Section 5 discusses queries evaluation for multiple agent network. Section 6 outlines an extension with priority levels of multiple agent logic and network.

2 Conditioning and Possibilistic Networks: Boolean Case

Conditioning is a crucial notion when dealing with possibilistic networks. Here we consider the elementary situation of a single agent and of two-valued possibility distributions. Possibilistic networks [3] are usually defined for non-dogmatic possibility distributions, i.e., taking only positive values in $(0, 1]$. However, in the two-valued case, the only non-dogmatic possibility distribution is the vacuous one with value 1 for all states. So we must use a definition of conditioning that makes sense for dogmatic possibility distributions. Conditioning in this case is defined in the following way: Let Ω be the universe of discourse (set of all interpretations). Then the interpretations known as possible are restricted by a subset $E \neq \emptyset$, $E \subset \Omega$, and the considered possibility measure Π is such that $\Pi(S) = 1$ if $E \cap S \neq \emptyset$ and $\Pi(S) = 0$ otherwise (the possibility distribution being the characteristic function of E). Conditioning obeys the equation:

$$\Pi(S \cap T) = \Pi(S|T) \wedge \Pi(T) \tag{1}$$

where $\wedge$ stands for Boolean conjunction. Then we define $\Pi(\cdot|T)$ as the possibility measure associated with the subset $E_T = T \cap E$ if $T \neq \emptyset$ and $E_T = T$ if $T \cap E = \emptyset$. E_T is the result of revising E by T, the minimally specific solution of the above equation under the success postulate $E_T \subseteq T$. Thus:

$$\begin{aligned} \Pi(S|T) = 1 \text{ if} & \begin{cases} S \cap E_T = S \cap T \cap E \neq \emptyset \ (\Pi(S \cap T) = \Pi(T) = 1) \\ S \cap E_T = S \cap T \neq \emptyset, \ T \cap E = \emptyset \ (\Pi(S \cap T) = \Pi(T) = 0) \end{cases} \\ = 0 & \qquad\qquad\qquad\qquad \text{otherwise } (\Pi(S \cap T) = 0, \Pi(T) = 1) \end{aligned}$$

A Boolean possibility distribution can be decomposed into a combination of conditional possibility distributions. This can be done by applying repeatedly the definition of conditioning. Indeed, taking an arbitrarily order of variables in set $V = \{X_1, \ldots, X_n\}$: $\pi(X_1, \ldots, X_n) = \pi(X_1|X_2, \ldots, X_n) \wedge \cdots \wedge \pi(X_n)$. This decomposition can be simplified when assuming some independence between variables. Graphically, it can be represented by a possibilistic network where each node represents a variable, edges represent the dependencies and conditional distributions define the associated tables.

Example 1. Consider 3 Boolean variables X, Y, Z and π defined by the two interpretations of $x \wedge y$. The possibilistic network associated to the ordering (X, Y, Z)

Table 1. Joint possibility distribution

XYZ	$\pi(Z \mid Y)$	$\pi(Y \mid X)$	$\pi(X)$	$\pi(XYZ)$
$\neg x \neg y \neg z$	1	1	0	0
$\neg x \neg y z$	1	1	0	0
$\neg x y \neg z$	1	1	0	0
$\neg x y z$	1	1	0	0
$x \neg y \neg z$	1	0	1	0
$x \neg y z$	1	0	1	0
$x y \neg z$	1	1	1	1
$x y z$	1	1	1	1

Table 2. Joint possibility distribution

XYZ	$\pi(Z \mid Y)$	$\pi(Y \mid X)$	$\pi(X)$	$\pi(XYZ)$
$\neg x \neg y \neg z$	1	0	0	0
$\neg x \neg y z$	0	0	0	0
$\neg x y \neg z$	1	1	0	0
$\neg x y z$	1	1	0	0
$x \neg y \neg z$	1	0	1	0
$x \neg y z$	0	0	1	0
$x y \neg z$	1	1	1	1
$x y z$	1	1	1	1

corresponding to this possibility distribution is given by columns 2, 3, 4 of Table 1. The original knowledge xy can be recovered from the joint distribution of Table 1 in the last column using the chain rule. In this network, Z is independent from X and Y.

Consider the same ordering of variables with the conditional tables given in Table 2. This illustrates the two first cases above in the definition of $\Pi(S \mid T)$. Note that here Y does not depend on X. The two networks have different tables but correspond to the same possibility distribution. The first network has conditional distributions less specific than the second one. So having fixed the ordering of variables, not only the conditional tables are not unique, but even the network topology is not unique.

3 Multiple Agent Representations

Multiple agent logic has been discussed in details in [2]. Formulas in this logic are pairs of the form (p, A), made of a proposition p and a subset of agents A. In this section, we explain the use of this logic for modeling preferences and present its graphical counterpart. *All* will denote the set of all the agents and capital letters, e.g., A, B, $A_i, \cdots$ denote subsets of *All*. Let $p, q, p_i, \cdots$ denote propositional formulas of a finite language.

3.1 Multiple Agent Logic

A possibilistic logic formula [6] of the form (p, α) is understood as $N(p) \geq \alpha$ (N is a necessity degree), where the higher α, the more imperative p. Multiple agent logic shares formal similarity with possibilistic logic in terms of inference rules, axioms, possibilistic measures and possibility distribution [2]. However, a multiple agent formula (p, A) is understood at the semantic level as a constraint of the form $\boldsymbol{N}(p) \supseteq A$ where $\boldsymbol{N}$ is a *set-valued* mapping that returns the set of agents for whom satisfying p is imperative. Therefore, the formula (p, A) means that at least all the agents in A find p imperative. Set-valued possibility measure and necessity measure are related via duality. Indeed, $\boldsymbol{\Pi}(p) = \overline{\boldsymbol{N}(\neg p)}$, which corresponds to the maximal set of agents for whom the falsity of p is not imperative, which could be expressed as "the truth of p is acceptable". $\boldsymbol{\Pi}(p) \cap \boldsymbol{\Pi}(\neg p)$ represents the set of agents that are indifferent to the truth value of p, and $\boldsymbol{N}(p) \cap \boldsymbol{N}(\neg p)$ represents a set of inconsistent agents, which may be empty or not. It can be checked that the set of agents who think that the truth of p is imperative is a subset of the set of agents who think that its falsity is not imperative, namely, $\boldsymbol{N}(p) \subseteq \boldsymbol{\Pi}(p)$ provided there is no inconsistent agent. The semantics of such a logic is defined by a so-called ma-distribution from a universe of discourse Ω to subsets of agents, formally, $\boldsymbol{\pi} : \Omega \to 2^{All}$. Subsets are partially ordered, which contrasts with a possibilistic logic distribution that maps to a totally ordered scale. A multiple agent formula (p_i, A_i) leads to the following semantic representation by the ma-distribution

$$\boldsymbol{\pi}_{(p_i,A_i)}(\omega) = \begin{cases} All & if\ \omega \models p_i \\ \overline{A_i}\ (= All \setminus A_i) & \text{otherwise.} \end{cases} \tag{2}$$

This expression indicates that agents not in A_i are indifferent to p_i, but agents in A_i find $\neg p_i$ unacceptable. More generally an ma-distribution should be interpreted as follows: $\boldsymbol{\pi}(\omega)$ is the set of all agents that do not find ω unacceptable.

A ma-logic base $\Gamma = \{(p_i, A_i)|i = 1, m\}$ is associated to an ma-distribution, s.t. $\boldsymbol{\pi}_\Gamma(\omega)$ is the intersection of sets of agents $\overline{A_i}$ that find the interpretation ω, for which all formulas p_i are false, acceptable.

$$\boldsymbol{\pi}_\Gamma(\omega) = \begin{cases} All & \text{if } \forall (p_i, A_i) \in \Gamma, \omega \models p_i \\ \bigcap\{\overline{A_i} : (p_i, A_i) \in \Gamma,\ \omega \models \neg p_i\} & \text{otherwise.} \end{cases} \tag{3}$$

Two types of normalization exist for $\boldsymbol{\pi}$: (i) The *ma-normalization* where $\exists\, \omega \in \Omega$ s.t. $\boldsymbol{\pi}(\omega) = All$. Thus, all agents are altogether consistent and have at least one common not unacceptable interpretation. This normalization entails the following one. (ii) the *i-normalization* where $\bigcup\{\boldsymbol{\pi}(\omega), \omega \in \Omega\} = All$. This means that each agent is consistent individually by having at least one interpretation that is not rejected. Yet, there may exist some contradictions between subgroups of agents, for instance $\Gamma = \{(p, A), (\neg p, \overline{A})\}$.

Example 2. Let us consider preferences of subsets of agents about drinks and their accompaniments. We consider that the agent population is described by two characteristics namely, being a *Woman* (W) or a *Man* (M) and being *Young* (Y) or *Old* (O). The variables are $\texttt{Drink} = \{\texttt{Tea}(t), \texttt{Coffee}(\neg t)\}$, $\texttt{Sugar} = \{\texttt{Yes}(s), \texttt{No}(\neg s)\}$. If we consider the ma-base: $\Gamma = \{(\neg t, M), (t, \overline{M \cap Y}), (\neg s, O), (s, Y)\}$, we can check that the ma-normalization is not verified. This is because $\boldsymbol{N}(\neg t) \supseteq M$ and $\boldsymbol{N}(t) \supseteq \overline{M \cap Y}$, hence $\boldsymbol{N}(t) \cap \boldsymbol{N}(\neg t) \supseteq \overline{M \cap Y} \cap M = M \cap O$. The old men demand tea and not tea.

3.2 Graphical Representation of Multiple Agent Preferences

Possibilistic networks are the graphical counterpart of possibilistic logic and one may go from one format to another while preserving semantics [3]. Likewise, given the close similarity between possibilistic and multiple agent logic, we propose a graphical reading of the latter. First, we introduce the multiple agent conditioning rule:

$$\boldsymbol{\Pi}(p \wedge q) = \boldsymbol{\Pi}(p|q) \cap \boldsymbol{\Pi}(q) \tag{4}$$

This means that the set of all agents for whom the truth of $p \wedge q$ is not unacceptable is equal to the intersection between the set of all agents for whom the truth of q is not unacceptable and the set of all agents for whom the truth of p is not unacceptable when q is true. It generalizes the conditioning of Boolean possibilities to multiple agents. As in standard possibilistic networks, the decomposition of a possibility distribution consists in expressing a joint possibility distribution as a combination of conditional possibility distributions, a process that in the two-valued possibility case, does not yield a unique result, even when fixing the ordering of the variables, as shown above. Let E be a subset of $All \times \Omega$ representing an ma-distribution $\boldsymbol{\pi}$, and let $E(a)$ the set of interpretations that agent $a \in All$ does not reject. The result of conditioning E by a set of interpretations B will be again defined as the minimally specific revision of $E(a)$ by B that agrees with the definition of conditioning (4), for each agent $a \in All$, namely $E_B(a) = E(a) \cap B$ if this intersection is not empty and B otherwise. Notice that the result differs from $E \cap (All \times B)$ even if this set is not empty. If B contains the set of models $[q]$ of q, then the characteristic function of E_B is denoted by $\boldsymbol{\Pi}(\cdot \mid q)$. The solution of Eq. (4) is then:

$$\boldsymbol{\Pi}(p|q) = \begin{cases} All \text{ if } \boldsymbol{\Pi}(p \wedge q) = \boldsymbol{\Pi}(q) \\ \boldsymbol{\Pi}(p \wedge q) \text{ otherwise.} \end{cases} \tag{5}$$

Let $V = \{X_1, \ldots, X_n\}$ be a set of variables, each variable X_i has a value domain $D(X_i)$. x_i denotes any value of X_i. In coherence with Eq. (4), we can use the chain rule:

$$\boldsymbol{\pi}(X_1, ..., X_n) = \boldsymbol{\pi}(X_1|X_2, ..., X_n) \cap .. \cap \boldsymbol{\pi}(X_{n-1}|X_n) \tag{6}$$

to decompose a joint ma-distribution into a conjunction of conditional possibility distributions. Now, we introduce a new graphical model for representing multiple agent preferences, called ma-net for short. This model shares similar graphical component and independence relations as possibilistic networks [3]. Formally,

Definition 1 (ma-net). *A multiple agent network $\mathcal{G}$ over a set of variables V consists of two components: (i) Graphical component composed of a directed acyclic graph (DAG). (ii) Numerical component associating to each node X_i a conditional multiple agent distribution for each the context u_i of its parents $Pa(X_i)$.*

Example 3. Let us use the same variables and sets of agents as in Example 2, plus variable `Cake` $= \{$`Yes`(c), `No`$(\neg c)\}$. The network `Drink` $\rightarrow$ `Cake` $\leftarrow$ `Sugar` and the following conditional distributions: $\boldsymbol{\pi}(t) = W$, $\boldsymbol{\pi}(\neg t) = M \cap Y$, $\boldsymbol{\pi}(s) = Y$, $\boldsymbol{\pi}(\neg s) = O$, $\boldsymbol{\pi}(c \mid ts) = M \cap Y$, $\boldsymbol{\pi}(c \mid t \neg s) = O$, $\boldsymbol{\pi}(c \mid \neg ts) = M \cap Y$, $\boldsymbol{\pi}(c \mid \neg t \neg s) = W$, $\boldsymbol{\pi}(\neg c \mid ts) = M \cap O$, $\boldsymbol{\pi}(\neg c \mid t \neg s) = W$, $\boldsymbol{\pi}(\neg c \mid \neg ts) = W$, $\boldsymbol{\pi}(\neg c \mid \neg t \neg s) = M \cap O$ represent conditional preferences of agents. Using the chain rule, we have the following ma-distribution: $\boldsymbol{\pi}(tsc) = \emptyset$, $\boldsymbol{\pi}(ts\neg c) = W \cap Y \cap O = \emptyset$, $\boldsymbol{\pi}(t \neg sc) = W \cap O$, $\boldsymbol{\pi}(t \neg s \neg c) = W \cap O$, $\boldsymbol{\pi}(\neg tsc) = M \cap Y$, $\boldsymbol{\pi}(\neg ts \neg c) = M \cap Y \cap W = \emptyset$, $\boldsymbol{\pi}(\neg t \neg sc) = M \cap Y \cap O \cap W = \emptyset$, $\boldsymbol{\pi}(\neg t \neg s \neg c) = M \cap Y \cap O \cap M \cap O = \emptyset$. In ma-logic, we can encode it by the following base: $\{(t \neg s) \vee (\neg tsc), All), (\neg t \vee s, M \cup Y), (t \vee \neg s \vee \neg c, W \cup O)\}$.

Let us reconstruct the ma-conditional distributions $\boldsymbol{\pi}$(`Cake`|`Drink`, `Sugar`) and the marginals $\boldsymbol{\pi}$(`Drink`), $\boldsymbol{\pi}$(`Sugar`) using the conditioning rule:

$\boldsymbol{\pi}(tsc) = \boldsymbol{\pi}(ts\neg c) = \boldsymbol{\pi}(ts) = \emptyset$ so $\boldsymbol{\pi}(c|ts) = \boldsymbol{\pi}(\neg c|ts) = All$.
$\boldsymbol{\pi}(t\neg sc) = \boldsymbol{\pi}(t\neg s\neg c) = \boldsymbol{\pi}(t\neg s) = W \cap O$ so $\boldsymbol{\pi}(c|t\neg s) = \boldsymbol{\pi}(\neg c|t\neg s) = All$.
$\boldsymbol{\pi}(\neg tsc) = \boldsymbol{\pi}(\neg ts) = M \cap Y$ so $\boldsymbol{\pi}(c|\neg ts) = All$.
But $\boldsymbol{\pi}(\neg ts\neg c) = \emptyset, \boldsymbol{\pi}(\neg ts) = M \cap Y$ so $\boldsymbol{\pi}(\neg c|\neg ts) = \emptyset$.
$\boldsymbol{\pi}(\neg t\neg sc) = \boldsymbol{\pi}(\neg t\neg s\neg c) = \boldsymbol{\pi}(\neg t\neg s) = \emptyset$ so $\boldsymbol{\pi}(c|\neg t\neg s) = \boldsymbol{\pi}(\neg c|\neg t\neg s) = All$.
It can be checked that $\boldsymbol{\pi}(s) = M \cap Y$, $\boldsymbol{\pi}(\neg s) = W \cap O$, $\boldsymbol{\pi}(t) = W \cap O$, $\boldsymbol{\pi}(\neg t) = M \cap Y$. We can easily check that even if this network has different conditional tables it again yields the same ma-distribution.

4 Bridging Logical and Graphical Multiple Agent Representations

Transformations between possibilistic graphical and logical representations [3] can be adapted to multiple agent representations.

4.1 Logical Encoding of a Multiple Agent Network

The main idea consists in considering the ma-net $\mathcal{G}$ as a combination of local multiple agent logic bases. Each node $X_i \in V$ is associated to a logic base Γ_{X_i} containing formulas of the form $(x_i \vee \neg u_i, \overline{A})$ and $(\neg x_i \vee \neg u_i', \overline{A'})$ where u_i, u_i' are instantiations of $Pa(X_i)$, and $\boldsymbol{\pi}(x_i|u_i) = A$, $\boldsymbol{\pi}(\neg x_i|u_i') = A'$ appear in the tables of $\mathcal{G}$ and $A, A' \neq All$. Each (conditional) possibility is viewed as a necessity formula expressing the material counterpart of the condition. Indeed, for a single agent $N(\neg p \mid q) = 1 - \Pi(p \mid q) = 1 - \Pi(p \wedge q) = N(\neg q \vee \neg p) = 1$ provided that $\Pi(p|q) = 0$. So, in the multiagent case we can replace $\boldsymbol{\pi}(x_i|u_i)$ by the clause $\neg x_i \vee \neg u_i$ when $A \neq All$. When considered separately, we can see that the conditional possibilities can be recovered from the local possibility distribution such that $\boldsymbol{\Pi}(x_i) = \bigcup_{\omega \models x_i} \boldsymbol{\pi}(\omega)$ since from $\boldsymbol{\Pi}(x_i \wedge u_i) = A$ and $\boldsymbol{\Pi}(u_i) = All$ we get $\boldsymbol{\Pi}(x_i|u_i) = A$ (by solving (4)). A multiple agent network is rarely normalized due to conflicting preferences (which contrasts with standard possibilistic networks), thus each conditional possibility distribution is represented by more than one formula. Combined together, it is clear that the resulting logic base is inconsistent with a degree equal to the intersection of all necessity values associated to formulas. Then, the multiple agent base associated with the ma-net $\mathcal{G}$ is $\Gamma_{\mathcal{G}} = \Gamma_{X_1} \bigcup \cdots \bigcup \Gamma_{X_n}$, $\forall X_i \in V$. The joint possibility distribution computed from the multiple agent network $\mathcal{G}$ by the chain rule is the intersection of the possibility distributions associated to each node. The possibility distribution associated to $\Gamma_{\mathcal{G}}$ is also an intersection of distributions associated to the formula(s) corresponding to each node. This explains why the two representations are represented by the same ma-distribution. This is the counterpart of the fact that the union of possibilistic logic bases corresponds to the min-based aggregation of their distributions [4]. Thus, the ma-net of Example 3 can be rewritten as the union of the bases $\Gamma_{\texttt{Cake}} = \{(t \vee \neg s \vee c, All)\}$, $\Gamma_{\texttt{Drink}} = \{(\neg t, M \vee Y), (t, W \vee O)\}$, $\Gamma_{\texttt{Sugar}} = \{(\neg s, W \vee O), (s, M \vee Y)\}$.

4.2 Transformation of a Multiple Agent Logic into a Graphical Structure

This converse transformation is more complex. Indeed, the independencies represented by the network are not explicit in logic bases. The transformation consists of two steps: (i) Constructing the network, thus detecting the dependencies, (ii) Computing the conditional possibilities. First, the logic base should be put into a special form, where tautologies are removed (by removing subsumed formulas) each formula should represent a disjunction of a variable value and an instance of all it parents. An algorithm performing this type of transformation is given in [3]. To adapt this algorithm the following definitions are useful:

Definition 2. *Let (p, A) be a formula in Γ. Then (p, A) is said to be subsumed by Γ if $\Gamma_{\supseteq A} \vdash p$, where $\Gamma_{\supseteq A}$ is composed of classical formulas that appear in Γ in association with sets of agents that include A or are equal to A.*

Removing subsumed formulas does not change the possibility distribution. This means that several syntactically different multiple agent logic bases may have the same possibility distribution as their semantic counterpart. For instance, $(x \vee y, A \cap B)$ is subsumed by (x, B), therefore $\Gamma = \{(x \vee y, A \cap B), (x, B)\} = \{(x, B)\}$.

Definition 3. *Let Γ be a multiple agent logic base in a clausal form, where* all *clauses involve an instance of a variable X. Let $\mathcal{Z}$ be the set of other variables appearing in the clauses of Γ. A* clausal completion *of Γ with respect to variable X, denoted by $E(\Gamma)$, is the set of clauses of the form $(x \vee \neg \mathbf{z}, A)$ where x is an instance of X, $\mathbf{z}$ is an instance of all variables in $\mathcal{Z}$, and $A = \bigcup\{A_i : (x \vee p_i, A_i) \in \Gamma, \mathbf{z} \models \neg p_i\}$, with $\bigcup(\emptyset) = \emptyset$.*

It can be proved that the two bases Γ and $E(\Gamma)$ are equivalent, i.e. correspond to the same possibility distribution.

The notions of subsumption and clausal completion are instrumental in the procedure (similar to the one in [3]) for finding the dependence graph from the multiple agent logic base. More precisely, for each X_i in V we execute these steps:

- Determination of the local base for X_i: Let $(x_i \vee p, A)$ be a clause of Γ s.t. x_i is an instance of X_i, and p is only built from $X_{i+1}, \ldots X_n$. If $(x_i \vee p, A)$ is subsumed, then remove it from Γ. If $\Gamma \vDash (p, A)$, then replace $(x_i \vee p, A)$ by (p, A). Let K_i be the set of clauses $(x_i \vee p,)$ in Γ s.t. p is only built from $X_{i+1}, \ldots, X_n$
- The parents of the variable X_i are $Pa(X_i) = \{X_j : \exists c \in K_i \ s.t. \, c$ contains an instance of $X_j\}$
- Compute the clausal completion of K_i: Replace in Γ, K_i by its clausal completion $E(K_i)$
- Remove incoherent data: For each $(x_i \vee p, A)$ of Γ (where p is built from $X_{i+1}, \ldots, X_n$ s.t. $\Gamma \vDash (p, A)$ replace $(x_i \vee p, A)$ by (p, A).

– Produce Γ_i: Let Γ_i be the set of clauses $(x_i \vee p, A)$ in Γ s.t. p is only built from $X_{i+1}, \ldots, X_n$.

At the end of the procedure, each node X_i of the constructed graph is associated to a local multiple agent base $\Gamma_{X_i} = \{(x_i \vee u_i) | x_i \in D(X_i)$ and u_i an instatiation of $Pa(X_i)\}$ containing only an instantiation of the node and its parents. These local bases are useful to compute the conditional possibilities such that:

$$\boldsymbol{\pi}_A(x_i|u_i) = \begin{cases} \overline{A} & \text{if } (\neg x_i \vee \neg u_i, A) \in \Gamma \\ All & \text{otherwise.} \end{cases} \tag{7}$$

For instance if $\Gamma = \{(x \vee y, A), (x \vee t, B)\}$, this base is equivalent to $\{(x \vee y \vee t, A \cup B), (x \vee \neg y \vee t, B), (x \vee y \vee \neg t, A)\}$, so, $\boldsymbol{\pi}(\neg x|\neg y \neg t) = \overline{A} \cap \overline{B}$, $\boldsymbol{\pi}(\neg x|\neg y t) = \overline{A}$, $\boldsymbol{\pi}(\neg x|y \neg t) = \overline{B}$, $\boldsymbol{\pi}(\neg x|yt) = All$.

5 Specializing Representations and Queries

Before handling queries, we discuss two types of specializations, performed equivalently on ma-nets and ma-logic bases, w.r.t. a subset of agents.

5.1 Sections and Restrictions of Networks and Logic Bases

In some cases, one may need to display preferences that are only related to a subset of agents. Two possible operations are conceivable.

First, one may extract the network with *common* preferences expressed by a subset of agents A, i.e. preferences approved by each element in A. This is called a *section*. The obtained network has the same structure (with possible deletion of nodes or edges) as the original ma-net and its conditional possibilities are computed such that: $\boldsymbol{\pi}^{\forall}_A(x_i|u_i) = A$ if $A \subseteq \boldsymbol{\pi}(x_i|u_i)$ and $\boldsymbol{\pi}^{\forall}_A(x_i|u_i) = \emptyset$ otherwise. Its logical counterpart Γ_A is a propositional logic base where only formulas weighted by A_i, such that $A \subseteq A_i$, are retained. This network can be represented by a Boolean one, the same for each agent in A. If the section Γ_A is inconsistent, then, all the interpretations have a possibility degree equal to 0. Second, one may restrain the set of agents to (subsets of) A, that is, forget about preferences of agents out of A. This is called a *restriction*. The corresponding network can be constructed as: $\boldsymbol{\pi}^{\downarrow}_A(x_i|u_i) = \boldsymbol{\pi}(x_i|u_i) \cap A$. Its logical reading corresponds to a multiple agent logic base containing multiple agent formulas of the form $(\neg x_i \vee \neg u_i, A_i \cap A)$ s.t. $A_i \cap A \neq \emptyset$.

Example 4. In Example 2, the logic base corresponding to the common preferences of the subset $W \cup O$ is $\Gamma_{W \cup O} = \{t\}$. However, the restriction of the multiple-agent base to subset $W \cup O$ corresponds to $\Gamma^{\downarrow}_{W \cup O} = \{(\neg t, M \cap O), (t, W \cup O), (\neg s, O), (s, W \cap Y)\}$.

5.2 Optimization, Dominance and Other Queries

Optimal configurations for group A of agents in an ma-net exist if the set of preferences of group of agents A is consistent, precisely, if for each node and depending on the parents instantiation, the set of agents represented by the conditional possibility is a superset of A. Finding an optimal configuration is straightforward and linear wrt the number of variables. Starting from the root nodes, we choose each time the value(s) x_i s.t. $A \subseteq \boldsymbol{\pi}(x_i)$. Then, depending on the parents instantiation, each time we again choose a value with a conditional possibility that includes or equals A. In case $\boldsymbol{\pi}(x_i)$ is not a superset of A for some i, then the algorithm stops and the set of agents A have inconsistent preferences. Note that under the ma-normalization, one is always sure to have at least one preferred configuration no matter the set A. In the Boolean setting, dominance queries just amount to testing if each of the two interpretations is accepted or rejected. Another possible query, is to search for the maximal set of agents that prefer a given interpretation. The answer can be obtained by sweeping through the ma-net starting from the roots with the set of agents initialized to *All*, performing, at each node, the intersection of the current evaluation with the ma-possibility corresponding to the value of the node variable for the given interpretation.

6 Extension to Graded Possibilistic Networks. A Brief Outline

Multi-agent possibilistic logic. We can extend multi-agent possibilistic logic to graded preferences of agents using fuzzy set-valued counterparts of the notions of possibility distribution, possibility measure, and necessity measure. Formulas in ma-π logic are of the form $(p, \alpha/A)$ (where α is a necessity measure and A is a subset of agents) expressing that, for at least all agents in A, it is imperative to satisfy p with a minimal priority degree α. Asserting $(p, \alpha/A)$ means that A is the maximal set of agents that tolerate the falsity of p with level at most $1-\alpha$, while the agents in $\overline{A}$ are indifferent to the truth or falsity of p, finding both tolerable at level 1. By duality, $\boldsymbol{\Pi}(p)$ is the fuzzy set of agents who do not require the truth of $\neg p$ imperatively. Each possibilistic ma-logic base Γ is associated to an ma-π distribution $\boldsymbol{\pi}_\Gamma$.

$$\boldsymbol{\pi}_\Gamma(\omega) = \begin{cases} 1/All & \text{if } \forall (p_i, \alpha_i/A_i) \in \Gamma, \omega \models p_i \\ \bigcap \{(1-\alpha_i)/A_i \cup 1/\overline{A_i} \mid (p_i, \alpha_i/A_i) \in \Gamma,\ \omega \models \neg p_i\} & \text{otherwise.} \end{cases} \tag{8}$$

where $\boldsymbol{\pi}_\Gamma(\omega) = \alpha/A$ means that at most all the agents in A find ω acceptable with a maximal satisfaction degree equal to α. The ma-normalization and the i-normalization defined above are still valid. Precisely, ma-normalization is still related to the consistency of the propositional logic base and means that $\exists\, \omega \in \Omega,\ \boldsymbol{\pi}(\omega) = 1/All$, where $1/All$ is clearly the same as *All*. Moreover, the i-normalization is still defined by $\boldsymbol{\Pi}(\Omega) = \bigcup_{\omega \in \Omega} \boldsymbol{\pi}(\omega) = All$, and means that all the agents are individually consistent.

Example 5. Let us consider a multiple agent possibilistic logic corresponding to the preferences over the variable `Drink` $\in \{t, \neg t\}$: $\Gamma = \{(\neg t, 0.9/\overline{W}), (t, 0.3/\overline{M \cap Y})\}$. The possibility distribution corresponding to this base is:
$\boldsymbol{\pi}(t) = ((0.1/\overline{W}) \cup (1/W)) \cap (1/All) = (0.1/\overline{W}) \cup (1/W)$,
$\boldsymbol{\pi}(\neg t) = (1/All) \cap ((0.7/\overline{M \cap Y}) \cup (1/M \cap Y)) = 0.7/\overline{M \cap Y}) \cup (1/M \cap Y)$.
$\boldsymbol{\pi}(t)$ indicates that women find a cup of tea fully acceptable and men find it tolerable at best to a very low level 0.1. The preference base for women is $\Gamma_W = \{(t, 0.3)\}$.

Multi-agent possibilistic networks. Based on the same conditioning (Eq. (4)) and the same chain rule (Eq. (6)), where intersection is extended to fuzzy sets, we can define multi-agent possibilistic networks (ma-π nets for short) an extension of the above-defined graphical counterpart of ma-logic, that have the same structure as ma-nets.

Example 6. Let us consider the ma-π tables of Table 3, associated to the network **Drink** $\rightarrow$ **Cake** $\leftarrow$ **Sugar**. We can see that the local possibility distribution associated to node 'Drink' corresponds to the logic base of Example 5. It is clear that the network is not ma-normalized and this can be verified on its associated possibility distribution. For instance, $\boldsymbol{\pi}(t \neg sc) = (1/W \cup 0.1/\overline{W}) \cap (1/O \cup 0.2/Y) \cap (1/O \cup 0.1/Y) = (1/W \cap O) \cup (0.1/M \cup Y)$. It is clear that non-sugared tea with cake ($t \neg sc$) is satisfactory at degree 1 only for old women ($W \cap O$).

Table 3. Conditional tables of an ma-π net

$\boldsymbol{\pi}(t)$	$\boldsymbol{\pi}(\neg t)$
$1/W \cup 0.1/\overline{W}$	$1/M \cap Y \cup 0.7/\overline{M \cap Y}$

$\boldsymbol{\pi}(s)$	$\boldsymbol{\pi}(\neg s)$
$1/Y \cup 0.6/O$	$1/O \cup 0.2/Y$

$\boldsymbol{\pi}(.\vert.)$	ts	$t\neg s$	$\neg ts$	$\neg t \neg s$
c	$(1/Y) \cup (0.3/O)$	$(1/O) \cup (0.1/Y)$	$(1/M \cap Y) \cup (0.2/\overline{M \cap Y})$	$(1/W) \cup (0.9/M)$
$\neg c$	$(1/O) \cup (0.5/Y)$	$(1/W) \cup (0.6/M)$	$(1/W) \cup (0.1/M)$	$(1/M \cap O) \cup (0.7/\overline{M \cap O})$

From an ma-π net to an instantiated π-Pref net. In contrast with ma-nets, ma-π nets enable us to express levels of preference. Indeed, preferences are no longer all or nothing. Then, the network pertaining to the preferences of a set of agents A, induced as a section of the ma-π net, corresponds to a possibilistic preference network (π-Pref net) with instantiated weights. Its structure is similar to the ma-π net and the local possibility distributions associated to A are defined by: $\pi_A(x_i|u_i) = \alpha$, $\forall A \subseteq B$ s.t. $\boldsymbol{\pi}_\Gamma(x_i|u_i) \subseteq \alpha/B$. Note that the induced net is not always normalized due to the possible lack of normalization of the ma-π net. Clearly, normalization states that the preferences of the set A of agents are consistent and at least one interpretation has a possibility degree equal to 1 for agents in A.

Example 7. Consider the ma-π net defined in Example 6. Its restriction to the set of agents $W \cap O$ are possibilistic tables: $\pi(t) = 1$, $\pi(\neg t) = 0.7$, $\pi(s) = 0.6$, $\pi(\neg s) = 1$,

$\pi(c|ts) = 0.3$, $\pi(c|t\neg s) = 1$, $\pi(c|\neg ts) = 0.2$, $\pi(c|\neg t\neg s) = 1$, $\pi(\neg c|ts) = \pi(\neg c|t\neg s) = \pi(\neg c|\neg ts) = 1$, and $\pi(\neg c|\neg t\neg s) = 0.7$. The resulting network is normalized. The joint possibility distribution too. It can be computed using the standard product-based chain rule, and $t\neg sc$ and $t\neg s\neg c$ are the best interpretations ($\pi(t\neg sc) = \pi(t\neg s\neg c) = 1$).

7 Related Work

Few models exist for representing multiple agent preferences. First, multi-agent CP-nets (mCP-nets) [8] are an extension of CP-nets in a multiple agent setting. They are made of several partial CP-nets representing the preferences of each agent, such that a partial CP-net is a CP-net where some variables may not be ranked when the agent is indifferent about the values of these variables. Graphically, the network is obtained by combining the partial CP-nets. We can reason about an mCP-net by querying each partial CP-net, and then deduce the answer using different voting concepts like Pareto optimality, lexicographic ordering, and quantitative ranking. Second, probabilistic CP-nets (PCP-nets) [5] enable a compact representation of a probability distribution over several CP-nets and stand for a summary of collective preferences. A PCP-net has the same graphical component as a CP-net. Lastly, generalized additive independence (GAI) nets [7] are quantitative graphical models where preferences of agents are expressed by utilities. In a multiple agent framework, each node is characterized by a utility vector where each of its elements represents the utility of the node given by an agent. An aggregation procedure is then applied to these utilities to find the optimal solution.

As shown here, ma-nets represent the collective preferences of agents with a single network, similarly to PCP-nets and GAI nets (and in contrast with mCP-nets), which facilitates the handling of preferences. Besides, it may handle the indifference and non consistency of some agents, and can deal with the agents based on their profiles and not only in terms of proportions contrarily to GAI nets and PCP-nets. The model can be extended to describe preference intensities by adding priorities, unlike mCP-nets and PCP-nets.

8 Concluding Remark

This paper calls for several lines of research. The handling of graded preferences has been only outlined. Algorithms for different types of queries have to be extended to this general case. We may also think of other requests such as identifying non consistent agents directly from multiple agent (possibilistic) networks. Besides, the full strength of the representation power of π-Pref nets comes from a symbolic handling of the priorities yet to be developed.

References

1. Ben Amor, N., Dubois, D., Gouider, H., Prade, H.: Graphical models for preference representation: an overview. In: Scalable Uncertainty Management - 10th International Conference, SUM 2016, Nice, France, 21–23 September 2016, Proceedings, pp. 96–111 (2016)
2. Belhadi, A., Dubois, D., Khellaf-Haned, F., Prade, H.: Multiple agent possibilistic logic. J. Appl. Non-Classic. Logics **23**, 299–320 (2013)
3. Benferhat, S., Dubois, D., Garcia, L., Prade, H.: On the transformation between possibilistic logic bases and possibilistic causal networks. Int. J. Approximate Reasoning **29**(2), 135–173 (2002)
4. Benferhat, S., Dubois, D., Prade, H.: From semantic to syntactic approaches to information combination in possibilistic logic. In: Aggregation and Fusion of Imperfect Information, Physica Verlag, pp. 141–151 (1997)
5. Bigot, D., Zanuttini, B., Fargier, H., Mengin, J.: Probabilistic conditional preference networks. In: Proceeding UAI'21 (2013)
6. Dubois, D., Prade, H.: Possibilistic logic: a retrospective and prospective view. Fuzzy Sets Syst. **144**(1), 3–23 (2004)
7. Dubus, J., Gonzales, C., Perny, P.: Choquet optimization using GAI networks for multiagent/multicriteria decision-making. In: Algorithmic Decision Theory, pp. 377–389 (2009)
8. Rossi, F., Venable, K., Walsh, T.: mCP-nets: representing and reasoning with preferences of multiple agents. In: Proceeding of AAAI, vol. 4, pp. 729–734 (2004)